

# Dispersion of fast ions in models of anisotropic plasma turbulence

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(results contained here were obtained at U Maryland)

with Professor William Dorland, U Maryland  
and Ingmar Brömstrup



# Generalized dispersion of particles

i.e. Metzler/Klafter Phys Rep 339 (2000) 1-77; Balescu book and articles

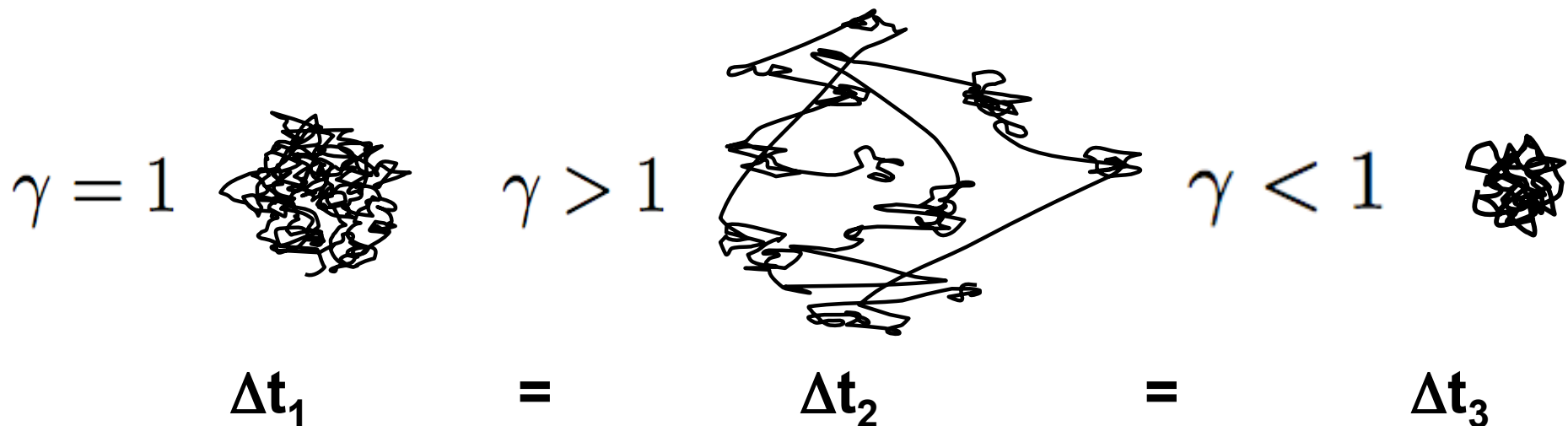
$$\delta x = x(t) - x(0)$$

$$\sigma^2 = \overline{(\delta x - \overline{\delta x})^2}$$

$$\sigma^2(t) \sim t^\gamma$$

- $\gamma < 1$  subdiffusive
- $\gamma = 1$  diffusive
- $\gamma > 1$  superdiffusive
- $\gamma = 2$  ballistic

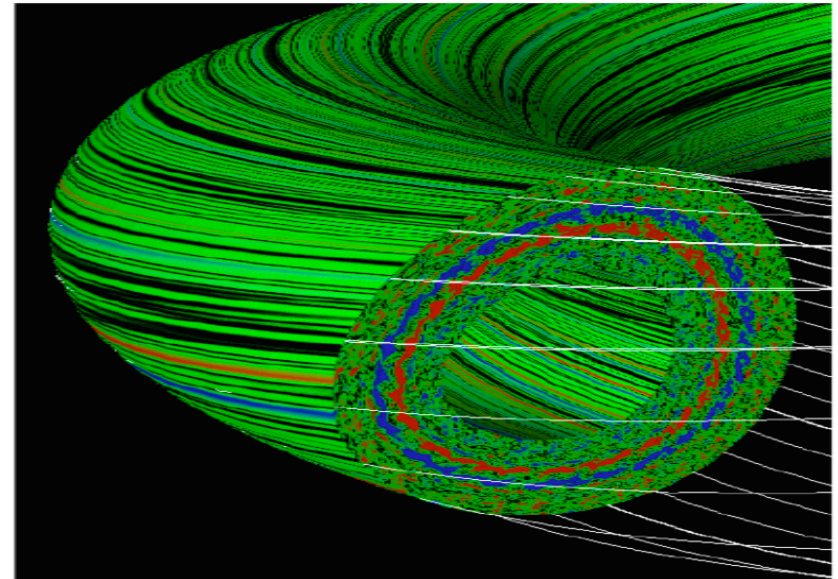
Brownian walk  $\subset$  continuous time random walks



# Random walks in turbulence

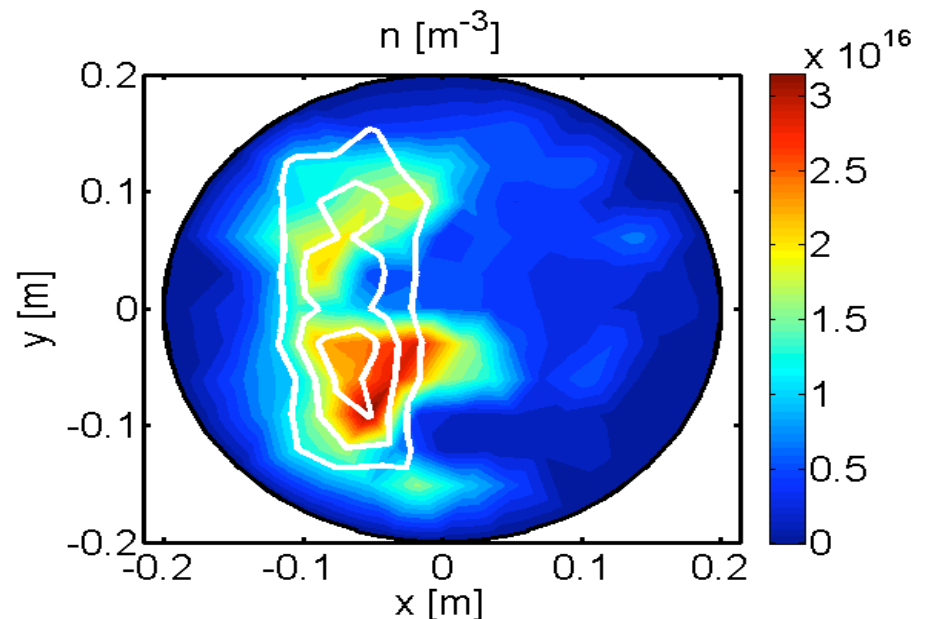
## 1) Microscale core turbulence with zonal flows

(thesis work, in preparation for publication)

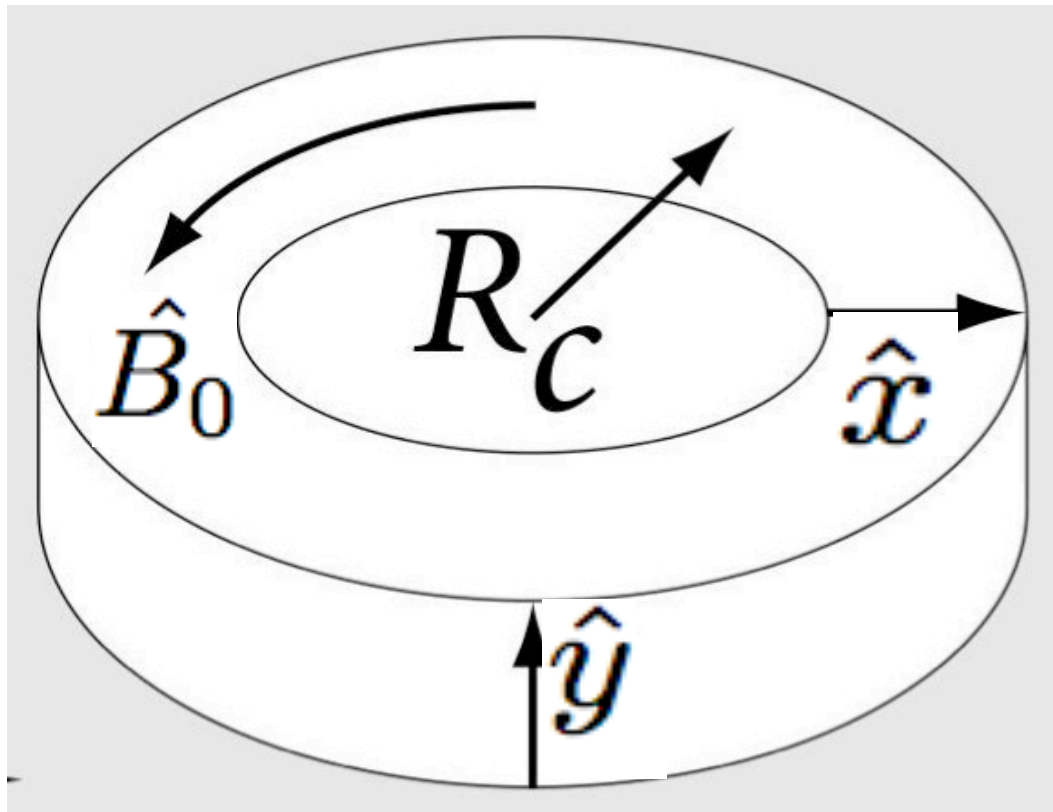


## 2) Macroscale edge turbulence with blobs

(new project)



# Z pinch geometry



- Magnetic field only toroidal,  $B \sim 1/r$  to edge.
- Gyroaveraged ExB drift produces radial (x) and axial (y) turbulent dispersion.
- Axial B drift  $\mathbf{v}_B$  is constant for each particle.

$$\Omega_s = \frac{q_s B}{m_s c} \quad \mathbf{v}_B = \frac{v_{\parallel}^2 + \frac{1}{2}v_{\perp}^2}{\Omega_s R_c} \hat{y}$$

# Pressure-gradient-driven instability in a gyrokinetic Z-pinch

Brömstrup PhD, U Maryland 2008; Ricci *et al* PRL 97 245001 (2006)

Our new gyrokinetic, local ( $1/L_n = -\nabla n/n$ ),  $\delta f$ , particle-in-cell code (GSP), provides markers as Lagrangian tracer probes.

