Cycling dynamics of the internal kink mode in non-linear two-fluid MHD simulations

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Introduction

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- Sawtooth oscillation are marked by sudden, periodic relaxations of the plasma core profiles
  - Reconnecting internal kink mode with $q = m/n = 1/1$ helicity
  - Heat, current, momentum, fast particles are redistributed during reconnection event taking place in 100μs timescale
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- Important for reactor operation, yet not fully understood
- The experiments show somewhat perplexing behavior
  - ”Mini-crashes”, snakes, helical states, partial magnetic reconnection...

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  - Improve the understanding of the cyclic behavior of sawteeth using three dimensional, fully non-linear fluid simulations
  - Characterize the steady-state (τ_η timescale) cyclic regimes of the internal kink respect to τ_η, ω_* to find diamagnetic thresholds for sawtoothing
- We will attempt to respect some of the experimental timescales set by plasma heat and current sources

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- Describing a sawtooth cycle (ramp, precursor, crash, ramp) requires switching between different instabilities and dynamic timescales at arbitrary mode amplitude
- This behavior is only tractable with numerical simulations !

Outline

Simulation model and setup

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Internal kink mode cyclic regimes

- Cyclic regimes in $S - \omega_*$ phase space

Diamagnetic thresholds

- Thresholds for cyclic regimes

Discussion

The simulations are carried out using the XTOR-2F code

The system evolved is a subset of the Braginskii two-fluid equations

$$\partial_t \rho = -\rho \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \rho - \alpha \nabla p_i \cdot \nabla \times \mathbf{B}/B^2 + \nabla \cdot D_\perp \nabla \left(\rho - \rho_{t=0}\right), \quad (1)$$

$$\rho \partial_t \mathbf{v} = -\rho \left(\mathbf{v} + \mathbf{v}_{*i}\right) \cdot \nabla \mathbf{v} + \mathbf{J} \times \mathbf{B} - \nabla p + \nu \nabla^2 \mathbf{v}, \quad (2)$$

$$\partial_t p = \Gamma p \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla p - \alpha \Gamma \frac{p}{\rho} \nabla p_i \cdot \nabla \times \mathbf{B}/B^2 + \nabla \cdot \chi_\perp \nabla_\perp \left(p - p_{t=0}\right) + \nabla \cdot \chi_\parallel \nabla_\parallel p \quad (3)$$

$$\partial_t \mathbf{B} = \nabla \times \left(\mathbf{v} \times \mathbf{B}\right) + \alpha \nabla \times \nabla_\parallel p_e/\rho - \nabla \times \eta \mathbf{J} \quad (4)$$

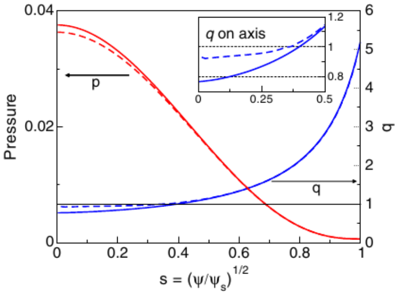
$$\mathbf{v}_i = \mathbf{v}_{\mathbf{E}\times\mathbf{B}} + \mathbf{v}_{\parallel i} + \mathbf{v}_{*i}, \mathbf{J} = e n_e (\mathbf{v}_i - \mathbf{v}_e),$$

$$\alpha = \left(\omega_{ci}\tau_a\right)^{-1} = \frac{c}{a\omega_{pi}}, \mathbf{v}_* \propto \alpha$$

Terms in red are corrections due to ω_* effects

Plasma equilibrium

- Equilibrium computed using CHEASE code
- Circular equilibrium, $A = \epsilon^{-1} = 2.7$, $\beta_p = 0.22$, $\partial_r \beta_p \approx 0$
- Parabolic q profile, $q_0 = 0.77$, $q_a = 5.2$, $(\psi/\psi_s)^{1/2}_{(q=1)} \approx 0.4$
- Warning : Initial equilibrium never recovered after first crash



