Vacuum Stability in Supersymmetric Effective Theories

THÈSE N^{O} 5192 (2011)

PRÉSENTÉE LE 18 NOVEMBRE 2011

À LA FACULTÉ SCIENCES DE BASE
GROUPE SCRUCCA
PROGRAMME DOCTORAL EN PHYSIQUE

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

POUR L'OBTENTION DU GRADE DE DOCTEUR ÈS SCIENCES

PAR

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ÉCOLE POLYTECHNIQUE
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Suisse
2011

Abstract

The main topics discussed in this thesis are supersymmetric low-energy effective theories and metastability conditions in generic non-renormalizable models with global and local supersymmetry.

In the first part we discuss the conditions under which the low-energy expansion in space-time derivatives preserves supersymmetry implying that heavy multiplets can be more efficiently integrated out directly at the superfield level. These conditions translate into the requirements that also fermions and auxiliary fields should be small compared to the heavy mass scale. They apply not only to the matter sector, but also to the gravitational one if present, and imply in that case that the gravitino mass should be small. We finally give a simple prescription to integrate out heavy chiral and vector superfields consisting respectively in imposing stationarity of the superpotential and of the Kähler potential; the procedure holds in the same form both for global and local supersymmetry.

In the second part we study general criteria for the existence of metastable vacua which break global supersymmetry in models with local gauge symmetries. In particular we present a strategy to define an absolute upper bound on the mass of the lightest scalar field which depends on the geometrical properties of the Kähler target manifold. This bound can be saturated by properly tuning the superpotential and its positivity therefore represents a necessary and sufficient condition for the existence of metastable vacua. It is derived by looking at the subspace of all those directions in field space for which an arbitrary supersymmetric mass term is not allowed and scalar masses are controlled by supersymmetry-breaking splitting effects. This subspace includes not only the direction of supersymmetry breaking, but also the directions of gauge symmetry breaking and the lightest scalar is in general a linear combination of fields spanning all these directions. Our purpose is to show that the largest value for the lightest mass is in general achieved when the lightest scalar is a combination of the Goldstone and the Goldstino partners.

We conclude by computing the effects induced by the integration of heavy multiplets on the light masses. In particular we focus on the sGoldstino partners and we show that heavy chiral multiplets induce a negative level-repulsion effect that tends to compromise vacuum stability, whereas heavy vector multiplets in general induce a positive-definite contribution.

Our results find application in the context of string-inspired supergravity models, where metastability conditions can be used to discriminate among different compactification scenarios and supersymmetric effective theories can be used to face the problem of moduli stabilization.

Keywords: Standard Model, Supersymmetry Breaking, Hidden Sector, Moduli, Supergravity, Effective Field Theories, Vacuum Stability.

Riassunto

I principali argomenti trattati in questo lavoro di tesi sono le teorie supersimmetriche effettive di bassa energia e le condizioni di metastabilità nell'ambito di generici modelli non rinormalizzabili con supersimmetria globale e locale.

Inizialmente discutiamo le condizioni per cui la supersimmetria viene preservata dallo sviluppo in derivate, in modo tale che i multipletti pesanti possano essere integrati via direttamente in supercampi. Le condizioni si traducono nel richiedere che anche i bilineari fermionici ed i campi ausiliari siano piccoli rispetto alla massa dei multipletti pesanti. Le stesse condizioni si applicano sia ai campi di materia che a quelli del settore gravitazionale, qualora esso sia presente; in quest'ultimo caso però, è necessario richiedere che anche la massa del gravitino sia piccola. Concludiamo definendo una procedura per integrare via i multipletti chirali e vettoriali che consiste nell'imporre rispettivamente la stazionarietà del superpotenziale e del potenziale di Kähler; la stessa procedura vale sia nel caso di supersimmetria rigida che di supergravità.

Nella seconda parte studiamo alcuni criteri generali per l'esistenza di vuoti metastabili che rompono la supersimmetria in modelli con simmetrie di gauge. In particolare, proponiamo una strategia per definire un limite superiore assoluto per la massa dello scalare più leggero che dipende dalle proprietà geometriche della varietà di Kähler. Questo limite può essere saturato fissando opportunamente i parametri del superpotenziale e pertanto, il fatto che esso sia positivo, costituisce una condizione necessaria e sufficiente per la metastabilità. Il limite è ottenuto considerando le direzioni nello spazio dei campi che non ammettono una massa supersimmetrica arbitrariamente grande e tali che le masse degli scalari associati siano interamente controllate da effetti di rottura di supersimmetria. Questo sottospazio include non soltanto la direzione di rottura della supersimmetria, ma anche le direzioni di rottura delle simmetrie di gauge e, in generale, lo scalare più leggero risulta essere una combinazione lineare di queste direzioni. Il nostro obbiettivo è di mostrare che il valore massimo per la massa più leggera si ottiene in generale quando lo scalare più leggero è una combinazione dei partner scalari associati al Goldstino e ai Goldstones.

Concludiamo studiando gli effetti indotti dall'integrazione di multipletti pesanti sulle masse di quelli leggeri; in particolare ci concentriamo sulla massa dei partner scalari del Goldstino e mostriamo che i multipletti chirali pesanti contribuiscono con un effetto negativo di level-repulsion che tende a compromettere la metastabilità mentre i multipletti vettoriali pesanti inducono in generale un contributo positivo.

I nostri risultati possono trovare applicazione nell'ambito dei modelli di supergravità derivanti dalla Teoria delle Stringhe, dove le condizioni di mestabilità possono essere usate per discriminare tra differenti scenari di compattificazione mentre le teorie effettive possono essere usate per affrontare il problema della stabilizzazione dei moduli.

KEYWORDS: Modello Standard, Rottura di Supersimmetria, Supergravità, Moduli, Settore Nascosto, Teorie di Campo Effettive, Stabilità del Vuoto.

A Eleonora, Francesca e Valentina

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"But precisely to the hero
is BEAUTY
the hardest thing of all.
Unattainable is beauty
by all ardent wills."

Friedrich Nietzsche
Thus Spake Zarathustra.

Introduction

We are entering a very exciting era for high energy physics! In November 2009 the Large Hadron Collider (LHC) at CERN in Geneva officially became the most powerful particle accelerator in the world. The energy scale which is going to be systematically explored by the LHC is the teraelectronvolt (TeV= 10^{12} eV) which is 11 orders of magnitude larger than the energy required to ionize a hydrogen atom. At these energies, physicists are going to probe the fundamental interactions between matter constituents up to distances of the order of 100 zeptometers (10^{-19} m) which is 9 orders of magnitude smaller than the typical atomic size! The theoretical framework describing physics at such scales is the *quantum theory* of *fields* which combines in a unified formalism the two most important physical theories of the 20th century namely *Special Relativity* and *Quantum Mechanics*.

The Standard Model is the universally accepted quantum theory of the fundamental particles and forces; it describes the physics of three fermionic generations of *quarks* and *leptons* and their interactions mediated by the exchange of *gauge bosons*. The Standard Model has been tested with extremely high accuracy up to energies of the order of 10^2 GeV and no significant deviations have been observed so far between theoretical predictions and experimental data. Despite this enormous success, one fundamental building block of the Standard Model is still missing: the *Higgs boson*. This particle is the elementary excitation of a fundamental field (the Higgs field) which is expected to trigger the spontaneous breaking of the electroweak symmetry and to give mass to both the gauge bosons of weak interactions and to the matter fermions. On top of that, the Higgs boson is a crucial ingredient of the Standard Model also because it ensures the unitarity of the scattering amplitudes of longitudinally polarized gauge bosons at energies of the order of 1 TeV. If the Higgs boson is not found in the near future, some new mechanism is expected to show up at the TeV scale, and this is exactly the energy range which is going to be extensively investigated by the LHC.

Besides the obvious interest for the Higgs sector, there exist other important motivations to expect very exciting physics at the TeV scale; this is because, even if the Higgs boson is discovered at the LHC, there are still several reasons to consider the Standard Model a partly unsatisfactory theory. The first problem we can mention is the fact that the Standard Model depends on too many free parameters (precisely 19) which must be fixed by experimental measurements; in particular the theory does not predict the values of the quark and lepton masses and there is no fundamental explanation for the different hierarchies among them. Another important aspect is that the Standard Model describes only three of the four fundamental interactions so far discovered in nature namely the *electromagnetic*, the *strong* and the *weak* interactions. The inclusion of *gravity* in this scheme presents some important difficulties. In particular it implies that the Standard Model is not a truly fundamental theory but just a low-energy effective description which is valid only up to the energy scale at which gravity becomes as strong as the other interactions. The energy scale at which this is expected to happen is the Planck scale which is of the order of 10^{28} eV, 16 orders of magnitude larger than the TeV scale! There exist, however, some potential problems in considering the Standard Model as a low-energy effective theory valid up to the Planck scale. The major difficulty comes from the fact that the Higgs boson mass receives very large quantum corrections from the exchange of virtual particles at the quantum level and these contributions are in general of the order of the Planck mass, which is very large compared to the expected order of magnitude of the Higgs physical mass. Such a small value can arise only as a consequence of bizarre cancellations of large unrelated contributions, achieved by an extremely accurate fine tuning of the parameters of the theory, and is then very unnatural. This problem associated to the naturalness of the Higgs mass is known as the *hierarchy problem* and it is the main theoretical argument to expect new physics beyond the Standard Model at the TeV scale. Other important limitations of the Standard Model emerge when we also consider cosmological observations; indeed the theory does not include a candidate sector for the *dark matter* and there is no natural explanation for the small value of the *cosmological constant*, introduced in Einstein's equations of *General Relativity* to explain the cosmological expansion of the Universe.

The quest for the high energy completion of the Standard Model and for the fundamental theory unifying gravity with the other interactions is the major challenge of modern high energy physics research. In the construction of realistic models of physics beyond the Standard Model, theoretical physicists are commonly guided and inspired, as in the case of the hierarchy problem, by aesthetic criteria like naturalness, elegance and simplicity; the most successful theories are considered to be those which, starting from the smallest number of hypotheses or assumptions, succeed in explaining the greatest number of empirical facts. There is nothing wrong in being inspired by subjective criteria like *beauty* or symmetry to formulate physical theories; on the other hand, the greatest mystery of science is probably the fact that nature seems to follow exactly the same criteria!

There are essentially two approaches to the study of physics beyond the Standard Model: on one hand one can try to guess what the theory describing all the interactions in an unified way at the Planck scale is and identify the low-energy features of the theory using consistency arguments. On the other hand one can look at the low-energy experimental facts which are not naturally explained in the Standard Model and search for a more natural explanation for them. It is very remarkable that these two approaches appear to converge in the same direction defining a special ingredient which is expected to characterize physics beyond the Standard Model: the idea of *supersymmetry* (SUSY). From the low-energy phenomenological perspective, supersymmetry represents a natural mechanism to explain the small value of the Higgs mass; the way this is achieved is by postulating the existence of a fundamental symmetry relating bosons and fermions which automatically enforces the miraculous cancellations among the large quantum corrections to the Higgs mass. On the other hand, from the purely theoretical high-energy perspective, supersymmetry appears also to be a necessary ingredient of *String Theory*, which is the only presently known candidate theory for a unified fundamental description of all the interactions at the Planck scale.

Despite the enormous appeal of supersymmetry, the construction of realistic supersymmetric models presents some very non-trivial aspects. The difficulties are associated to the fact that SUSY cannot exist as an exact symmetry of nature since in that case it would predict a degenerate spectrum of fermion and boson masses which is not experimentally observed. On the other hand, if one expects supersymmetry to be the mechanism responsible for the stabilization of the Higgs mass, it cannot be arbitrarily broken. Most of the difficulties then arise because it is known from some general sum rules that the spontaneous breaking of supersymmetry must involve a completely new sector, called *hidden sector*, which interacts with the Standard Model particles only through suppressed interactions and whose physics is a priori completely unknown.

A very common paradigm is to assume that the hidden sector contains the *moduli* sector of String Theory. Moduli fields are neutral scalar fields whose vacuum expectation values determine the geometrical properties of the compactification manifold and which interact with Standard Model fields only through gravitational interactions suppressed by inverse powers of the Planck mass. The fact that moduli are in general expected to be stabilized with non-vanishing vacuum expectation values makes them very natural candidates for triggering the spontaneous breaking of supersymmetry. In this scenario, gravity is assumed to be the principal mechanism by which supersymmetry-breaking effects are transmitted to Standard Model superpartners. This suggests that the most natural theoretical framework to study supersymmetry breaking is actually *supergravity* (SUGRA).

In this thesis work we will review and extend some useful tools that can be used to simplify the study of the moduli sector physics in string-inspired models. There are two fundamental difficulties that one has to face in this context. The first one is the fact that there exists a very large variety of models which correspond to different compactification scenarios, but not all of them are expected to give a realistic description of our universe. The second problem is the fact that for each particular model there is a proliferation of moduli fields and in general an analytical study of the full dynamics is totally prohibitive. The first complication requires the study of some general criteria to efficiently discriminate among different scenarios. As a matter of fact, the only strong condition one can impose on realistic models is the existence of (at least) one metastable vacuum which breaks supersymmetry with a small and positive cosmological constant. Tackling the second difficulty requires the possibility of reducing the number of moduli fields that are effectively important in the study of the low-energy supersymmetry breaking dynamics. This can be done by integrating out all the heavy moduli that are stabilized with large supersymmetric masses and constructing a low-energy effective field theory describing the dynamics of the remaining light moduli. To summarize, the main objective of this thesis is to develop some useful strategies to tackle the two problems mentioned above; metastability conditions and supersymmetric effective theories are the principal instruments we introduce to achieve our purpose and are the main topics we are going to extensively study in this work.

This thesis is structured as follows. In the first two chapters we review the main ideas and results in global and local supersymmetry that are relevant for our discussions; the remaining three chapters represent the original core of this work and collect our main results on supersymmetric effective theories and vacuum stability. To be more precise, in Chapter 1 we first discuss more in depth the fundamental arguments motivating the study of supersymmetry; the second part of the chapter is a technical review of the most general non-renormalizable models containing matter and gauge fields in SUSY and SUGRA. Particular attention is given to the derivation of the supergravity Lagrangian which, for later convenience, is performed in the superconformal formalism. In Chapter 2 we discuss in some detail supersymmetry breaking focusing our attention on the constraints imposed by the supertrace formula and the hidden sector paradigm; we then present two important transmission mechanisms, namely gravity and gauge mediation. We conclude by discussing the characteristics of the hidden sector in string-inspired models and by better defining the problematics which inspire this work.

Chapter 3 is dedicated to the study of low-energy effective theories and the consistent supersymmetric integration of heavy multiplets in global and local supersymmetry. This part is based on the results presented in our paper *Brizi, Gomez-Reino* and *Scrucca*, 2009 [1]. Our main contribution relates to the integration of heavy multiplets in supergravity theories, since the case of rigid supersymmetry has already been extensively studied in the literature both for chiral and vector multiplets. In the case of SUGRA we find that one can use the same procedure valid in the rigid case to integrate out heavy multiplets at the superfield level provided that the mass of the gravitino, or equivalently the cosmological constant, is small compared to the heavy field mass scale.

Chapter 4 is devoted to the study of metastability conditions in general non-linear σ-models including both chiral and vector multiplets. This part is based on the results presented in our paper *Brizi* and *Scrucca*, 2011 [2]. Our main contribution consists in this case in clarifying the role of Goldstone partners in defining the strongest upper bound on the mass of the lightest scalar in models in which the superpotential can be arbitrarily varied while the Kähler potential and the gauged isometries are assumed to be fixed. This work extends and improves some previous studies in which only the Goldstino partners were taken into account to define necessary conditions for the existence of metastable vacua.

In Chapter 5 we combine the general ideas of the previous two chapters and study the effects induced by the integration of heavy multiplets on the masses of the light scalars and on the metastability conditions; this is done in the special case for which all vector multiplets are heavy and the only potentially dangerous modes are the Goldstino partners. This part is based on the results presented in our paper *Brizi* and *Scrucca*, 2010 [3]. In this chapter we show that the correction to the effective sGoldstino mass induced by heavy chiral multiplets is always negative and tends to compromise vacuum metastability, whereas the contribution from heavy vector multiplets is always positive and tends, on the contrary, to reinforce it.

In the section dedicated to the conclusions we present a detailed summary of the main results achieved in this thesis and discuss some possible future directions.

Chapter 1

Supersymmetry and Supergravity

In this chapter we present a review of the basic ideas and tools in global and local $N = 1$ supersymmetry that will be useful for future discussions; in particular we focus our attention on the class of generic non-renormalizable models called *non-linear σ-models.*

In the first part we review the most important arguments singling out supersymmetry as one of the most fascinating conjectured feature of physics beyond the Standard Model. We show that there exist several hints, mostly based on phenomenological and on purely theoretical arguments, which suggest that supersymmetry may play a relevant role in describing physics at the TeV scale and beyond. This first part has the form of a brief non-technical review of the main motivations for studying supersymmetry.

In the following sections we review in a pragmatic way the structure of non-linear σ-models both in SUSY and SUGRA, focusing our attention on the derivation of the scalar potential and the mass matrices. In the rigid case we schematically recall the derivation of the full Lagrangian and the masses of scalar, spinor and gauge fields; we will use these expressions to revisit the supertrace formula which imposes strong constraints on the possibility of realizing realistic scenarios for supersymmetry breaking. This part is presented as a brief technical review of the main formulas that we will need in the following chapters; since this topic is quite standard and well established we will focus on the main concepts avoiding too many details.

In the case of supergravity theories, we will present in some detail the construction of the most general Lagrangian. This part is less standard since in the literature there exist several different approaches to the subject; for this reason we will perform a more careful and detailed analysis. It turns out that the most suitable framework for our purposes is the *superconformal supergravity* formalism. We therefore revisit the main steps and arguments followed in the construction of the supergravity Lagrangian in this approach and we recall the expressions of the scalar potential and the scalar masses which will be extensively used in the following chapters.

1.1 Effective Field Theories and Natural Hierarchies

At present time there exist several indications suggesting that supersymmetry [4–6] should be considered as a plausible guiding principle for physics beyond the Standard Model [7–9]. Before discussing the main arguments in favor of this hypothesis, let us first review the modern point of view on the Standard Model and why it is expected to be an incomplete theory.

The very first consideration we can do is that the Standard Model cannot be a fundamental theory because it does not include a truly fundamental description of gravitational interactions at the quantum level. More precisely, the canonical quantization of General Relativity produces a non-renormalizable quantum field theory with a dimensionful coupling $1/M_P^2$ proportional to the inverse of the Planck mass. From a modern perspective, the fact the Standard Model plus General Relativity is a non-renormalizable quantum field theory means that it is an effective description which is valid only for energies much smaller than the cut-off scale M_P at which the effective coupling E^2/M_P^2 becomes of order one. At the Planck scale this picture is expected to break down and should be replaced by a more fundamental theory which includes new degrees of freedom. A priori there are no serious motivations to believe that this ultimate theory is a renormalizable quantum field theory; in fact, most attempts to construct a truly fundamental quantum theory of gravitational interactions are based on completely new paradigms (e.g. String Theory). We conclude that renormalizability should not be considered as a fundamental principle in quantum field theory model building; non-renormalizable theories are perfectly fine as long as we consider them as low energy effective descriptions of more fundamental theories; it is exactly in this sense that we consider the Standard Model as an incomplete (or not fundamental) theory.

Renormalizable quantum field theories are very peculiar theories; technically renormalizability corresponds to the possibility of extrapolating long range physics to small distances without encountering new degrees of freedom. In this sense renormalizable quantum field theories can be truly fundamental descriptions of nature. However, more in general, effective Lagrangians do not contain only renormalizable operators of dimension $d_i \leqslant 4$ but include also a tower of higher-dimensional operators whose couplings are suppressed by the mass scale Λ at which new physics shows up:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{d_i \leqslant 4} + \sum_{d_i > 4} \lambda_{d_i} \frac{\mathcal{O}_{d_i}}{\Lambda^{d_i - 4}} , \tag{1.1}$$

where $\mathcal{O}_{d_i}$ are operators of dimension d_i and λ_{d_i} are dimensionless couplings; $\mathcal{L}_{d_i \leqslant 4}$ is the renormalizable Lagrangian. At tree level, the effect of each coupling can be tracked by simple dimensional analysis; in particular only the dimensionless effective couplings $\tilde{\lambda}_{d_i}(E) \sim \lambda_{d_i} (E/\Lambda)^{d_i - 4}$ can enter in the definition of observable amplitudes. This analysis shows that in the infrared region $E \ll \Lambda$ only the operators in the renormalizable Lagrangian are important whereas the contributions coming from the higher dimensional ones flow to zero. Operators with mass dimension $d_i < 4$ are called *relevant* since they always give important contributions in the infrared; the ones with $d_i > 4$ are called *irrelevant* since their effects disappear in the low-energy regime; finally operators with $d_i = 4$ are called *marginal* and the associated tree -level effects are independent of the energy scale. Despite the infinite tower of higher-dimensional operators, non-renormalizable Lagrangians conserve a predictive power. Indeed at each finite order $(E/\Lambda)^n$, only a finite number of operators contribute to the amplitudes; this is in fact not too restrictive since theoretical predictions must be matched with experimental observations which have finite precision. Quantum corrections introduce some technical subtleties in this analysis but do not spoil the general picture. This can be seen by choosing a regularization scheme (such as Dimensional Regularization) which does not exhibit power-like divergencies; in that case, simple dimensional analysis considerations hold also true at the quantum level. It is also possible to see that renormalizable Lagrangians are stable under loop corrections in the sense that no new higher-dimensional operator is generated at the quantum level. On the other hand, if a non-renormalizable operator is included at tree level, infinitely many higher-dimensional operators are generated by quantum effects.

The Standard Model, by itself, is a renormalizable theory, and a priori it is thus a good candidate to be a fundamental theory; however, as we have seen, when gravity is included this is not true anymore. Actually, even without introducing gravity, there is no reason to assume that there does not exist any new physical effect arising at some high energy scale smaller than the Planck mass, since the Standard Model has been tested only up to energies of order 10^2 GeV. From a low energy perspective, what we can do is to experimentally estimate the scale Λ at which new physics can appear. At present time no significant deviations from Standard Model predictions have been experimentally observed; moreover the accuracy achieved in past experiments is sufficiently high to conclude that new physics effects are strongly suppressed and the new physics scale should be very large. More precisely, we observe for example that dimension-six four-fermion operators violating baryon number are suppressed by a scale of order [10]

$$\Lambda_{\not{B}} \gtrsim 10^{15}\,\mathrm{GeV}\,. \tag{1.2}$$

Other constraints on new physics come from flavor-violating processes and the associated operator are found to be suppressed by a scale of order [10]

$$\Lambda_{\not{F}} \gtrsim 10^{6}\,\mathrm{GeV}\,. \tag{1.3}$$

Despite the good agreement between Standard Model predictions and observations, there are still some unsatisfactory aspects of the model which motivate theoretical particle physicists to expect that some new physics should actually show up at smaller scales. In particular, some problematic aspects of the Standard Model, as we are going to see, require a solution in terms of new physics in the TeV region. In the following we are going to discuss the so called *hierarchy problem* (or naturalness problem) [11, 12], which is one of the most significant theoretical drawbacks of the Standard Model. It has inspired most of the modern scenarios for physics beyond the Standard Model and, more relevantly, it is the most important phenomenological motivation to study supersymmetry.

The hierarchy problem is associated with the theoretical difficulty in explaining in a natural way the small value of the Higgs mass that is suggested by precision electroweak measurements [10]:

$$m_h \lesssim 150\text{GeV}. \tag{1.4}$$

As we are going to discuss in a moment, this value is very unnatural if the energy scale Λ associated to the new physics is much larger than the TeV scale. It is worth to stress that the hierarchy problem is not a theoretical inconsistency of the Standard Model; it is however a well motived question about the naturalness of one of its parameters which appears to be unnaturally adjusted. The existence of this fine-tuning suggests that, very likely, some new underlying mechanism is "conspiring" to produce such an unexpected value.

The problem consists in the fact that a large hierarchy between the Higgs mass and the physical cut-off is not automatically stable under quantum corrections; more precisely, loop corrections to the Higgs mass are quadratic in Λ:

$$(m_h^2)_{\text{eff}} = m_h^2(\Lambda) + c\,\Lambda^2 + ... \tag{1.5}$$

and tend to destroy the hierarchy unless the tree-level mass is unnaturally tuned. The amount of the necessary fine tuning increases with the scale at which new physics effects become sizable and for $\Lambda \simeq M_P$ it turns out to be a formidable task to naturally explain such a huge hierarchy between the Higgs mass and the Planck scale.

It is natural to expect that if a particle has a mass which is much smaller than Λ there should exist a symmetry (at least an approximate one) under which the mass term is forbidden; in this case we say that the mass is "protected" by a symmetry [1]. In ordinary quantum field theory, many examples of naturally small masses protected by symmetries are known. For example, the photon is naturally massless since gauge invariance prevents quantum corrections from generating a mass term for it. Similarly, chiral symmetry forbids a mass term for Dirac fermions implying that quantum corrections are not proportional to the large cut-off but to the fermion mass itself. Scalar particles, on the other hand, can be naturally light if they are (pseudo) Goldstone bosons of a spontaneously broken (approximate) global symmetry. In this case shift symmetry forbids a mass term and the scalar field has only derivative couplings.

In the Standard Model, none of the above mentioned mechanisms prevents large quantum corrections to the tree-level mass of the Higgs scalar. In absence of any symmetry principle we then expect $m_h \simeq \Lambda$.

[1]This naturalness criterium has been rigorously formulated by 't Hooft [13].

1.2 SUSY as a Solution to the Hierarchy Problem and Beyond

As we anticipated, supersymmetry offers a natural explanation for the small value of the Higgs mass; this is achieved by protecting the mass of scalar fields by a new (unconventional) symmetry relating bosons and fermions. Supersymmetry is not an ordinary symmetry in the sense that its algebra is a graded Lie algebra; besides the ordinary (commuting) bosonic generators of the internal and Poincaré symmetries, it contains anticommuting fermionic generators Q implementing transformations of boson into fermions and vice-versa; schematically:

$$Q\,|B\rangle = |F\rangle\,, \qquad\qquad Q\,|F\rangle = |B\rangle\,. \tag{1.6}$$

Global supersymmetry requires that particles belonging to the same supermultiplet are degenerate in mass; this implies that the same chiral symmetry which forbids fermion mass terms protects also scalar masses from large quantum corrections. Technically, the way in which this is achieved is through the cancellation of the dangerous loop diagrams among superpartners of different spins; such cancellations are enforced by the peculiar structure of the dimensionless couplings required by supersymmetry invariance. More precisely, the large quadratic quantum corrections to the Higgs mass associated to loops of heavy fermions (the top or some new heavy particle) are cancelled by loop diagrams of the corresponding scalar superpartners:

$$\Delta m_h^2 \propto \frac{1}{8\pi^2}(\,\lambda_\phi - |\lambda_\psi|^2\,)\,\Lambda^2 + \ldots\,; \tag{1.7}$$

the cancellation is possible thanks to the special relations between the couplings which hold in supersymmetry and which guarantee that:

$$\lambda_\phi = |\lambda_\psi|^2\,. \tag{1.8}$$

However, as explained more extensively in Chapter 2, supersymmetry cannot be an exact symmetry of nature and must be broken by some mechanism in order to explain the non-observation of the predicted degeneracy between Standard Model particles and superparticles. On the other hand, if supersymmetry is the mechanism responsible for the stabilization of the electroweak scale, it cannot be broken in an arbitrary way. In the next chapter we will discuss in some detail *soft* supersymmetry breaking; here we limit ourselves to mention the fact that supersymmetry breaking terms should not spoil the relation between dimensionless couplings which ensure the cancellation of the dangerous quantum corrections to all orders in perturbation theory. This means that supersymmetry-breaking terms should be relevant operators with dimensionful couplings. If we call m_{soft} the scale associated to these operators we have that:

$$\Delta m_h^2 \propto m_{soft}^2\,, \tag{1.9}$$

which implies that m_{soft} cannot exceed too much the TeV range to avoid fine-tuning problems.

Supersymmetry is not the only mechanism which has been investigated to solve the hierarchy problem. A more conventional mechanism to generate naturally large hierarchies is by dimensional transmutation of dimensionless couplings in asymptotically-free non-Abelian gauge theories (as for example in QCD). The basic idea is to start with a theory which has no dimensionful couplings; the fundamental scale Λ_{IR} of the theory is then dynamically generated at the quantum level by the anomalous breaking of the scale invariance. More precisely, the low energy scale Λ_{IR} at which the coupling becomes of order one and perturbation theory breaks down is given by:

$$\frac{\Lambda_{IR}}{\Lambda} = e^{\frac{-8\pi^2}{b\, g_0^2}} , \tag{1.10}$$

where g_0 denotes the value of the gauge coupling at the large ultra-violet cut-off scale Λ. At the scale Λ_{IR}, gauge interactions become strong and the creation of fermion condensates $\langle\bar{\psi}\psi\rangle$ with a scale of order Λ_{IR} can take place. An example of such condensates in QCD are pions. These mesons are three pseudo-Goldstone bosons associated with chiral symmetry breaking and have masses which are much smaller than the other hadronic resonances. The important aspect is that the presence of light scalar mesons does not give rise to any fine-tuning problem since a large hierarchy between Λ_{IR} and Λ can be achieved in a natural way without need to tune the coupling g_0 with an extremely high accuracy. Models based on this mechanism are called Technicolor [14, 15]; in these models the Higgs is not a fundamental particle but (effectively) a composite one, similarly to the case of pions in QCD. In this case however the scale at which the postulated new interaction becomes strong is much larger than the one associated to strong interactions ($\Lambda_{QCD} \simeq 200\,\text{MeV}$) and it is of the order of $\Lambda_{TC} \simeq 500\,\text{GeV}$. Without entering any further into the details of these models, we just mention the fact that the minimal realization of the Technicolor scenario as a scaled version of QCD does not work; qualitatively speaking, this is because the six-dimensional operators which generate quark masses is as relevant as the dangerous higher-dimensional (4-quarks) operators which produce anomalous flavor changing effects.