We extract a subset of these tracers and compare the absolute dispersion in the nonlinear phase of the turbulence.

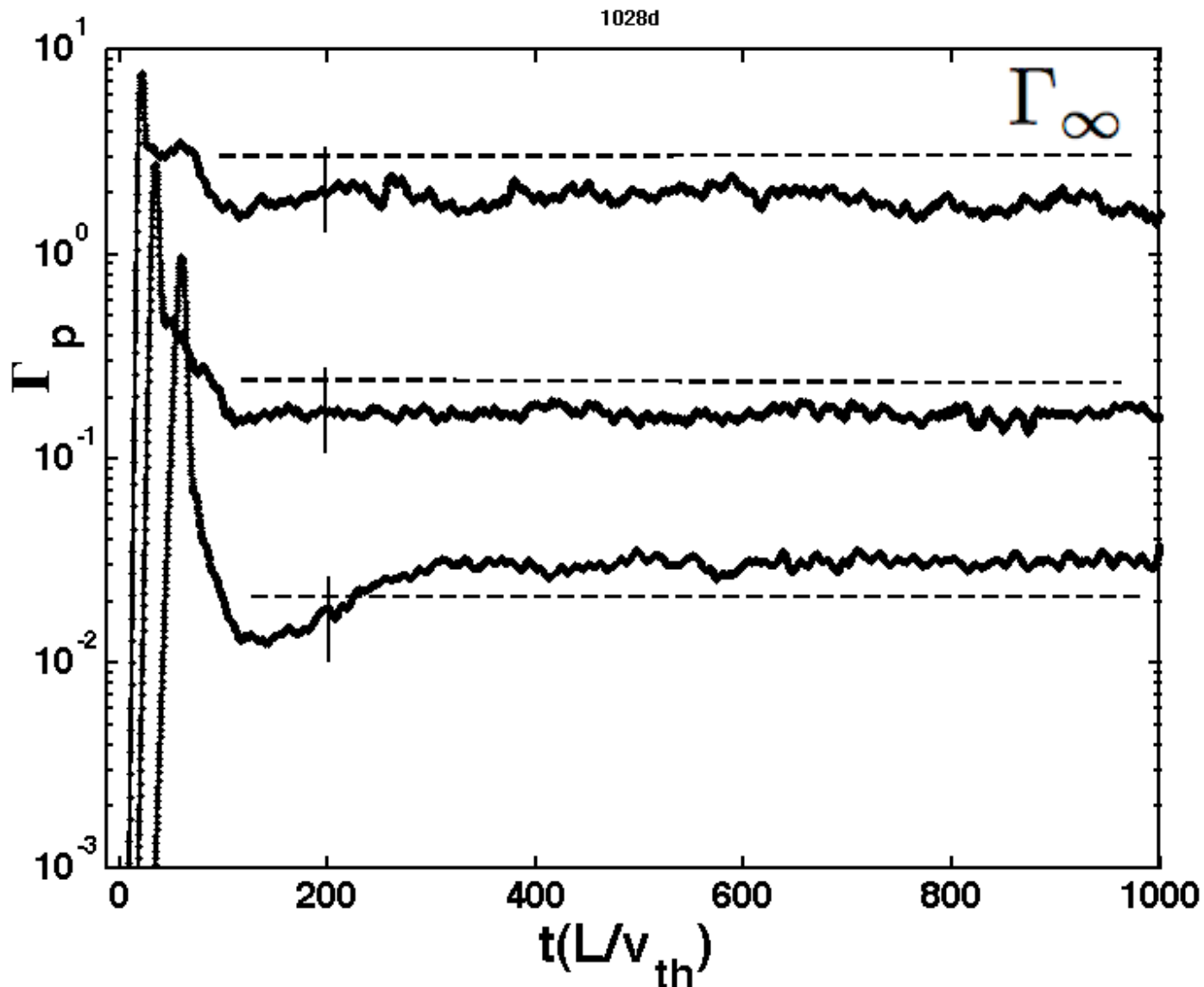
Turbulence driven by density gradient ( $1/L_T = 0$ ) and curvature, unstable for  $2/7 < L_n/R < \pi/2$

# Results from nonlinear gyrokinetic simulations

- Benchmarked fluxes with GS2
- Axially: superdiffusive to ballistic
- Radially: diffusive for several density gradients
- Comparing diffusion coefficients from test-particles and flux/gradient relation
- Dependence of test-particle diffusion coefficient on gyroradius

# Particle flux compared with GS2

Ricci *et al* PRL 97 245001 (2006) for GS2 data



$$\Gamma_p = \sum_i^{N_{part}} \delta f_1 \langle v_x \rangle_R$$

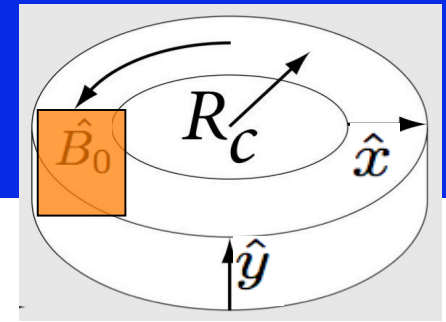
Top to bottom:  
strong to weak  
gradient;

Dashed lines:  
published GS2  
values

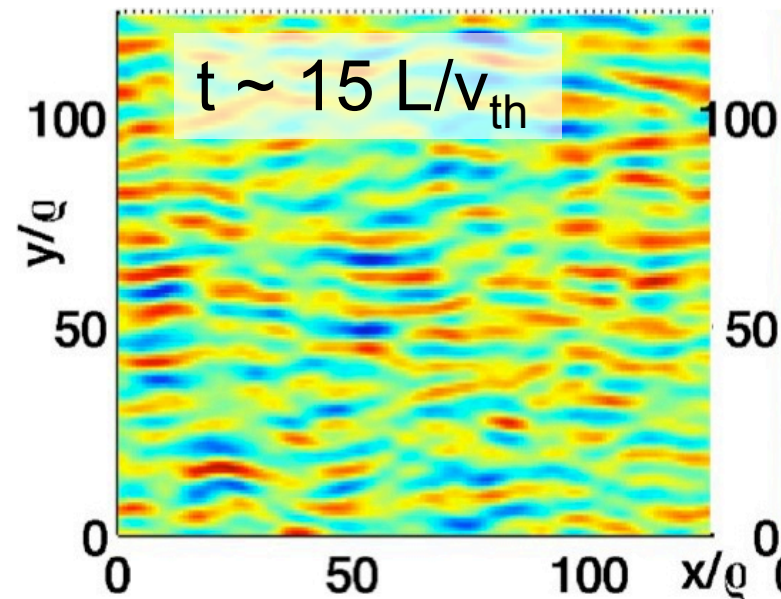
Units for  $\Gamma_p$  :