Simulation setup

- Simulations must be advanced until the cycle period and amplitude stabilizes or until cycles stop
- Retained toroidal harmonics have $n = 0, 1, 2, 3$, with $n - 4 \leq m \leq n + 7$ for $n = 1, 2, 3$

Internal kink timescales

- The internal kink cycles are affected by the interplay between :
  - $S = \tau_\eta = 1/\eta = 10^6 - 10^7$ (resistive time)
  - $\tau_\eta = 30\tau_{\chi_\perp}$, $\chi_\parallel/\chi_\perp \approx 10^7$ (energy diffusion times)
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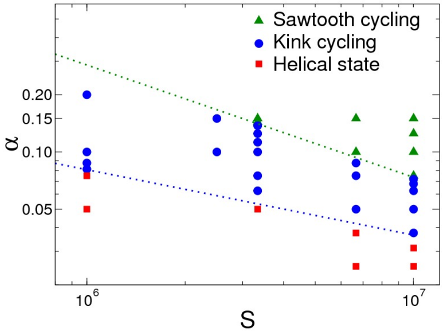
- Pressure dynamics follows magnetic field lines
  - Parallel temperature perturbations are strongly damped $\nabla_\| T \approx 0$, so $\omega_{*i} \approx 9\omega_{*e}$

Cyclic regimes

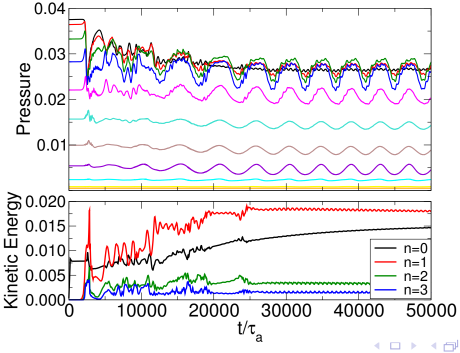
Distribution of cyclic regimes in $S - \omega_*$ parameter space :



We now describe briefly each regime...

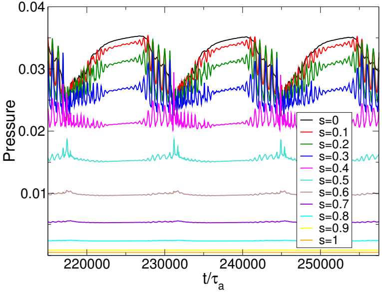
$m/n = 1/1$ helical states

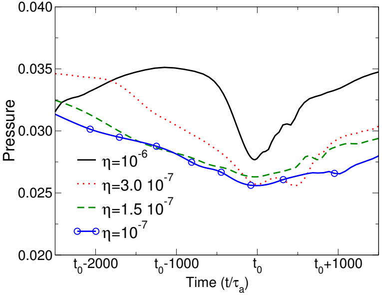
- First regime : Equilibrium due to low-shear saturated kink (axisymmetric boundary and $m/n = 1/1$ helical core) [Internal kink : Waelbrock, Phys.Fluids **31**, 1217 (1988)] [Equilibrium state : Cooper, NF **51** 072002 (2011)]



Resistive kink cycles (Kadomtsev's sawteeth)

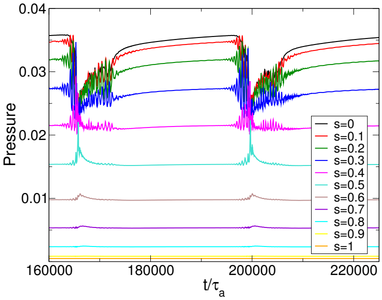
- Diamagnetic stabilization allows access to cycling regime
- They are characterized by slow, collisional crashes ($\tau_{\mathrm{crash}} \sim S^{-1/2}$) [Baty et al., Phys.Fluids B **5**, 1213 (1993)]
- The ramp is never quiescent, large $m/n = 1/1$ island present