We conclude this general and not completely exhaustive discussion [2] on the possible solutions to the hierarchy problem by mentioning a third class of models which assumes the existence of large extra dimensions [17]. In this scenario the large hierarchy between the electroweak scale and the Planck scale is completely removed and gravity becomes strong at the TeV scale, which is assumed to be the only fundamental scale in the theory. The observed weakness of gravity at distances larger than the millimeter range is due to the existence of (at least 2) new compact spatial dimensions which are large compared to the weak scale; Standard Model particles are supposed to be constrained by some mechanism to live on a four-dimensional manifold whereas gravity can also propagate over all the extra dimensions. The phenomenological signature of this scenario is the

[2] For a more complete review at the same level of details see for example [16].

appearance of quantum gravity effects at the TeV scale.

Even though we introduced supersymmetry as a very plausible solution to the hierarchy problem, this is not the only reason to be interested in it. There exist in fact at least two other remarkable indications which suggest that supersymmetry is likely to play a relevant role for physics beyond the Standard Model. These are indirect evidences coming from theoretical speculations about physics at very high energy scales (much larger than the TeV scale). In this context, the main paradigm guiding the theoretical investigation is the idea of *unification* of gauge interactions (Grand Unified Theories, GUT) and, at higher energy, the unification of all the interactions, including gravity (String Theories). It is a remarkable fact that, in both cases, supersymmetry seems to emerge as the fundamental ingredient which must be taken into account. In the case of GUTs [18], the unification of gauge couplings within the Standard Model is inconsistent with the observations at LEP (see for instance [19]). This rules out any minimal GUTs which break directly to the Standard Model gauge group with only ordinary matter field content. The inclusion of additional particles or intermediate steps in the symmetry breaking pattern may improve the situation but a large amount of model dependence is unavoidably introduced in this way. On the contrary, in the case of supersymmetric GUTs case (see [20] for an extended review), unification is achieved with a very good precision within the minimal supersymmetric extension of the Standard Model (see Chapter 2); the grand unification scale is predicted to be at $M_G \simeq 10^{16}\,\text{GeV}$, quite close to M_P which is considered as the scale at which also gravity is expected to unify with all the other interactions.

As we anticipated, supersymmetry is also intimately connected to String Theory [21, 22]; in this context its role seems to be even more fundamental. More precisely, any of the five known String Theories admits an effective low-energy description in terms of a supergravity theory in 10 space-time dimension; moreover some of these supergravity models can be obtained from dimensional reduction of $N = 1$ supergravity in 11 dimensions, which is supposed to be the low energy limit of an even more fundamental theory called M-theory. Most of these topics go beyond the principal objectives of this thesis and they will not be developed any further in the following. However, we think it is worth to mention them to stress again the fact that the hierarchy problem should not be considered as the unique motivation to study supersymmetric models; the most relevant hints indicating supersymmetry as a very appealing ingredient of physics beyond the Standard Model come not only from bottom-up analyses based on phenomenological observations but also from more abstract theoretical considerations in a top-bottom approach.

With these considerations, we conclude this non-technical review of the main arguments motivating the study of supersymmetry; in the remaining sections of this chapter we will review in some detail the more technical aspects of globally and locally supersymmetric models.

1.3 Global Supersymmetry

To start we review some basic ingredients of supersymmetric quantum field theories. The most compact and efficient framework to represent the supersymmetry algebra on fields is the formalism of *superfields* in *superspace*; for an exhaustive introduction to the subject see [23–26]. The concept of superspace emerges very naturally in the context of the usual coset construction as an extension of Minkowski space-time and requires the introduction of four extra fermionic coordinates (Grassmannian coordinates) θ^α and $\bar\theta^{\dot\alpha}$. The fermionic nature of these new coordinates implies that any series expansion of superfields along these extra directions will involve a finite (small) number of ordinary fields. The basic building blocks used to construct supersymmetric Lagrangians are *chiral superfields* and *vector superfields* which contain respectively matter and gauge fields, as well as their superpartners and auxiliary fields. Chiral superfields have the following content in terms of ordinary fields [3]:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) = \phi(x) + i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\phi(x) + \frac{1}{4}\,\theta^2\bar\theta^2\,\Box\,\phi(x)\\ + \sqrt{2}\,\theta\psi(x) - \frac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi(x)\sigma^\mu\bar\theta + \theta^2\,F(x)\end{aligned} \tag{1.11}$$

and satisfy the supersymmetric covariant constraint $\bar D_{\dot\alpha}\Phi = 0$, where $\bar D_{\dot\alpha}$ is the supercovariant derivative. Vector superfields are given by:

$$V(x,\theta,\bar\theta) = -\theta\sigma^\mu\bar\theta\,A_\mu(x) + i\,\theta^2\,\bar\theta\bar\lambda(x) - i\,\bar\theta^2\,\theta\lambda(x) + \frac{1}{2}\,\theta^2\bar\theta^2\,D(x) \tag{1.12}$$

and satisfy the reality condition $V = V^\dagger$. In fact the expression (1.12) is not the most general definition and is obtained from the general expression of the vector superfield by gauge fixing to zero two scalars (C and N) and one spinor (ξ_α) corresponding respectively to the θ^0, θ^2 and θ^α components; this gauge choice is known as the Wess-Zumino gauge. The superfield formalism is very practical to construct supersymmetric Lagrangians since the tensor product of representations of the SUSY algebra (supermultiplets) simply reduces to the product of superfields; supersymmetric invariant actions can then be constructed by properly integrating arbitrary functions of superfields over the whole superspace.

1.3.1 Models with only Chiral Multiplets

In this section we recall the structure and properties of non-linear σ-models containing only chiral superfields [27, 28]; we restrict ourselves to models which contain the minimal number of space time derivatives, which means two derivatives acting on scalar fields, one derivative acting on fermion fields and no derivatives acting on auxiliary fields. This assumption corresponds to require that each field propagates the minimal

[3] In this work we adopt the same conventions as [25].

amount of degrees of freedom (d.o.f.'s), which means two for each complex scalar, two for each Weyl fermion and zero for auxiliary fields. The most general Lagrangian that satisfies these properties is parametrized by two functions, the *Kähler potential* $K(\Phi, \bar{\Phi})$ and the holomorphic *superpotential* $W(\Phi)$, and can be expressed in the compact form:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi}) + \int d^2\theta\, W(\Phi) + \text{h.c.}. \tag{1.13}$$

The requirement of minimal number of space-time derivatives implies that K and W cannot depend on supersymmetric covariant derivatives D_α. Indeed, suppose for example that W depends also on the chiral superfield

$$\bar{D}^2\bar{\Phi} = -4\,\bar{F} - 4\sqrt{2}\, i\theta\sigma^\mu\partial_\mu\bar{\psi} - 4\,\Box\bar{\phi}\,\theta^2\,; \tag{1.14}$$

in this case it is easy to verify that the Lagrangian contains terms which are second order in space-time derivatives acting on the spinor fields and are controlled by second derivatives of superpotential.[4]

The situation is even worse if we suppose that also K depends on the same chiral superfields $D^2\bar{\Phi}$; in this case the Lagrangian contains also terms with higher derivatives acting on scalar fields which are controlled by second derivatives of the Kähler potential. On top of that, also the auxiliary fields F^i get kinetic terms and become dynamical in order to compensate for the fact that spinor fields are now propagating more degrees of freedom.

The off-shell Lagrangian in component fields is easily found to be:

$$\begin{aligned}\mathcal{L} = &- g_{i\bar{\jmath}}\,\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{\jmath}} - i\,g_{i\bar{\jmath}}\,\bar{\psi}^{\bar{\jmath}}\bar{\sigma}^\mu\left(\partial_\mu\psi^i + \Gamma^i_{kl}\,\partial_\mu\phi^k\psi^l\right)\\ &+ g_{i\bar{\jmath}}\,F^i\bar{F}^{\bar{\jmath}} + \left[\,F^i\,(W_i - \frac{1}{2}\,g_{i\bar{\jmath}}\,\Gamma^{\bar{\jmath}}_{\bar{k}\bar{l}}\,\bar{\psi}^{\bar{k}}\bar{\psi}^{\bar{l}}) + h.c.\,\right]\\ &- \left[\,\frac{1}{2}W_{ij}\,\psi^i\psi^j + h.c.\right] + \frac{1}{4}\,g_{i\bar{\jmath},k\bar{l}}\,\psi^i\psi^k\,\bar{\psi}^{\bar{\jmath}}\bar{\psi}^{\bar{l}}\end{aligned} \tag{1.15}$$

where $g_{i\bar{\jmath}}$ and Γ^i_{jk} are respectively the Kähler metric and the Levi-Civita connection of the Kähler manifold associated to the target space spanned by the scalar fields. Notice that we adopt the short notation in which the derivatives with respect to chiral and antichiral superfields are denoted by lower indices i and $\bar{\imath}$ which are raised through the inverse of the Kähler metric.

The Lagrangian is invariant under the action of SUSY transformations on component fields:

$$\delta\phi^i = \sqrt{2}\,\epsilon\,\psi^i\,, \tag{1.16}$$

$$\delta\psi^i = \sqrt{2}\,\epsilon\,F^i + i\,\sqrt{2}\,\sigma^\mu\bar{\epsilon}\,\partial_\mu\phi^i\,, \tag{1.17}$$

$$\delta F^i = i\,\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^\mu\partial_\mu\psi^i. \tag{1.18}$$

[4] Note that a superpotential linear in $\bar{D}^2\bar{\Phi}$ of the form $W(\Phi) = f(\Phi) + g(\Phi)\,\bar{D}^2\bar{\Phi}$ is perfectly fine; however we can rewrite the second term as a total $\bar{D}^2$ derivative and interpret it as a correction to the Kähler potential $K'(\Phi,\bar{\Phi}) \equiv K(\Phi,\bar{\Phi}) + \Phi\,\bar{g}(\bar{\Phi}) + \bar{\Phi}\,g(\Phi)$.

A remarkable property of the Lagrangian (1.15) is that it is not only of leading order in space-time derivatives, as expected by construction, but also quadratic in the auxiliary fields F^i and in fermion-bilinears $\psi^i\psi^j$; this is again a simple consequence of requiring that the Kähler potential and the superpotential do not depend on supersymmetric covariant derivatives D_α. From this analysis we learn a very important lesson that will be useful for future discussions on supersymmetric low energy-effective theories: supersymmetric models with a limited number of space-time derivatives contain limited powers of auxiliary fields and fermion bilinears.

The on-shell Lagrangian can be obtained by solving the algebraic equations of motion of the auxiliary fields F^i, which give:

$$F^i = -g^{i\bar{\jmath}}\,\bar{W}_{\bar{\jmath}} + \frac{1}{2}\Gamma^i_{jk}\,\psi^j\psi^k\,. \tag{1.19}$$

Substituting back this relation into $\mathcal{L}$ we finally obtain:

$$\begin{aligned}\mathcal{L} = &- g_{i\bar{\jmath}}\,\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{\jmath}} - i g_{i\bar{\jmath}}\,\psi^i\bar{\sigma}^\mu\big(\partial_\mu\bar{\psi}^{\bar{\jmath}} + \Gamma^{\bar{\jmath}}_{\bar{m}\bar{n}}\,\partial_\mu\bar{\phi}^{\bar{m}}\bar{\psi}^{\bar{n}}\big) + \frac{1}{4}R_{i\bar{\jmath}k\bar{l}}\,\psi^i\psi^k\bar{\psi}^{\bar{\jmath}}\bar{\psi}^{\bar{l}}\\ &- \frac{1}{2}\nabla_i W_j\,\psi^i\psi^j + \text{h.c.} - V_{\rm S}\,,\end{aligned} \tag{1.20}$$

where $V_{\rm S}$ is the scalar potential which has the form:

$$V_{\rm S} = g^{i\bar{\jmath}}\,W_i\bar{W}_{\bar{\jmath}}\,. \tag{1.21}$$

A vacuum is defined by constant values of the scalars ϕ^i and vanishing values of the fermions ψ^i, such that $V_{\rm S}$ is stationary; the stationarity condition then implies:

$$\nabla_i W_j\,F^j = 0\,. \tag{1.22}$$

The masses for the scalar and fermion fields describing fluctuations around the vacuum are then found to be given by:

$$(m_0^2)_{i\bar{\jmath}} = \nabla_i W_k \nabla_{\bar{\jmath}}\bar{W}^k - R_{i\bar{\jmath}k\bar{l}}\,F^k\bar{F}^{\bar{l}}\,, \tag{1.23}$$

$$(m_0^2)_{ij} = -\nabla_i\nabla_j W_k\,F^k\,, \tag{1.24}$$

and

$$(m_{1/2})_{ij} = \nabla_i W_j\,. \tag{1.25}$$

From the expressions of the SUSY transformations discussed above, we see that the vacuum is invariant and supersymmetry is preserved as long as all the auxiliary fields get vanishing vacuum expectation values (v.e.v's). On the contrary supersymmetry is broken whenever one of the auxiliary fields F^i is non-vanishing on the vacuum and in this case the only non-vanishing SUSY variation is: $\delta\psi^i = \sqrt{2}\,\epsilon\,\langle F^i\rangle$. The direction $\langle F^i\rangle$ in field space is special. For fermions it defines at any stationary point the spinor field

$$\eta = \sqrt{2}\,\langle\bar{F}_i\rangle\,\psi^i\,, \tag{1.26}$$

which transforms inhomogeneously and can then be identified with the Goldstino field associated to the spontaneous breaking of supersymmetry. As expected, we can easily verify that the Goldstino has a vanishing mass $m_\eta = 0$ by using the stationarity condition (1.22) and the expression of the fermion mass matrix (1.25). For scalar fields, the direction $\langle F^i \rangle$ defines instead the supersymmetric partner of the Goldstino, the sGoldstino

$$\varphi = \sqrt{2}\, \langle \bar{F}_i \rangle\, \phi^i, \tag{1.27}$$

which transforms under SUSY as $\delta\varphi = \sqrt{2}\, \epsilon\, \eta$. The complex sGoldstino field describes two real scalar fields whose masses are entirely controlled by supersymmetry breaking effects; we will see in the following chapters that these special modes play an important role in discussing the stability properties of the scalar potential.

We conclude this section by recalling the expression of the supertrace of the tree level mass matrices defined as [29, 30]:

$$\mathrm{sTr}[m^2] \equiv \mathrm{Tr}[m_0^2] - 2\,\mathrm{Tr}[m_{1/2}^2] = 2\, R_{i\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}}\,. \tag{1.28}$$

This formula and its generalization to include vector multiplets (see next section) will be used in Chapter 2 to review the main obstructions in constructing realistic models in which supersymmetry is spontaneously broken within the minimal supersymmetric extension of the Standard Model.

1.3.2 Models with Chiral and Vector Multiplets

In this section we generalize the previous model to include also vector superfields V^a associated to gauge symmetries [31, 32]. We proceed by first considering a model with only chiral superfields which is invariant under some group $\mathcal{G}$ of global symmetries; note that a group transformation is a symmetry of the action only if it leaves the Kähler metric invariant or, in other words, if it is an isometry of the Kähler manifold. The generators of such isometries are holomorphic Killing vectors $X_a(\Phi) \equiv X_a^i\, \partial_i$ and, at the infinitesimal level, a generic transformation can be written in terms of some real parameters λ^a: $\delta = \lambda^a\, (X_a^i\, \partial_i + \bar{X}_a^{\bar{\imath}}\, \partial_{\bar{\imath}})$. It is important to note that the Killing vectors are not restricted to be linear functions of the superfields; this is because, in general, at any arbitrary point of the scalar manifold, only the the stabilizer subgroup of $\mathcal{G}$ admits a linear realization [33] whereas generic group transformations may act non-linearly on a subset of fields (on the Goldstone bosons for example). Under isometry transformations, the Kähler potential is demanded to be invariant at least up to a Kähler transformation: $\delta K = \lambda^a\, [\, p_a(\Phi^i) + \bar{p}_a(\bar{\Phi}^{\bar{\imath}})\,]$; this implies that:

$$X_a^i K_i + \bar{X}_a^{\bar{\imath}} K_{\bar{\imath}} = p_a(\Phi^i) + \bar{p}_a(\bar{\Phi}^{\bar{\imath}})\,. \tag{1.29}$$

By taking two sequential covariant derivatives of this expression we obtain Killing's equations, which express the invariance of the Kähler metric under Lie dragging:

$$\nabla_i X_{a\bar{\jmath}} + \nabla_{\bar{\jmath}} \bar{X}_{ai} = 0\,. \tag{1.30}$$

The next step to construct a gauge invariant non-linear σ-model consists, as usual, in promoting the global symmetry to a local one. First of all we need to promote each constant parameters λ^a to be a function of the superspace coordinates $(x^\mu, \theta, \bar{\theta})$; more precisely, in order to preserve the chiral nature of matter superfields under gauge transformations, each group parameter must be promoted to a chiral superfield Λ^a, which implies that the gauge group must be complexified. Moreover, we need to introduce a vector superfield V^a for each group generator and to require that they properly transform under gauge transformations in order to make the action invariant under local symmetry transformations. The Kähler potential must be generalized to include a dependence on vector fields and it must be invariant under gauge transformation, at least up to a Kähler transformation:

$$\delta K(\Phi, \bar{\Phi}, V) = \Lambda^a\, p_a(\Phi) + \bar{\Lambda}^a\, \bar{p}_a(\bar{\Phi})\,. \tag{1.31}$$

Gauge transformations must form a Lie group with an algebra defined by some structure constants $f_{ab}{}^c$:

$$[X_a,\, X_b] = -f_{ab}{}^c\, X_c\,, \tag{1.32}$$

and the action of gauge transformation on fields at leading order in Λ^a is:

$$\delta\Phi^i = \Lambda^a X^i_a(\Phi)\,, \tag{1.33}$$

$$\delta V^a = -\frac{i}{2}\big(\Lambda^a - \bar{\Lambda}^a\big) + \frac{1}{2} f_{bc}{}^a \big(\Lambda^b + \bar{\Lambda}^b\big)\, V^c + \mathcal{O}(V^2)\,. \tag{1.34}$$

When the symmetry is linearly realized these expressions reduce to ordinary gauge transformations with $X^i_a(\Phi) = -i\,(T_a\Phi)^i$, where the generators T_a satisfy the Lie algebra $[T_a, T_b] = i\, f_{ab}{}^c\, T_c$; note that the transformation laws for V^a do not depend on the way the symmetry is realized on the chiral fields (whether linearly on non-linearly).

The most general non-renormalizable Lagrangian with leading number of derivatives is given by:

$$\mathcal{L} = \int d^4\theta \Big[K(\Phi, \bar{\Phi}, V)\Big] + \int d^2\theta \Big[\, W(\Phi) + \frac{1}{4}\, H_{ab}(\Phi)\, W^{a\alpha} W^b_\alpha \Big] + \text{h.c.}\,, \tag{1.35}$$

where, in addition to the standard potentials K and W we introduced a holomorphic gauge kinetic function H_{ab} multiplying the kinetic term of gauge fields; for simplicity we also exclude Fayet-Iliopoulos terms since such terms are not guaranteed to be compatible with gravitational interactions.

To write down the previous Lagrangian in terms of ordinary fields, it is useful to fix the Wess-Zumino gauge. In this gauge we can expand K in powers of V and use the fact that $V^3 = 0$:

$$K(\Phi, \bar{\Phi}, V) = K(\Phi, \bar{\Phi}) + K_a(\Phi, \bar{\Phi}) V^a + \frac{1}{2} K_{ab}(\Phi, \bar{\Phi}) V^a V^b\,. \tag{1.36}$$

The functions K_a and K_{ab} are not arbitrary and are constrained by gauge invariance to be functions of the holomorphic Killing vectors. To derive K_a we can use the relation (1.31) at leading order in Λ^a evaluated in $V^a = 0$ and we obtain:

$$K_a = -2i\, X^i_a K_i + 2i\, p_a\,. \tag{1.37}$$

Taking one derivative of the previous expression with respect to anti-chiral superfields, we obtain:

$$X_a^i = \frac{i}{2}\, g^{i\bar{\jmath}} K_{a\bar{\jmath}}\,; \tag{1.38}$$

this expression shows that $-\frac{1}{2}K_a$ can be identified with the Killing potentials for the Killing vectors X_a^i. To obtain K_{ab} we can use the fact that the imaginary part of (1.37) is actually approximately satisfied also for $V^a \neq 0$ up to the second order:

$$X_a^i K_i - \bar{X}_a^{\bar{\jmath}} K_{\bar{\jmath}} - iK_a = p_a - \bar{p}_a + \mathcal{O}(V^2)\,. \tag{1.39}$$

By taking a derivative of this equation with respect to vector superfields and evaluating it at $V^a = 0$ we finally obtain:

$$K_{ab} = 4\, g_{i\bar{\jmath}}\, X^i_{(a} \bar{X}^{\bar{\jmath}}_{b)}\,. \tag{1.40}$$

Another important relation can be obtained by imposing the gauge invariance of the superpotential:

$$X_a^i\, W_i = 0\,. \tag{1.41}$$

Gauge invariance of the gauge kinetic Lagrangian implies that the variation of the gauge kinetic function must cancel the variation of W_α^a transforming in the adjoint. This implies that:

$$X_a^i H_{bci} = -2 f_{a(b}{}^{d} H_{c)d}\,. \tag{1.42}$$

Finally an important relation can be obtained by exploiting the equivariance condition (1.32) on the Killing vectors, which guarantees that the Killing potentials can be chosen in the adjoint representation, so that:

$$g_{i\bar{\jmath}} X^i_{[a} \bar{X}^{\bar{\jmath}}_{b]} = \frac{i}{4}\, f_{ab}{}^{c} K_c\,. \tag{1.43}$$

We are now ready to recall the expression for the full Lagrangian in terms of component fields; the computation shows that also in this case the Lagrangian contains, by construction, the minimal number of space-time derivatives and it is quadratic in fermion-bilinears and in the auxiliary fields F^i and D^a. This time, for simplicity, we give directly the expression of the final Lagrangian in which the auxiliary fields F^i and D^a have been replaced by their equations of motion.

$$\begin{aligned}
\mathcal{L} = &- g_{i\bar{\jmath}}\, D_\mu \phi^i D^\mu \bar{\phi}^{\bar{\jmath}} - \frac{1}{4} h_{ab}\, F^a_{\mu\nu} F^{b\mu\nu} + \frac{1}{4}\theta_{ab}\, F^a_{\mu\nu} \tilde{F}^{b\mu\nu} - i g_{i\bar{\jmath}}\, \psi^i \big(\not{D} \bar{\psi}^{\bar{\jmath}} + \Gamma^{\bar{\jmath}}_{\bar{m}\bar{n}}\, \not{D} \bar{\phi}^{\bar{m}} \bar{\psi}^{\bar{n}} \big) \\
&- \frac{i}{2} h_{ab}\, \lambda^a \not{D} \bar{\lambda}^b + \text{h.c.} + \frac{1}{\sqrt{2}} h_{abi}\, \lambda^a \sigma^{\mu\nu} \psi^i F^b_{\mu\nu} + \text{h.c.} \\
&- \frac{1}{2}\Big[\nabla_i W_j\, \psi^i \psi^j - g^{i\bar{\jmath}} h_{abi} \bar{W}_{\bar{\jmath}}\, \lambda^a \lambda^b + \sqrt{8}\big(g_{i\bar{\jmath}} \bar{X}^{\bar{\jmath}}_a + \frac{i}{4} h^{bc} h_{abi} K_c \big) \psi^i \lambda^a \Big] + \text{h.c.} \\
&+ \frac{1}{4} R_{i\bar{\jmath}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^{\bar{\jmath}} \bar{\psi}^{\bar{l}} - \frac{1}{4} g^{i\bar{\jmath}} h_{abi} h_{cd\bar{\jmath}}\, \lambda^a \lambda^b \bar{\lambda}^c \bar{\lambda}^d - \frac{1}{2} h^{cd} h_{aci} h_{bd\bar{\jmath}}\, \psi^i \lambda^a \bar{\psi}^{\bar{\jmath}} \bar{\lambda}^b \\
&+ \frac{1}{4}\Big[\nabla_i h_{abj}\, \psi^i \psi^j \lambda^a \lambda^b + h^{cd} h_{aci} h_{bdj} \psi^i \lambda^a \psi^j \lambda^b \Big] + \text{h.c.} - V_{\mathrm{S}}\,,
\end{aligned} \tag{1.44}$$

where V_S is the scalar potential defined as:

$$V_S = g^{i\bar{\jmath}} W_i \bar{W}_{\bar{\jmath}} + \frac{1}{8} h^{ab} K_a K_b \,. \tag{1.45}$$

In these expressions D_μ is the covariant derivative acting as $D_\mu \phi^i = \partial_\mu \phi^i + A^a_\mu X^i_a$, $D_\mu \psi^i = \partial_\mu \psi^i + A^a_\mu \partial_j X^i_a \psi^j$ and $D_\mu \lambda^a = \partial_\mu \lambda^a + f_{bc}{}^a A^b_\mu \lambda^c$, whereas $F^a_{\mu\nu}$ is the field-strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{bc}{}^a A^b_\mu A^c_\nu$ and h_{ab} and θ_{ab} denote the real and imaginary parts of H_{ab}.

The equations of motion of the auxiliary fields are given by:

$$F^i = -g^{i\bar{\jmath}} \bar{W}_{\bar{\jmath}} + \frac{1}{2} \Gamma^i_{jk} \psi^j \psi^k + \frac{1}{2} g^{i\bar{\jmath}} h_{ab\bar{\jmath}} \bar{\lambda}^a \bar{\lambda}^b \,, \tag{1.46}$$

$$D^a = -\frac{1}{2} h^{ab} K_b - \frac{i}{\sqrt{2}} h^{ab} h_{bci} \psi^i \lambda^c + \text{h.c.} \,. \tag{1.47}$$

The previous Lagrangian is not invariant under ordinary SUSY transformations; this is because the Wess-Zumino gauge is not preserved by SUSY transformations and a compensating gauge transformation is required to restore the gauge choice. The additional gauge transformation has the effect of changing ordinary derivatives into covariant derivatives in the variation of matter fields whereas it has no effects on the transformation laws of the vector multiplets:

$$\delta \phi^i = \sqrt{2}\, \epsilon\, \psi^i \,, \tag{1.48}$$

$$\delta \psi^i = \sqrt{2}\, \epsilon\, F^i + \sqrt{2} i\, \not{D} \phi^i\, \bar{\epsilon} \,, \tag{1.49}$$

$$\delta A^a_\mu = i \epsilon\, \sigma_\mu \bar{\lambda}^a - i \lambda^a \sigma_\mu\, \bar{\epsilon} \,, \tag{1.50}$$

$$\delta \lambda^a = i \epsilon\, D^a + \sigma^{\mu\nu} \epsilon\, F^a_{\mu\nu} \,. \tag{1.51}$$

Note finally that the Wess-Zumino gauge does not fix completely the gauge freedom and the Lagrangian (1.44) is still invariant under ordinary gauge transformations with real parameter $\lambda^a \equiv \mathrm{Re}\, \Lambda^a|_{\theta\,=\,0}$.

A vacuum is defined by constant values of the scalars ϕ^i and vanishing values of the fermions ψ^i, λ^a and the vectors A^a_μ, such that V_S is stationary. The stationarity conditions $\nabla_i V_S = 0$ imply that:

$$\nabla_i W_j\, F^j + \frac{1}{2} h_{abi} D^a D^b + i \bar{X}_{ai} D^a = 0 \,. \tag{1.52}$$

One further important relation can be derived by taking a particular linear combination of the stationarity conditions. We can contract the previous system of equations with the Killing vectors X^i_a; the real part of $X^i_a\, V_{S,i}$ vanishes once we use the relations (1.42)-(1.43) as we expect from the gauge invariance of the scalar potential under real gauge transformations. On the contrary, the imaginary part does not vanish automatically and it represents a constraint relating F-type and D-type auxiliary fields. Using (1.41) and its derivatives as well as (1.42), one finds the following relation:

$$i \nabla_i X_{a\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} - g_{i\bar{\jmath}}\, X^i_{(a} \bar{X}^{\bar{\jmath}}_{b)}\, D^b + \frac{1}{2} f_{ab}{}^d \theta_{dc}\, D^b D^c = 0 \,. \tag{1.53}$$

This relation is very interesting since it shows that the values of the auxiliary fields F^i and D^a are not completely independent and we will use it to discuss some important consequences for spontaneous supersymmetry breaking.