$$(\rho_i/R)^2 n_0 v_{th_i}$$

# Gyrokinetic turbulence

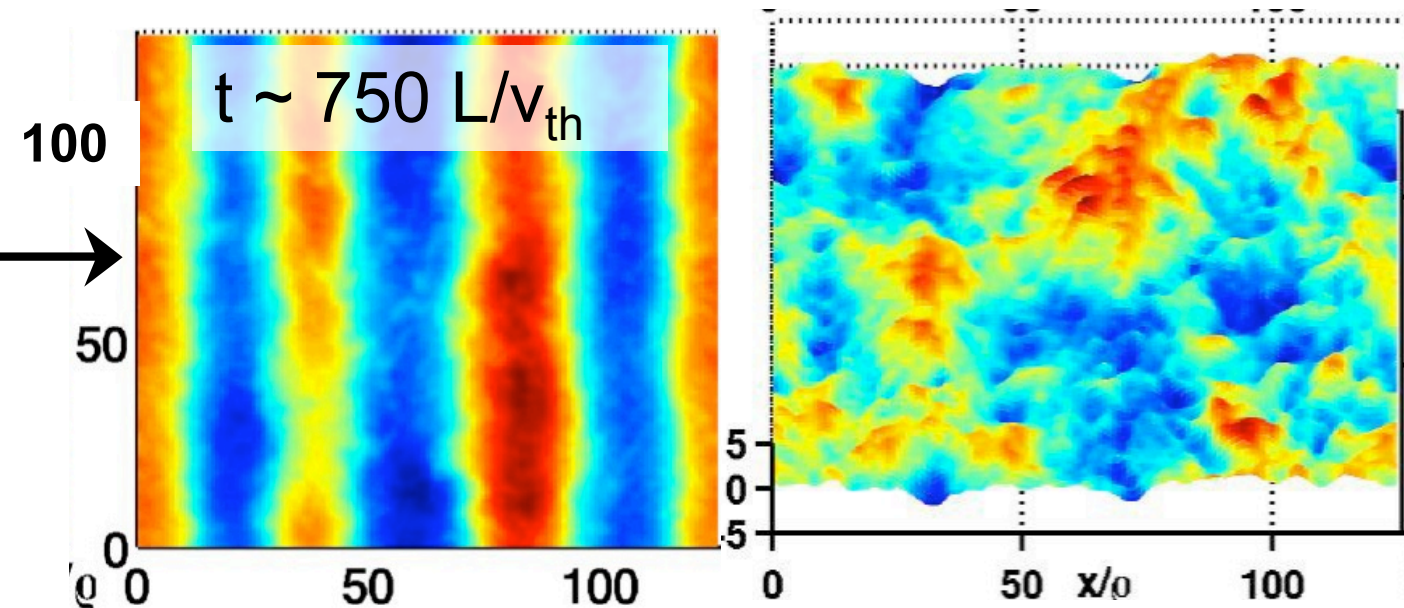


Linear phase -  
growing  $k_y$   
modes →



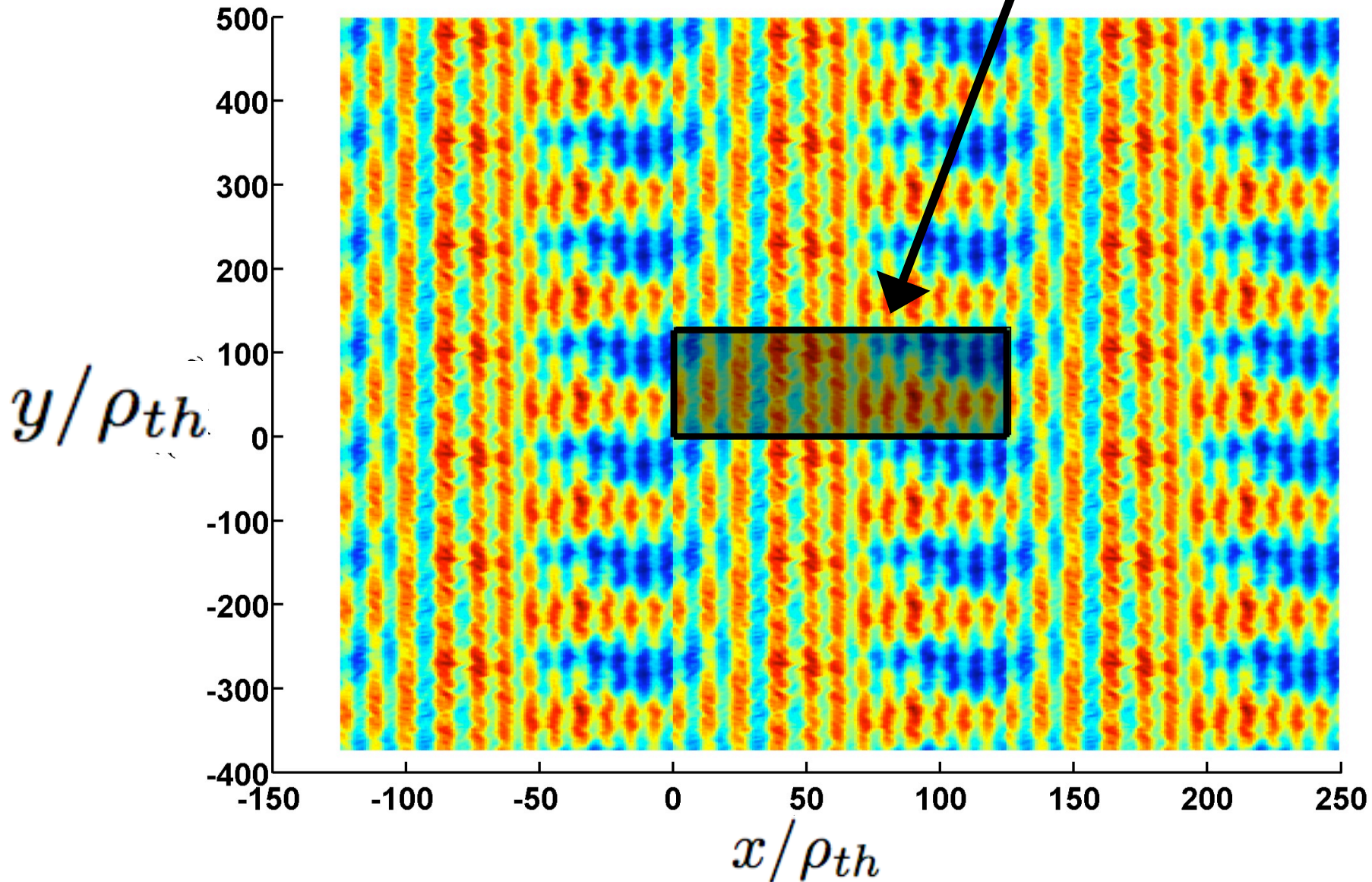
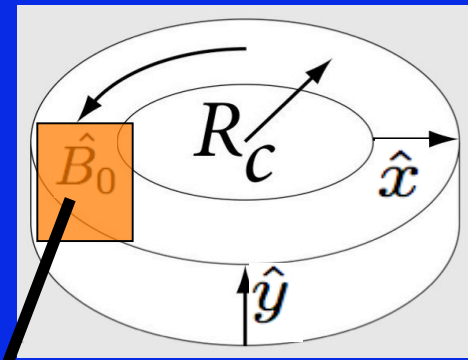
Surface plots  
of electrostatic  
potential,  $\Phi(x,y)$