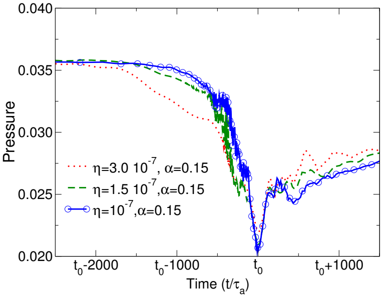




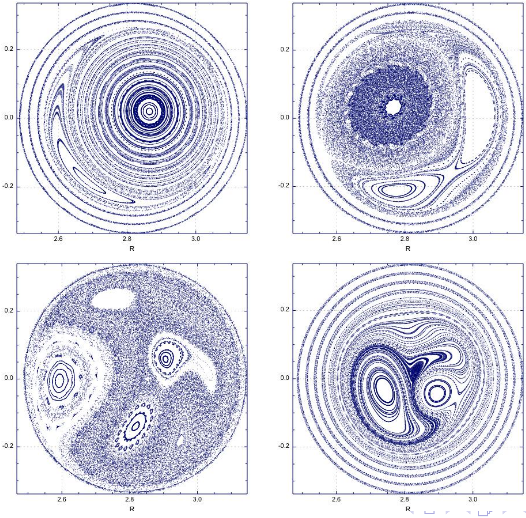
Sawtooth cycles

- Cycles have quiescent ramps, precursor and postcursor modes
- Fast, collisionless crashes (weak scaling of τ_{crash} vs S)
- Sometimes a "mini-crash" is observed





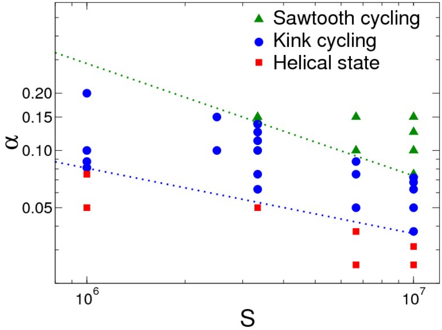
Magnetic field cross sections



Diamagnetic thresholds for internal kink cyclic regimes

Critical diamagnetic stabilization thresholds have the form

$$\alpha_{\text{crit},1} = \alpha_1 S^{-0.34}$$
$$\alpha_{\text{crit},2} = \alpha_2 S^{-0.60}$$



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- Transition at $\alpha_{\mathrm{crit},1}$: Stabilization of resistive branch of internal kink with $\gamma \sim S^{-1/3} - \alpha$
- Transition at $\alpha_{\mathrm{crit},2}$: Stabilization of deep-ideal-MHD-stable branch of internal kink with $\gamma \sim S^{-3/5} - \alpha$ (tearing like)

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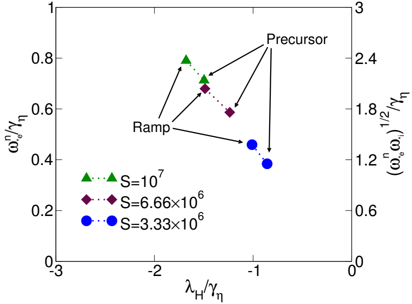
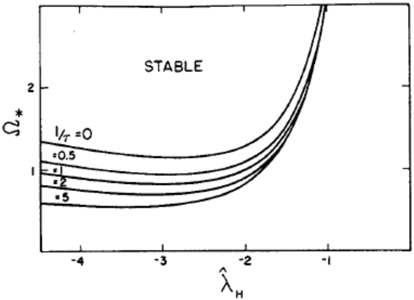
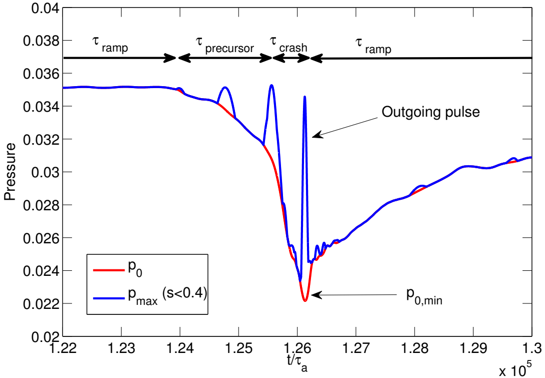
Instability regimes appear to inhabit different regions of stability diagram during the ramp :