The masses of the scalars and fermions describing fluctuations around the vacuum are found to be given by:

$$\begin{aligned}(m_0^2)_{i\bar{\jmath}} =& \nabla_i W_k \nabla_{\bar{\jmath}} \bar{W}^k - R_{i\bar{\jmath}k\bar{l}}\, F^k \bar{F}^{\bar{l}} + h^{ab} \bar{X}_{ai} X_{b\bar{\jmath}} + h^{ab} h_{aci} h_{bd\bar{\jmath}}\, D^b D^c \\ &+ \left(i\nabla_i X_{a\bar{\jmath}} - i\, h^{bc} h_{abi} X_{c\bar{\jmath}} + i\, h^{bc} h_{ab\bar{\jmath}} \bar{X}_{ci}\right) D^a\,, \end{aligned} \tag{1.54}$$

$$\begin{aligned}(m_0^2)_{ij} =& -\nabla_i \nabla_j W_k\, F^k - h^{ab} \bar{X}_{ai} \bar{X}_{bj} - \frac{1}{2}\left(\nabla_i h_{abj} - 2h^{cd} h_{aci} h_{bdj}\right) D^a D^b \\ &+ 2i\, h^{bc} h_{ab(i} \bar{X}_{cj)} D^a\,, \end{aligned} \tag{1.55}$$

and

$$(m_{1/2})_{ij} = \nabla_i W_j\,, \tag{1.56}$$

$$(m_{1/2})_{ab} = h_{abi}\, F^i\,, \tag{1.57}$$

$$(m_{1/2})_{ia} = \sqrt{2}\, \bar{X}_{ai} - \frac{i}{\sqrt{2}} h_{abi}\, D^b\,, \tag{1.58}$$

whereas the masses of vector fields are given by:

$$(m_1^2)_{ab} = 2\, X^i_{(a} \bar{X}_{b)i}\,. \tag{1.59}$$

Form the expressions of the SUSY transformations we see that the vacuum is invariant as long as all the F and D auxiliary fields vanish; on the contrary SUSY is spontaneously broken whenever some F^i and/or D^a is non-vanishing on the vacuum and in this case we have $\delta\psi^i = \sqrt{2}\,\epsilon\, F^i$ and $\delta\lambda^a = i\,\epsilon D^a$. As discussed before, however, the values of the F and D auxiliary fields are not completely independent and using eq. (1.53) we can conclude that supersymmetry breaking scenarios in which $D \neq 0$ while $F = 0$ cannot be realized in models in which the Kähler potential is strictly invariant under gauge transformations (1.31). More precisely, this is because $F^i = 0$ implies that either D^a or X^i_a should vanish in eq. (1.53) but $X^i_a = 0$ implies $D_a = 0$ by equation (1.37) when $p_a = 0$; we will discuss in more detail this aspect in Chapter 4 where we will derive an accurate inequality involving F and D in the case of renormalizable models. From now on we exclude for simplicity the possibility of non-zero variations of the Kähler potential under gauge transformations and we assume

$$p_a = 0 \tag{1.60}$$

since other situations are not guaranteed to be compatible with a coupling to gravity and can also not emerge in low-energy effective descriptions of microscopic theories where the variations were strictly vanishing (see [34] for a recent discussion of this point). In particular, this is also the motivation to exclude Fayet-Iliopoulos terms from the beginning.

The directions $\langle F^i \rangle$ and $\langle D^a \rangle$ in the fermion field space are special since at any stationary point they define the Goldstino spinor

$$\eta = \sqrt{2}\,\langle \bar{F}_i \rangle\, \psi^i - i\,\langle D_a \rangle \lambda^a\,, \tag{1.61}$$

which is massless and represents the Goldstone mode of broken supersymmetry. The identification of the scalar superpartners of the Goldstino turns out to be more complicated than in the pure chiral case; indeed, we can see that there does not exist any simple linear combination of scalar fields ϕ^i which is trivially mapped into the Goldstino fermion under SUSY transformations, the main complications arising in reproducing the term which involves the gauginos. These difficulties are associated to the fact that we have fixed a non-supersymmetric gauge (the Wess-Zumino gauge); to better understand which scalar fields should be associated to the gauginos λ^a it is easier to work in a supersymmetric gauge. When the gauge symmetry is spontaneously broken it is practical to fix the so called "Fayet gauge" [35] in which one chiral field for each broken generator is frozen to an arbitrary scale: $\Phi_a \equiv \langle \bar{X}_{ai} \rangle\, \Phi^i = M_a$ with $a = 1, ...n_B$ and n_B is the number of broken generators. Note that we are allowed to gauge away an entire superfield for each broken generator as consequence of the fact that in supersymmetric models the gauge parameter is promoted to a chiral superfield.

In the Fayet gauge the model contains n_B massive vector superfields; the dynamics of the chiral superfields we gauged away reappears in each massive vector superfield through new propagating d.o.f's: the real scalars C^a, the spinors ξ^a , the longitudinal polarizations of A^a_μ and new complex auxiliary fields N^a. In this gauge it is natural to identify the fields C^a as the scalar partners of the gauginos and the ambiguity that we described above is eliminated.

In the Wess-Zumino gauge, however, the identification of the scalar partners is more involved. The longitudinal polarizations of the gauge fields A^a_μ is associated to the real scalar fields:

$$\sigma^a = \mathrm{Re}\,\langle \bar{X}_{ai} \rangle\, \phi^i\,; \tag{1.62}$$

these fields are associated with zero modes of the scalar mass matrix and they correspond to unphysical would-be Goldstone fields of the spontaneous gauge symmetry breaking. This can be seen by using the gauge invariance of the scalar potential under ordinary gauge transformations parametrized by the real parameter λ^a; we have:

$$\delta V_{\mathrm{S}} = \lambda^a \left(X^j_a\, V_j + \bar{X}^{\bar{j}}_a\, V_{\bar{j}} \right) = 0 \;\;\Rightarrow\;\; V_{ij}\, X^j_a + V_{i\bar{j}}\, \bar{X}^{\bar{j}}_a = 0\,, \tag{1.63}$$

where the second expression on the right hand side is evaluated at stationary points. These real scalar fields are non-physical and can be gauged away by exploiting the residual gauge symmetry left by the Wess-Zumino gauge choice; the model that we obtain has the same d.o.f's of the gauge-fixed Lagrangian in the Fayet supersymmetric gauge that we discussed above. Indeed it can be shown that in general the two Lagrangians are equivalent up to a non-linear field redefinition [35]. The real physical scalars representing the extra d.o.f.'s of massive vector supermultiplets are then naturally identified with the fields:

$$\rho^a = \mathrm{Im}\,\langle \bar{X}_{ai} \rangle\, \phi^i\,; \tag{1.64}$$

it is easy to verify that these fields, in the supersymmetric limit in which all the F and D auxiliary fields vanish, have the same masses of the gauge vector fields. It is also interesting to note that, when the gauge symmetry is not broken and all the $\langle X^i_a \rangle$ vanish, we have that $\sigma^a = \rho^a = 0$ as is to be expected since in this case A^a_μ's have only two d.o.f and there are no physical propagating scalars associated to vector multiplets. Notice that a priori the gauge part in the Goldstino field (1.61) may be different from zero even when the gauge symmetry is not broken; in this case there still remains a problem related to the identification of the scalar partner of the Goldstino since the previous analysis does not apply. However we have seen that this situation can be realized only if there are non-trivial holomorphic functions p_a in the gauge transformations of the Kähler potential and we have excluded this possibility.

The previous analysis shows that in the Wess-Zumino gauge the supersymmetric structure of massive vector multiplets is not manifest in the Lagrangian and this complicates the identification of the scalar partner of the Goldstino; in particular it is not easy to find a linear combination of scalars which transforms into η under supersymmetry transformations. We can however define a projected Goldstino $\eta' = \sqrt{2}\,\langle \bar{F}_i \rangle\, \psi^i$ to which we can associate, without ambiguity, the projected sGoldstino:

$$\phi = \sqrt{2}\,\langle \bar{F}_i \rangle\, \phi^i\,. \tag{1.65}$$

Under SUSY transformations one then finds $\delta\varphi = \sqrt{2}\,\epsilon\,\eta'$ and also in this case the complex sGoldstino field describes two real scalar fields whose masses are entirely controlled by supersymmetry breaking effects. We will see in Chapter 4 that these special modes play a relevant role for discussing the stability properties of the scalar potential. The previous analysis, however, suggests that also the special direction

$$\mathrm{Im}\,\langle D^a \bar{X}_{ai} \rangle \tag{1.66}$$

should be somehow associated to the scalar superpartners of the Goldstino. This suggests that the value of the scalar mass matrix projected on this direction should not be completely arbitrary. This argument will be discussed in more detail in Chapter 4 where the role of the directions $\langle F^i \rangle$ and $\langle X^i_a \rangle$ for the study of vacuum stability is extensively analyzed.

We conclude this section by recalling the expression of the supertrace for non-linear gauged σ-models [29]:

$$\begin{aligned} \mathrm{sTr}[m^2] \;&\equiv\; \mathrm{Tr}[m_0^2] - 2\,\mathrm{Tr}[m_{1/2}^2] + 3\,\mathrm{Tr}[m_1^2] \\ &=\; 2\big(R_{i\bar{\jmath}} - h^{ac}h^{bd}h_{abi}h_{cd\bar{\jmath}}\big)F^i\bar{F}^{\bar{\jmath}} \\ &\quad+\; i\,\big(\nabla_i X_{a\bar{\jmath}} + 2\,h^{bc}h_{abi}X^i_c\big)\,D^a + \mathrm{h.c.}\,. \end{aligned} \tag{1.67}$$

The implications of this formula for SUSY phenomenology are discussed in the next chapter.

1.4 Local Supersymmetry

In this section we will review in some detail the basic ideas and procedures to construct locally supersymmetric invariant models. For simplicity we start as before by considering models containing only chiral multiplets and then we discuss the generalization including gauge symmetries and vector multiplets.

1.4.1 Models with only Chiral Multiplets

There exist several different versions of $N = 1$ supergravity; each formulation is characterized by a different choice of the auxiliary fields that are included in the gravitational sector together with the *graviton* and the *gravitino*. Here will focus on the so called "*old minimal supergravity*" [36, 37] in which the off-shell supergravity multiplet, in addition to the graviton e^a_μ and the gravitino ψ^α_μ fields, contains two auxiliary fields namely one complex scalar F_ϕ and one vector field A_μ :

$$\left\{ e^a_\mu, \psi^\alpha_\mu, A_\mu, F_\phi \right\} . \tag{1.68}$$

Supergravity can be obtained as a gauge theory of the Super-Poincaré group, provided that certain constraints are implemented to remove non-physical gauge d.o.f's in the gravitational sector. Since the concept of supergravity as a gauge theory plays a fundamental role in our derivation of the Lagrangian, we shall clarify this point by briefly recalling how General Relativity can be obtained as a gauge theory of the Poincaré group. This idea is implemented by the so called Cartan formalism [5] which consists in introducing a *vierbein* e^a_μ, which is the gauge field associated to local translations P_a and a *spin connection* ω^{ab}_μ, which is the gauge field associated to local Lorentz transformations M_{ab}. General Relativity can be obtained by gauging the Poincaré group and by imposing the torsion-free constraint, which fixes the spin connection as a function of the vierbein. The reason why this constraint must necessarily be imposed to recover ordinary General Relativity can be understood in the following way: the generator P_a is associated to local translations and satisfies the algebra $[\delta^1_P, \delta^2_P] = 0$; however General Relativity requires invariance under general coordinate transformations (diffeomorphisms) which in general do not commute. The standard procedure to convert local translations to general coordinate transformations is to impose that the torsion tensor vanishes. In this case the group algebra must be modified since the constraint turns out to be non-invariant under local translations. In other words, we need to define new transformation laws $\delta_{\hat{P}} \equiv \delta_P + \delta'$ to compensate the non vanishing transformation of the constraint and, in general, the new transformations will not commute as expected for diffeomorphisms. We will see that this construction can be generalized also to derive the supergravity Lagrangian.

As for the case of ordinary gauge theories, one can choose among three different approaches to construct locally supersymmetric theories; let us recall them:

[5] See [38] for a comprehensive introduction to Cartan's formalism.

1. The Noether approach, which is the standard procedure by which the invariance under local transformations is implemented, at the level of the component Lagrangian, by coupling the gauge vectors to the conserved currents of global symmetries; when necessary, gauge invariance must be enforced by adding extra terms to compensate for the non-vanishing variation of the action [39–41] (see also [42]).

2. The superspace approach, which generalizes the geometric techniques of gauge theories to superspace. For ordinary gauge theories, it reduces to introduce a gauge connection $A^a_{\{\mu,\alpha,\dot{\alpha}\}}$ for each generator of the symmetry group in order to define supersymmetric gauge covariant derivatives and field strengths. For gravitational interactions, however, this approach implies further complications since it requires the study of curved superspaces geometry; this implies also that one has to generalize ordinary superfields to be functions of curved superspace coordinates [25, 43].

3. The tensor calculus approach, in which the ordinary (rigid) SUSY tensor calculus gets properly "covariantized". In this case, one works with standard (global) superfields in which space-time derivatives are replaced by covariant derivatives and the component fields transform covariantly under both gauge and SUSY transformations [44, 45].

The first approach, even if didactically more enlightening, turns out to be quite cumbersome; this is because in general the procedure must be implemented several times and at each step new terms must be introduced to compensate for the variation of the action. In the second approach, the gauge group is maximally extended in order to implement gauge transformations on the whole superspace; this introduces several new gauge fields which must finally be fixed by performing appropriate gauge choices and by imposing several covariant constraints. This procedure, though more appealing from a geometrical point of view, is rather tedious and not particularly enlightening.

In this work we will use the third formalism to construct the supergravity Lagrangian. This approach may appear less elegant than the one in superfields but it has the advantage that it requires to manipulate only the minimal set of physical (gravitational) gauge fields. Moreover, the tensor calculus is only slightly modified with respect to the rigid one and it is possible to exploit all the powerful machinery of ordinary superfields. Finally we anticipate that, in general, also in this formalism it is necessary to impose some constraints to discard unphysical gauge d.o.f.'s but, in our opinion, the few constraints that one has to impose are less arbitrary and more easy to be solved than those of the curved superspace approach. These constraints are essentially of the same nature as the torsion-free constraint in ordinary General Relativity described above. Summarizing, in the following we adopt the tensor calculus approach to construct SUGRA as a gauge theory since, from a technical point of view, it is the most efficient for our purposes.

Historically, there exist two different formalisms based on the tensor calculus approach. The references we gave at the point 3 above refer to the construction of supergravity as a gauge theory of the super-Poincaré group; on the other hand, one can also obtain minimal supergravity as a gauge theory of the superconformal group [46–49]. This alternative is not peculiar of supersymmetry; also ordinary General Relativity can be obtained as a gauged-fixed version of a scale invariant theory. Indeed, we can promote General Relativity to be scale invariant by adding a new real scalar field ϕ called *compensator* which transforms in a suitable way under local scale (Weyl) transformations:

$$\mathcal{L} = \frac{1}{2}\int d^4x\,\sqrt{-g}\,[\,\frac{1}{6}\phi^2\,R + \partial_\mu\phi\partial^\mu\phi\,]\,, \tag{1.69}$$

where the transformation laws of the metric and compensator are:

$$\delta g_{\mu\nu}(x) = -2\lambda(x)\,g_{\mu\nu}(x)\,, \qquad \delta\phi(x) = \lambda(x)\,\phi(x)\,. \tag{1.70}$$

General Relativity in the Einstein frame is then obtained after gauge-fixing the extra symmetry by imposing $\phi = \sqrt{3/4\pi G_N}$. One may wonder why it is convenient to opt for the superconformal approach, which a priori looks more complicated; indeed, the conformal group has 15 generators instead of 10 and it will be necessary to impose more constraints to fix the extra gauge symmetries. The answer to this question is related to the intimate nature of the scalar auxiliary field F_ϕ in the minimal SUGRA multiplet. The main feature of the superconformal approach is that, in this formalism, F_ϕ is disentangled from the rest of the gravitational multiplet. It belongs to a different supermultiplet, called *conformal compensator* multiplet, which has the same function of the compensator ϕ that we discussed above. F_ϕ is the only field in the gravitational sector that can affect the scalar potential; for this reason, having a formalism in which F_ϕ can be manipulated avoiding all the complications introduced by the other gravitational fields, is very attractive. As we are going to see, the computation of the scalar potential and the scalar masses in this formalism can be done in a very economical way.

Superconformal Supergravity

Let us start by reviewing the main ingredients that are necessary to construct the supergravity Lagrangian in the superconformal formalism. In this analysis we will follow the notation and the arguments of [48] (see also [50] for a review). The group of superconformal transformations is generated by the 15 bosonic generators of the conformal group: P_a for local translations, M_{ab} for local Lorentz transformations, K_a for local special conformal transformations and D for local dilatations. In addition there are 2 kinds of fermionic supersymmetry generators Q_α and S_α and one generator T_R associated to a local $U(1)_R$ R-symmetry. The gauge fields associated to each transformation are summarized in Tab. 1.1, where we use greek letters for global indices and latin

P_a	M_{ab}	D	K_a	Q_α	S_α	T_R
e_μ^a	ω_μ^{ab}	b_μ	f_μ^a	ψ_μ^α	φ_μ^α	A_μ

Table 1.1: Superconformal generators and gauge fields

letters for frame indices. As usual the transformation laws for the gauge vectors are given by:

$$\delta_{\text{gauge}}(\epsilon)\, h_\mu^A = \partial_\mu \epsilon^A + h_\mu^B \epsilon^C f_{CB}{}^A\,, \tag{1.71}$$

where h_μ^A and ϵ^A collectively denote the gauge fields and the parameters of the various transformations whereas $f_{CB}{}^A$ are the structure constants of the algebra. The gauge theory associated to this setup is still very different from minimal Poincaré supergravity. Indeed, there are still 7 independent gauge fields whereas in minimal SUGRA there are only 3: e_μ^a, ψ_μ^α and A_μ. Actually this gauge theory is not yet a gravity theory (in the Einsteinian sense), not even a superconformal version of SUGRA, since the translation generators do not behave as expected for a gravitational theory. In this case, as carefully discussed in [46], one has to impose 3 constraints in order to promote P_a to be the generators of general coordinate transformations. These constraints can be solved by expressing ω_μ^{ab}, f_μ^a and φ_μ^α in terms of:

$$\left\{\, e_\mu^a,\, \psi_\mu^\alpha,\, A_\mu,\, b_\mu \right\}. \tag{1.72}$$

Once these constraints are imposed the algebra must be deformed; indeed, as in General Relativity, the constraints are not invariant under the whole symmetry group and the transformation laws of the gauge fields need to be properly modified in order to preserve the constraints. Notice finally that the independent gauge fields in (1.72) correspond to the gravitational multiplet of conformal supergravity.

The next step of the procedure consists in constructing the superconformal generalization of the action (1.69). One starts by considering the general Lagrangian (1.13) in rigid supersymmetry:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\, K(\Phi^i, \bar{\Phi}^{\bar{\imath}}) + \int d^2\theta\, W(\Phi^i) + \text{h.c.} \\ &= \left[\, K(\Phi^i, \bar{\Phi}^{\bar{\imath}})\, \right]_{\mathrm{D}} + \left[\, W(\Phi^i)\, \right]_{\mathrm{F}} + \text{h.c.}\,,\end{aligned} \tag{1.73}$$

and modifies it in such a way to make it invariant under global superconformal transformations. For this, one needs to introduce the conformal compensator multiplet, which will be finally used to gauge fix the extra superconformal symmetries that are not in the super-Poincaré group, namely D, T_R, K_a and the S-supersymmetry. Note that there is a certain freedom regarding the choice of the compensator multiplet; minimal supergravity is obtained by taking a chiral multiplet $\Phi = \{\,\phi\,, \chi_\phi\,, F_\phi\}$ as conformal compensator.

The coupling of the conformal compensator to ordinary matter is completely fixed by its transformation laws under scale and $U(1)_R$ symmetries. For the following analysis we refer to [51]. The Lagrangian (1.73) must have *conformal weight* (dimension) $d(\mathcal{L}) = 4$ and *chiral weight* (R-charge) $R(\mathcal{L}) = 0$. Since $d(\theta^\alpha) = -\frac{1}{2}$ and $R(\theta^\alpha) = 1$, this implies that:

$$d(K) = 2\,, \qquad d(W) = 3\,; \tag{1.74}$$

$$R(K) = 0\,, \qquad R(W) = 2. \tag{1.75}$$

To find the right powers of the compensator field which should be inserted in each term of the supersymmetric Lagrangian it is useful to assume, without loss of generality, that matter fields have vanishing conformal weight $d(\phi^i) = 0$ and vanishing chiral weight $R(z) = 0$, whereas the compensator has $d(\phi) = 1$; this fixes also $R(\phi) = 2/3$ since it is possible to show that the conformal and the chiral weights are not independent for chiral multiplets. Note that having matter chiral superfields with zero mass dimension is not a problem since, in the end, it is possible to perform a field redefinition to restore the right dimensions.

From the previous analysis, one can guess the proper coupling of the conformal compensator to matter fields in superconformal supergravity:

$$\mathcal{L} = \left[\, K(\Phi^i, \bar{\Phi}^{\bar{\imath}})\, \bar{\Phi}\Phi \,\right]_{\mathfrak{D}} + \left[\, W(\Phi^i)\, \Phi^3 \,\right]_{\mathfrak{F}} + \text{h.c.}\,. \tag{1.76}$$

Note that we did not express the Lagrangian in terms of integrals over (rigid) spinor coordinates θ^α. This is because ordinary D and F terms cannot be used to construct covariant actions of the type $\int d^4x\, e\,[\;\;]_{\mathrm{D}} + \int d^4x\, e\,[\;\;]_{\mathrm{F}}$, where e is the determinant of the vierbein. The problem is that ordinary F and D terms do not transform properly under local supercovariant transformations, and some new terms must be added to obtain invariant actions. As explained in [48] (see also [52] for conventions compatible with ours), the right expression for the superconformal F-term that should be used to construct locally superconformal invariant action is :

$$\int d^4x\; [\,\Sigma\,]_{\mathfrak{F}}\,, \qquad [\,\Sigma\,]_{\mathfrak{F}} \;=\; e\Big(\, F - i\,\sqrt{2}\,\chi\sigma^\mu\bar{\psi}_\mu - z\,\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu \Big)\,, \tag{1.77}$$

where $\Sigma = \{z, \chi, F\}$ is a chiral multiplet with conformal weight $d(\Sigma) = 3$ as required in the Lagrangian (1.76). The expression for the the superconformally invariant D-term is slightly more complicated; we just recall the most interesting terms:

$$\int d^4x\; [\,\Omega\,]_{\mathfrak{D}}\,, \qquad [\,\Omega\,]_{\mathfrak{D}} \;=\; e\Big(\,\frac{1}{2}\,D - \frac{1}{2}\,(\,\lambda\sigma^\mu\bar{\psi}_\mu - i\,\xi\sigma^{\mu\nu}\mathcal{D}^c_\mu\psi_\nu + \text{h.c}\,) + \frac{C}{3}\,(\,\frac{1}{2}\,R - \frac{1}{e}\mathcal{L}^0_{\mathrm{RS}}\,)\Big) + \dots\,, \tag{1.78}$$

where $\Omega = \{C, \xi, M, B_\mu, \lambda, D\}$ is a vector multiplet with conformal weight $d(\Omega) = 2$; $\mathcal{L}^0_{RS}$ represents the standard kinetic term of the Rarita-Schwinger action for the gravitino ψ^α_μ, whereas the ellipsis represents terms quadratic and cubic in the gravitino multiplied by the vector and spinorial components of Ω which do not contribute to the gravitino mass. Note finally that the function K in the superconformal Lagrangian

(1.76) is not what is called Kähler potential in the supergravity literature; in order to recover the canonical form of the kinetic terms of the matter multiplets one has to perform the redefinition $K \to -3\, M_P^2\, e^{-K/3M_P^2}$.[6] Setting $M_P^2 = 1$ one finally obtains the Lagrangian of superconformal supergravity:

$$\mathcal{L} = \left[-3\, e^{-K/3}\, \bar{\Phi}\Phi \right]_{\mathfrak{D}} + \left[W\, \Phi^3 \right]_{\mathfrak{F}} + \text{h.c.}\,. \tag{1.79}$$

Notice that, in order to obtain the full Poincaré supergravity Lagrangian in component fields, one has now to gauge-fix the extra-symmetries in (1.79) by using the compensator superfield. As a matter of fact, however, it is often convenient to work with the full superconformally invariant Lagrangian, manipulating the compensator superfield as an ordinary matter chiral superfield, and substitute its gauge-fixed expression only at the very end of the computations. This is because, as argued in [48] (see also [53]) the superconformal tensor calculus is substantially simpler than the super-Poincaré tensor calculus. It turns out that under ordinary SUSY transformations generated by Q^α, superconformal multiplets transform in the usual way (1.16)-(1.18) with the only change that ordinary space-time derivatives must be replaced by superconformal covariant derivatives. Also the multiplet components and the tensor calculus are easily covariantized by using the same technique. This makes it is possible to use rigid SUSY superfields to represent a superconformal chiral multiplets $\Phi^i = \{\phi^i, \chi^i, F^i\}$ in the following way:

$$\begin{aligned}\Phi^i(x,\theta,\bar\theta) = \; & \phi^i + \sqrt{2}\,\theta\chi^i + \theta^2 F^i \\ & + i\,\theta\sigma^\mu\bar\theta\,\mathcal{D}^c_\mu\phi^i - \frac{i}{\sqrt{2}}\,\theta^2\,\mathcal{D}^c_\mu\chi^i\sigma^\mu\bar\theta + \frac{1}{4}\,\theta^2\,\mathcal{D}^c_\mu\mathcal{D}^{c\mu}\,\phi^i\,;\end{aligned} \tag{1.80}$$

where $\mathcal{D}^c_\mu = \partial_\mu - h^a_\mu T_a$ is the conformal covariant derivative and the sum over the group generators T_a excludes the local translations P_a. A similar (rigid-like) covariantized expression can be defined also for vector multiplets. The results of superconformal tensor calculus can be smartly reproduced by multiplying the above-defined "covariantized" superfields; for example the highest components of the real multiplet obtained by taking the product of one chiral and one anti-chiral multiplet is given by:

$$\Phi^i\bar\Phi^{\bar\imath} = \;...\; + \theta^2\bar\theta^2\,[\,F^i\bar F^{\bar\imath} - \mathcal{D}^c_\mu\bar\phi^{\bar\imath}\,\mathcal{D}^{c\mu}\phi^i - i\,\bar\chi^{\bar\imath}\bar\sigma^\mu\mathcal{D}^c_\mu\chi^i\,] + \text{ tot. deriv.}\,. \tag{1.81}$$

Using these results, it is easy to understand which is the fundamental simplification introduced by this formalism. Indeed, if one focuses only on the scalar potential, it is possible to discard the gravitational gauge fields e^a_μ, ψ^α_μ and A_μ, and the expressions for the supercovariant F-terms (1.77) and D-terms (1.78) then reduce to the usual ones in rigid supersymmetry. Moreover, in this case also the expressions (1.80) and (1.81) reduce to the ordinary ones, so that the tensor calculus works exactly as in rigid superspace. One can then rewrite (1.79) in terms of rigid superspace integrals:

$$\mathcal{L} = \int d^4\theta\, \left(-3\, e^{-K/3}\, \bar\Phi\Phi\right) + \int d^2\,\theta\, W\, \Phi^3 + \text{h.c.} + \;...\,, \tag{1.82}$$

[6]A more detailed discussion of this part can be found for example in [50].

where the ellipsis indicates the gravitational terms which do not admit a simple expression in terms of integral over the rigid superspace and are usually expressed directly in components fields.[7]

The last aspect we need to discuss is the gauge-fixing of the extra symmetries of the superonformal group [54] (see also [52]). The special conformal transformations K_a can be fixed by setting the dilatation gauge field to zero:

$$b_\mu = 0\,. \tag{1.83}$$

In order fix D, T_R and S_α symmetries, it is useful to redefine the compensator multiplet in the following way:

$$\Phi = \phi\,\{\,1\,,\,\chi_\phi\,,\,U\}\,, \qquad \phi = |\phi| e^{i\sigma}\,, \tag{1.84}$$

where we defined the normalized auxiliary field U as:

$$F_\phi = \phi\, U\,. \tag{1.85}$$

One common gauge choice consists in using the modulus and the phase of the scalar ϕ to fix D and T_R, and the spinor χ_ϕ to fix the S-supersymmetry; the field U is left unfixed and, together with A_μ, it is an auxiliary field of the Poincaré gravitational multiplet (1.68). The actual values at which the fields are fixed are chosen in order simplify as much as possible the resulting Lagrangian. $|\phi|$ is fixed in such a way that the kinetic term of graviton is canonically normalized (Einstein frame); by inspection of (1.78), one is led to chose:

$$C \equiv -3\,|\phi|^2\, e^{-K/3} = -3 \quad \Rightarrow \quad |\phi| = e^{K/6}\,. \tag{1.86}$$

The phase of ϕ can be fixed in such a way that the gravitino mass is real; the relevant term to consider is $-z\,\bar\psi_\mu \bar\sigma^{\mu\nu}\bar\psi_\nu$. This suggests the following choice:

$$\mathrm{Im}\, z \equiv \mathrm{Im}(\,\phi^3\, W\,) = 0 \quad \Rightarrow \quad \sigma = \frac{i}{6}(\log W - \log \bar W)\,. \tag{1.87}$$

Finally χ_ϕ is fixed in such a way to cancel the non-canonical mixing $i\,\xi\sigma^{\mu\nu}\partial_\mu\psi_\nu$ in the gravitino kinetic term. This implies:

$$\xi \equiv 3i\sqrt{2}\,|\phi|^2\, e^{-K/3}\left(\chi_\phi - \frac{1}{3}K_i\chi^i\right) = 0 \quad \Rightarrow \quad \chi_\phi = \frac{1}{3}\,K_i\,\chi^i\,. \tag{1.88}$$

After the gauge fixing we then get:

$$\Phi = \exp\left[\frac{1}{6}(K - \log W + \log \bar W)\right] \cdot \left\{1\,,\,\frac{1}{3}\,K_i\,\chi^i\,,\,U\right\}\,. \tag{1.89}$$

[7]Examples of applications in which one is not allowed to naively ignore these terms are discussed in [52]; in such situations, the authors propose to use a smarter gauge choice which produces some useful simplifications.

This gauge, proposed in [54], is very useful since it avoids the field redefinitions which are usually needed to recover the canonical kinetic terms and masses for the graviton and the gravitino.

The compensator field and the SUGRA Lagrangian (1.79) are usually rewritten in a more compact form by exploiting the invariance of the action under generalized Kähler transformations. Under these transformations the Kähler potential, the superpotential and the compensator field transform in the following way:

$$(\,\Phi\,,\,K\,,\,W\,) \to (\,\Phi\, e^{Y/3}\,,\,K+Y+\bar{Y}\,,\,W\, e^{-Y}\,)\,. \tag{1.90}$$

By choosing $Y = \log W$ we obtain:

$$\mathcal{L} = \big[-3\,e^{-G/3}\,\bar{\Phi}\Phi\,\big]_{\mathfrak{D}} + \big[\,\Phi^3\,\big]_{\mathfrak{F}} + \text{h.c} \tag{1.91}$$

where:

$$G(Z,\bar{Z}) = K(Z,\bar{Z}) + \log W(Z) + \log \bar{W}(\bar{Z}) \tag{1.92}$$

and the new compensator multiplet is given by:

$$\Phi = e^{G/6}\cdot\left\{\,1\,,\,\frac{1}{3}\,G_i\,\chi^i\,,\,U\,\right\}. \tag{1.93}$$

We conclude this discussion on the superconformal gauge fixing by recalling that the ordinary supersymmetry transformations of Poincaré SUGRA emerge as a combination of supersymmetry and extra conformal transformations which preserves the superconformal gauge choice; this is similar to what happens in ordinary supersymmetric gauge theories, in which supersymmetry transformations must be modified in order to preserve the Wess-Zumino gauge choice, see eq. (1.48)-(1.51).