Nonlinear  
phase - steady  
zonal flow with  
background  
turbulence →



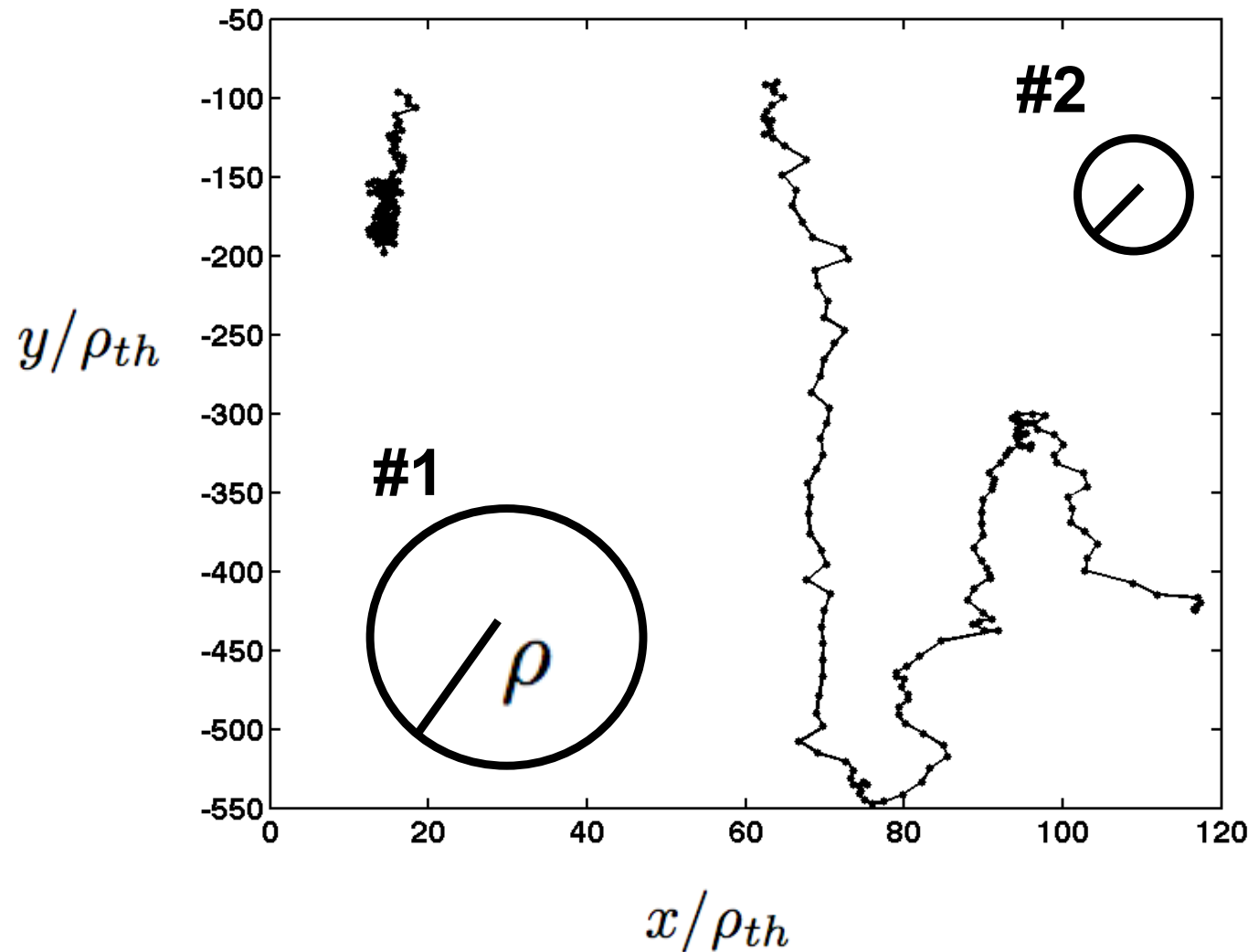


# Periodic copies for tracers to follow



# Sample gyrocenter trajectories

Marker particles distributed on  $v_{\perp}$  grid



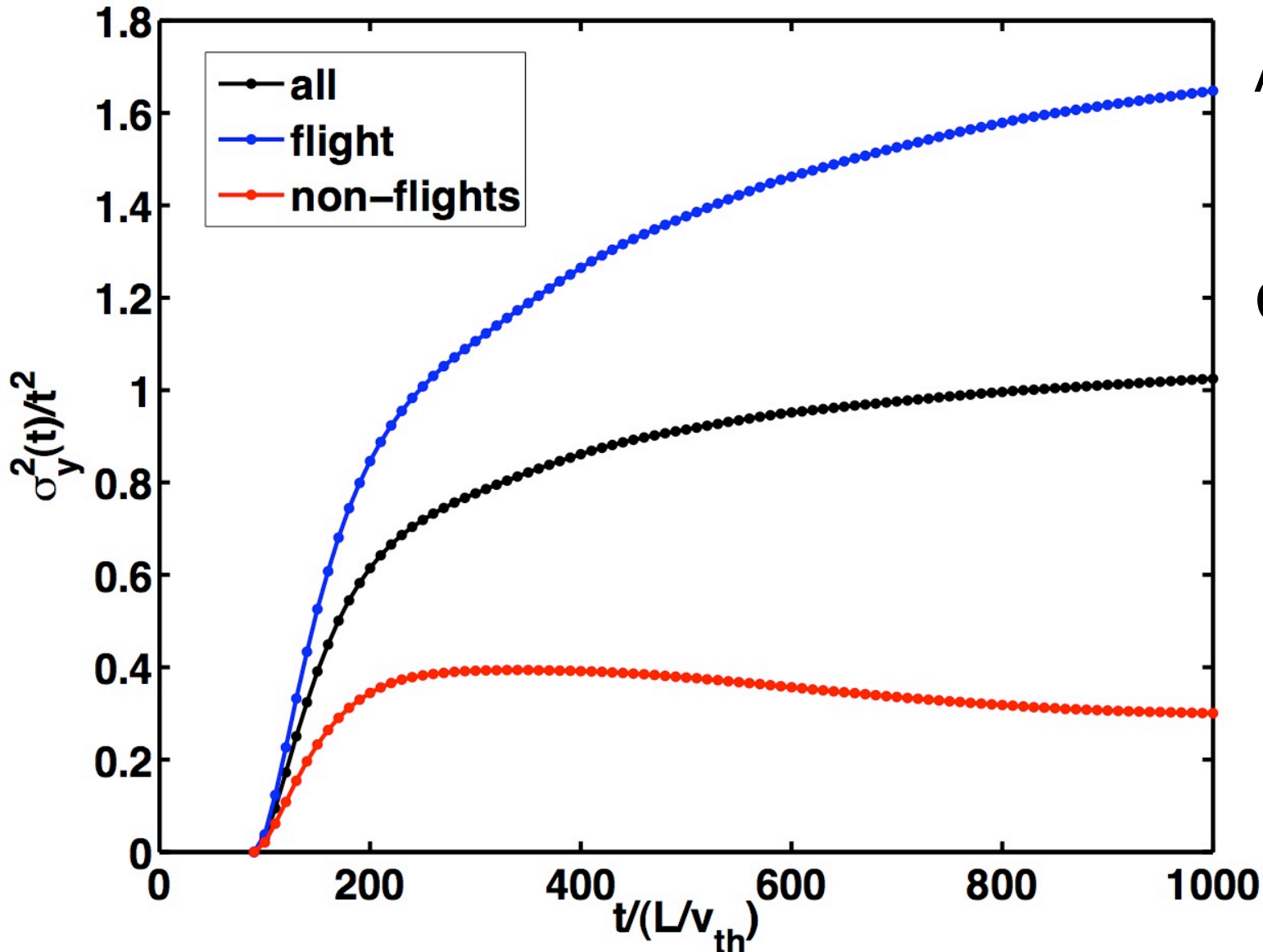
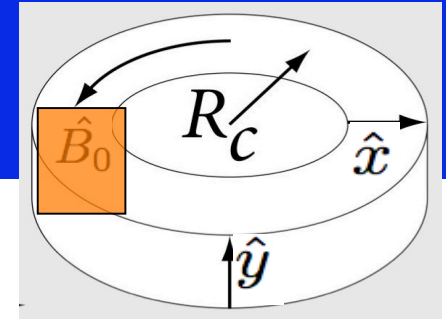
$$\Delta x_1 \sim 5 \rho_{th}$$

$$\Delta x_2 \sim 60 \rho_{th}$$

Larger  $v_{\perp}$  implies more trapping in  $E \times B$  vortices (for a given turbulence scale)

# Axial displacements

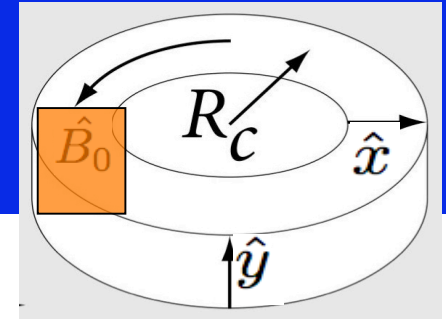
Gustafson, del Castillo Negrete, Dorland, PoP 102309 (2008)



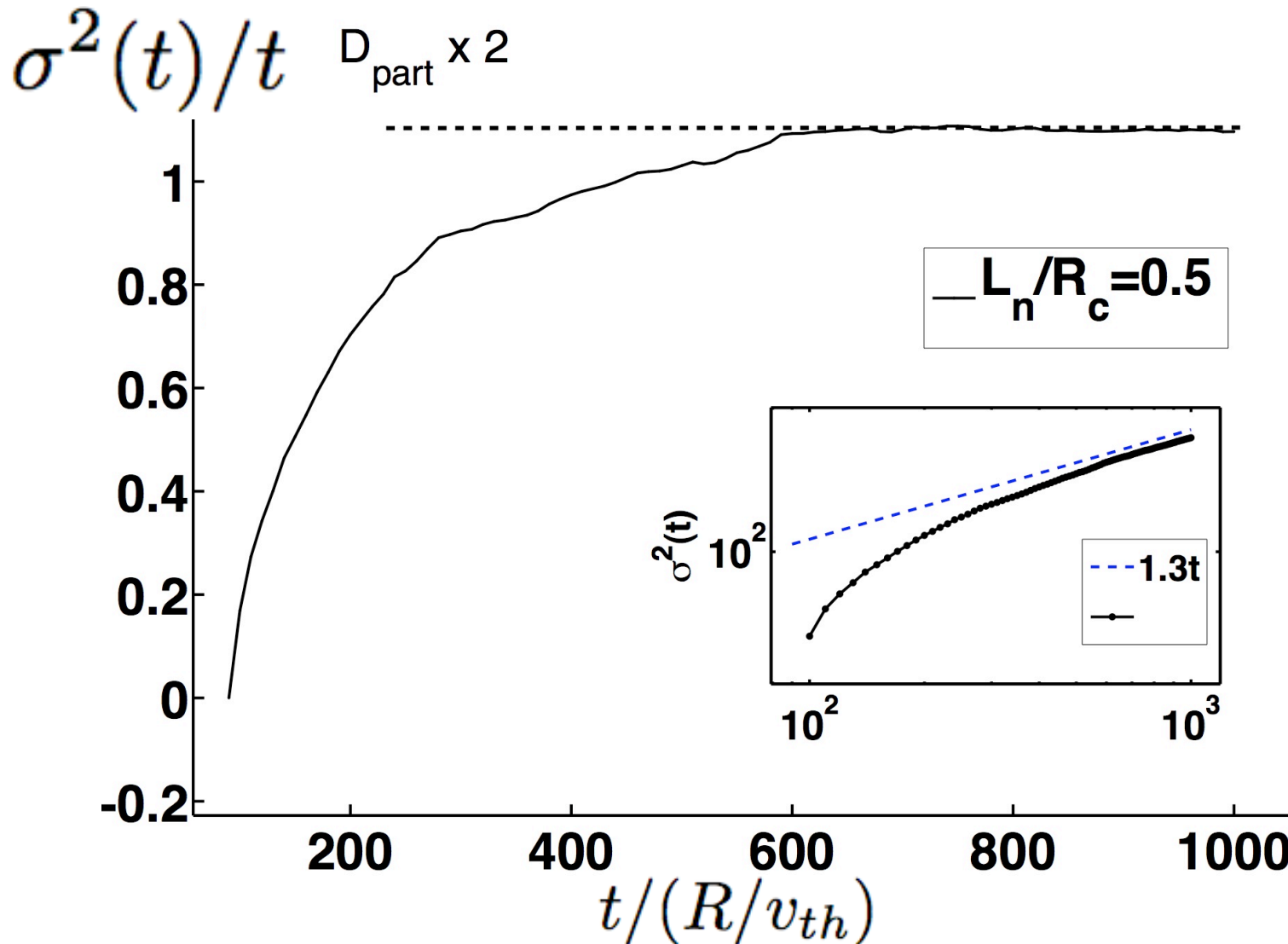
Approaching  
ballistic ( $\gamma \sim 2$ )

Consistent with  
our study of a  
prescribed  
vortex  
chain in shear  
flow (see ref.)