- Kink cycles have $\lambda_H/\gamma_\eta \sim -1$, move toward $\lambda_H = 0$
- Sawteeth have more strongly negative λ_H/γ_η
- Compare to [Migliuolo, NF 33 (1993) 1721] :

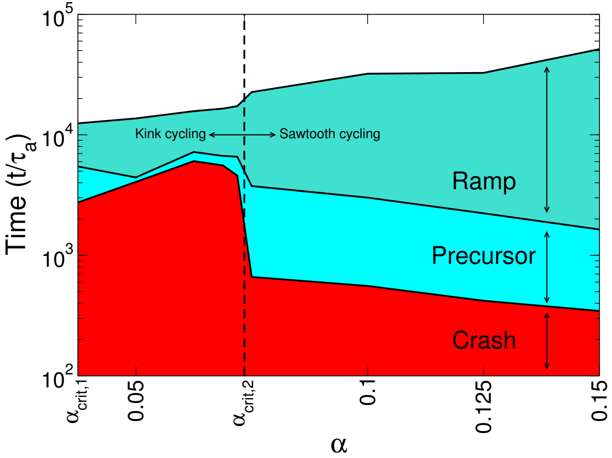




Measuring ramp, precursor, and crash times



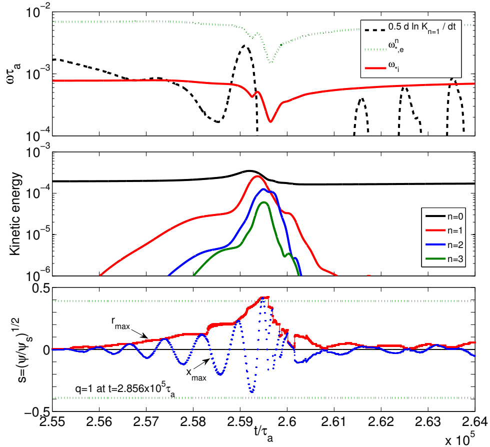
Ramp, precursor, crash timescales



- $\tau_{\rm ramp}$, $\tau_{\rm precursor}$, $\tau_{\rm crash}$ are shown for cases with $S = 10^7$

Role of diamagnetic stabilizations at crash onset

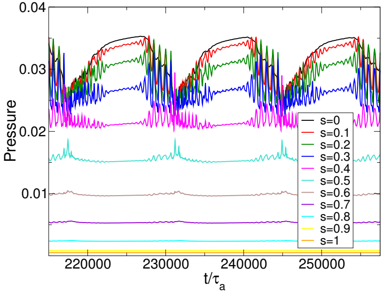
Well within the "sawtooth" regime :

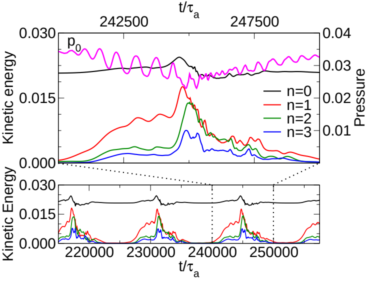


Role of diamagnetic stabilizations at crash onset

Just below the diamagnetic threshold :

- The crash time is increasing, with $\tau_{\text{crash}} + \tau_{\text{precursor}} \approx \tau_{\text{ramp}}/2$
- Rate of energy release accelerates, without any effect on the crash time





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- Access to sawtoothing regime requires that all three conditions are fulfilled

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- In a two-fluid model with realistic S and ω_*, sawtooth cycles should have a quiescent ramp and a crash in the 100μs scale