Let us now recall the expressions for the scalar potential and the mass matrix of scalar fields in the non-linear σ-model coupled to gravity [55, 56]. As already discussed, these quantities can be efficiently computed by using the superfield version of the SUGRA Lagrangian (1.91):

$$\mathcal{L} = \int d^4\theta \left[-3\,e^{-G(\Phi^i,\bar{\Phi}^{\bar{\imath}})/3}\,\bar{\Phi}\Phi\right] + \int d^2\theta\,\Phi^3 + \text{h.c} + \dots\,. \tag{1.94}$$

After eliminating the auxiliary fields of the matter and the compensator chiral multiplets, one finally finds:

$$V_{\text{S}} = e^G\,(\,G^k G_k - 3\,)\,. \tag{1.95}$$

A vacuum is defined by constant values of the scalar fields for which V_S is stationary whereas spinor and tensor fields must have vanishing expectation values in order to preserve Lorentz invariance. The stationarity conditions have the form:

$$G_i + G^k\,\nabla_i\,G_k = 0\,, \tag{1.96}$$

and at any stationary point the auxiliary fields of the compensator and of chiral matter multiplets are given respectively by the expressions:

$$U = e^{G/2}\,(\,1 - \frac{1}{3}\,G^k\,G_k\,)\,, \tag{1.97}$$

$$F^i = -e^{G/2}\,g^{i\bar{\jmath}}\,G_{\bar{\jmath}}\,. \tag{1.98}$$

The expectation value $\langle V_{\rm S}\rangle$ defines the *cosmological constant*, which must be almost vanishing ($\simeq 10^{-3}$eV), as suggested by cosmological observations [57]; this implies that the space-time must be approximately Minkowski. Note that a small value of the cosmological constant is not stable against quantum corrections and can be achieved only by a severe fine tuning of the parameters in the Lagrangian. This is the so called *cosmological constant problem* [58]; it has the same nature of the hierarchy problem that we discussed in the first section of this chapter but in this case the amount of fine-tuning is dramatically larger: 120 order of magnitudes! We see that the condition for a vanishing cosmological constant is given by:

$$V_{\rm S} = 0 \quad \Rightarrow \quad G^i\,G_i = 3\,. \tag{1.99}$$

The masses of scalar fields are found to be:

$$(m_0^2)_{i\bar{\jmath}} = \ e^G\,\nabla_i G_k\,\nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}k\bar{l}}\,F^k\,F^{\bar{l}} + m_{3/2}^2\,g_{i\bar{\jmath}}\,, \tag{1.100}$$

$$(m_0^2)_{ij} = -e^{G/2}\,F^k \nabla_{(i}\nabla_{j)}\,G_k + 2\,m_{3/2}^2\,\nabla_{(i}\,G_{j)}\,, \tag{1.101}$$

where we have introduced the gravitino mass given by:

$$m_{3/2} = e^{G/2}\,. \tag{1.102}$$

The mass mixing term involving the gravitino and matter fermions has instead the form:

$$-\frac{i}{2}\sqrt{2}\,\langle e^{G/2}\,G_i\rangle\,\chi^i\,\sigma^\mu\,\bar{\psi}_\mu + \text{h.c.}\,. \tag{1.103}$$

From this expression we recognize that the Goldstino is defined by the direction $\langle G_i\rangle$ in the spinor field space. This can also be seen by using the SUSY transformation laws of matter fields; indeed it is possible to show that the only non-homogeneous transformation on the vacuum is [25]:

$$\delta\chi^i = -\sqrt{2}\,\epsilon\,\langle e^{G/2}\,G_i\rangle = \sqrt{2}\epsilon\,\langle F^i\rangle\,. \tag{1.104}$$

This expression shows that SUSY is broken in SUGRA whenever $\langle G_i\rangle \neq 0$. In supergravity, however, the Goldstino is not a physical degree of freedom and it can be gauged away by a proper gauge choice; in common language one says that it is "eaten" by the massive gravitino through the super-Higgs mechanism [59]. When the cosmological constant vanishes we can define the normalized Goldstino as:

$$\eta = \frac{\sqrt{2}}{\sqrt{3}}\,m_{3/2}\,\langle G_i\rangle\,\chi^i. \tag{1.105}$$

The associated scalar superpartner, the sGoldstino, is defined as:

$$\varphi = \frac{\sqrt{2}}{\sqrt{3}} m_{3/2} \langle G_i \rangle \phi^i . \tag{1.106}$$

This transforms under SUSY as $\delta\varphi = \sqrt{2}\,\epsilon\,\eta$.

1.4.2 Models with Chiral and Vector Multiplets

Let us now recall the main ingredients of the non-linear gauged σ-model coupled to gravity [60, 49]; for an exhaustive review of this part see also [61]. The generalization of the previous construction to include ordinary gauge symmetries and vector multiplets does not present any important complication. The suitable coupling of the compensator multiplet to the gauge fields can be guessed by performing the same analysis as for chiral multiplets, which was based on the transformation properties of the compensator under superconformal transformations. It turns out that there is no dependence on Φ in the gauge kinetic Lagrangian; this is because assuming $d(V) = 0$ and $R(V) = 0$ it is easy to see that:

$$d(W^{\alpha a}\, W^a_\alpha) = 3\,, \qquad R(W^{\alpha a}\, W^a_\alpha) = 2\,. \tag{1.107}$$

Superconformal invariance then implies that a coupling between the compensator and the vector multiplets can only arise in the non-holomorphic part of the Lagrangian. It is easy to promote the Lagrangian (1.35) to be superconformal invariant:

$$\mathcal{L} = \left[-3\, e^{-K/3}\, \bar{\Phi}\Phi \right]_{\mathfrak{D}} + \left[W\, \Phi^3 \right]_{\mathfrak{F}} + \frac{1}{4} \left[H_{ab}(\Phi^i)\, W^{\alpha a}\, W^b_\alpha \right]_{\mathfrak{F}} + \text{h.c}\,, \tag{1.108}$$

where $[\ \ ]_{\mathfrak{D}}$ and $[\ \ ]_{\mathfrak{F}}$ have the same expressions we gave in the previous section. In particular, it is still true that the computation of the scalar potential and the mass matrix of the scalar fields can be more efficiently performed by rewriting the previous lagrangian as an integral over rigid superspace:

$$\begin{aligned} \mathcal{L} = &\int d^4\theta \Big[-3\, \exp\Big\{ -\frac{1}{3} G(\Phi^i, \bar{\Phi}^{\bar{\imath}}, V) \Big\} \Big]\, \bar{\Phi}\Phi + \Big(\int d^2\theta\, \Phi^3 + \text{h.c.} \Big) \\ &+ \Big(\int d^2\theta\, \frac{1}{4}\, H_{ab}(\Phi^i)\, W^{a\alpha} W^b_\alpha + \text{h.c.} \Big) + \dots\,, \end{aligned} \tag{1.109}$$

where, again, the terms that we discard depend on the non-scalar fields of the gravitational supermultiplet. In addition, the same gauge fixing of the extra symmetries of the superconformal group can be applied [54] and the compensator multiplet has the same form:

$$\Phi = e^{G/6} \cdot \Big\{ 1\,,\ \frac{1}{3} G_i\, \chi^i\,,\ U \Big\}\,. \tag{1.110}$$

where $G(\Phi^i, \bar{\Phi}^{\bar{\imath}})$ and its derivative are evaluated at $V^a = 0$.

As for pure chiral models, the gauge vectors are associated to the gauging of the isometry group of the target space of scalar fields; in this case one must require that $G(\Phi^i, \bar{\Phi}^{\bar{\imath}}, V)$ is strictly invariant under gauge transformations generated by the holomorphic Killing vectors X_a^i associated to the isometries of the metric $G_{i\bar{\jmath}}$. In order to easily manipulate the Lagrangian (1.109), it is useful to fix the Wess-Zumino gauge for the internal gauge symmetries; we can then develop the integrand of the non-holomorphic part of the lagrangian $\Omega = -3e^{-\frac{1}{3}G}\bar{\Phi}\Phi$ as:

$$\Omega = -3\, e^{-\frac{1}{3}G}\, \bar{\Phi}\Phi + G_a\, e^{-\frac{1}{3}G}\, \bar{\Phi}\Phi\, V^a + \frac{1}{2}\left(G_{ab} - \frac{1}{3}\, G_a\, G_b\right) e^{-\frac{1}{3}G}\, \bar{\Phi}\Phi\, V^a\, V^b \qquad (1.111)$$

where G and its derivatives with respect to vector fields are now evaluated at $V^a = 0$. As before, the relevant functions $G_a(\Phi^i, \bar{\Phi}^{\bar{\imath}})$ and $G_{ab}(\Phi^i, \bar{\Phi}^{\bar{\imath}})$ are determined by using the invariance condition $\delta\, G(\Phi^i, \bar{\Phi}^{\bar{\imath}}, V) = 0$ and taking multiple derivatives. We finally obtain:

$$X_a^i\, G_i - \frac{i}{2}\, G_a = 0\,, \qquad (1.112)$$

$$X_a^i = \frac{i}{2}\, g^{i\bar{\jmath}}\, G_{a\bar{\jmath}}\,, \qquad (1.113)$$

$$G_{ab} = 4\, g_{i\bar{\jmath}}\, X_{(a}^i\, \bar{X}_{b)}^{\bar{\jmath}}\,. \qquad (1.114)$$

The second expression shows that the Killing potentials for X_a^i can be identified with $-\frac{1}{2}\, G_a$ The transformation properties of the gauge kinetic function are fixed by requiring the gauge invariance of the kinetic Lagrangian of the vector multiplets. This implies:

$$X_a^i H_{bci} = -2 f_{a(b}{}^d H_{c)d}\,. \qquad (1.115)$$

The last useful relation comes from the equivariance condition of the Killing vectors, which guarantees that the Killing potentials can be chosen in the adjoint representation:

$$g_{i\bar{\jmath}} X_{[a}^i \bar{X}_{b]}^{\bar{\jmath}} = \frac{i}{4}\, f_{ab}{}^c\, G_c\,. \qquad (1.116)$$

Again we avoid to present the full expression of the Lagrangian and we focus on the scalar potential and the masses, which can be easily computed from the Lagrangian (1.109) by exploiting the superfield machinery. In particular, as in the previous case, we can manipulate the compensator as an independent chiral superfield and substitute its gauge-fixed expression (1.110) only at the very end of the computation. Moreover, since we are interested mainly in the scalar part of the Lagrangian, the spinor components of matter, gauge and compensator superfield can be set to zero..

The scalar potential is found to be:

$$V_{\rm S} = e^G \left(G^k\, G_k - 3\right) + \frac{1}{8}\, h^{ab} G_a G_b\,, \qquad (1.117)$$

where h^{ab} represents the real part of the gauge kinetic function H^{ab} and we used the scalar parts of the equations of motion of the auxiliary fields F^i and D^a, which give:

$$F^i = -e^{G/2}\, g^{i\bar{\jmath}}\, G_{\bar{\jmath}}\,, \tag{1.118}$$

$$D^a = -\frac{1}{2}\, h^{ab}\, G_b\,. \tag{1.119}$$

The vacuum is defined by constant values of the scalar fields for which V_S is stationary. As already discussed, for phenomenological reasons we shall require a vanishing vacuum energy (flatness condition); this gives the relation:

$$-3 + G^i G_i + \frac{1}{8}\, e^{-G} G^a G_a = 0\,. \tag{1.120}$$

The stationarity conditions are then given by:

$$G_i + G^k \nabla_i G_k + \frac{1}{4}\, e^{-G}\Big[G^a\Big(\nabla_i - \frac{1}{2}\, G_i\Big) G_a - \frac{1}{2}\, h_{abi} G^a G^b\Big] = 0\,. \tag{1.121}$$

In this case too, one can deduce from the stationarity conditions a relation between F and D auxiliary fields which is valid at any stationary point. Following [61] we multiply the stationarity condition by X^i_a and consider the two independent relations given by the real and imaginary parts. The real part is trivial and vanishes because of gauge invariance of the scalar potential under ordinary (real) gauge transformations. The imaginary part insted does not vanish automatically and gives a "dynamical" constraint between F and D terms which is valid at any stationary point. Using the relations (1.112) and (1.113),[8] one finds [61–64]:

$$i\nabla_i X_{\bar{\jmath}a} F^i \bar{F}^{\bar{\jmath}} - g_{i\bar{\jmath}}\, X^i_a \bar{X}^{\bar{\jmath}}_b\, D^b + \frac{1}{2}\, f_{ab}{}^{d}\, \theta_{dc} D^b D^c - (F_i \bar{F}_i - m^2_{3/2}) D_a\,. \tag{1.123}$$

where $m_{3/2}$ is the gravitino mass which is defined as in the previous section as:

$$m_{3/2} = e^{G/2}\,. \tag{1.124}$$

In SUGRA, however, there exists another "kinematical" constraint between F and D auxiliary fields which is valid, contrarily to the previous one, at any point of the scalar field space. It follows directly from (1.112), which gives:

$$D^a = i\,\frac{h^{ab}}{m_{3/2}}\,\bar{X}_{bi} F^i = -i\,\frac{h^{ab}}{m_{3/2}}\, X_{b\bar{\imath}} \bar{F}^{\bar{\imath}}. \tag{1.125}$$

This constraint is purely gravitational and it becomes trivial in the rigid limit $M_P \to \infty$ and $m_{3/2} \to 0$ whereas the "dynamical" constraint reduces to expression (1.53) in the

[8] One also needs to use the relation

$$X^i_a + X^j \nabla_i G_k + G_j \nabla_i X^j_a = 0\,, \tag{1.122}$$

which can be obtained by taking one covariant derivative of the equation that expresses the invariance of $G(\Phi^i, \bar{\Phi}^{\bar{\imath}})$ under isometry transformations.

same limit. The rigid limit is performed by sending also $m_{3/2}$ to zero. This corresponds, in a flat space, to keep the SUSY breaking scale fixed. In this case too, it is possible to use the above constraints to derive interesting bounds on the ratio between F and D auxiliary fields, which have important consequences for SUSY breaking scenarios.

Using the stationarity and the flatness conditions, it is possible to show that the mass matrices of scalar fields are given by [61]:

$$\begin{aligned}
(m_0^2)_{i\bar{\jmath}} \;=\; & e^G\,\nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}k\bar{l}} F^k \bar{F}^{\bar{l}} + h^{ab}\bar{X}_{ai}X_{b\bar{\jmath}} + m_{3/2}^2\, g_{i\bar{\jmath}} \\
& - \frac{1}{2}\left(g_{i\bar{\jmath}} - G_i G_{\bar{\jmath}}\right) D^a D_a + \left(G_{(i}h_{ab\bar{\jmath})} + h^{cd}h_{aci}h_{bd\bar{\jmath}}\right) D^a D^b \\
& + \left(i\nabla_i X_{a\bar{\jmath}} - i\, h^{bc}h_{abi}X_{c\bar{\jmath}} + i\, h^{bc}h_{abj}\bar{X}_{ci} \right) D^a \\
& - \left(i\, G_i X_{a\bar{\jmath}} - i\, G_{\bar{\jmath}}\bar{X}_{ai} \right) D^a\,,
\end{aligned} \tag{1.126}$$

$$\begin{aligned}
(m_0^2)_{ij} \;=\; & -e^{G/2}\, F^k\, \nabla_{(i}\nabla_{j)} G_k - h^{ab}\bar{X}_{ai}\bar{X}_{bj} + 2\, m_{3/2}^2\, \nabla_{(i}G_{j)} \\
& - \frac{1}{2}\left(\nabla_i h_{abj} - 2h^{cd}h_{aci}h_{bdj} - 2G_{(i}h_{abj)} \right) D^a D^b \\
& - \frac{1}{2}\left(\nabla_{(i}G_{j)} - G_i G_j\right) D^a D_a + \; 2\, i\, h^{bc}h_{ab(i}\bar{X}_{cj)} D^a \\
& + 2\, i\, G_{(i}\bar{X}_{aj)}\, D^a\,.
\end{aligned} \tag{1.127}$$

The fermion mass term that is relevant to discuss SUSY breaking is the mixing between gravitino with matter and gauge fermions. The mixing is given by [25]:

$$-\frac{i}{\sqrt{2}}\,\langle e^{G/2} G_i\rangle\, \chi^i \sigma^\mu \bar{\psi}_\mu - \frac{1}{4}\,\langle G_a\rangle\, \lambda^a \sigma^\mu \bar{\psi}_\mu\,. \tag{1.128}$$

In this case we see that there are two relevant directions in the spinor field space, namely $\langle G_i\rangle$ and $\langle G_a\rangle$, which define the proper linear combination of fermions associated to the would-be Goldstino of the super-Higgs mechanism. This can again also be seen by studying the transformation laws of matter and gauge fermions at the vacuum, which read

$$\delta\chi^i = -\sqrt{2}\epsilon\,\langle e^{G/2}\, g^{i\bar{\jmath}}\, G_{\bar{\jmath}}\rangle = \sqrt{2}\,\epsilon\,\langle F^i\rangle\,, \tag{1.129}$$

$$\delta\lambda^a = \frac{i}{2}\epsilon\,\langle G_b\, h^{ba}\rangle = -i\,\epsilon\,\langle D^a\rangle\,. \tag{1.130}$$

We verify that, as expected, χ^i and λ^a transform by a shift. In flat space we can then define the normalized Goldstino as:

$$\eta = i\,\frac{\sqrt{2}}{\sqrt{3}}\, e^{G/2}\langle G_i\rangle\chi^i - \frac{1}{2\sqrt{3}}\,\langle G_a\rangle\lambda^a\,. \tag{1.131}$$

In this case too there are some subtleties related to the identification of the scalar superpartners of the Goldstino. Again, the difficulties come from the term containing the gaugino. Following the same arguments discussed in the rigid case, we can conclude that the physical scalars associated to heavy vector multiplets are:

$$\rho^a = \mathrm{Im}\left[\,\langle \bar{X}_{ai}\rangle \phi^i\,\right]\,. \tag{1.132}$$

Finally, we define also in this case the projected Goldstino to which we associate without ambiguity the sGoldstino field:

$$\varphi = \,i\,\frac{\sqrt{2}}{\sqrt{3}}\,e^{G/2}\langle G_i\rangle \chi^i\,. \tag{1.133}$$

We are going to discuss the relevance of this mode for the study of vacuum stability in Chapter 4.

Chapter 2

Supersymmetry Breaking

In the previous chapter we have mostly studied the formal and technical aspects of supersymmetric models without discussing the phenomenological constraints which should be taken into account. In this chapter we present a review of the main ideas developed to construct realistic supersymmetric extensions of the Standard Model. We start by briefly reviewing the structure of the Minimal Supersymmetric Standard Model (MSSM) and then we discuss the breaking of supersymmetry and the implications of the sum rules that we introduced in the previous chapter. This analysis brings us to the formulation of the *hidden sector* paradigm. We then recall two possible mechanisms to transmit supersymmetry breaking effects to the visible sector, namely gravity and gauge mediation, and we discuss the origin of the *soft terms*.

We conclude this chapter by reviewing the main features of the hidden sector in supergravity models inspired by String Theory. In particular we remark that, in string compactification scenarios, the hidden sector naturally includes the *moduli* sector. This part is not supposed to be a technical review of string compactification nor of moduli stabilization; its scope is to discuss the main motivations which inspire the forthcoming discussions and to define the context in which our results can have interesting applications.

2.1 The Minimal Supersymmetric Standard Model

Let us start by reviewing the main ingredients of the minimal supersymmetric extension of the Standard Model (MSSM). Since this topic is quite standard and widely studied in literature, we will focus on those aspects which are most relevant for our discussions. A good pedagogical introduction to the subject can be found in [65] (see also [51]) and in our presentation we mostly follow these references.

The building blocks of supersymmetric models, as we have seen in the previous chapter, are chiral and vector multiplets. To construct a supersymmetric extension of the Standard Model, each observed fundamental particle should properly fit into a supermultiplet and should have a superpartner with spin differing by 1/2 unit. Matter

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
(×3 families)	u^c	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	d^c	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
(×3 families)	e^c	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_u	$(H^+_u \;\; H^0_u)$	$(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$
	H_d	$(H^0_d \;\; H^-_d)$	$(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$

Table 2.1: Chiral supermultiplets in the MSSM.

particles, namely quarks and leptons, can be arranged into chiral multiplets; on the contrary vector multiplets are used to describe gauge fields, namely the photon, the gluon and the three weak gauge bosons. The Higgs scalar boson must be included into a chiral multiplet since it has spin 0. Actually, it turns out that two Higgs superfields are necessary to give masses to quarks through Yukawa couplings. This is essentially a consequence of the holomorphicity of the superpotential and the fact that only a Higgs scalar H_u with hypercharge $Y = +1/2$ can couple to up-type quarks (u,c,t) whereas only a Higgs scalar H_d with hypercharge $Y = -1/2$ can couple to down-type quarks (d,s,b). Another less trivial reason for demanding two Higgs superfields is anomaly cancellation, which is spoiled if we include only one fermion superpartner associated to the Higgs scalar. The neutral scalar which corresponds to the physical standard model Higgs boson is a linear combination of the neutral components of the Higgs doublets H^0_u and H^0_d.

The Supersymmetry algebra imposes that fields which belong to the same supermultiplet must have the same Standard Model quantum numbers; this strongly constrains the number of realistic possibilities of arranging Standard Model particles into supermultiplets. In particular, it turns out that it is not possible to fit two Standard Model particles into the same supermultiplet [1] and that all supersymmetric particles must be truly new (undiscovered) particles. In Tab. 2.1 we summarize the chiral multiplets of MSSM and their transformation properties under the Standard Model gauge group. The superpartners of Standard Model particles are indicated as usual with an extra tilde ($\sim$) over the Standard Model symbol; we adopt the convention that all the chiral superfields are expressed in terms of left-handed Weyl spinors. The vector multiplets associated to gauge fields are summarized in Tab. 2.2.

Having defined the particle content and the gauge symmetries of the MSSM, we

[1]As explained in [65], early attempts in this sense do not work; for example a neutrino cannot be taken to be the superpartner of a Higgs scalar.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\mathbf{1},\ \mathbf{1}\ ,\ 0)$

Table 2.2: Gauge supermultiplets in the MSSM.

can now write down the most general Lagrangian compatible with these assumptions. In particular, following the standard paradigm, we expect that at sufficiently low energies, physics is described by a renormalizable Lagrangian in which only relevant and marginal operators compatible with all the postulated symmetries appear. Supersymmetry allows very little arbitrariness in the choice of the Lagrangian and most of the couplings are obtained as a supersymmetric generalization of the Standard Model couplings. The relevant operators admitted by supersymmetry and gauge symmetries are:

$$\int d^2\theta\,\left(\mu\, H_u H_d + k_i\, L^i H_u\right) + \text{h.c.}\,, \tag{2.1}$$

The μ-term is a supersymmetric mass mixing between the Higgs multiplets whereas the other terms are mixings between Higgs and leptons doublets; note that the mixing terms are admitted because H_d and L^i have the same quantum numbers. These last terms are dangerous since they violate lepton number conservation. In the Standard Model, lepton number as well as baryon number turn out to be accidental $U(1)$ global symmetries of the renormalizable Lagrangian which quite remarkably explain the observed stability of the proton. Operators which violate lepton and baryon numbers in the Standard Model are irrelevant operators and as discussed in the previous chapter they are suppressed by a large (new physics) mass scale. On the contrary, this is not the case in the MSSM since dangerous relevant operators are in fact admitted by the symmetries. Actually, as we are going to see, there exist some other dangerous interactions which can be included in the Lagrangian; we will discuss in a moment how to get rid of these problematic operators. The marginal operators include all the kinetic terms of chiral and vector superfields as well as the standard Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = \int d^2\theta\,\left(\,u^c\,\lambda_u\,QH_u + d^c\,\lambda_d\,QH_d + e^c\,\lambda_e\,LH_d\,\right)\,, \tag{2.2}$$

where each term has been expressed for simplicity in compact form in which the color index a, the weak index α and the family index i have not been displayed. In full notation we have for example:

$$u^c\,\lambda_u\,QH_u \equiv (u^c)^{ia}(\lambda_u)_i{}^j Q_{ja\alpha}(H_u)_\beta\,\epsilon^{\alpha\beta}\,. \tag{2.3}$$

In addition to these standard couplings it is possible to include the following dangerous marginal operators:

$$\mathcal{L}_{\text{dangerous}} = \int d^2\theta \, (\, d^c \, \lambda_{LQQ} \, QL + e^c \, \lambda_{LLL} \, LL + \lambda_{QQQ} u^c d^c d^c \,) \, . \tag{2.4}$$

The first two operators violate lepton number by one unit whereas the last operator violate baryon number by one unit. As discussed in the first chapter, experimental bounds on couplings which violate lepton and/or baryon number are quite stringent and imply that they must be strongly suppressed with respect to other interactions. The only natural way to achieve this suppression is to assume that these operators are forbidden by some new global symmetry. Quite remarkably, all the dangerous interactions can be excluded in an economical way by postulating just one new discrete symmetry called *R-parity* or (matter parity); under this symmetry ordinary Standard Model particles have charge +1 whereas their supersymmetric partners have charge −1. At the superfield level, this can be achieved by the following transformation laws:

$$\Phi(\theta) \to \pm\Phi(-\theta) \quad V \to +V(-\theta, -\bar{\theta}) \, , \tag{2.5}$$

where the minus sign in the transformations of chiral fields holds for $L, Q, \bar{u}, \bar{d}$ while the sign plus holds for H_u and H_d. Postulating R-parity conservation has two major phenomenological signatures: in collider experiments, sparticles can only be produced in pairs and the lightest supersymmetric particle (LSP) must be absolutely stable. This last feature in particularly interesting since it provides a natural candidate for Dark matter.

So far we have only discussed the structure of the SUSY-preserving part of the MSSM Lagrangian but, as already discussed, supersymmetry breaking terms must be included to construct a realistic model. The SUSY breaking Lagrangian contains the most interesting operators from the point of view of phenomenology but, as we are going to see in the following sections, it turns out to be the major source of arbitrariness in the MSSM.

2.2 The Hidden Sector Paradigm and Soft SUSY Breaking

It is obvious that supersymmetry cannot be an exact symmetry of nature because none of the supersymmetric partner of the Standard Model particles has been observed so far in collider experiments. Supersymmetric particles must have a mass larger than $\sim$ 100GeV in order to be consistent with the experimental bounds; however, as we discussed in the previous chapter, their mass cannot exceed too much the TeV range if we expect that supersymmetry is the mechanism that is responsible for the natural stabilization of electroweak scale. Any satisfactory supersymmetric extension of the Standard Model must then include a supersymmetry breaking sector which naturally explains the non-observation of supersymmetric particles and which is consistent with all the experimental bounds.

It is natural to try to construct a model in which supersymmetry is broken by tree-level effects and communicated to MSSM fields by renormalizable interactions; as an obvious analogy we can take the $SU(2) \times U(1)$ electroweak symmetry of the Standard Model whose breaking is triggered by the Higgs field v.e.v. and transmitted to fermions through Yukawa interactions and to vector bosons through gauge interactions. However, as anticipated, there exists a major difference between electroweak and supersymmetry breaking which prevents us to push this analogy any further. Indeed, in supersymmetric models some non-trivial constraints associated to the supertrace formulas reviewed in the previous chapter rule out any scenario in which supersymmetry breaking is communicated to MSSM multiplets by tree level renormalizable couplings. Let us analyze more in detail this aspect. The supertrace formula (1.67) for a renormalizable model and linearly realized symmetries (as the MSSM) reduces to:

$$\mathrm{sTr}[m^2] = 2\,\mathrm{Tr}[\,T_a\,]\,D^a\,. \tag{2.6}$$

This trace formula holds separately for each set of conserved quantum numbers, namely electric charge, color, baryon and lepton number; this is due to the fact the mass matrices cannot have elements which connect particles with different values of these quantum numbers. The trace of T_a vanishes automatically, unless T_a is a $U(1)$ generator; moreover, the cancellation of the gravitational anomaly imposes that also all the $U(1)$ generators must be traceless. This finally implies that the supertrace is always vanishing in any renormalizable supersymmetric model. This result is a disaster from the phenomenological point of view since it implies that at least one of the superparticles must be lighter than its fermionic superpartner, and this is absolutely excluded by experimental observations.

It is actually not too hard to overcome these difficulties, since the supertrace formula (2.6) is only valid for renormalizable models and, furthermore, it does not take into account radiative corrections. The standard paradigm to construct realistic supersymmetric scenarios is to postulate that the sector responsible for supersymmetry breaking has no renormalizable tree-level couplings with the MSSM fields. Following the standard conventions, we call this sector the *hidden sector* in order to distinguish it from the *observable sector* which contains ordinary matter, gauge and Higgs fields as well as their superpartners. Supersymmetry-breaking effects are communicated to the MSSM supermultiplets by *messenger* fields, which interact with both observable and hidden sector fields. In this scenario, the restrictions encountered in the minimal setup can be avoided through a non-vanishing supertrace for the non-renormalizable low energy effective theory which describes the dynamics of the observable sector fields. The hidden sector paradigm allows to solve the severe constraints imposed by renormalizability in the trace formula but, as a draw back, it introduces a large amount of arbitrariness since the field content of the hidden sector and the interactions responsible for the transmission of SUSY-breaking effects are unspecified. Concerning the identification of the transmission mechanism, there are essentially two natural candidates: gravity and gauge interactions.