# Radial displacements



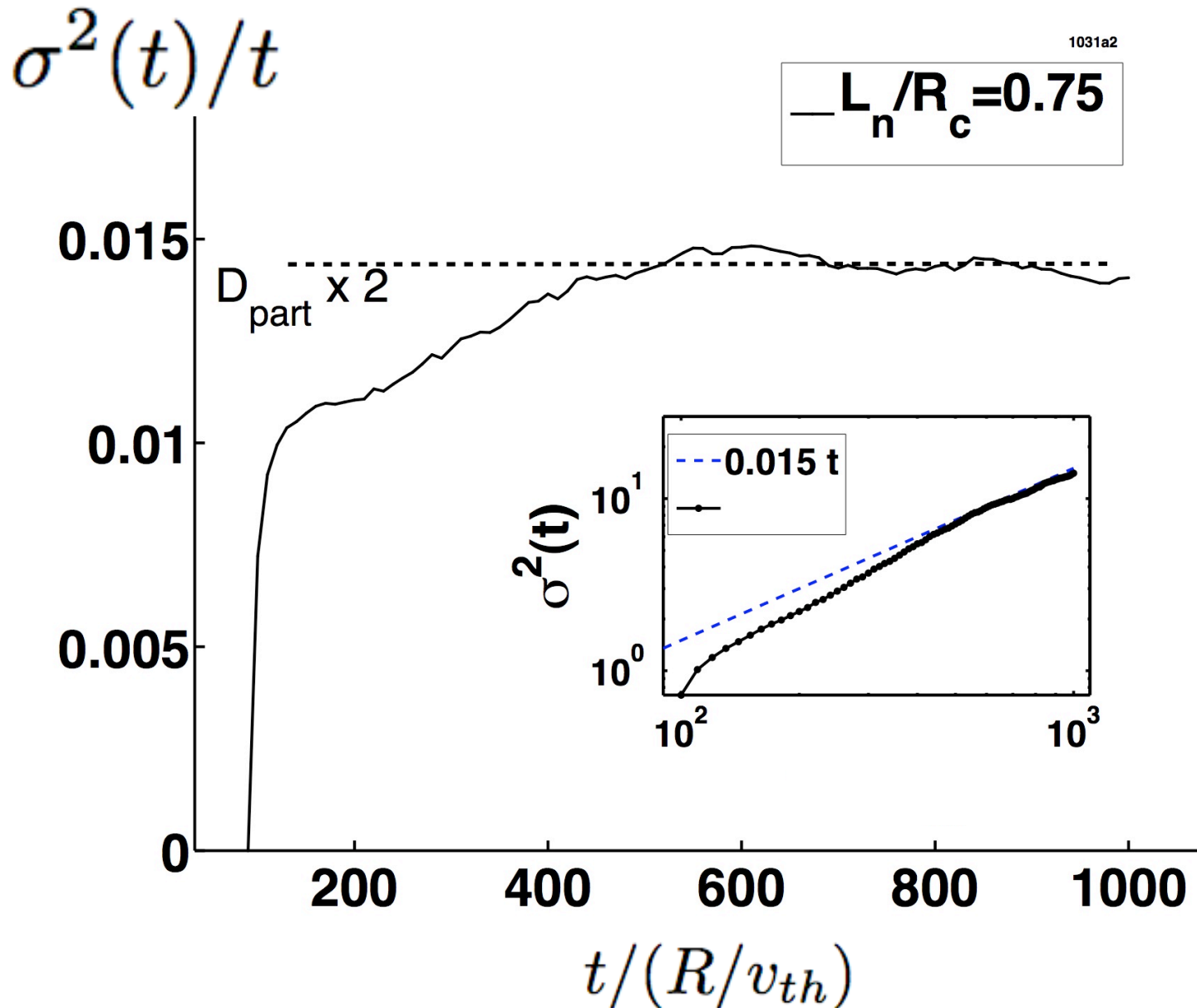
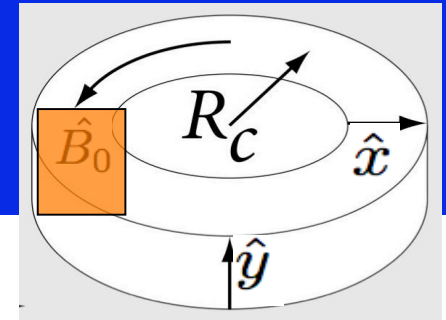
c.f. Manfredi & Dendy PRL 1996; Zhang *et al* PRL 2008;  
 Sanchez *et al* PRL 2008; Hauff *et al* PRL 2009



Strongest  
 gradient tested

Clearly diffusive  
 after  
 $t = 600 R/v_{th}$ .

# Radial displacements



Medium  
gradient ( $L_n$ )

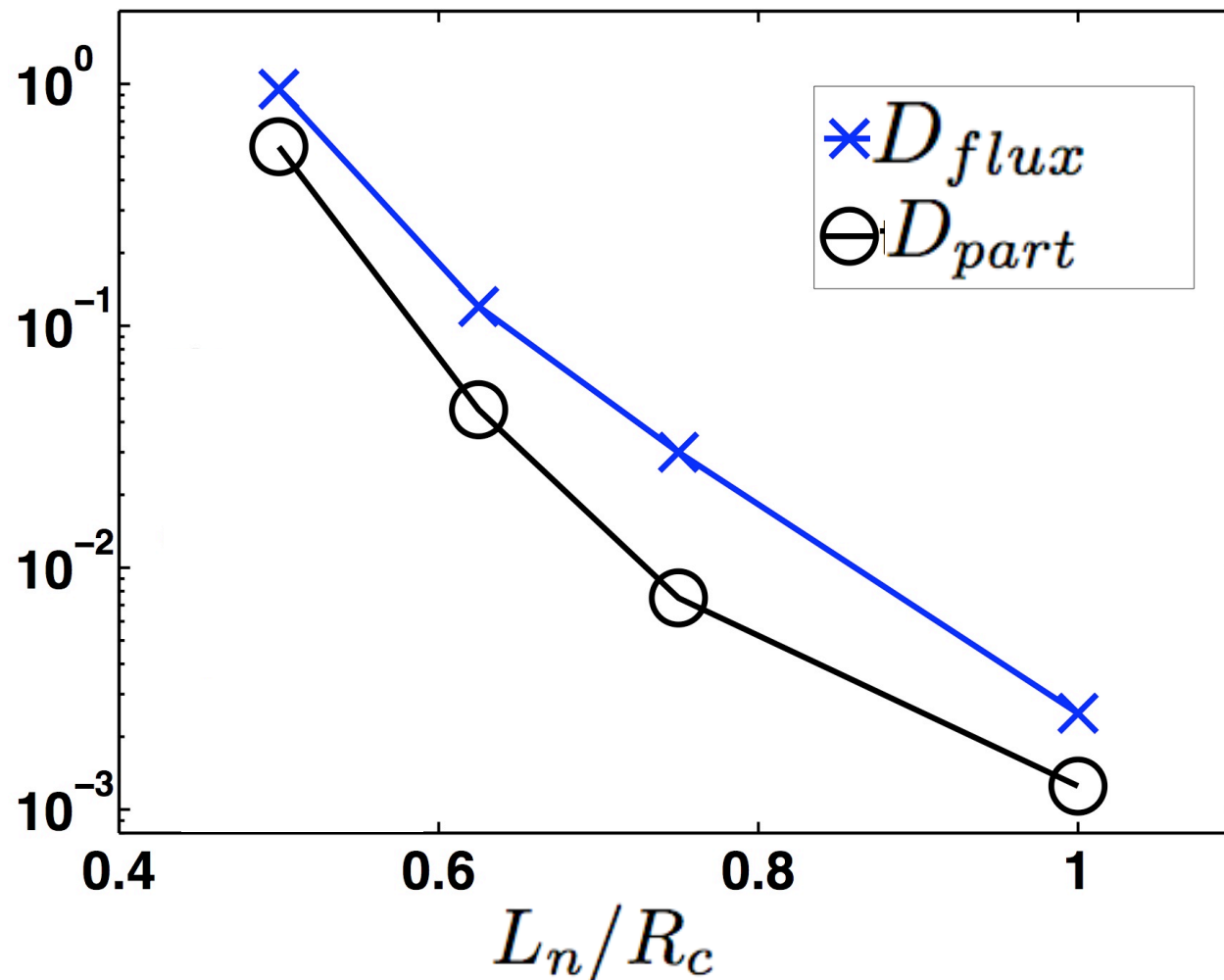
Clearly diffusive  
after  
 $t = 600 R/v_{th}$ .