In gravity-mediated supersymmetry breaking [66–68], the hidden sector and the observable sector communicate only through gravitational interactions, which are assumed to be the strongest interaction connecting the two sectors. In this case the complete theory is intrinsically non-renormalizable, in such a way that the supertrace over the whole mass spectrum is non-vanishing.

On the contrary, gauge mediation [69] assumes that the high-energy microscopic theory describing the dynamics of both hidden and observable sector is a renormalizable theory. At tree level the theory has a vanishing supertrace and no mass splitting within the observable sector. However non-canonical kinetic terms for both chiral and vector supermultiplets are generated at the quantum level by radiative corrections and the supertrace is non-vanishing in the effective Lagrangian.

In general, from a low energy perspective, it is not of fundamental importance to understand in detail the hidden sector dynamics. In first approximation we may then be interested in simplified models which can be used to study the main features of the low-energy effective theory obtained by integrating out the hidden sector dynamics. With this purpose in mind we can briefly study the following two benchmark models of gravity and gauge mediation, which lead to a characteristic structure for the supersymmetry-breaking Lagrangian.

2.2.1 Gravity Mediation

The minimal model for the hidden sector we can imagine is a neutral chiral field X, which interacts with MSSM supermultiplets only through gravitational interactions suppressed by the Planck mass M_P and breaks supersymmetry by a non-vanishing v.e.v. for its auxiliary field $\langle F_X \rangle \neq 0$. For simplicity we can assume that the hidden sector is characterized by only one mass scale defining the amount of supersymmetry breaking in the hidden sector: $\langle X \rangle = M_S^2\, \theta^2$. Without loss of generality the v.e.v. of the lowest component has been taken equal to zero. Since there are no relevant and marginal interactions between the hidden and the observable sector fields, the leading interactions allowed by the $SU(3) \times SU(2) \times U(1)$ gauge symmetry and R-parity are schematically given by:

$$\begin{aligned}\mathcal{L}_{SUSY} = \int d^4\theta \Bigg\{ & \frac{Z^Q_{i\bar{j}}}{M_P^2} X\bar{X} Q^i \bar{Q}^{\bar{j}} + (\, Q \leftrightarrow u, d, L, e, H_u, H_d\,) \\ & + \frac{b}{M_P}\bar{X} H_u H_d + \frac{b'}{M_P^2} X\bar{X} H_u H_d + \text{h.c.} \Bigg\} \\ + \int d^2\theta\, & \frac{S^a}{M_P} X W^\alpha_a W_{a\alpha} + \text{h.c.} \\ + \int d^2\theta\, & \left\{ \frac{A^u_{ij}}{M_P} X\,(u^c)^j\, Q^i H_u + \frac{A^d_{ij}}{M_P} X\,(d^c)^j\, Q^i H_d + \frac{A^e_{ij}}{M_P} X\,(e^c)^j\, L^i H_d + \text{h.c.} \right\} ,\end{aligned} \tag{2.7}$$

where Z, S and A are dimensionless parameters. By expanding the Lagrangian in components and substituting the v.e.v of the X superfield we can read out the most significant SUSY breaking effects in the effective theory (see [70, 71] for a complete discussion). From the first line we obtain a Hermitian flavor-mixing mass term for the squarks:

$$(m^2_{\widetilde{Q}})_{i\bar{\jmath}}\, \widetilde{Q}^i \widetilde{Q}^{*\bar{\jmath}}\,, \qquad (m^2_{\widetilde{Q}})_{i\bar{\jmath}} \sim \frac{M_S^4}{M_P^2}\, Z^Q_{i\bar{\jmath}}\,, \tag{2.8}$$

as well as soft masses for sleptons and the two scalar Higgses. In the second line, the first term produces a supersymmetric mass term for the Higgs supermultiplets (the μ-term we discussed above) whereas the second term produces a mixing for the two Higgs scalars known as $B\mu$-term:

$$\int d^2\theta\, \mu H_d\, H_u\,, \qquad \mu \sim \frac{M_S^2}{M_P}\, b\,, \tag{2.9}$$

$$B\mu\, H_u H_d\,, \qquad B\mu \sim \frac{M_S^4}{M_P^2}\, b'\,. \tag{2.10}$$

The third line produces a mass term for the gluinos, the winos and the bino:

$$\frac{1}{2} M_g\, \widetilde{g}\, \widetilde{g}\,, \qquad M_g \sim \frac{M_S^2}{M_P}\, S^g\,. \tag{2.11}$$

Finally the last line gives cubic terms containing two sfermions and one Higgs scalar; in the literature these terms are known as A-terms:

$$\widetilde{\bar{u}}^j (a_u)_{ij}\, \widetilde{Q}^i H_u\,, \qquad (a_u)_{ij} \sim \frac{M_S^2}{M_P}\, A_{ij}\,. \tag{2.12}$$

Notice that we have not introduced linear terms in X multiplying matter supermultiplets in the $d^4\theta$ term because they are redundant. Indeed, once the auxiliary fields of matter supermultiplets are eliminated through their equation of motion, these terms would produce contributions of the same form as the sfermion masses $(m^2_{\widetilde{Q}})$ and the A-terms.

From the above analysis we conclude that the effects induced by supersymmetry breaking are encoded in a finite number of relevant operators. This aspect is remarkable because, as already discussed, the fact that no dimensionless SUSY breaking coupling appears is of fundamental importance to preserve the supersymmetric stabilization of the Standard Model Higgs mass. In fact, the terms we discussed above, even if derived in the context of this simple model, are more in general the only admitted supersymmetry breaking operators that do not re-introduce quadratic divergencies at loop level. These terms have been carefully classified (see for example [72]) and are known in literature as *soft terms*. Any realistic microscopic supersymmetric model must look in the low-energy limit like softly broken supersymmetric Lagrangian. Actually, there exists a further class of soft terms, called C-terms, which includes non-holomorphic cubic scalar terms, which can are only if the theory does not admit singlet fields. In the MSSM there are no singlet fields, but C-terms are usually not included since in most SUSY breaking scenarios they are not generated with sizable coefficients. In our situation, these terms are generated by operators of the type:

$$\Delta\mathcal{L}_C = \int d^4\theta \, \frac{X\bar{X}}{M_P^3} \, u^c \, \bar{H}_d Q \,, \tag{2.13}$$

which are suppressed by extra powers of M_S/M_P.

Summarizing, in this model all the soft terms are characterized by the same scale:

$$m_{\rm soft} \sim \frac{M_S^2}{M_P} \,, \tag{2.14}$$

which defines the effective scale of supersymmetry breaking in the observable sector; as we have already anticipated, m_{soft} must not exceed the TeV range in order to avoid fine tuning problems. An interesting feature of this model is that it contains a natural mechanism to generate a μ-term and a $B\mu$-term of the same order of magnitude as the other soft masses; this is very important since for phenomenological reasons these terms are expected to be of the order of the weak scale.

Finally we can compute the energy scale at which supersymmetry is expected to be broken in the hidden sector. Taking $m_{\rm soft}$ of the order of $1\,$TeV we deduce that in the hidden sector supersymmetry must be broken at the intermediate scale:

$$\sqrt{\langle F_X \rangle} = M_S \sim 10^{11}{\rm GeV} \,. \tag{2.15}$$

The main difficulty associated to gravity-mediated supersymmetry breaking is that the soft masses and the A-terms which are generated by this mechanism can violate flavor. The A-terms arise from holomorphic operators containing one power of X and we can imagine to control them by imposing some extra symmetry on X; however the soft masses arise from terms involving $X\bar{X}$, which are invariant under all possible symmetries. In this scenario there is then no natural reason to expect that the soft masses of squarks and sleptons should be almost flavor-diagonal, unless we postulate the existence of flavour symmetries at the Planck scale.

Given their phenomenological importance, we conclude this section by recalling the general expression of soft scalar masses in supergravity models. Ignoring D-type effects and restring to the case of vanishing cosmological constant, these masses can be obtained from expression (1.100) by distinguishing between observable and hidden sector indices (respectively u, v and i, j). The last term in the expression can be discarded; this is a consequence of the fact that the observable sector fields have vanishing v.e.v.'s and do not have holomorphic quadratic invariants. In this situation, the soft masses are given by:

$$(m^2_{\tilde{Q}})_{u\bar{v}} = -R_{u\bar{v}i\bar{j}} F^i \bar{F}^{\bar{j}} + g_{u\bar{v}} \, m^2_{3/2} \,. \tag{2.16}$$

2.2.2 Gauge Mediation

In gauge mediated supersymmetry breaking (see [73] for an extensive review on the subject), the microscopic (high energy) Lagrangian which describes the dynamics of both observable and hidden sector fields is renormalizable; in this scenario, the difficulties related to the supertrace formula are overcome by the fact that at low energies, quantum effects produce a non-vanishing supertrace for the effective theory. The main ingredients in gauge mediation are: a hidden sector responsible for supersymmetry breaking, which has no renormalizable interactions with observable fields, and a messenger sector composed by fields charged under Standard Model symmetries. Messengers interact by Yukawa interactions with the (neutral) hidden sector fields and by gauge interactions with observable fields. In general, tree-level Yukawa couplings between messenger and observable fields can also be included, but in many interesting applications one does not consider these kind of interactions to avoid any new source of flavor breaking (see [74] for more details).

The dynamics of the hidden and the messenger sectors is a priori not known and it represents the main source of model dependence. Nevertheless, as in the gravity mediation paradigm, the main features and the general predictions of the model can be analyzed by studying some simplified model. In this case we are interested in the case in which the messenger characteristic mass scale M and the supersymmetry breaking scale M_S are fixed by the v.e.v. of one chiral field X of the hidden sector:

$$\langle X \rangle = M + \theta^2 M_S^2 \,. \tag{2.17}$$

The messenger scale is assumed to be large: $M \gg 1\text{TeV}$, but still sufficiently small with respect to M_P to avoid comparable flavor violating effects induced by gravity. The X field gives a supersymmetric mass M to the messenger supermultiplets Φ and Φ^c by the Yukawa coupling:

$$W = \int d^2\theta X \Phi \Phi^c \,. \tag{2.18}$$

We consider the situation in which the supersymmetry breaking splittings induced on messenger field are small with respect to the messenger scale: $F_X = M_S^2 \ll M^2$. In this limit, messenger supermultiplets are stabilized in an approximately supersymmetric way and can be integrated out directly at superfield level (an extended review of this technique in a more general context and several new developments are the main topics of Chapter 3). In this simplified situation, it is possible to efficiently study the structure of soft terms in the effective theory by using superspace techniques. By direct computation it is possible to show that gaugino masses are generated at one loop level whereas sfermion masses are generated at two loops. The characteristic scale of soft terms is in this case:

$$m_{soft} \sim g^2 \frac{M_S^2}{M} \,. \tag{2.19}$$

A very elegant derivation of the soft SUSY breaking terms in gauge mediation has been given in [74]; let us briefly review this procedure, which is based on Renormalization Group (RG) techniques. At low energies $E \ll M$, the dynamics of the observable-sector fields can be described by a Wilsonian effective Lagrangian at the renormalization scale $\mu \ll M$, defined by integrating out at the quantum level all the modes with momentum larger than μ. The messenger fields can be completely integrated out and, because of the non-renormalization of superpotential (see [75] for a smart derivation of this result), the effective action is parametrized only by the wave function $Z(X, \bar{X}, \mu)$ and the holomorphic gauge kinetic functions $S_g(X, \mu)$:

$$\mathcal{L}_{eff} = \int d^4\theta \, Z(X, \bar{X}, \mu) \, Q\bar{Q} + \int d^2\theta \, S_g(X, \mu) \, W^{\alpha g} W^g_\alpha + \dots \, . \tag{2.20}$$

Contrarily to the gravity mediation scenario, in this case the wave function Z is automatically flavor-diagonal, as required to avoid new sources of flavor violation. As anticipated, the functional dependence of S_g and Z on the X superfield are obtained by solving the RG equations for $S_g(M, \mu)$, $Z_i(M, \mu)$ and by substituting back X to M in a proper way. Gaugino and sfermion masses, as well as A-terms, can then be easily found to be given by:

$$M_g(\mu) = -\frac{1}{2} \frac{\partial \ln S_g(X, \mu)}{\partial \ln X}\bigg|_{X=M} \frac{M_S^2}{M} \, , \tag{2.21}$$

$$m^2_{\widetilde{Q}}(\mu) = -\frac{\partial^2 \ln Z(X, \bar{X}, \mu)}{\partial \ln X \partial \ln \bar{X}}\bigg|_{X=M} \frac{M_S^4}{M^2} \, , \tag{2.22}$$

$$A_i(\mu) = \frac{\partial \ln Z_i(X, X^\dagger, \mu)}{\partial \ln X}\bigg|_{X=M} \frac{M_S^2}{M} \, . \tag{2.23}$$

From a technical point of view this method significantly simplifies the computation of the soft terms and drastically reduces the number of Feynman diagrams to be computed.

Without describing any further the details of the mechanisms which communicate supersymmetry breaking effects to the observable sector, let us summarize what we learnt from the previous examples. We have seen that the supertrace formula forbids scenarios in which the supersymmetry breaking sector interacts with the MSSM supermultiplets through renormalizable interactions. This implies that the minimal supersymmetric generalization of the Standard Model must be extended to include a new non-standard sector whose physics is mostly unknown and is the main source of arbitrariness in SUSY phenomenology. From a pragmatic point of view, we can parametrize our ignorance by breaking explicitly SUSY in the MSSM through the addition of soft terms that preserve the main features of SUSY as a solution of the hierarchy problem. Any realistic microscopic theory which includes supersymmetry must look in the infrared like a softly broken SUSY Lagrangian. In addition we have seen that phenomenology imposes important constraints on the form and on the relative size of soft terms. The structure of soft terms is mostly related to the mechanism by which supersymmetry breaking is communicated to the observable sector and several phenomenological constraints can be exploited to characterize the possible viable scenarios. In our discussion we emphasized that the most stringent constraints come from flavor physics. We have also stressed the fact that the structure of the soft terms does not depend too much on the details of the hidden sector physics and that the most relevant features of the soft SUSY breaking Lagrangian can be easily captured by studying the simple models discussed above. This implies that from the phenomenological point of view, very few low-energy constraints can be imposed on the hidden sector physics. In other words, from the low-energy perspective, the hidden sector looks like a "black box"and the only strong constraints that can be imposed on the supersymmetry-breaking dynamics is demanding that the scalar potential for the hidden sector fields admits a sufficiently long-lived metastable vacuum and that the associated cosmological constant is positive and almost vanishing. Other constraints come for instance from Big Bang nucleosynthesis [76], which is not compatible with the existence of light scalars with masses smaller than 1 TeV.

In the last decades, many attempts have been made to describe the physics of the hidden sector in the context of String Theory models. Indeed, as we are going to discuss in the next section, a general feature of the low energy effective models derived from String Theory is the existence of several neutral fields (moduli) which are natural candidates to constitute the hidden sector responsible for supersymmetry breaking.

2.3 Hidden Sector in String-Inspired Models

In this section we present a qualitative review of the main topics in string phenomenology which are relevant for our discussions (see [77] for a concise review in the spirit of this section). The problematics we are going to discuss can be considered as the principal motivation and inspiration for the studies presented in the next chapters. Nevertheless, our results apply to more general contexts and, in practice, depend only marginally on the arguments presented in this section.

At the present time, String Theory is the most accredited candidate for a truly fundamental description of all interactions and, in particular, it is the most important candidate theory to describe gravity at the quantum level. There exist five different String Theories and all of them are consistent in 10 space-time dimensions (see [21, 22]). A common feature of these theories is that the string spectrum contains a massless spin 2 particle, the graviton $G_{\mu\nu}$, a singlet massless scalar, the dilaton and antisymmetric tensors of different ranks depending on the theory. In the low energy limit, obtained by discarding all the massive excitations in the string spectrum, these theories are described by ordinary supergravity models in 10D. Despite the uniqueness of the 10 dimensional theories, a large amount of arbitrariness is introduced when the six extra dimensions are compactified (à la Kaluza-Klein) in order to obtain four-dimensional theories. Most of the model dependence comes from the choice of the compactification manifold; the requirement of having $N = 1$ supersymmetry in the four-dimensional theory constrains the manifold to be a Calabi-Yau manifolds, which is a 6D Ricci flat complex manifold.

The major problem in string phenomenology is that there exists a huge number of topologically inequivalent Calabi-Yau manifolds (see for instance [22]) whose geometry is controlled by the expectation values of moduli fields. These fields arise from the dimensional reduction of the higher-dimensional graviton and other tensor fields; by construction they are associated to the massless levels of the Kaluza-Klein tower. The number of moduli that characterize each Calabi-Yau manifold is given by a set of topological numbers known as Hodge numbers; they correspond to the number of independent harmonic forms that can be defined on the compactification manifold. There exist essentially two classes of moduli fields:

1. The Complex Structure moduli U, which characterize the shape of the compactification manifold;

2. The Kähler moduli T, which control the size of the compact manifold.

Beside these fields and independently from the details of the compactification, each 4D models contains the dilaton field S which fixes the string coupling.

We see that in String Theory there is then a twofold degeneracy on the "space" of possible theories: the first one is a "discrete" degeneracy associated to the choice among the topologically different Calabi-Yau manifold ,whereas the second one is a "continuous" degeneracy, in the sense that, for each manifold, there is a set of massless fields which can be freely varied and whose vacuum expectation values fix the shape and the size of the compact manifold. This huge degeneracy can be visualized as a *landscape* of string vacua [78] where each vacuum corresponds to a particular 4D model. In order for String Theory to have any chance to be a realistic and predictive description of our universe, it must incorporate a mechanism which explains how a particular vacuum is selected or, in other words, how the compactification manifold is dynamically fixed. This is one of the most challenging problems in string phenomenology and it is strongly related to the problem of finding a natural mechanism to stabilize the moduli fields.

In the basic string constructions, the moduli fields U, T and S are massless fields associated to flat directions of the scalar potential. In the last years, two main mechanisms have been explored to generate a non-trivial dynamics for moduli fields and lift the associated flat directions. Let us recall them.

1. *Gaugino condensation* (see for example [79–83]). Roughly speaking, in this case, a non-trivial superpotential for certain moduli fields is generated by non-perturbative effects due to the fact that they control also the gauge couplings through the gauge kinetic function. If the gauge sector contains asymptotically-free gauge interactions, gaugino condensation can naturally occur at the dynamically generated scale $\Lambda \sim e^{-1/g}$. Through this mechanism a non perturbative moduli-dependent superpotential à la ADS [84] (see also [85]) is generated.

2. *Flux compactication* (see for example [86] for a detailed review). In this case a tree-level moduli dynamics is generated by the addition of quantized background fluxes. This mechanism stabilizes the moduli with a supersymmetric mass of the order of the string scale, which is large with respect to the intermediate supersymmetry breaking scale. One may then integrate out these heavy moduli and work with a low energy supersymmetric effective theory describing the dynamics of the remaining light moduli.

It is important to remark at this point that the quest for a natural mechanism for moduli stabilization is essentially connected with the problem of finding a realistic mechanism to break supersymmetry in the hidden sector; more precisely, it is natural to expect that the same mechanism that is responsible for supersymmetry breaking also produces a non-trivial dynamics for the light moduli fields. In many situations that have been explored, none of the previous mechanisms is completely satisfactory and in general, a combination of tree-level flux-induced dynamics and non-perturbative effects is necessary to fully stabilize all the moduli and break supersymmetry. One famous example in this sense is the so called KKLT scenario [87]. The KKLT proposal consists in a two steps moduli stabilization. In the first step fluxes are used to induce a non-trivial potential for the dilaton S and the Complex Structure moduli U; these fields are stabilized with a supersymmetric mass which is assumed to be much larger than the Kähler moduli masses. The author suggests that under these assumptions it is possible to freeze S and U to their vacuum expectation values. The Kähler moduli T, on the other hand, remain unstabilized. In the second step, non-perturbative (gaugino condensation) effects are invoked to stabilize T at a supersymmetric AdS minimum. Finally, the authors use a brane sector to generate an uplifting (fine-tuned) potential that breaks supersymmetry and leads to a local minimum with a small positive cosmological constant.

Many concerns have been raised regarding the validity of the KKLT procedure (see for instance [88]). The weak point consists in the fact that the stabilization of S and U fields is performed before including the non perturbative potential and without taking into account the dynamics of T fields. The main question to be answered is whether one is allowed to freeze the heavy moduli to their vacuum expectation values and decouple them from the low energy dynamics. A more careful analysis would consist in properly integrate them out in the complete supergravity theory describing the dynamics of all the S, U and T fields.

This question is the main motivation for the study presented in Chapter 3 where, in a completely general context, we discuss under which conditions it is possible to integrate out heavy supermultiplets that are stabilized with large masses in order to define an effective theory for the light modes. This problem is well understood in the rigid limit whereas, as we will see, a more careful analysis is necessary in the supergravity case. The importance of effective theories is evident in many research fields in physics; in the case of moduli stabilization however it seems really crucial, since it represents a fundamental tool to deal efficiently with the may moduli fields that arise in most of the string compactification scenarios.

The physics of the hidden sector in string-inspired supergravity models is also the main motivation for the second topic we are going to discuss in the forthcoming chapters which, as anticipated, is metastability. The "continuous" degeneracy of string vacua that we discussed above can be faced with the use of effective field theories; however the "discrete" degeneracy associated to the choice among topologically different compactification manifolds needs some new and more refined tool to be investigated. In Chapter 4 we will derive some general criteria that allow to establish whether a supergravity model can admit realistic metastable vacua depending on the geometrical properties of the associated scalar (Kähler) manifold. In general, the geometry of the scalar target space is completely determined by the details of the compactification and having a model-independent procedure to discriminate among different compactification scenarios can be very helpful.

Chapter 3

Supersymmetric Effective Field Theories

Low-energy effective field theories are a very useful tool which can be used to simplify the study of complicated systems involving a large number of fields. Whenever there exist large hierarchies in the mass spectrum one expects that the dynamics of the light degrees of freedom can be more efficiently described by a simpler low-energy macroscopic Lagrangian in which heavy modes do not explicitly appear and the small effects induced by their dynamics are encoded into a new set of effective parameters. In string-inspired supersymmetric models, effective field theory is a particularly useful instrument to study the dynamics of moduli fields of the hidden sector. In that case, the large number of fields makes it prohibitive to study analytically the vacuum structure in the microscopic Lagrangian and a drastic simplification is required.

In this chapter we address the problem of how to construct low-energy effective theories in globally and locally supersymmetric theories by integrating out heavy fields. We consider general non-linear sigma models with chiral and vector multiplets and we study under which conditions the low-energy effective theory turns out to be approximately supersymmetric. These conditions translate into the requirements that all the derivatives, fermions and auxiliary fields should be small in units of the heavy mass scale. They apply not only to the matter sector, but also to the gravitational one if present, and imply in that case that the gravitino mass should be small. We then argue that in this limit the ordinary procedure, which applies also for non-supersymmetric models, can be replaced by a more efficient one which consists in integrating out heavy fields directly at the superfield level. We show how to determine the unique exactly supersymmetric theory that approximates this effective theory at the lowest order in the counting of derivatives, fermions and auxiliary fields, by working both at the superfield level and with component fields. As a result we give a simple prescription for integrating out heavy superfields in an algebraic and manifestly supersymmetric way, which turns out to hold in the same form both for globally and locally supersymmetric theories, meaning that the process of integrating out heavy modes commutes with the process of switching on gravity. More precisely, for heavy chiral and vector multiplets one has to impose respectively stationarity of the superpotential and the Kähler potential. We will mainly discuss the tree-level (classical) integration of heavy fields; a brief qualitative analysis of quantum corrections is discussed in the last part of the chapter. This chapter is based on our paper [1].

3.1 Integrating Out Heavy Fields: General Setup

In supersymmetric theories there exist essentially two approaches to construct low-energy effective theories by integrating out heavy fields. The superspace formalism suggests that it should be possible to do this directly at the superfield level by solving the superfield equations of motion of the heavy multiplets. On the other hand, working with the extended Lagrangian written in component fields, one is induced to integrate each heavy field by separately solving the corresponding equations of motion, and in general this turns out to be a more involved procedure. The questions we want to address in this chapter are then the following: are these two procedures equivalent? Under which conditions is it possible to apply the superfield approach and in which limit do the two effective Lagrangians coincide? To answer these questions we need to discuss the possible scenarios that may arise for the stabilization of heavy modes. In general, the heavy fields will be stabilized at values implying a spontaneous breakdown of supersymmetry, and the low-energy effective theory will consequently be non-supersymmetric. In such a case, the best thing that one can do is to proceed in the same way as for ordinary effective theories. In particular, at the leading order in derivatives the effective theory is obtained by determining the heavy fields in terms of the light ones by requiring stationarity of the potential with respect to the heavy fields. However, it may happen that the heavy fields are stabilized in an approximately supersymmetric way, with vacuum expectation values that break only very little or not at all supersymmetry. In this limit we expect that the manifestly supersymmetric approach in superfields differs from the actual effective theory only by small effects. It is then of general interest to understand more precisely under which conditions such a situation can arise and to develop a systematic procedure to construct the supersymmetric low-energy effective theory.

The case of theories with global supersymmetry is well understood, both for chiral [84, 89] and vector multiplets [90–92], but we will nevertheless review it in some detail. In this case one arrives very naturally at a simple procedure allowing to integrate out heavy superfields directly at the superspace level, and thus in a manifestly supersymmetric way. One particularly relevant situation where this procedure can be very usefully employed is that of supersymmetric Grand Unified Theories, with a high scale of gauge symmetry breaking yielding large masses for several fields [90, 91]. The case of supergravity theories, on the other hand, seems to be less understood, and the main aim of this chapter is to clarify how one should proceed in that case. For chiral multiplets, the question has been investigated some time ago in [93], and some difficulties seem to appear, whereas for vector multiplets the situation seems to be simpler [92] (see also [94, 61, 95]). We will however show that also in this case under suitable conditions one arrives at a simple prescription for integrating out heavy superfields in a manifestly supersymmetric way. The main result of our analysis is that the procedure to integrate out heavy multiplets in supergravity is the same as in the rigid case once the condition of small gravitino mass is satisfied.

The basic reason why gravity does not affect the way in which one integrates out heavy fields at the leading order in the low-energy expansion is due to the fact that when requiring also gravity to be described at the two-derivative level, its couplings are essentially fixed. This is true in general, for any theory with fields of spin 0, 1/2 and 1, independently of whether it is supersymmetric or not. It can be understood through the following argument. A generic two-derivative theory without gravity is entirely parametrized by a potential V and some wave-function factors Z defining the kinetic terms, which are functions of the fields. To get the effective theory at the two-derivative level, one can then integrate out the heavy fields ξ^h by using as equations of motion $\partial_h V = 0$ and completely neglecting space-time derivatives. As a matter of fact, this correctly determines not only the effective potential, but also the effective wave-function factors. The reason is that the corrections to the equation $\partial_h V = 0$ involve derivatives of the fields. Their effect can then be neglected in the wave function Z, since this would give terms with more than two derivatives in the action. It is easy to see that their effect can also be neglected in V. The reason for this is that only the leading linear effect can produce a term with two or less derivatives, but this term is proportional to $\partial_h V$ evaluated on the approximate solution and therefore vanishes. When switching on gravity, the potential and kinetic terms get covariantized in a unique way, and the only new allowed term is an Einstein-Hilbert kinetic term for gravity, multiplied by a function Ω of the fields. One can then repeat exactly the same reasoning as without gravity, treating Ω in a similar way as Z, and arrive again to the conclusion that one can use the simple equation $\partial_h V = 0$ to define the effective theory at the two-derivative level. The case of supersymmetric theories is then just a special case of this. For heavy chiral multiplets Φ^h, the Kähler potential K plays a role similar to Z, whereas the superpotential W corresponds essentially to V. For heavy vector multiplets V^x, it is instead the gauge kinetic function H that plays the role of Z and the Kähler potential K that plays the role of V. In the case where such heavy chiral and vector superfields are stabilized in an approximately supersymmetric way, the analogs of the equation $\partial_h V = 0$ turn then out to be respectively $\partial_h W = 0$ and $\partial_x K = 0$. For exactly the same reasons as before, these equations allow to correctly compute not only the effective potential but also the wave function factors, and turn out to be valid also in the presence of gravity. The only assumption behind this is that gravity can be treated at the two-derivative level, and we shall see that this implies that the gravitino mass should be small.

As anticipated in the previous chapter, the study we are going to present is particularly relevant in the context of the effective supergravity description of string models, where some of the moduli fields may be stabilized in a supersymmetric way with a large mass, like for example in the scenarios of [96, 87]. There has been some debate on the circumstances in which it is justified to freeze such heavy moduli to constant values [88, 97, 98] (see also [99, 100]), and although this issue has recently been settled in [101–103], it is important to know the procedure to integrate them out in general.

3.1.1 The Derivative Expansion

In quantum field theories, one can get rid of heavy degrees of freedom at tree level by integrating the equations of motion of the associated fields at leading order in space-time derivatives. To illustrate this point and in order to better understand the considerations done in the last part of the previous section, let us consider a simple non-supersymmetric model containing two sectors of real scalar fields with a large hierarchy in the mass spectrum. We indicate with ϕ^i the light fields, with ϕ^α the heavy ones and with $M_{\alpha\beta}$ the heavy-field mass scale:

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi^i\partial^\mu\phi^i + \frac{1}{2}\,\partial_\mu\phi^\alpha\partial^\mu\phi^\alpha - V(\phi^i,\phi^\alpha)\,, \tag{3.1}$$

with

$$V(\phi^i,\phi^\alpha) = \frac{1}{2}\,M^2_{\alpha\beta}\,\phi^\alpha\phi^\beta + \tilde{V}(\phi^i,\phi^\alpha)\,. \tag{3.2}$$

At low energy we can solve perturbatively the equations of motion of the heavy fields:

$$\phi^\alpha = (\Box + M^2)^{-1\alpha\beta}(-\tilde{V}_\beta) \approx -M^{-2\alpha\beta}\,\tilde{V}_\beta + \mathcal{O}\left(\frac{\Box\phi^i}{M^2}\right)\,. \tag{3.3}$$

At leading order in space-time derivatives, we can replace ϕ^α in the microscopic Lagrangian (3.1) by :

$$\phi^\alpha \to \phi^\alpha_0(\phi^i) + \mathcal{O}\left(\frac{\Box\phi^i}{M^2}\right)\,, \tag{3.4}$$

where $\phi^\alpha_0(\phi^i)$ is defined as the solution of the algebraic approximate equations of motion $M^2_{\alpha\beta}\,\phi^\beta_0(\phi^i) + \tilde{V}_\alpha(\phi^i,\phi^\alpha_0(\phi^i)) = 0$, which gives:

$$V_\alpha(\phi^i,\phi^\alpha_0(\phi^i)) = 0\,. \tag{3.5}$$

The effective Lagrangian at the two-derivative level is then given by:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\,Z_{ij}(\phi^i)\,\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi^i,\phi^\alpha_0(\phi^i)) \tag{3.6}$$

with

$$Z_{ij}(\phi^i) = \delta_{ij} + \sum_\alpha \frac{\partial\phi^\alpha_0}{\partial\phi^i}\,\frac{\partial\phi^\alpha_0}{\partial\phi^j}\,, \qquad \frac{\partial\phi^\alpha_0}{\partial\phi^i} = -\sum_\beta V^{\alpha\beta}_{\text{inv}}\,V_{\beta i}\,. \tag{3.7}$$

The relation on the right side can be derived from (3.5) by taking a derivative with respect to light fields and $V^{\alpha\beta}_{\text{inv}}$ denotes the inverse of $V_{\alpha\beta}$ as a matrix. We see now explicitly that in the effective Lagrangian we are allowed to keep the corrections to the wave function since the next-to-leading contributions of order $\mathcal{O}\left(\Box\phi^i/M_\alpha^2\right)$ in the scalar potential are proportional to $V_\alpha(\phi^i, \phi_0^\alpha(\phi^i))$, which automatically vanishes.

The crucial point to take into account when dealing with supersymmetric low-energy effective theories is that the usual expansion in number of derivatives does not preserve order by order supersymmetry. Any truncation on the number of space-time derivatives spoils then supersymmetry, unless some other measure is taken. This point has already been discussed in Sections 1.3.1 and 1.3.2 where we studied the most general non-renormalizable supersymmetric models in rigid SUSY; we have seen that a restriction on the number of space-time derivatives implies also a restriction on the number of fermions and auxiliary fields. As a matter of fact, this feature turns out to be valid more in general also for SUGRA models and it is essentially due to the general form taken by supersymmetry transformations.

When heavy multiplets are integrated out to define a low-energy effective theory valid below a certain mass scale M, infinitely many terms with arbitrarily large number of auxiliary fields and fermions are generated and these terms are suppressed by inverse powers of M. One may then decide to retain only those terms with the leading number of space-time derivatives, auxiliary fields and fermions in order to preserve supersymmetry, but this truncation is justified solely when not only derivatives but also the auxiliary fields and fermions bilinears are small in units of M.

This means physically that the modes that are integrated out should not only be heavy, but also be stabilized in a way that approximately preserves supersymmetry, with small values for the fermions and auxiliary fields, implying in particular small mass splittings. The supersymmetric low-energy effective theory defined in this way, by truncating the total number of derivatives, fermion bilinears and auxiliary fields, is then different from the standard low-energy effective theory, obtained by truncating only the number of derivatives, and the two approximately coincide only in those regions of field space where fermions and auxiliary fields are small. One can summarize this reasoning by simply saying that a multiplet of fields can be integrated out in a supersymmetric way only if it has a large supersymmetric mass.

In the following sections we study in detail how to supersymmetrically integrate out heavy multiplets, both by working in components and by using the superfield approach.

3.2 Integrating Out Heavy Chiral Multiplets in Global SUSY

Let us first consider the simplest case of a globally supersymmetric theory with light chiral multiplets Φ^i and heavy chiral multiplets Φ^α, denoted collectively by Φ^I. As we have seen in Section 1.3.1 the most general Lagrangian containing the leading number of space-time derivatives is:

$$\mathcal{L} = \int d^4\theta\, K(\Phi^I, \bar{\Phi}^{\bar{I}}) + \int d^2\theta\, W(\Phi^I) + \int d^2\bar{\theta}\, \bar{W}(\bar{\Phi}^{\bar{I}})\,, \tag{3.8}$$

where K and W are not allowed to depend on supercovriant derivatives D_α. It is useful to define a weight n which counts the number of space-time derivatives n_∂, auxiliary fields n_F and fermion bilinears n_ψ of each term in the component Lagrangian and which is preserved by supersymmetry. More precisely, by inspection of the supersymmetry transformations of chiral multiplets (1.16)-(1.18), we see that we can assign $n[\phi^I] = 0$ to scalar fields and $n[\partial_\mu \phi^I] = 1$ to their derivatives. In order for $\delta\psi^I$ to have a well defined n, we then need to assign $n[F^I] = 1$ also to the auxiliary fields. Finally to assign the correct weight to ψ we should observe that the expression for $\delta\phi^I$ implies that the operator δ carries the same weight as ψ^I, or schematically $n[\delta] = n[\psi^I]$. Using again the transformation law $\delta\psi^I$, we deduce that $2\,n[\psi] = 1$, which implies $n[\psi] = 1/2$. By this analysis we are then led to define the following parameter:

$$n = n_\partial + \frac{1}{2}\, n_\psi + n_F\,. \tag{3.9}$$

Using this definition, we see that restricting to a supersymmetric Lagrangian with at most 2 space-time derivatives translate into requiring $n \leqslant 2$. But this, as already discussed, produces also a limitation on the number of spinor and auxiliary fields. We will use this weight to establish which terms give the leading contributions in the supersymmetric limit of the low energy effective theory.

To work at the superfield level, it is more useful to define a weight p which just counts the number of supercovariant derivatives:

$$p = \frac{1}{2}\, n_{D_\alpha} + \frac{1}{2}\, n_{\bar{D}_{\dot{\alpha}}}\,. \tag{3.10}$$

By rewriting any integral on the superspace as supercovariant derivatives plus vanishing boundary terms, we can verify that this definition is consistent with the previous one, in the sense that to any term with a given weight p in superfield correspond terms with weight $n = p$ in components; more precisely, we see that all the terms coming from the $d^4\theta$ integral have $n = 2$ whereas all the terms coming from the $d^2\theta$ integral have $n = 1$. In superfield language, restricting to two-derivatives Lagrangians then consistently translates in requiring $p \leqslant 2$.

Superfield approach

Let us start by studying in detail how to supersymmetrically integrate out the heavy multiplets using the superfield approach. The exact superfield equation of motion for Φ^α is obtained by first rewriting the first term in eq. (3.8) as an F-term by making use of supercovariant derivatives, and then varying $\mathcal{L}$ with respect to the unconstrained chiral superfield Φ^α. This yields:

$$W_\alpha - \frac{1}{4}\bar{D}^2 K_\alpha = 0\,. \tag{3.11}$$

The presence of a large supersymmetric mass for Φ^α means that around the value Φ_0^α at which the superfield is stabilized, the superpotential W has a large second derivative $W_{\alpha\beta}(\Phi^i, \Phi_0^\alpha)$ setting the mass scale M. This implies that the first term in eq. (3.11) dominates over the second and that, at leading order in $1/M$, we can integrate out the heavy chiral multiplets by replacing Φ^α in the microscopic Lagrangian (3.8) by:

$$\Phi^\alpha \to \Phi_0^\alpha(\Phi^i) + \mathcal{O}\left(\bar{D}^2\bar{\Phi}^i/M\right)\,, \tag{3.12}$$

where $\Phi_0^\alpha(\Phi^i)$ is determined by the algebraic equation $W_\alpha(\Phi^i, \Phi_0^\alpha) = 0\,.$

It turns out that the corrections $\mathcal{O}\left(\bar{D}^2\bar{\Phi}/M\right)$ can be completely neglected in our approximation, as the leading contributions that they would give to the effective action would have $p = 3$. This statement is obvious for the terms coming from K, which gives terms with $p = 2$ in the absence of extra supercovariant derivatives. For the terms coming from W, which gives terms with $p = 1$ in the absence of extra supercovariant derivatives, this is on the other hand due to the fact that the leading correction is proportional to W_α, and therefore vanishes on the leading order solution. Summarizing, one can thus integrate out the superfields Φ^α by using the simple chiral superfield equation

$$W_\alpha = 0\,. \tag{3.13}$$

This equation determines in an algebraic way the heavy chiral superfields in terms of the light chiral superfields:

$$\Phi^\alpha = \Phi_0^\alpha(\Phi^i)\,. \tag{3.14}$$

The effective theory for the Φ^i is then obtained by plugging back this solution into K and W. This yields:

$$K^{\text{eff}}(\Phi^i, \bar{\Phi}^{\bar{\imath}}) = K(\Phi^i, \bar{\Phi}^{\bar{\imath}}, \Phi_0^\alpha(\Phi^i), \bar{\Phi}_0^{\bar{\alpha}}(\bar{\Phi}^{\bar{\imath}}))\,, \tag{3.15}$$

$$W^{\text{eff}}(\Phi^i) = W(\Phi^i, \Phi_0^\alpha(\Phi^i))\,. \tag{3.16}$$

Component approach

It is instructive to rederive these results by using component fields. The Lagrangian has the usual form $\mathcal{L} = T - V$ and it is given by expression (1.15). The kinetic part is

$$T = -K_{I\bar{J}}\big(\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}} + i\bar{\psi}^{\bar{J}}\bar{\sigma}^\mu D_\mu\psi^I\big)\,, \tag{3.17}$$

with $D_\mu\psi^I = \partial_\mu\psi^I + \Gamma^I_{MN}\partial_\mu\phi^M\psi^N$, and the potential is given by

$$\begin{aligned} V = &- W_I F^I - \bar{W}_{\bar{J}}\bar{F}^{\bar{J}} + \frac{1}{2}W_{IJ}\psi^I\psi^J + \frac{1}{2}\bar{W}_{\bar{I}\bar{J}}\bar{\psi}^{\bar{I}}\bar{\psi}^{\bar{J}} \\ &- K_{I\bar{J}}F^I\bar{F}^{\bar{J}} + \frac{1}{2}K_{I\bar{J}\bar{K}}F^I\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{K}} + \frac{1}{2}K_{\bar{J}MN}\bar{F}^{\bar{J}}\psi^M\psi^N - \frac{1}{4}K_{I\bar{J}P\bar{Q}}\psi^I\psi^P\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{Q}}\,. \end{aligned} \tag{3.18}$$

Recall that the capital indices run over all fields whereas we reserve i and α indices to refer respectively to light and heavy fields. Recall also that the auxiliary fields are actually determined by their algebraic equations of motion, and are given by:

$$F^I = -K^{I\bar{J}}\Big(\bar{W}_{\bar{J}} - \frac{1}{2}K_{\bar{J}MN}\psi^M\psi^N\Big)\,. \tag{3.19}$$

We can now derive the exact equations of motion of F^α, ψ^α and ϕ^α. These correspond to the θ^0, θ^α and θ^2 components of (3.11) and determine respectively the values of the auxiliary fields F^α, the wave equation for ψ^α and the wave equation for ϕ^α. One finds, without needing to use eq. (3.19), the following equations:

$$W_\alpha + K_{\alpha\bar{J}}\bar{F}^{\bar{J}} - \frac{1}{2}K_{\alpha\bar{I}\bar{J}}\bar{\psi}^{\bar{I}}\bar{\psi}^{\bar{J}} = 0\,, \tag{3.20}$$

$$W_{\alpha I}\psi^I + K_{\alpha I\bar{J}}\psi^I\bar{F}^{\bar{J}} - \frac{1}{2}K_{\alpha I\bar{J}\bar{K}}\psi^I(\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{K}}) + iK_{\alpha\bar{J}}\sigma^\mu D_\mu\bar{\psi}^{\bar{J}} = 0\,, \tag{3.21}$$

$$\begin{aligned} &W_{\alpha I}F^I - \frac{1}{2}W_{\alpha IJ}\psi^I\psi^J + K_{\alpha I\bar{J}}F^I F^{\bar{J}} - \frac{1}{2}K_{\alpha I\bar{J}\bar{K}}F^I\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{K}} - \frac{1}{2}K_{\alpha\bar{J}MN}F^{\bar{J}}\psi^M\psi^N \\ &\quad + \frac{1}{4}K_{\alpha I\bar{J}P\bar{Q}}\psi^I\psi^P\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{Q}} + K_{\alpha\bar{J}}\Box\bar{\phi}^{\bar{J}} + K_{\alpha\bar{J}\bar{K}}\partial_\mu\bar{\phi}^{\bar{J}}\partial^\mu\bar{\phi}^{\bar{K}} = 0\,. \end{aligned} \tag{3.22}$$

Under supersymmetry transformations, these equations get mapped into each other and remain thus satisfied.

In the situation in which the fields ϕ^α and ψ^α have a large supersymmetric mass M, there must be a quadratic term in W leading to a second derivative $W_{\alpha\beta}$ of order M. The equations of motion (3.21) and (3.22) for ψ^α and ϕ^α are then dominated by the first terms, which involve second derivatives of W. Similarly, in the equation of motion (3.20) for F^α, the first term is expected to dominate, since the other two do not involve W at all. In the brutal limit in which one takes $M \to \infty$ one would find that ϕ^α is determined by the condition $W_\alpha(\phi^\alpha) = 0$ whereas ψ^α and F^α vanish. However, this brutal approximation does not preserve supersymmetry. One therefore needs to look at the subleading terms and check which ones should be kept in order to get a set of equations that is supersymmetric. The appropriate criterion to do so is related to the counting of the total number n of derivatives, fermion bilinears and auxiliary fields. Indeed, in order to obtain an effective theory with $n \leq 2$, each of the equations used to integrate out the heavy fields in terms of the light ones should involve terms with the same minimal value of n. Looking at eqs. (3.20)–(3.22), it is easy to see that the terms depending on W have a value of n that is one unit less than the terms depending on K and are therefore the dominant ones. One may then drop all the terms involving K and find the following set of approximate equations:

$$W_\alpha = 0\,, \tag{3.23}$$

$$W_{\alpha I}\psi^I = 0\,, \tag{3.24}$$

$$W_{\alpha I}F^I - \frac{1}{2}W_{\alpha IJ}\psi^I\psi^J = 0\,. \tag{3.25}$$

It is easy to check that these are now exactly supersymmetric. More precisely, under supersymmetry transformations each equation transforms into a combination of its space-time derivative and one of the other equations. These equations are in fact the non-trivial components of a chiral superfield equation, which is nothing but eq. (3.13). The first of them is now understood as determining ϕ^α, the second ψ^α and the third F^α. The bottom line is that the appropriate equation to be used to integrate out the scalar fields is indeed the naive one, whereas for the fermion and auxiliary fields supersymmetry forces us to keep some subleading terms suppressed by the mass scale M.

Let us finally spell out more concretely the content of the three components (3.23)–(3.25) of the superfield equation (3.13) by explicitly expressing heavy fields in terms of the light ones. It is useful to introduce the following notation for the supersymmetric masses of the heavy fields and the light fields, their mixings and ratios:

$$M_{\alpha\beta} = W_{\alpha\beta}\,, \quad m_{ij} = W_{ij}\,, \quad \mu_{\alpha i} = W_{\alpha i}\,, \quad \epsilon^\alpha_i = -M^{-1\alpha\beta}\,\mu_{\beta i}\,. \tag{3.26}$$

The other relevant parameters are the cubic couplings in the superpotential involving heavy chiral multiplets, namely:

$$\lambda_{\alpha ij} = W_{\alpha ij}\,, \quad \lambda_{\alpha\beta j} = W_{\alpha\beta j}\,, \quad \lambda_{\alpha\beta\gamma} = W_{\alpha\beta\gamma}\,. \tag{3.27}$$

The first equation (3.23), compared to (3.20), states that the scalar components ϕ^α of the heavy chiral multiplets must adjust to values compatible with the assumption that all the ψ^I and F^I vanish in first approximation:

$$\phi^\alpha_0(\phi^i): \quad \text{solution of } W_\alpha(\phi^i, \phi^\alpha_0) = 0\,. \tag{3.28}$$

The second equation (3.24) tells us instead that the heavy fermions ψ^α are not exactly zero but proportional to the light ones, ψ^i, through a coefficient given by the ratio between the supersymmetric mass mixing μ between light and heavy fields and the mass M of the heavy fields:

$$\psi^\alpha_0(\phi^i, \psi^i) = -(M^{-1}\mu)^\alpha{}_i\,\psi^i = \epsilon^\alpha{}_i\,\psi^i\,. \tag{3.29}$$

Finally, the third equation implies that the F^α are not exactly zero, but proportional to the F^i, plus some terms quadratic in the ψ^i, again through coefficients involving the ratio between μ and M:

$$\begin{aligned} F^\alpha_0(\phi^i, \psi^i) &= \epsilon^\alpha{}_i\,F^i \\ &\quad + \frac{1}{2}\left(M^{-1\alpha\beta}\lambda_{\beta ij} + 2\,M^{-1\alpha\beta}\lambda_{\beta\gamma i}\,\epsilon^\gamma{}_j + M^{-1\alpha\beta}\lambda_{\beta\gamma\delta}\,\epsilon^\gamma{}_i\,\epsilon^\delta{}_j\right)\psi^i\psi^j\,. \end{aligned} \tag{3.30}$$

In summary, we see that this procedure automatically keeps track of the fact that the heavy superfields have small but yet non-vanishing fermion and auxiliary field components. The effects of these suppressed components are in general relevant and cannot be neglected. The final result is then a supersymmetric effective theory that is accurate at leading order in ∂^μ/M, $\psi^I/M^{3/2}$ and F^I/M^2, but a priori not limited to small ϕ^I/M.

We have checked in a variety of examples that the supersymmetric effective theory defined by the superfield equation $W_\alpha = 0$ and the standard effective theory defined by the ordinary equation $V_\alpha = 0$ do indeed approximately coincide under the above assumptions. Focusing for concreteness on the scalar potential, the region in the space of scalar fields ϕ^i where the two theories match is defined by the following two conditions:[1]

$$m(\phi^i), \mu(\phi^i) \ll M\,, \quad F^i(\phi^i), F^\alpha(\phi^i) \ll M^2\,. \tag{3.31}$$

3.3 Integrating Out Heavy Chiral Multiplets in Supergravity

Let us consider next the case of a locally supersymmetric theory with light chiral multiplets Φ^i and heavy chiral multiplets Φ^α, denoted collectively by Φ^I, as well as the gravitational multiplet. As anticipated in Section 1.4 we will work in the superconformal formalism; in this framework it is possible to rewrite as superspace integrals all the relevant terms of the Lagrangian, which contain the couplings between the auxiliary field F_ϕ and the matter and gauge fields. Schematically we can write:

$$\mathcal{L} = \int d^4\theta \left(-3\, e^{-K/3}\, \bar\Phi\Phi\right) + \int d^2\,\theta\, W\, \Phi^3 + \text{h.c.} \;+\; \text{gravity}(e^a_\mu, \psi_\mu, A_\mu)\,. \tag{3.32}$$

The superspace part of the SUGRA Lagrangian turns out to be the most significant one for our purposes and in our analysis we will focus on it. The omitted terms, as we have already discussed in Chapter 1, are completely fixed by covariance and for this reason we will not keep track of them in our computations. In this formalism, the total number n of derivatives, fermion bilinears and scalar auxiliary fields corresponds again to half the number of supercovariant derivatives. Requiring $n \le 2$ amounts then to work at leading order in space-time derivatives, auxiliary fields and fermion bilinears in both matter and gravitational sector.

[1]Note that in general the whole supersymmetric mass matrix, including all the blocks m, μ and M, is field dependent. One has therefore to make sure that not only m but also the mixing term μ stay small compared to M (see also [104] regarding this point). One can however focus on the supersymmetric part of the mass matrix, since the F^I are independently assumed to be small.

Superfield approach

Working as in the rigid case, we can integrate out the heavy chiral multiplets by solving their approximate equations of motion. The exact superfield equations are given by:

$$W_\alpha - \frac{1}{4}\bar{D}^2(K_\alpha\, e^{-K/3}\bar{\Phi})\Phi^{-2} = 0\,. \tag{3.33}$$

We assume that as before the presence of a large supersymmetric mass means that around the value Φ_0^α at which the heavy superfields Φ^α are stabilized, the superpotential W has a large second derivative $W_{\alpha\beta}(\Phi^i, \Phi_0^\alpha)$ setting the mass scale M. The equations of motion are then dominated by the first term, and we can integrate out the heavy chiral multiplets, at leading order in $1/M$, by replacing Φ^α in the microscopic Lagrangian (3.32) by:

$$\Phi^\alpha \to \Phi_0^\alpha(\Phi^i) + \mathcal{O}\left(\bar{D}^2\bar{\Phi}^i/M,\; D^2\bar{\Phi}/M\right)\,, \tag{3.34}$$

where $\Phi_0^\alpha(\Phi^i)$ is determined by the algebraic equation $W_\alpha(\Phi^i, \Phi_0^\alpha) = 0$.

As before, the sub-leading corrections $\mathcal{O}\left(\bar{D}^2\bar{\Phi}^i/M,\; D^2\bar{\Phi}/M\right)$ can be neglected, since they would give corrections with $n > 2$.[2] The heavy chiral superfields can thus be integrated out by using the same simple chiral superfield equation as in the rigid case, namely

$$W_\alpha = 0\,. \tag{3.35}$$

As before, the solution of this equation determines the heavy chiral fields in terms of the light chiral fields:

$$\Phi^\alpha = \Phi_0^\alpha(\Phi^i)\,. \tag{3.36}$$

The effective theory for the Φ^i is then obtained by plugging back this solution into K and W. This yields:

$$K^{\rm eff}(\Phi^i, \bar{\Phi}^{\bar{\imath}}) = K(\Phi^i, \bar{\Phi}^{\bar{\imath}}, \Phi_0^\alpha(\Phi^i), \bar{\Phi}_0^{\bar{\alpha}}(\bar{\Phi}^{\bar{\imath}}))\,, \tag{3.37}$$

$$W^{\rm eff}(\Phi^i) = W(\Phi^i, \Phi_0^\alpha(\Phi^i))\,. \tag{3.38}$$

Notice now that, as discussed in Section 1.4, the microscopic theory involving all the fields has a Kähler symmetry acting as $(\Phi, K, W) \to (\Phi\, e^{Y/3}, K + Y + \bar{Y}, We^{-Y})$, where $Y(\Phi^I)$ is an arbitrary holomorphic function of the matter chiral superfields. On the other hand, the superfield equation (3.35) defining the effective theory is not manifestly invariant under such a transformation for generic Y. More precisely, it is invariant if Y depends only on the Φ^i, corresponding to Kähler transformations within the effective theory. But it is not invariant if Y depends also on the Φ^α. The reason for this is that we have assumed in our derivation that the large mass scale M of the heavy fields is associated only with a large quadratic term in W, and no large term in K. This is clearly a gauge-dependent assumption and it selects a restricted subclass of

[2]A similar reasoning has also been used in [102, 103] in the special case of effective theories describing string models with fluxes.

Kähler gauges, which is particularly well-suited to work out the effective theory.

One may wonder at this point whether it is really justified to neglect supercovariant derivatives acting on the compensator, and try to see what is the outcome when one keeps such terms and neglects only those where supercovariant derivatives act on the other chiral superfields. Proceeding in this way, eq. (3.33) would not reduce to eq. (3.35), but rather to

$$W_\alpha - \frac{1}{4}\Phi^{-2} K_\alpha e^{-K/3} \bar{D}^2 \bar{\Phi} = 0\,. \tag{3.39}$$

In order to get rid of the dependence on the compensator, one can now use the exact superfield equation of motion of Φ. From the Lagrangian (3.32), one finds that this equation of motion is given by

$$W + \frac{1}{4}\bar{D}^2\Big(e^{-K/3}\bar{\Phi}\Big)\Phi^{-2} = 0\,. \tag{3.40}$$

For the same reasons as before, all the terms involving supercovariant derivatives acting on K can certainly be neglected. However, one should keep the terms where the supercovariant derivatives act on the compensator. Eq. (3.40) then becomes

$$-\frac{1}{4}\bar{D}^2\bar{\Phi}\Phi^{-2} = e^{K/3} W\,. \tag{3.41}$$

Plugging this relation back into eq. (3.39) allows finally to eliminate completely the dependence on the compensator, and the final equation simply reads:

$$W_\alpha + K_\alpha W = 0\,. \tag{3.42}$$

This equation can also be derived in a more direct way by choosing from the beginning a Kähler gauge defined by $Y = \ln W$. In this way one does not need to use the compensator equation of motion, but the derivation still implicitly assumes that $W \neq 0$ and $\bar{D}^2\bar{\Phi} \neq 0$. More in detail, from the Lagrangian (1.94) we can derive the exact superfield equation of motions for the heavy superfields:

$$\bar{D}^2(G_\alpha\, e^{-G/3}\bar{\Phi}) = 0\,. \tag{3.43}$$

By keeping only the term in which the supercovariant derivative acts on the compensator superfield we obtain:

$$G_\alpha\, e^{-G/3}\bar{D}^2\bar{\Phi} = 0\,, \tag{3.44}$$

which is equivalent to (3.42) if the above-mentioned assumptions are satisfied.

Notice that eq. (3.42) reduces to the equation $W_\alpha = 0$ in the rigid limit, and is moreover manifestly invariant under Kähler transformations. However, a closer look shows that it cannot possibly be the correct equation. An obvious problem is that it is a vector and not a chiral superfield equation. This means that it cannot be solved as a superfield equation by just setting the Φ^α to some functions of the Φ^i, due to the fact that there are more component equations than component fields. On the other hand, the original exact equation of motion (3.33) for Φ^α is chiral, and it is by dropping only part of the terms involving supercovariant derivatives that one arrives at an equation which is no longer chiral. Thus, the new equation must somehow also be approximately chiral, meaning that only its chiral components should really be significant, the non-chiral ones being approximately satisfied in an automatic way. This means that the equation $W_\alpha + K_\alpha W = 0$ cannot be used as an exact equation to define a manifestly supersymmetric approximate version of the low-energy effective field theory, and that the appropriate equation should instead be $W_\alpha = 0$, as already argued. Through this argument, which clarify the problems raised in [93], we have moreover learned that neglecting terms involving supercovariant derivatives acting on the compensator amounts to neglect W compared to M, i.e. to have approximately

$$W \simeq 0\,. \tag{3.45}$$

This equation should however not be imposed as an exact superfield equation as it comes from a reasoning on the compensator superfield Φ, for which most of the components can be gauged away. More precisely, in the formulation where the superconformal symmetry is gauge-fixed to the super Poincaré symmetry, only the lowest component of this equation, corresponding to the equation coming from the auxiliary field of the compensator (3.40), should be considered. Finally, it should also be emphasized that although W must be neglected in the equation that is used to integrate out the Φ^α, one should a priori not neglect terms involving W in the Lagrangian where the solution for the Φ^α is substituted to obtained the effective theory for the Φ^i.

The crucial point behind this extra difficulty that one encounters in the gravitational case is that space-time derivatives and supersymmetry-breaking auxiliary fields must be small also in the gravitational sector. This brings up a new condition that needs to be fulfilled in order to be in the situation in which an approximate two-derivative supersymmetric low-energy effective theory is expected to exist: the compensator auxiliary field F_ϕ should be much smaller than M:

$$F_\phi \ll M\,. \tag{3.46}$$

Once all the other auxiliary fields F^I are also assumed to be small, $F^I \ll M^2$, this condition implies that:

1. the gravitino mass (and therefore W) is small, $m_{3/2} \ll M$;
2. the cosmological constant is small, $V_{\rm S} \ll M^4$.

The first statement is justified by the fact that $m_{3/2}$ turns out to be a linear combination of F^I and F_ϕ auxiliary fields; to see this we can use the equation of motion of F_ϕ evaluated at the vacuum, which can be derived as the lowest component of the superfield equations (3.40). We then obtain:

$$m_{3/2} = |W|\,e^{K/2} = \left|\,e^{-K/6}\,F_\phi - \frac{1}{3}\,K_I F^I\,\right| \ll M\,. \tag{3.47}$$

The second statement can be verified by rewriting the scalar potential (1.95) in a more suitable way:

$$V_{\rm S} = g_{i\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} \,-\, 3\, m_{3/2}^2 \ll M^4\,. \tag{3.48}$$

The effect induced by $F_\phi \neq 0$ is to produce a splitting Δm among the masses of fields belonging to the same multiplet; we understand then that the reason for requiring F_ϕ to be small is twofold. In a flat background it can represent a genuine supersymmetry breaking effect and then it should be small in order to not induce too large mass splittings. On the other hand, when SUSY is unbroken, it represents the expected mass splitting in an AdS background, which is proportional to the inverse of the curvature radius, and we must require

$$\Delta m \propto 1/R_{\rm AdS} \ll M\,, \tag{3.49}$$

in order to justify a two-derivative small-curvature approximation for the graviton. We will discuss in more detail later on these aspects when we will study the role of F_ϕ on mass splittings.

It should be finally emphasized that this further condition $F_\phi \ll M$ (or equivalently $m_{3/2} \ll M$) can in general not be achieved in a natural way, but must instead be implemented through an adjustment of parameters in the Lagrangian. Notice however that for phenomenological applications it is anyhow necessary to eventually tune this cosmological constant to a yet smaller value in the low-energy effective theory. This step does therefore not represent a really severe restriction. Nevertheless, it is not possible to define a locally supersymmetric two-derivative low-energy effective theory below M without making sure that this condition is satisfied.