Smaller  
diffusivity  
consistent with  
observed  
change in flux

# Diffusivity two ways

Basu *et al* Phys Plasmas, 10 2003

$$D_{part} = \frac{\sigma_x^2(t)}{2t} \stackrel{?}{=} D_{flux} = \Gamma_\infty L_n$$



Conservation of potential vorticity,

$\Pi = \nabla^2 \phi - n + x$ ,  
in the inviscid local  $L_n$  limit implies these should be equal.

Disagreement : artificial dissipation and Krook collision operator, or new version of potential vorticity is needed.

# Diffusivity depends on gyroradius

see Hauff & Jenko PoP 102309 (2006) and references therein

$$L_n/R_c = 0.5$$

$D_{part}$

Strong gradient

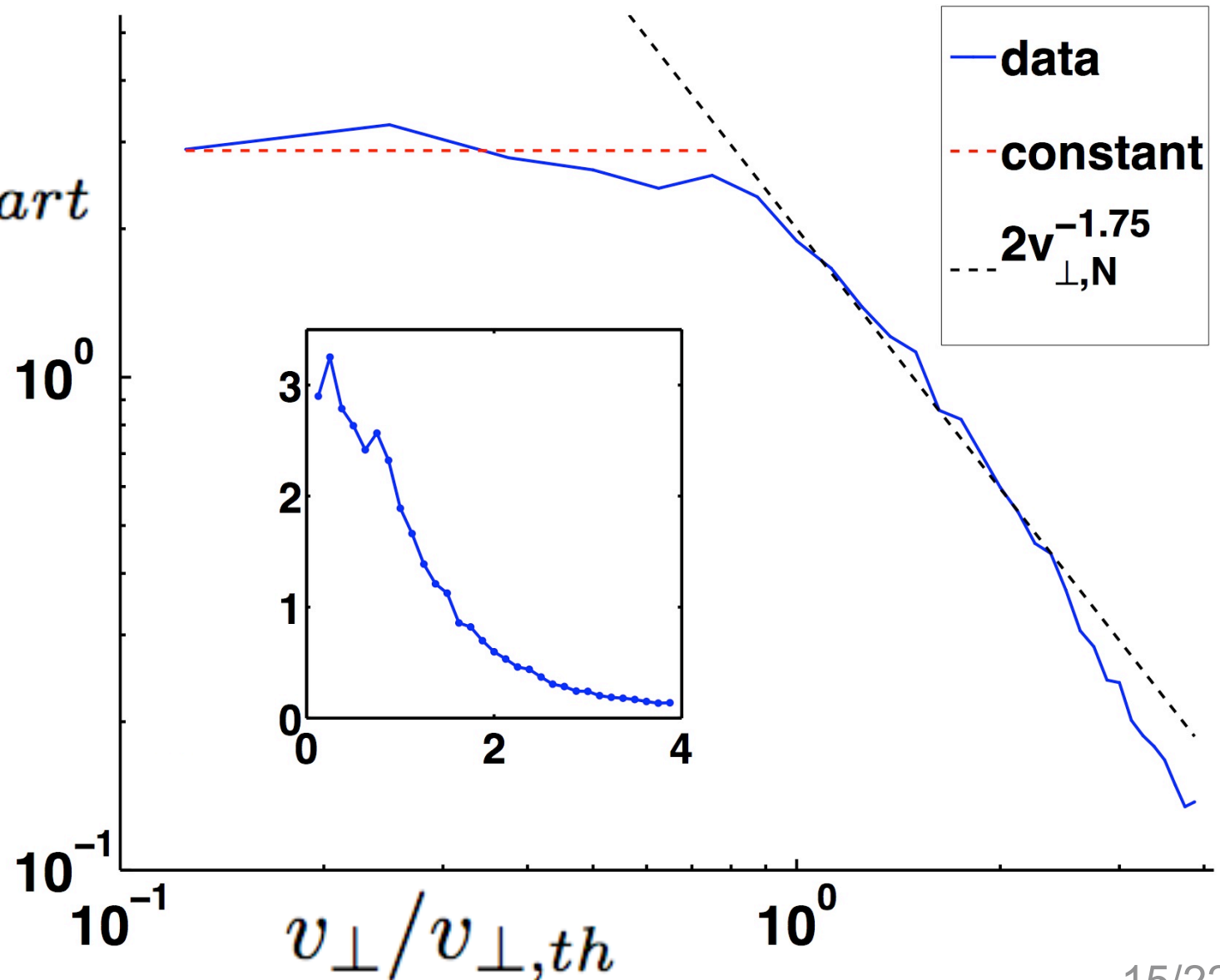
→

turbulent  
scales

comparable  
to  $\rho_{th}$

→

decay of  
diffusivity sets  
in later



# Diffusivity depends on gyroradius

$$L_n/R_c = 0.75$$

$D_{part}$

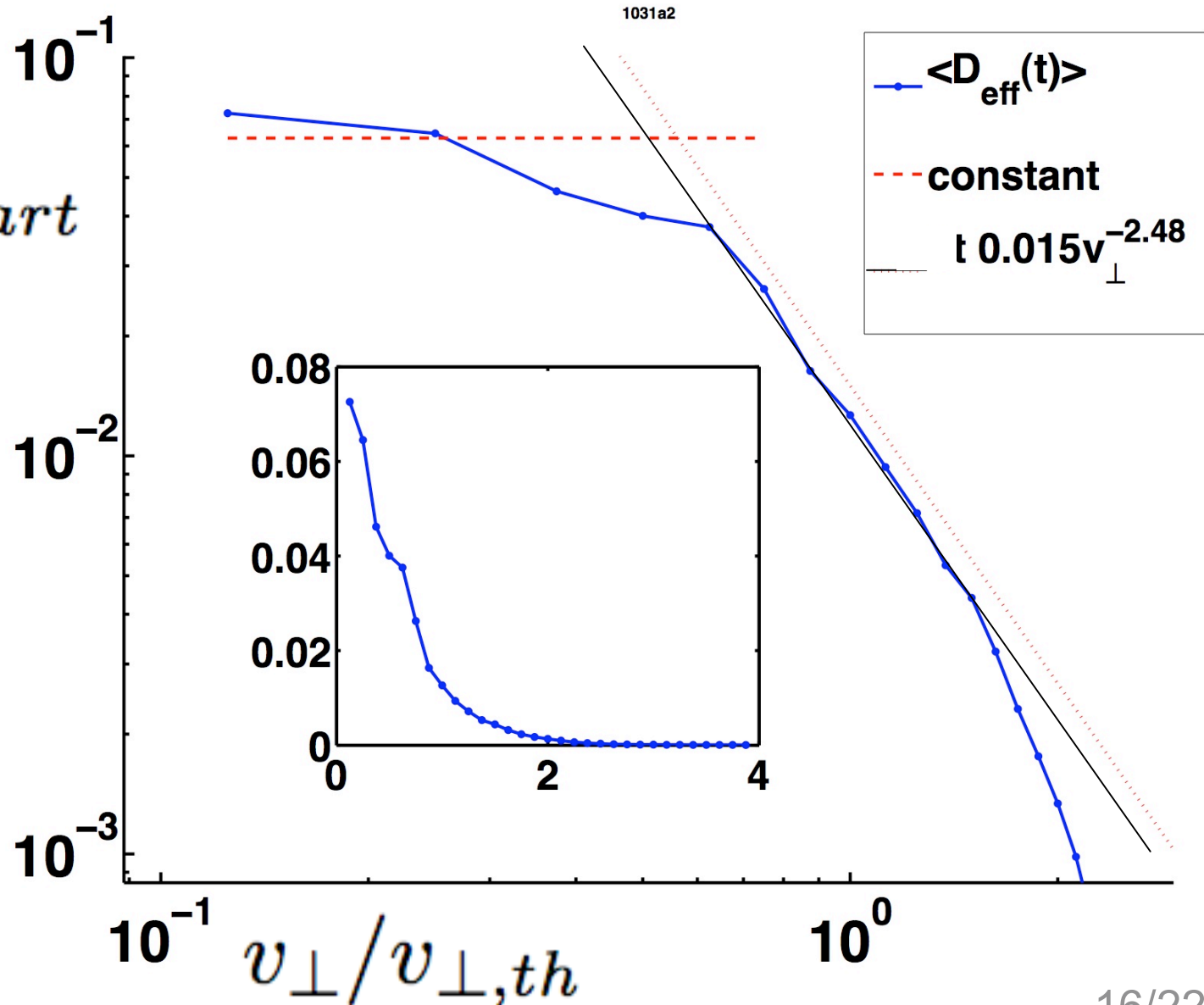
Weaker gradient

→

smaller scales  
of turbulence

→