Component approach

One can derive the same results using component fields in the ordinary formulation of supergravity. We will work in the Einstein frame defined by the gauge fixing (1.89) of the Lagrangian (1.82); in this case we find it useful to reabsorb the factorized phase associated to W through a field redefinition. In addition, since we are not interested in keeping track of the fermionic couplings, we will set the spinorial component of the compensator multiplet to zero. We finally have:

$$\Phi = e^{K/6} \cdot \{1, 0, U\}\,. \tag{3.50}$$

This gauge choice has the advantage of getting rid of the mentioned phase but has the disadvantage that the component Lagrangian is not manifestly invariant under Kähler transformations. As explained in [25] Kähler invariance is restored if Kähler transformations are accompanied by a redefinition of the gravitino and the chiral fermions. We finally remark that, as before, we will discard the dependence on all the gravitational fields except U.

With these assumptions, the Lagrangian has the usual form $\mathcal{L} = T - V$, with a kinetic term that is the same as in global supersymmetry,[3]

$$T = -K_{I\bar{J}}\,\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}}\,, \tag{3.51}$$

and a potential taking the following form:

$$\begin{aligned} V = &- W_I F^I e^{K/2} - \bar{W}_{\bar{J}}\bar{F}^{\bar{J}}e^{K/2} - 3\,WUe^{K/2} - 3\,\bar{W}\bar{U}e^{K/2} \\ &- \Big(K_{I\bar{J}} - \frac{1}{3}K_I K_{\bar{J}}\Big)F^I\bar{F}^{\bar{J}} - K_I F^I\bar{U} - K_{\bar{J}}\,\bar{F}^{\bar{J}}U + 3\,U\bar{U}\,. \end{aligned} \tag{3.52}$$

The auxiliary fields F^I and U are determined by their algebraic equations of motion, which give:

$$F^I = -K^{I\bar{J}}\big(\bar{W}_{\bar{J}} + K_{\bar{J}}\bar{W}\big)e^{K/2}\,, \tag{3.53}$$

$$U = \Big(1 - \frac{1}{3}K_I K^{I\bar{J}}K_{\bar{J}}\Big)\bar{W}e^{K/2} - \frac{1}{3}K_I K^{I\bar{J}}\bar{W}_{\bar{J}}e^{K/2}\,. \tag{3.54}$$

From these equations it follows that:

$$We^{K/2} = \bar{U} - \frac{1}{3}K_{\bar{J}}\bar{F}^{\bar{J}}\,, \tag{3.55}$$

$$W_I e^{K/2} = -\Big(K_{I\bar{J}} - \frac{1}{3}K_I K_{\bar{J}}\Big)\bar{F}^{\bar{J}} - K_I\bar{U}\,. \tag{3.56}$$

We can now derive the equations of motion of F^α and ϕ^α. These correspond to the θ^0 and θ^2 components of the exact equations of motion after performing the superconformal gauge fixing on the compensator. One finds:

$$W_\alpha e^{K/2} + \Big(K_{\alpha\bar{J}} - \frac{1}{3}K_\alpha K_{\bar{J}}\Big)\bar{F}^{\bar{J}} + K_\alpha\bar{U} = 0\,, \tag{3.57}$$

$$\begin{aligned} &W_{\alpha I}F^I e^{K/2} + \Big(K_{\alpha I\bar{J}} - \frac{1}{3}\big(K_I K_{\alpha\bar{J}} + K_{\bar{J}}K_{\alpha I}\big) - K_\alpha K_{I\bar{J}} + \frac{1}{3}K_\alpha K_I K_{\bar{J}}\Big)F^I\bar{F}^{\bar{J}} \\ &\quad + \Big(K_{\alpha I} - K_\alpha K_I\Big)F^I\bar{U} - 2K_{\alpha\bar{J}}\bar{F}^{\bar{J}}U + K_{\alpha\bar{J}}\Box\bar{\phi}^{\bar{J}} + K_{\alpha\bar{J}\bar{K}}\partial_\mu\bar{\phi}^{\bar{J}}\partial^\mu\bar{\phi}^{\bar{K}} = 0\,. \end{aligned} \tag{3.58}$$

In order to arrive at this last equation, we have used the relations (3.56) and (3.55) that follow from eqs. (3.53) and (3.54).

To define an approximate low-energy effective theory, we can now neglect in each of these equations those terms which are subleading in the counting of the total number n of derivatives and auxiliary fields. In this way we get:

$$W_\alpha = 0\,, \tag{3.59}$$

$$W_{\alpha I}F^I = 0\,. \tag{3.60}$$

[3]Note that the Kähler covariant derivative emerges only after taking into account the couplings to the vector auxiliary field that remains in the gravitational multiplet after superconformal gauge fixing.

We recognize now that these equations correspond indeed to the θ^0 and θ^2 components of the superfield equation (3.35) obtained in the superfield approach.

To check the effect of the compensator auxiliary field $F_\phi = e^{K/6}U$, one may re-do the same analysis without considering it as an auxiliary field but rather as an ordinary scalar field. This can be easily done by first eliminating the field U from the two equations for ϕ^α and F^α by using its equation of motion $U = \bar{W}e^{K/2} + 1/3K_IF^I$. Using also eq. (3.53), and dropping then in (3.57) and (3.58) only terms that are subleading in the total number of derivatives and matter auxiliary fields, one would find the following two equations:

$$\big(W_\alpha + K_\alpha W\big)e^{K/2} = 0\,, \tag{3.61}$$

$$\big(W_{\alpha I} + K_{\alpha I}W + K_\alpha W_I\big)F^I e^{K/2} - 2\,K_{\alpha\bar{J}}\bar{W}\bar{F}^{\bar{J}}e^{K/2} = 0\,. \tag{3.62}$$

These should correspond to the θ^0 and θ^2 components of eq. (3.42). As a matter of fact, this is indeed the case if one discards the last term in eq. (3.62). This is related to the fact that (3.42) is a vector superfield equation which is only approximately chiral and has, as already argued, also some $\bar{\theta}^2$ and $\theta^2\bar{\theta}^2$ components that must somehow be automatically satisfied within our approximations. Its $\bar{\theta}^2$ component, in particular, implies that the quantity $K_{\alpha\bar{J}}\bar{W}\bar{F}^{\bar{J}}$ should be discarded. Under this assumption, the above equations correspond then indeed to the chiral components of eq. (3.42). As already argued, this equation cannot be taken as an exact superfield equation, and this shows up here through the fact that the above set of component equations is not preserved by supersymmetry transformations.

Let us finally study a bit more in detail, as promised, the role of the compensator auxiliary field in the mass splitting between scalars and pseudo-scalars. In the limit in which all the F^I and U are small, the only term that survives at second order in the expansion of the scalar potential around the vacuum is the supersymmetric mass $e^K\, W_{IP}\bar{W}_{\bar{J}\bar{N}}K^{P\bar{N}}\,\phi^I\bar{\phi}^{\bar{J}}$. On the other hand, if we only discard F^I and we assume that U cannot be neglected we obtain:

$$\mathcal{L}_{\rm mass} = -\big(N_{IP}K^{P\bar{Q}}\bar{N}_{\bar{Q}\bar{J}} - 2K_{I\bar{J}}|U|^2\big)\phi^I\bar{\phi}^{\bar{J}} + \frac{1}{2}N_{IJ}U\phi^I\phi^J + \frac{1}{2}\bar{N}_{\bar{I}\bar{J}}\bar{U}\bar{\phi}^{\bar{I}}\bar{\phi}^{\bar{J}}\,, \tag{3.63}$$

where

$$N_{IJ} = e^{K/2}W_{IJ} + (K_{IJ} - K_IK_J)\bar{U}\,. \tag{3.64}$$

The physical masses are then no-longer degenerate in pairs, but display now a splitting between scalars and pseudo-scalars, of the order of the off diagonal elements $N_{IJ}U$ in eq. (3.63). As anticipated, if supersymmetry is unbroken and the background geometry is AdS, the mass splittings coincide with those required by the supersymmetry algebra in AdS space. U represents then a curvature scale and more precisely the inverse of the radius $R_{\rm AdS}$ of AdS. In this case one must require that the Compton wave length $1/M$ of the heavy fields should be much smaller that this curvature length L, in order to be able to integrate out these states in the small curvature approximation. This implies in particular $U \ll M$. If on the other hand supersymmetry is broken and the background geometry is Minkowski, the mass splittings represent a soft supersymmetry breaking effect. U corresponds then to an effective supersymmetry breaking scale. In this case one must require that the square mass M^2 of the heavy fields should be much larger than the mass splittings of order MU and U^2 arising in eq. (3.63). This implies again $U \ll M$. Notice finally that if the condition $U \ll M$ is not satisfied, it is impossible for any light chiral multiplets to heave both its scalar and pseudo scalar components with a mass much smaller than M, [4] and the gravitino is also not light.

The content of the superfield equation (3.35) is the same as the one displayed in eqs. (3.28)–(3.30) for the rigid case. The first equation states again that the ϕ^α must adjust to values compatible with the assumption that all the ψ^α and F^α vanish in first approximation, whereas the second and the third equations tell that ψ^α and F^α must actually have small but non-vanishing values. As before, these suppressed components are important and cannot be neglected. The final result is then a supersymmetric effective theory that is accurate at leading order in ∂^μ/M, $\psi^I/M^{3/2}$, F^I/M^2 and U/M, but again a priori not limited to small ϕ^i/M.

We have checked in a number of examples that the supersymmetric effective theory defined by the superfield equation $W_\alpha = 0$ and the standard effective theory defined by the ordinary equations $V_\alpha = 0$ do indeed approximately coincide under the assumptions mentioned above. For the scalar potential, in particular, the region in the space of scalar fields ϕ^i where the two theories match is now defined by three conditions:[5]

$$m(\phi^i), \mu(\phi^i) \ll M\,, \quad F^i(\phi^l), F^\alpha(\phi^i) \ll M^2\,, \quad m_{3/2}(\phi^i) \ll M\,. \tag{3.65}$$

In this case, it is not possible to perform analytic checks. The reason for this is that the validity of the approximation requires not only the $F^i(\phi^i)$ to be small, but also $U(\phi^i)$ (corresponding to $m_{3/2}(\phi^i)$) to be negligible as compared to the mass scale M. One has then one more condition than scalar fields, and this makes it impossible to re-express the deviation between the two effective potentials as a function of $F^i(\phi^i)$ and $U(\phi^i)$ instead of ϕ^i. This reflects the fact that, as already mentioned, there generically exists a domain in field space where all the $F^i(\phi^i)$ are small, but in order to have in addition that also $U(\phi^i)$ is small in a non-empty portion of this domain, one needs in general to adjust some coefficients in the theory. Nevertheless, we performed a numerical point-by-point check for several non-trivial examples and verified that indeed our general conclusions hold true.

[4] A point similar to this last observation was already made in [98].

[5] The first two conditions are as before required to make sure that there is indeed a hierarchy between the light and heavy eigenvalues of the full supersymmetric mass matrix. The last additional condition is, as already explained, equivalent to the condition $U(\phi^i) \ll M$.

3.4 Integrating Out Heavy Vector Multiplets in Global SUSY

Let us consider again the case of global supersymmetry, but including both chiral multiplets Φ^I and vector multiplets V^A, which we split into light ones V^a and heavy ones V^x (we will use the latin indices $a, b, c, ...$ for light vectors and $x, y, w, z, ...$ for heavy ones). For simplicity we restrict to Abelian gauge fields, but the generalization to the non-Abelian case is straightforward. As we discussed in Section 1.3.2, the most general Lagrangian with the leading number of space-time derivatives is given by (1.35), which we rewrite here in a slightly different form for convenience:

$$\mathcal{L} = \int d^4\theta \left[K(\Phi, \bar{\Phi}, V)\right] + \int d^2\theta \left[W(\Phi) + \frac{1}{64} H_{AB}(\Phi)\, \bar{D}^2 D^\alpha V^A \bar{D}^2 D_\alpha V^B\right] + \text{h.c.}\,. \tag{3.66}$$

The new important aspect to take into account when vector multiplets are introduced is the fact that the counting of the total number of derivatives, fermion bilinears and auxiliary fields gets modified; this is essentially related to the fact that vector multiplets have mass-dimension 0 rather than 1 and this implies that in the superfield Lagrangian there may appear operators with a higher number of supercovariant derivatives. We can naively generalize the weight n to include vector multiplets by assigning to A_μ^A, λ^A and D^A respectively $n = 0, 1/2, 1$; to the additional components C^A, χ^A and N^A arising in non-Wess-Zumino gauges must then be assigned $n = -1, -1/2$ and 0 to preserve supersymmetry. This counting guarantees that the minimal Lagrangian has still $n \leqslant 2$ but it has two major disadvantage: first of all, it does not preserve gauge invariance (as one can easily verify by looking for example at the gauge transformation of A_μ^A or the covariant derivatives); second, it is not true anymore that to each superfield expression with a fixed weight p correspond terms with $n = p$. Indeed, we can see that the $d^4\theta$ integral which has $p = 2$ produces also terms with n equal to 0 (e.g. from $K_{AB}|_{\theta=0}(V^A V^B)|_{\theta^2\bar{\theta}^2}$) and 1 (e.g. from $K_A|_{\theta=0} V^A|_{\theta^2\bar{\theta}^2}$). More remarkably, with this definition the kinetic term for gauge fields has $p = 4$ but the associated terms developed in components have $n = 2$. This raises then the question of whether one should in this case keep subleading terms with a higher number of covariant derivatives acting on chiral and vector superfields. We will see that it turns out that this is again not necessary, but for a slightly less trivial reason than the previous case.

Let us start our analysis by first studying the integration of heavy chiral multiplets in the presence of light vector multiplets, both in global and local supersymmetry. We will finally concentrate on the integration of heavy vector multiplets .

Heavy Chiral Multiplets in Gauged Models

Before to start studying the integration of heavy matter fields in supersymmetric gauge theories, let us briefly discuss how this can be done in non-supersymmetric models; this analysis will be useful as a guideline for the supersymmetric generalization. Consider for simplicity the following Abelian gauge theory with light complex scalar fields ϕ^i, a light gauge vector A_μ and heavy scalar fields indicated by ϕ^α. The Lagrangian is given by:

$$\mathcal{L} = D_\mu \phi^I \, D^{*\mu} \bar{\phi}^{\bar{I}} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(\phi, \bar{\phi}) \,, \quad \text{with} \quad D_\mu + i A_\mu \,. \tag{3.67}$$

The equations of motion of the heavy fields are given by:

$$D^2 \phi^\alpha + \frac{\partial V}{\partial \bar{\phi}^{\bar{\alpha}}} = 0 \,. \tag{3.68}$$

We want to integrate the heavy fields at tree level by solving their approximate equations of motions.

The new aspect that one has to consider at this point is the fact that a naive truncation on the number of ordinary space-time derivatives spoils gauge invariance in the low energy effective theory. What one can do in this case to preserve gauge invariance is to perform a truncation on the number of covariant derivatives instead of ordinary derivatives; one may then ask under which conditions the additional terms involving gauge vectors can be safely discarded. It is easy to verify that such terms are controlled by the ratio between the mass of the gauge vector and the heavy mass scale of the scalar fields, which is assumed to be small in the region of validity of the low-energy effective theory. In this situation one is then allowed to neglect subleading corrections depending on covariant derivatives in the solution of (3.68) and heavy scalar fields can be integrated out exactly as in the non-gauged case by solving the algebraic equation $V_\alpha = 0$.

The same analysis can now be generalized to supersymmetric models with heavy chiral multiplets in the presence of light vectors multiplets. Working at superfield level, the equations of motion for heavy chiral multiplets are:

$$W_\alpha - \frac{1}{4} \bar{D}^2 K_\alpha(\Phi, \bar{\Phi}, V) + \frac{1}{64} \, H_{ab\alpha}(\Phi) \, \bar{D}^2 D^\beta V^a \bar{D}^2 D_\beta V^b = 0 \,. \tag{3.69}$$

As we have already discussed in this chapter, in order to preserve supersymmetry in the low-energy effective theory, we need to solve these equations perturbatively in the supercovariant derivatives. At leading order one obtains $\Phi^\alpha = \Phi_0^\alpha + \mathcal{O}(\bar{D}^2 \bar{\Phi}^i / M, \bar{D}^2 V / M)$, where Φ_0^α is the solution of $W_\alpha = 0$. It is easy to verify that the subleading terms involving supercovariant derivatives acting on chiral superfields produce only terms with $n \geqslant 3$ when substituted into the superfield Lagrangian (3.66); we can then reasonably discard these terms in first approximation. On the contrary, as anticipated in the final part of the previous subsection, subleading terms involving supercovariant derivatives of vector superfields are more subtle since they may generate terms in the effective Lagrangian with $n \leqslant 2$ and should in principle be kept. On the other hand, discarding only supercovariant derivatives of chiral superfields while keeping the ones of vector superfields spoils the gauge invariance of the low-energy effective theory. Indeed, at the leading order and keeping only supercovariant derivatives of vector superfields (3.69) becomes:

$$W_\alpha - \frac{1}{4} (K_{\alpha ab} \bar{D} V^a \bar{D} V^b + K_{\alpha a} \bar{D}^2 V^a) + \mathcal{O}(\bar{D}^2 \bar{\Phi}^i / M, \bar{D}^4 V / M^2) = 0 \,. \tag{3.70}$$

The perturbative solution is:

$$\Phi^\alpha = \Phi_0^\alpha + \Phi_{1\,a}^\alpha \bar{D}^2 V^a + \Phi_{1\,ab}^\alpha \bar{D} V^a \bar{D} V^b + \mathcal{O}(\bar{D}^2\bar{\Phi}^i/M, \bar{D}^4 V/M^2)\,, \tag{3.71}$$

where the coefficients of the sub-leading terms are found to be:

$$\Phi_{1\,a}^\alpha(\Phi^i) = \frac{1}{4} M^{-1\alpha\beta} K_{\beta a}\,, \tag{3.72}$$

$$\Phi_{1\,ab}^\alpha(\Phi^i) = \frac{1}{4} M^{-1\alpha\beta} K_{\beta ab}\,. \tag{3.73}$$

One can verify that the supercovariant derivative part in expression (3.71) spoils gauge covariance of the solution and when substituted back into the original Lagrangian it breaks explicitly gauge invariance of the effective theory. However, exact gauge invariance is restored if one neglects also supercovariant derivatives of vector superfields; from expressions (3.72) and (3.73) we discover that these terms can be consistently discarded if one assumes that:[6]

$$|X_a^I| \ll M\,. \tag{3.74}$$

This corresponds to require, as in the non-supersymmetric case, that the gauge vectors must have small masses in order for gauge invariance to be preserved in the low-energy effective theory. When this extra condition is satisfied, the exactly supersymmetric gauge invariant effective theory can be constructed as in the pure chiral case by solving the algebraic equation $W_\alpha = 0$ obtained from eq. (3.71) by neglecting also the leading corrections in supercovariant derivatives of the vector superfields.

In the more general case in which the theory contains also heavy vector superfields, one should first integrate them out and then integrate out heavy chiral fields following the prescription that we have just described. Let us then pass to study how to properly integrate out heavy vector multiplets.

Heavy Vector Multiplets in Global Supersymmetry

The integration of heavy vector superfields does not introduce new complications. Let us start as usual by studying the case of non-supersymmetric Abelian gauge theories and then generalize it to the supersymmetric case. Let us consider the same Lagrangian (3.67), but this time assuming that all the scalar fields are light and all the vector fields are assumed heavy. Since each gauge field is massive, it propagates one extra degree of freedom and one real scalar becomes unphysical. One usually gets rid of this unphysical mode by fixing the unitary gauge. We can however work without fixing the gauge freedom and check that the unphysical mode is automatically decoupled in the low-energy effective theory.

The equation of motion of A_μ is:

$$\partial^\nu F_{\nu\mu} + 2|\phi^i|^2 A_\mu + i\,(\,\phi^i\partial_\mu\bar{\phi}^{\bar{\imath}} - \bar{\phi}^{\bar{\imath}}\partial_\mu\phi^i\,) = 0\,. \tag{3.75}$$

[6] Use the fact that $K_{a\alpha} = 2i\, g_{I\bar{J}}\, \bar{X}_a^{\bar{J}}$ and $K_{ab\alpha} = 4\, g_{I\bar{J}} \nabla_\alpha X_{(a}^I \bar{X}_{b)}^{\bar{J}}$.

At leading order in space-time derivatives, we can neglect the term involving the field strength; the subleading corrections contain 3 derivatives and can be discarded. We then see that there are no subtle contributions to be discussed in this case and the solution of the approximate equation of motion is given by:

$$A_\mu = -\frac{i(\,\phi^i\partial_\mu\bar{\phi}^{\bar{\imath}} - \bar{\phi}^{\bar{\imath}}\partial_\mu\phi^i\,)}{2|\phi^i|^2}\,. \tag{3.76}$$

The only non-trivial feature to discuss is the fact that when we substitute this solution back we obtain a non-invertible effective wave function, which admits a zero mode:

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\,\left(\partial_\mu\bar{\phi}^{\bar{\imath}}\,,\partial_\mu\phi^i\,\right)\begin{pmatrix}P_{\perp\bar{\imath}j} & P_{\|\bar{\imath}\bar{\jmath}}\\ P_{\|ij} & P_{\perp i\bar{\jmath}}\end{pmatrix}\left(\begin{array}{c}\partial^\mu\phi^j\\ \partial^\mu\bar{\phi}^{\bar{\jmath}}\end{array}\right), \tag{3.77}$$

where

$$P_{\perp\bar{\imath}j} = \delta_{\bar{\imath}j} - \frac{1}{2}\,\frac{\bar{\phi}_{\bar{\imath}}\,\phi_j}{|\phi|^2}\,, \qquad P_{\|ij} - \frac{1}{2}\,\frac{\phi_i\phi_j}{|\phi|^2}\,. \tag{3.78}$$

The direction associated to the vanishing eigenvalue of the kinetic mode is $(\phi^i, \bar{\phi}^{\bar{\imath}})$ and it corresponds to the unphysical would-be Goldstone, which is automatically projected out.

The same analysis can now be generalized to supersymmetric models. For convenience in this case we prefer however to fix the gauge symmetries associated to heavy vector multiplets, even if as discussed this is not really necessary. The most convenient type of gauge fixing is the one in which some charged chiral superfield is fixed to some reference scale (the Fayet gauge discussed in Chapter 1). In such a gauge, the corresponding vector multiplet becomes a general real vector multiplet, with all its components being physical. This way of proceeding allows to integrate out the heavy vector superfields at the superfield level. The exact superfield equations of motion for the heavy vector superfields V^x are obtained by first rewriting the last two terms of the Lagrangian (3.66) as D-terms by dropping two supercovariant derivatives, and then varying $\mathcal{L}$ with respect to V^x. This gives:

$$K_x + \frac{1}{8}D^\alpha\Big(H_{xA}\bar{D}^2 D_\alpha V^A\Big) + \frac{1}{8}\bar{D}_{\dot{\alpha}}\Big(\bar{H}_{xA}D^2\bar{D}^{\dot{\alpha}}V^A\Big) = 0\,. \tag{3.79}$$

The presence of large supersymmetric mass for V^x means in this case that around the value V_0^x at which it is stabilized, the Kähler potential K has a large second derivative $K_{xy}(V_0^x)$ proportional to M^2. The first term in eq. (3.79) then dominates over the others, and V^x is approximately determined by the simple equation $K_x(V_0^x) = 0$. The departure from this approximate solution is in this case found to be $\Delta V_0^x \sim \mathcal{O}(D^4V^a/M^2,\, D^4\Phi^m/M^2)$, where Φ^m denotes all the chiral multiplets that have not been frozen by gauge-fixing conditions. In our approximation, this correction can be neglected, since it would contribute only terms with $n \geq 3$. For the terms coming from H, which already lead to terms with $n = 2$ without extra supercovariant derivatives, this is obvious. On the other hand, for the terms coming from K, which can now lead to terms with $n = 1$ due to the vector superfields, this is due to the fact that the leading correction is proportional to K_x, which vanishes on the approximate solution. Summarizing, one can thus integrate out the superfields V^x by using the simple vector superfield equation

$$K_x = 0\,. \tag{3.80}$$

This equation determines the heavy vector superfields as real functions of the light vector superfields and the chiral superfields plus their conjugates:

$$V^x = V_0^x(V^a, \Phi^m, \bar{\Phi}^{\bar{m}})\,. \tag{3.81}$$

The effective theory for the V^a and Φ^m is then obtained by plugging back this solution into the original Lagrangian. In the particular case in which there are no light vector multiplets one needs to consider only K and W, and one finds:

$$K^{\text{eff}}(\Phi^i, \bar{\Phi}^{\bar{\imath}}) = K(\Phi^i, \bar{\Phi}^{\bar{\imath}}, V_0^x(\Phi^i, \bar{\Phi}^{\bar{\imath}}))\,, \tag{3.82}$$

$$W^{\text{eff}}(\Phi^i) = W(\Phi^i)\,. \tag{3.83}$$

In the case where there are also light vector multiplets, one can also get new effects from the gauge kinetic terms. In particular, the effective gauge kinetic function is easily found to be:

$$H_{ab}^{\text{eff}} = H_{ab} - H_{ax} K^{-1xy} K_{yb} - H_{bx} K^{-1xy} K_{xb} + H_{xy} K^{-1xz} K^{-1xw} K_{za} K_{wb}\,. \tag{3.84}$$

As in the case of chiral multiplets, the components of the superfield equation (3.80) have a simple interpretation. To spell it out, let us first notice that the supersymmetric mass matrix for the heavy vector superfields, the light ones and their mixing are given by:

$$M_{xy}^2 = 2\,K_{xy}\,,\quad m_{ab}^2 = 2\,K_{ab}\,,\quad \mu_{xa}^2 = 2\,K_{xa}\,. \tag{3.85}$$

The other object that enters is the coupling between one heavy vector multiplet and two chiral multiplets:

$$Q_{xi\bar{\jmath}} = -\frac{1}{2}\,K_{xi\bar{\jmath}}\,. \tag{3.86}$$

Notice next that eq. (3.80) makes sense in any gauge. In order to interpret its components in physical terms, the most convenient choice is a supersymmetric gauge. In this way, one finds that the real scalar component C^x must adjust its value in a way compatible with the approximate vanishing of D^x, whereas the other components are related to corresponding components of the light superfields through coefficients suppressed by inverse powers of the heavy mass. One may also go to the Wess-Zumino gauge to simplify the component expansion. The first few components of (3.80) imply then restrictions on the charged chiral multiplets fields. In particular, their scalar fields must adjust in such a way that the tadpole for D^x cancels, whereas their auxiliary fields are subject to a linear relation corresponding to the gauge invariance of the superpotential. The higher components of (3.80) imply on the other hand that the non-trivial components of the vector superfield can be re-expressed in terms of components of the charged chiral-multiplets. In particular, one finds that

$$D^x = -(M_0^{-2}\mu_0^2)^x{}_a D^a + (M_0^{-2}Q_0)^x_{i\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}}\,. \tag{3.87}$$

This equation coincides with the exact equation of motion of the complex partner of the would-be Goldstone boson eaten by the gauge boson (see equation (1.53)). Notice that the first term has $n = 1$ and is always relevant whereas the second gives $n = 2$ and can thus give a relevant contribution only in K.

3.5 Integrating Out Heavy Vector Multiplets in Supergravity

Let us finally consider the case of local supersymmetry, but including both chiral multiplets Φ^I and vector multiplets V^A that are split into light ones V^a and heavy ones V^x. Using the superconformal superspace formalism, the requirement $n \leq 2$ corresponds as before to simply neglect any dependence on supercovariant derivatives, except the ones in the kinetic terms for the gauge fields. The theory can then again be parametrized in terms of a real Kähler potential $K = K(\Phi^I, \bar{\Phi}^{\bar{I}}, V^A)$, a holomorphic superpotential $W = W(\Phi^I)$ and a holomorphic gauge kinetic function $H_{AB}(\Phi^I)$ [49, 105, 60]. The Lagrangian takes in this case the form

$$\begin{aligned}\mathcal{L} = &\int d^4\theta \left(-3\, e^{-K/3}\right)\bar{\Phi}\Phi + \int d^2\theta\, W\Phi^3 + \int d^2\bar{\theta}\, \bar{W}\bar{\Phi}^3 \\ &+ \frac{1}{64}\int d^2\theta\, H_{AB}\, \bar{D}^2 D^\alpha V^A \bar{D}^2 D_\alpha V^B + \frac{1}{64}\int d^2\bar{\theta}\, \bar{H}_{AB}\, D^2 \bar{D}_{\dot{\alpha}} V^A D^2 \bar{D}^{\dot{\alpha}} V^B\,.\end{aligned} \tag{3.88}$$

As in the rigid case, we shall fix the local gauge symmetry associated to each heavy vector superfield, and the most convenient way to do this is to set a charged chiral superfield to some reference value. The exact superfield equations of motion for the heavy vector superfields are then obtained as before, and read:

$$K_x + \frac{1}{8} e^{K/3}(\bar{\Phi}\Phi)^{-1}\Big[D^\alpha\Big(H_{xA}\bar{D}^2 D_\alpha V^A\Big) + \bar{D}_{\dot{\alpha}}\Big(\bar{H}_{xA} D^2 \bar{D}^{\dot{\alpha}} V^A\Big)\Big] = 0\,. \tag{3.89}$$

The presence of a large supersymmetric mass implies again that around the values V_0^x at which the heavy superfields V^x are stabilized, the Kähler potential K has a large second derivative $K_{xy}(V_0^x)$ proportional to M^2. The first term in eq. (3.89) dominates then over the others, and V^x is approximately determined by the equation $K_x(V_0^x) = 0$. As before, the departure from this approximate solution is found to be $\Delta V_0^x \sim \mathcal{O}(D^4 V^a/M^2,\ D^4\Phi^m/M^2)$, and can be neglected. Summarizing, one can thus integrate out the superfields V^x by using the same simple vector superfield equation as in the rigid case, namely

$$K_x = 0\,. \tag{3.90}$$

Note that this equation is automatically and trivially invariant under Kähler transformations, since these are not allowed to depend on the vector superfields.