turbulent  
trapping  
smooths away  
at smaller  $\rho_{th}$



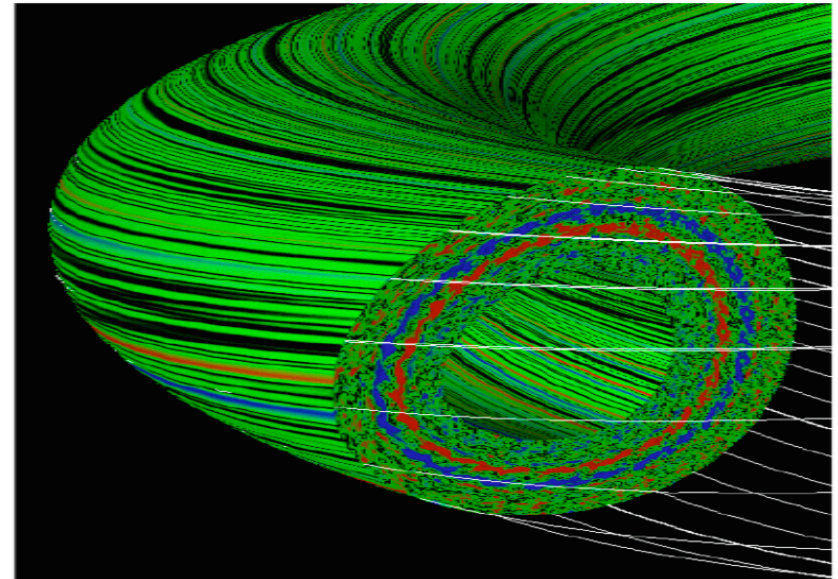


# Summary of these results

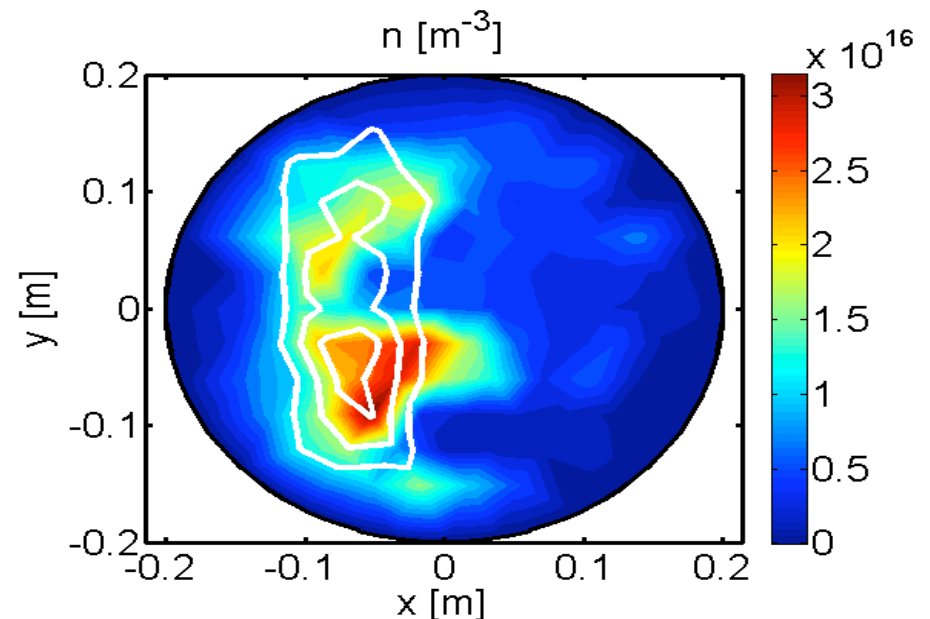
- New gyrokinetic PIC code, valid at large  $k_{\perp}\rho$ , is benchmarked with GS2 and convenient for studying particle dispersion
- Zonal flows in a local gyrokinetic simulation give diffusive radial test-particle transport at the ion Larmor radius scale.
- Test-particle diffusivity,  $D_{\text{part}}$ , is related to Fick's law diffusivity,  $D_{\text{flux}}$ .
- Energy dependence of diffusivity shows clear trends for several values of the density gradient.

# Random walks in turbulence

1) Microscale turbulence with zonal flows



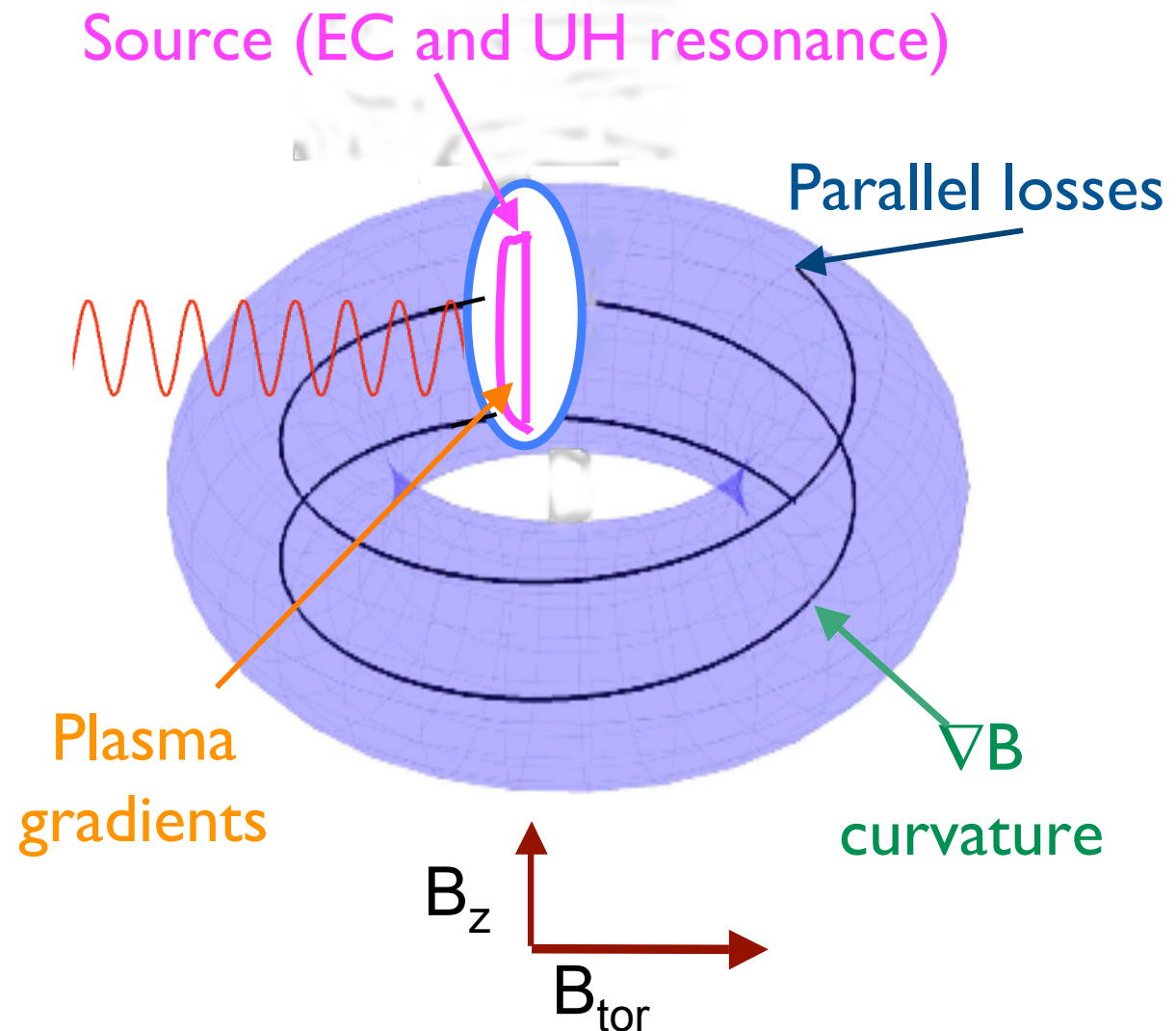
2) Macroscale turbulence with blobs



# New project: fast ions in TORPEX

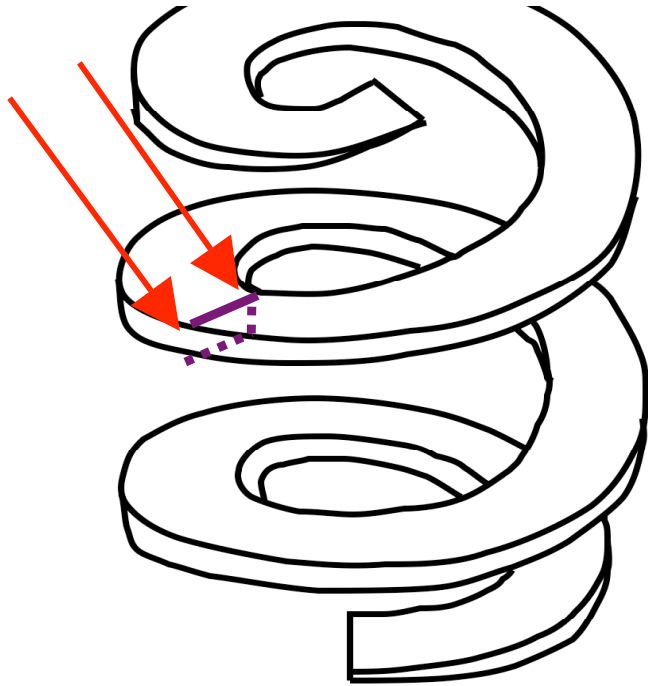
A. Fasoli, *et al*, Phys. Plasmas 13, 055902 2006

- Experimental model of scrape-off-layer - simple magnetized torus.
- Well-characterized blobs could drive nondiffusive transport

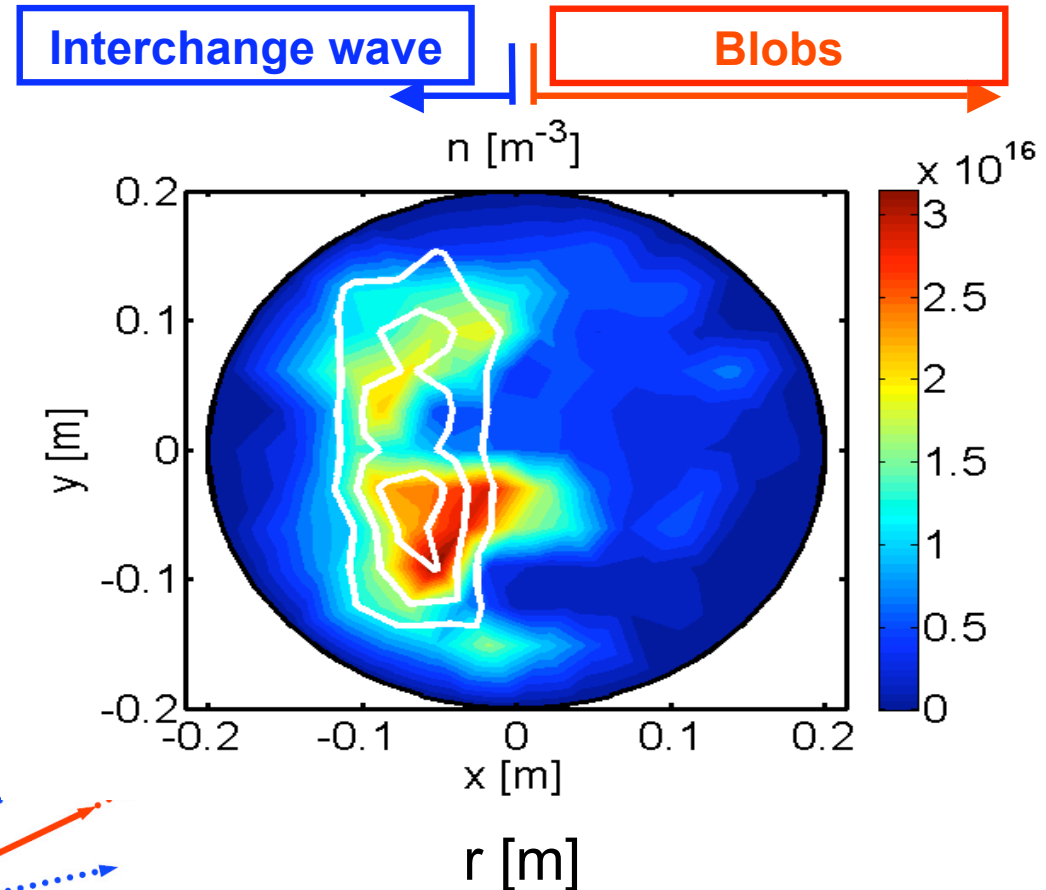
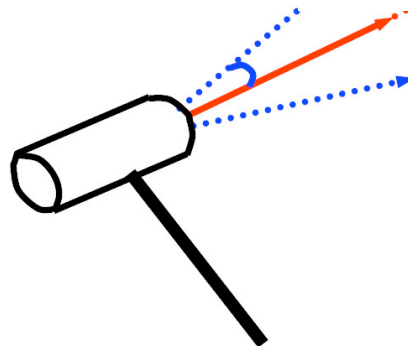


# Simulated fast ions in turbulent E-field: drift-reduced Braginskii equations

Periodic boundary conditions



$k_{\parallel}=0$   
2D simulation

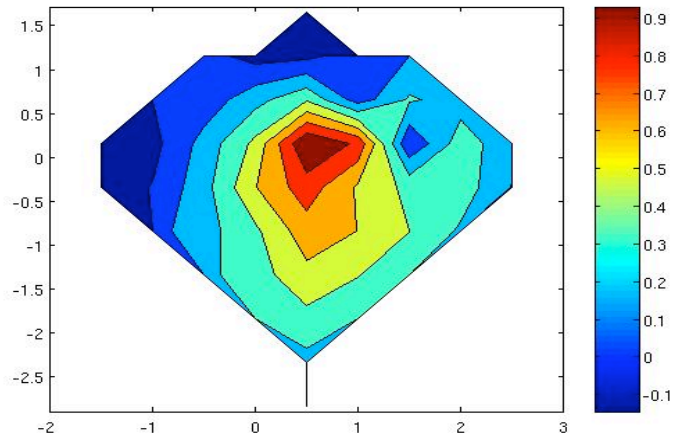


Tracer particle source with realistic  
spread in energies (10%) and in  
angular distribution (0.2rad)

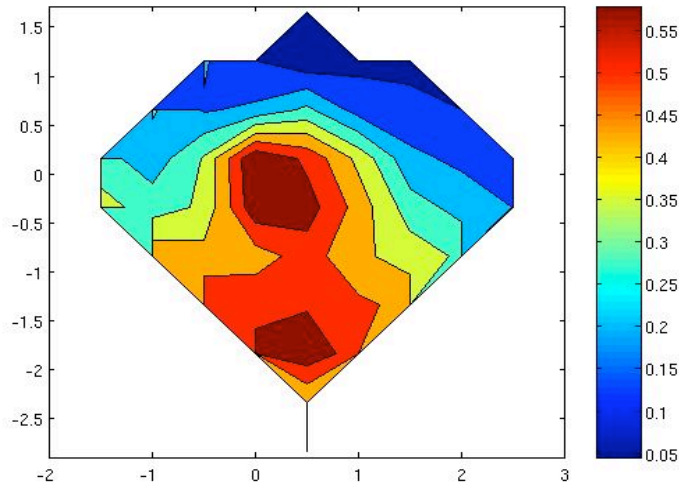
# Fast ions: early results

Data

Synthetic diagnostic

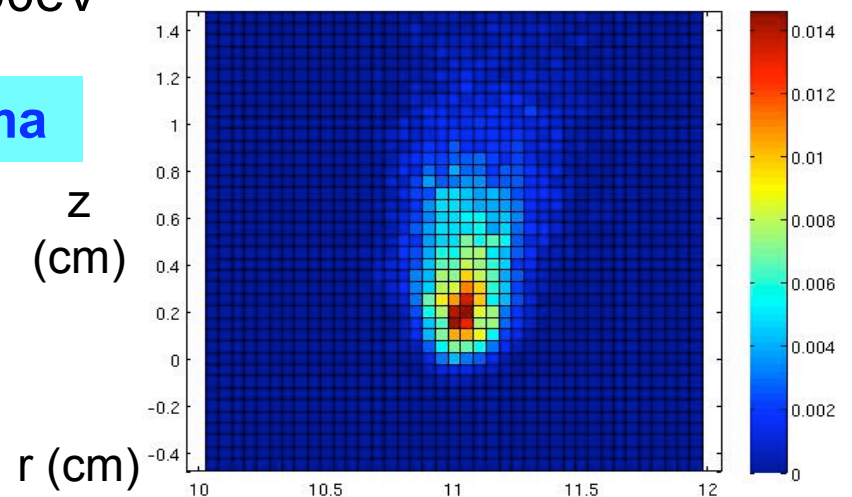
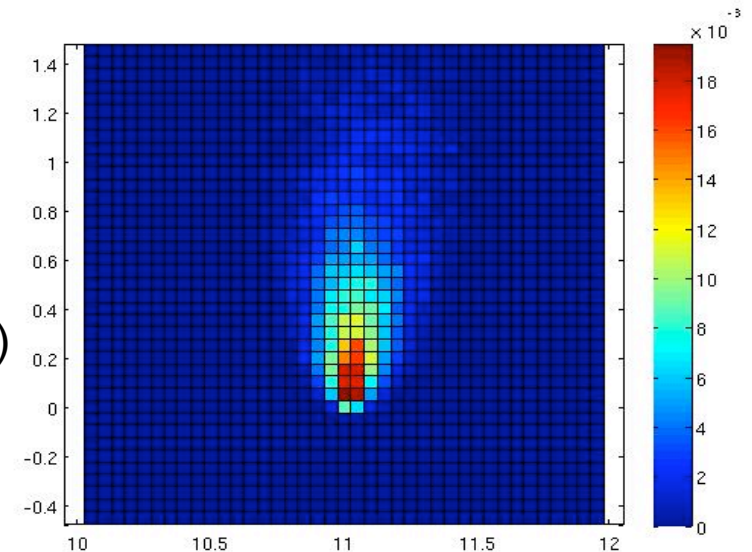


Without plasma



With plasma

$E_{\text{fast}} = 300\text{eV}$



# Asymptotic modeling for high and low energy ions

- To be characterized in terms of the variance and a separable continuous time random walk: trapping time and flight length distributions
- Radial trapping increases with particle energy because averaging effects make radial streaming ineffective