The components of this superfield equation admit exactly the same interpretation as in the global case. This makes sense as long as the auxiliary field of the compensator is small, implying $m_{3/2} \ll M$. In particular, eq. (3.87) still holds true, but comparing it with the exact equation of motion of the complex partner of the would-be-Goldstone mode eq. (1.123) (see for instance [61, 62, 64, 63] and also [94, 95]), one finds that it agrees with it only in the limit where $m_{3/2} \ll M$ and also $F^i \ll M$, which are indeed satisfied in our approximation.

3.6 Summary

In this chapter, we have addressed the general question of understanding under which conditions it is possible to define a two-derivative supersymmetric low-energy effective theory by integrating out a heavy superfield with mass M, and we defined a procedure to explicitly construct it. We studied the cases of chiral and vector multiplets, both in global and in local supersymmetry. Concerning the conditions for the existence of such a theory, we have argued that one has to require that all the derivatives, fermion fields and auxiliary fields should be small in units of M. In the global case, this means $\partial^\mu \ll M$ on all the fields, $\psi^I, \lambda^A \ll M^{3/2}$ for the chiralini and gaugini, and $F^I, D^A \ll M^2$ for the chiral and vector auxiliary fields. In the local case, one has in addition to impose $\partial^\mu \ll M$ on all the gravitational fields, $\psi^\mu_\alpha \ll M^{3/2}$ for the gravitino and $U \ll M$ for the gravitational scalar auxiliary field. This implies that M should correspond to a supersymmetric mass, that comes from W for chiral multiplets and from K for the vector fields. We have then shown that under the above conditions the superfield equations allowing to integrate out heavy chiral and vector superfields Φ^α and V^x in terms of light chiral and vector superfields Φ^i and V^a are respectively the stationarity of the superpotential and the Kähler potential W and K:

$$\begin{aligned} &\partial_\alpha W(\Phi^i, \Phi^\alpha) = 0 & & & &\Phi^\alpha = \Phi^\alpha_0(\Phi^i)\,, \\ & & &\Rightarrow & & \\ &\partial_x K(\Phi^i, \bar\Phi^{\bar\imath}, \Phi^\alpha, \bar\Phi^{\bar\alpha}, V^a, V^x) = 0 & & & &V^x = V^x_0(\Phi^i, \bar\Phi^{\bar\imath}, V^a)\,. \end{aligned} \tag{3.91}$$

The fact that these equations are exactly the same in globally and locally supersymmetric theories is a consequence of the assumption that higher-derivative terms should be negligible also in the gravitational sector. This implies that the gravitino mass should be much smaller than the supersymmetric mass M of the superfield to be integrated out: $m_{3/2} \ll M$. One is then in a situation where the coupling to gravity is minimal and essentially dictated by the space-time symmetries, except for the Einstein term, which can however be canonically normalized in a universal way by going to the Einstein frame from the start. As a result, the operations of integrating in/out heavy fields and switching on/off gravitational interactions commute. Exactly the same thing is true also for a generic non-supersymmetric theory, where the two-derivative effective theory can be deduced by integrating out the heavy fields by imposing stationarity of the potential.

In general, integrating out heavy superfields induces relevant corrections to the dynamics of the light superfields, which cannot be ignored in many interesting situations. In principle, to compute these corrections one simply needs to solve the superfield equations (3.91), which is a simple algebraic problem. In practice this may however be a non-trivial task, for example due to non-linearities or due to the proliferation of fields. It is then of interest to understand in which cases the effect of integrating out heavy superfields is trivial, in the sense that it is equivalent to freezing these to some constant values independent of the light superfields. According to the above superfield equations (3.91), we see that for chiral superfields this is the case when W is separable, $W = W_L(\Phi^i) + W_H(\Phi^\alpha)$, which still allows for non-trivial heavy-light interactions in K. On the other hand, for vector superfields one would need K to be separable (for supersymmetric gauges), $K = K_L(\Phi^m, \bar{\Phi}^{\bar{m}}, V^a) + K_H(V^x)$, which implies that there are no heavy-light interactions at all since vector superfields are not allowed to appear in W. Actually, as far as the effective potential is concerned, this still approximately works even in the case where W or K respectively consist of a dominant term depending only on the heavy superfields and an other one depending also on the light ones but suppressed by some small parameter ϵ, provided that the gravitino mass is at most of the same order ϵ.[7] The reason is that W and K are stationary with respect to the approximate solution for the heavy chiral and vector fields and corrections can thus arise only at second order. This property was already derived in a different way in [101], and further generalized in [106]. Notice however that there may be cases in which W and/or K are not separable but can be made separable after a superfield redefinition. In that case, the integration of heavy superfields will also be trivial, but only in the new superfield basis. Using the original field basis, one would find non-trivial corrections for the light field dynamics, but these clearly simply implement in an automatic way the field redefinition to the clever basis of light fields. On the other hand, in a generic effective theory a heavy field can be integrated out in a exactly trivial way only if the potential V is separable, at least at the point where the heavy fields are stabilized. For supersymmetric theories, to have such an exact trivialization one needs the stronger conditions that both K and W are separable in the rigid case, and that K is separable and W factorizable in the local case [107], again at least at the point where the heavy fields are frozen [108, 99, 100].

As a final remark, let us emphasize that although the above results were derived at the classical level, similar considerations apply also at the quantum level. In particular, it is always true that superfields with large supersymmetric masses can be integrated out at the level of superfields to define a two-derivative supersymmetric low-energy effective action. Due to the non-renormalization theorem for W, these loop corrections

[7]Note that this generically implies that $m_{3/2} \sim m$, which is a stronger condition than the restriction $m_{3/2} \ll M$ that is needed to be able to define a supersymmetric effective theory.

affect only K. See for example [109] and [110] for explicit examples in globally and locally supersymmetric theories.

Chapter 4

Vacuum Stability and Bound on the Lightest Scalar

In this chapter we study some general criteria for the existence of metastable vacua with spontaneously broken global supersymmetry in generic supersymmetric models with local gauge symmetries. In particular we derive an absolute upper bound on the mass of the lightest scalar field which depends essentially on the geometrical properties of the scalar manifold and its gauged isometries. This bound can be saturated by properly tuning the superpotential and its positivity therefore represents a necessary and sufficient condition for the existence of metastable vacua. It is derived by looking at the subspace of all those directions in field space for which an arbitrary supersymmetric mass term is not allowed and scalar masses are controlled by supersymmetry-breaking splitting effects. This subspace includes not only the direction of supersymmetry breaking, but also the directions of gauge symmetry breaking and the lightest scalar is in general a linear combination of fields spanning all these directions. We explicitly present analytic results for the simplest case of globally supersymmetric theories with a single Abelian gauge symmetry. For renormalizable gauge theories, the lightest scalar is a combination of the Goldstino partners and its square mass is always positive. For more general non-linear sigma models, on the other hand, the lightest scalar can involve also the Goldstone partner and its square mass is not always positive. The generalization of this analysis to local supersymmetry does not present new conceptual obstructions even though it appears to be more involved from the technical point of view; we qualitatively discuss this topic at the end of the chapter. This chapter is based on our paper [2].

4.1 General Criteria for Metastability

In Chapter 2 we have seen that there exist several phenomenological arguments constraining the structure of the soft masses and the mechanism by which supersymmetry breaking effects are transmitted to the visible sector. On the contrary, the only simple constraints one can impose on the actual mechanism responsible for supersymmetry breaking in the hidden sector are the metastability of the vacuum and the value of the cosmological constant.[1] Even though these constraints may appear to be weak, they can be used to define some relevant criteria to discriminate among different scenarios for hidden sector physics. The important aspect to take into account is the fact that the structure of the mass matrices in supersymmetric models is strongly constrained and the spontaneous breaking of supersymmetry allows to split the masses of bosons and fermions but not to achieve totally arbitrary mass matrices. In general, these mass matrices consist of a supersymmetric contribution that is common to all the states of a multiplet plus a non-supersymmetric contribution splitting the masses of these states within each multiplet.

The first source of constraints is that the various non-supersymmetric contributions to the masses are correlated among each other. A simple consequence of these correlations is expressed by the celebrated supertrace formula that we discussed in Chapter 1. When computing this quantity, the supersymmetric contributions to masses drop out and the non-supersymmetric contributions combine into a remarkably simple result. This then constrains to some extent the relative masses that can be achieved for bosons and fermions and has, as we have seen, important implications in phenomenological model building.

The second source of constraints, which is the most relevant for our discussion, consists in the fact that some of the supersymmetric contributions to the mass matrices are constrained by symmetry arguments. More precisely, there exist particular directions in field space which are associated to supersymmetric masses that cannot be made arbitrarily large; such directions are dangerous in the sense that supersymmetry breaking effects can make them unstable. One important example is the supersymmetric contribution to the mass of the Goldstino chiral multiplet which must vanish, since the fermion of this multiplet is constrained by Goldstone's theorem to have vanishing mass. As a result, the two scalar partners of this fermion have masses that are entirely controlled by splitting effects. Similarly, the supersymmetric contribution to the mass of the vector multiplets is fixed by the expectation value of the Killing vectors, since the vector boson masses arise through the Higgs mechanism. As a result, the real scalar partner of each massive gauge boson has a mass that differs from the gauge boson mass only by splitting effects, and this can also be viewed as the statement that the would-be Goldstone chiral multiplet has a constrained mass in the supersymmetric limit.

A remarkable consequence of this second class of constraints is that there exists an upper bound on the mass of the lightest scalar, even if the superpotential is freely tuned. More importantly, the direction associated to the lightest scalar is in general not arbitrary and it belongs to the subspace spanned by the Goldstino and the Gold-

[1] Other constraint from cosmological arguments is the existence of viable inflationary trajectories ensuring a "slow-roll" motion of the inflaton field. See for instance [111–114]. In this work however we will not take into account this class of constraints.

stone partners. This suggests that to establish whether a model can admit metastable vacua breaking supersymmetry, it is not really necessary to study the whole scalar mass matrix, since the relevant information concerning metastability is contained in some special sub-blocks associated to the dangerous directions discussed above. In this chapter we want to develop this idea to derive some general necessary and sufficient conditions for the existence of metastable vacua in models in which the Kähler geometry and the gauged isometries are fixed whereas the superpotential can be arbitrarily varied in order to saturate the bound on the mass of the lightest scalar. This analysis can be relevant for string-inspired supersymmetric models obtained by compactification since, as we discussed in Chapter 2, the structure of the Kähler potential is in general determined by the details of the compactification whereas the structure of the superpotential is more difficult to be determined. Since a priori there exist a plethora of different compactification manifolds which are admitted by String Theory, it is very useful to have some general criteria to discriminate among scenarios with different Kähler geometries independently of the form of the superpotential, and to be able to characterize the possible geometries which allow for realistic vacua.

The case of theories with only chiral multiplets and no gauge symmetries is well understood. What matters in this case is the two-dimensional sub-block of the scalar mass matrix restricted to the two Goldstino partners. For renormalizable models, the two eigenvalues of this matrix are equal and opposite, and the best situation that can occur is that both vanish. This implies the presence of two pseudo-moduli fields with vanishing mass, which actually represent flat directions of the classical potential with peculiar properties [115, 116]. For more general non-renormalizable chiral non-linear sigma-models, one similarly finds that the two eigenvalues are split around an average value that is fixed by the Riemann curvature of the Kähler manifold, and in the best situation one has two scalars with identical masses given by this value [108, 117]. Similar results also hold in supergravity theories, and these give a useful guideline towards the ingredients that are needed to achieve metastable de Sitter vacua in string models [118, 119]. More precisely, in the case in which supersymmetry breaking is dominated by chiral multiplets, the requirement of metastability and positivity of the cosmological constant can be operatively translated into constraints on the parameters of the theory and it is possible to implement a well-defined procedure to locally reconstruct a scalar potential which admits metastable de Sitter vacua (see [120]). The idea of using metastability criteria to characterize supersymmetry breaking scenarios in string-inspired models has been proven to be very powerful in many situations; for instance, it has been applied to prove in a simple and sharp way the fact that supersymmetry breaking dynamics cannot be dominate by a single Kähler Modulus or by the dilaton unless subleading corrections to the Kähler potential are considered [121–123, 108]

The case of theories involving also vector multiplets and local gauge symmetries is more complicated and less understood (see for example [124, 125]). As anticipated, one should in principle look at a higher-dimensional sub-block of the scalar mass matrix that includes not only the two Goldstino partners but also the Goldstone partners. In this case the complexity of the analysis increases with the number of generators of the symmetry transformations and studying the relevant sub-blocks of the mass matrix may turn out to be a non-trivial task as much as studying the whole mass matrix. It has been argued in [61] that the presence of D-type in addition to F-type supersymmetry breaking tends to alleviate the metastability condition which can be derived by looking only at the Goldstino direction. But a full analysis including also the Goldstone partners was still missing and, in particular, it is not obvious that the improving effects associated to gauge vectors can always be used to achieve metastability.

The purpose of this chapter is to perform a detailed study of the scalar mass matrix of generic theories with rigid N=1 supersymmetry and local gauge symmetries, and to derive an upper bound on the value of its lightest eigenvalue. The main improvement that we aim to achieve compared to previous analyses is to obtain the strongest possible bound, with the property that it should be possible to saturate it by adjusting only the superpotential. To achieve this goal, we will need not only to consider the effect of the vector multiplets on the two Goldstino partners, but also to include in the analysis the Goldstone partners, and focus our attention on the full dangerous sub-block of the scalar mass matrix for which supersymmetric effects are constrained. The aim of this analysis is to show that in general situations the directions associated to the Goldstone partners play a relevant role in discussing metastability.

4.2 Structure of the Scalar Mass Matrix

Let us consider a generic globally supersymmetric model with n chiral multiplets Φ^i and k vector multiplets V^a defined by the Lagrangian (1.35):

$$\mathcal{L} = \int d^4\theta \Big[K(\Phi,\bar\Phi,V)\Big] + \int d^2\theta \Big[W(\Phi) + \frac{1}{4} H_{ab}(\Phi)\, W^{a\alpha} W^b_\alpha\Big] + \text{h.c.}\,. \tag{4.1}$$

As we did in the previous chapters, we exclude for simplicity the possibility of non-zero variations of the Kähler potential under gauge transformations. In particular, we thus exclude Fayet-Iliopoulos terms. In the following, we shall also restrict for simplicity to the special case where the gauge kinetic function H_{ab} is constant, so that $H_{abi} = 0$. This does not represent a very big conceptual limitation, but it leads to a substantial simplification of the theory. We shall on the other hand retain the possibility of having a generic Kähler potential K and generic Killing vectors X_a^i defining the non-constant matrices

$$Q_a{}^i{}_j = i\,\nabla_j X_a^i\,. \tag{4.2}$$

The particular case of renormalizable gauge theories corresponds to choosing $K = \delta_{ij}\Phi^i\bar\Phi^{\bar\jmath}$, $X_a^i = -i\,T_a{}^i{}_j\Phi^j$ and $Q_a{}^i{}_j = T_a{}^i{}_j$, with constant $T^{ai}{}_j$.

For later convenience, we shall furthermore introduce an arbitrary gauge coupling constant g, although this could be reabsorbed in the normalization of H_{ab}; this amounts to slightly modify some relevant expressions that we encountered in Section 1.3.2 and in particular the relation following from gauge invariance:

$$X_a^i K_i = \frac{i}{2} g^{-1} K_a \,, \tag{4.3}$$

$$K_{ai} = 2ig\bar{X}_{ai} \,, \tag{4.4}$$

$$g_{i\bar{\jmath}} X^i_{[a} \bar{X}^{\bar{\jmath}}_{b]} = \frac{i}{4} g^{-1} f_{ab}{}^c K_c \,. \tag{4.5}$$

The values of the auxiliary fields F^i and D^a are fixed by their equations of motion and read:

$$F^i = -g^{i\bar{\jmath}} \bar{W}_{\bar{\jmath}} \,, \quad D^a = -\frac{1}{2} h^{ab} K_b = igh^{ab} X_b^i K_i = -igh^{ab} \bar{X}_b^{\bar{\imath}} K_{\bar{\imath}} \,. \tag{4.6}$$

The vacuum energy V is still given by

$$V = g_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} + \frac{1}{2} h_{ab} D^a D^b \,. \tag{4.7}$$

The stationarity condition $V_i = 0$ implies that

$$\nabla_i W_j \, F^j + ig\bar{X}_{ai} D^a = 0 \,. \tag{4.8}$$

We recall the fact that contracting the stationarity conditions with the Killing vector and taking the imaginary part one obtains a very useful relation constraining F and D term, which in this case reads:

$$Q_{ai\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} - \frac{1}{2} g^{-1} M^2_{ab} D^b = 0 \,. \tag{4.9}$$

Finally, with these assumptions the masses of scalar fields (1.54) and (1.55) are simplified in the following way:

$$m^2_{i\bar{\jmath}} = g^{k\bar{l}} \nabla_i W_k \nabla_{\bar{\jmath}} \bar{W}_{\bar{l}} - R_{i\bar{\jmath}k\bar{l}} \, F^k \bar{F}^{\bar{l}} + g^2 h^{ab} \bar{X}_{ai} X_{b\bar{\jmath}} + g \, Q_{ai\bar{\jmath}} D^a \,, \tag{4.10}$$

$$m^2_{ij} = -\nabla_i \nabla_j W_k \, F^k - g^2 h^{ab} \bar{X}_{ai} \bar{X}_{bj} \,. \tag{4.11}$$

Let us now study more in detail the structure of the whole mass matrix of scalar fields. Since the two real components of each complex scalar field are allowed to split, one has to consider the space of all the independent real modes. This can be described by $2n$-dimensional vectors ϕ^I built out of the n fields ϕ^i and their complex conjugates $\bar{\phi}^{\bar{\imath}}$:

$$\phi^I = \begin{pmatrix} \phi^i & \bar{\phi}^{\bar{\imath}} \end{pmatrix} , \quad \phi^{\bar{J}} = \begin{pmatrix} \bar{\phi}^{\bar{\jmath}} \\ \phi^j \end{pmatrix} . \tag{4.12}$$

With this parametrization,[2] the quadratic Lagrangian for the scalar fields can be written in the following form:

$$\mathcal{L} = \frac{1}{2} g_{I\bar{J}} \partial_\mu \phi^I \partial^\mu \phi^{\bar{J}} - \frac{1}{2} m^2_{I\bar{J}} \phi^I \bar{\phi}^{\bar{J}} \,, \tag{4.13}$$

[2] Notice that in this chapter capital indices are exceptionally used with another meaning respect to the previous chapter.

with wave-function and square-mass matrices given by

$$g_{I\bar{J}} = \begin{pmatrix} g_{i\bar{\jmath}} & 0 \\ 0 & g_{\bar{\imath}j} \end{pmatrix}, \quad m^2_{I\bar{J}} = \begin{pmatrix} m^2_{i\bar{\jmath}} & m^2_{ij} \\ m^2_{\bar{\imath}\bar{\jmath}} & m^2_{\bar{\imath}j} \end{pmatrix}. \tag{4.14}$$

To obtain the physical masses, one can then proceed as follows. First, one choses a parametrization of the fields such that the wave-function $g_{I\bar{J}}$ locally trivializes to the identity matrix and the kinetic terms are canonically normalized. This corresponds to choose normal coordinates around the vacuum point. Next, one diagonalizes the Hermitian matrix $m^2_{I\bar{J}}$ to find the mass eigenvalues $m^2_{(I)}$. Equivalently, one can consider the matrix $m^2_{I\bar{J}}$ in a new basis defined by a set of vectors $v^I_{(K)}$ that are orthonormal with respect to the metric $g_{I\bar{J}}$. The eigenvalues of the new matrix defined by all the matrix elements of $m^2_{I\bar{J}}$ on the basis of vectors $v^I_{(K)}$ then yield directly the physical masses. This is the approach that we will use.

To make progress in our quest for an interesting bound on the physical mass eigenvalues, and in particular the minimal physical eigenvalue $m^2_{\rm min}$, we will use some standard results in linear algebra. The basic point is that the value of the matrix $m^2_{I\bar{J}}$ along any particular direction must be larger that $m^2_{\rm min}$. A slight generalization of this fact is that the eigenvalues of any sub-block of the matrix $m^2_{I\bar{J}}$, corresponding for example to the subspace spanned by a set of several particular directions, must similarly be all larger than $m^2_{\rm min}$. This means that we can find an upper bound to $m^2_{\rm min}$ by computing the smallest eigenvalue of any principal sub-matrix of $m^2_{I\bar{J}}$. In general, the obtained bound improves in quality by considering larger and larger sub-matrices, and the exact value of $m^2_{\rm min}$ can be obtained only by considering the full matrix. Nevertheless, there is a well-defined limiting situation in which the bound derived by considering a finite diagonal block actually saturates $m^2_{\rm min}$. This happens when the complementary diagonal block has eigenvalues that are very large compared to the elements of the off-diagonal block. For this reason, to detect the obstructions against making $m^2_{\rm min}$ large it is enough to study the mass matrix along those directions where its values cannot be made arbitrarily large by adjusting the superpotential.

Each direction defined by a unit vector v^i in the space of complex scalar fields ϕ^i defines a plane in the space of real scalar fields ϕ^I, which can be described by a basis of two orthonormal unit vectors v^I_+ and v^I_- defined as follows:

$$v^I_+ = \frac{1}{\sqrt{2}}\big(v^i \;\; \bar{v}^{\bar{\imath}}\big)\,, \quad v^I_- = \frac{1}{\sqrt{2}}\big(iv^i \;\, -i\bar{v}^{\bar{\imath}}\big)\,. \tag{4.15}$$

Strictly speaking, the vector space of all real scalar fields is a real vector space, and one is therefore allowed to perform only real orthogonal transformations. However, for the problem of studying the eigenvalues of the mass matrix $m^2_{I\bar{J}}$, which is Hermitian, one may also consider complex unitary transformations, because such more general transformations still preserve these eigenvalues. For a given complex direction v^i, one may then also use as alternative basis the two orthonormal vectors $v_{A,B} = \frac{1}{\sqrt{2}}(v^I_+ \mp i v^I_-)$, which take the form:

$$v_A^I = \big(v^i \;\; 0\big)\,, \;\; v_B^I = \big(0 \;\; \bar{v}^{\bar{\imath}}\big)\,. \tag{4.16}$$

From the discussion of previous section, we know that there are two kinds of special complex directions along which the mass matrix displays particular restrictions. These are the supersymmetry-breaking Goldstino direction F^i and the gauge-symmetry-breaking Goldstone directions X_a^i. In all the other orthogonal directions, one can have arbitrary supersymmetric contributions to the mass. Taking these to be large one can then forget about these extra directions altogether, as already explained. Let us then focus on the subspace defined by the complex directions F^i and X_a^i. We already know that F^i is always orthogonal to all the X_a^i, as a consequence of the gauge invariance of the superpotential. On the other hand, the X_a^i are in general not orthogonal to each other, and the matrix of their scalar products defines in fact the vector mass matrix. We may however perform an orthogonal transformation in the space of vector multiplets, to go to a basis where at the vacuum all the X_a^i are orthogonal to each other and the vector mass matrix is diagonal. The norms of the vectors F^i and X_a^i define respectively the supersymmetry breaking scale $\sqrt{|F|}$ in the chiral multiplet sector and the masses M_a of the vector fields. More precisely, these quantities are defined as follows:

$$|F| = \sqrt{g_{i\bar{\jmath}}F^i\bar{F}^{\bar{\jmath}}}\,, \qquad M_a = \sqrt{2}g\sqrt{g_{i\bar{\jmath}}X_a^i\bar{X}_a^{\bar{\jmath}}}\,. \tag{4.17}$$

One then finds:

$$g_{i\bar{\jmath}}F^i\bar{F}^{\bar{\jmath}} = |F|^2\,, \qquad g_{i\bar{\jmath}}X_a^i\bar{X}_b^{\bar{\jmath}} = \frac{1}{2}g^{-2}M_aM_b\,\delta_{ab}\,, \;\; g_{i\bar{\jmath}}F^i\bar{X}_b^{\bar{\jmath}} = 0\,. \tag{4.18}$$

We may finally define the following normalized vectors:

$$f^i = \frac{F^i}{\sqrt{F^k\bar{F}_k}} = \frac{F^i}{F}\,, \qquad x_a^i = \frac{X_a^i}{\sqrt{X_a^k\bar{X}_{ak}}} = \sqrt{2}g\frac{X_a^i}{M_a}\,. \tag{4.19}$$

These form an orthonormal basis for the subspace of complex directions we want to study, and satisfy:

$$g_{i\bar{\jmath}}\,f^i\bar{f}^{\bar{\jmath}} = 1\,, \;\; g_{i\bar{\jmath}}\,x_a^i\bar{x}_b^{\bar{\jmath}} = \delta_{ab}\,, \;\; g_{i\bar{\jmath}}\,f^i\bar{x}_b^{\bar{\jmath}} = 0\,. \tag{4.20}$$

Following our general discussion on the map between a complex direction in the space of complex scalars and a basis of two independent directions in the space of real scalars, we now introduce the following orthonormal basis of real directions:

$$f_+^I = \frac{1}{\sqrt{2}}\big(f^i \;\; \bar{f}^{\bar{\imath}}\big)\,, \qquad f_-^I = \frac{1}{\sqrt{2}}\big(if^i \; -i\bar{f}^{\bar{\imath}}\big)\,, \tag{4.21}$$

$$x_{a+}^I = \frac{1}{\sqrt{2}}\big(x_a^i \;\; \bar{x}_a^{\bar{\imath}}\big)\,, \qquad x_{a-}^I = \frac{1}{\sqrt{2}}\big(ix_a^i \; -i\bar{x}_a^{\bar{\imath}}\big)\,. \tag{4.22}$$

Alternatively, we may as already explained also use the alternative but less physical basis defined by

$$f_A^I = \big(f^i \;\; 0\big)\,, \;\; f_B^I = \big(0 \;\; \bar{f}^{\bar{\imath}}\big)\,, \tag{4.23}$$

$$x_{aA}^I = \big(x_a^i \;\; 0\big)\,, \;\; x_{aB}^I = \big(0 \;\; \bar{x}_a^{\bar{\imath}}\big)\,. \tag{4.24}$$

The directions f_+^I and f_-^I describe the two real scalar partners of the massless Goldstino fermion. Due to the symmetric roles of these two modes, it will in fact be convenient to use the alternative description in terms of f_A^I and f_B^I. In the limit of unbroken supersymmetry, the modes defined by f_+^I and f_-^I would both belong to the same multiplet as the massless Goldstino fermion and would thus be massless too. As a result, their masses can be non-zero only because of splitting effects. The directions x_{a+}^I and x_{a-}^I describe instead two different kinds of real scalars which are respectively the unphysical would-be Goldstone modes, which correspond to fake null vectors of the mass matrix that we should discard, and their partners, which we should instead consider. Due to the asymmetric roles of these two kinds of modes, it will not be convenient to use the alternative description in terms of x_{aA}^I and x_{aB}^I. In the limit of unbroken supersymmetry, the modes x_{a-}^I would belong to the same multiplet as the massive vector bosons and would thus be massive too. As a result, their mass can differ from that of the gauge fields only by splitting effects. We thus find a total of $2+k$ scalar modes which are dangerous for metastability: the 2 modes associated to $f_\pm^I$ and alternatively described by $f_{A,B}^I$, whose masses are equal to zero plus supersymmetry breaking effects, and the k modes associated to x_{a-}^I, whose masses are equal to the gauge boson masses plus supersymmetry breaking effects.

Let us then look at the mass matrix $m^2_{I\bar{J}}$ in the $(2+k)$-dimensional subspace spanned by the vectors $f_A^I = (f^i \;\; 0)$, $f_B^I = (0 \;\; \bar{f}^{\bar{\imath}})$ and $x_{a-}^I = (ix_a^I \;\; -i\bar{x}_a^{\bar{\imath}})$, which form an orthonormal set. More precisely, we need to compute the matrix elements $m^2_{\alpha\bar{\beta}} = m^2_{I\bar{J}} v_\alpha^I \bar{v}_{\bar{\beta}}^{\bar{J}}$, where v_α^I can be either f_A^I, f_B^I or x_{a-}^I. Exploiting gauge invariance, we can rewrite most of the contributions coming from the non-Hermitian blocks m^2_{ij} and $m^2_{\bar{\imath}\bar{\jmath}}$ in terms of the Hermitian blocks $m^2_{i\bar{\jmath}}$. Indeed, Goldstone's theorem implies that $m^2_{ij} x_a^j = -m^2_{i\bar{\jmath}} \bar{x}_a^{\bar{\jmath}}$ at a stationary point. One then finds that the $(2+m)$-dimensional sub-matrix $m^2_{\alpha\bar{\beta}}$ takes the form

$$m^2_{\alpha\bar{\beta}} = \begin{pmatrix} m^2_{f\bar{f}} & \Delta & -\sqrt{2}i\, m^{2*}_{f\bar{x}_b} \\ \Delta^* & m^2_{f\bar{f}} & \sqrt{2}i\, m^2_{f\bar{x}_b} \\ \sqrt{2}i\, m^2_{f\bar{x}_a} & -\sqrt{2}i\, m^{2*}_{f\bar{x}_a} & 2\, m^2_{x_a\bar{x}_b} \end{pmatrix}\,, \tag{4.25}$$

where

$$m^2_{f\bar{f}} = m^2_{i\bar{\jmath}} f^i \bar{f}^{\bar{\jmath}}\,, \;\; m^2_{f\bar{x}_b} = m^2_{i\bar{\jmath}} f^i \bar{x}_b^{\bar{\jmath}}\,, \;\; m^2_{x_a\bar{x}_b} = m^2_{i\bar{\jmath}} x_a^i \bar{x}_b^{\bar{\jmath}}\,, \tag{4.26}$$

and

$$\Delta = m^2_{ij} f^i f^j\,. \tag{4.27}$$

It is important to emphasize that the above structure is completely general, since it depends only on the gauge invariance of the theory and not on the detailed structure of the masses.

It is a straightforward exercise to compute the entries $m^2_{f\bar{f}}$, $m^2_{f\bar{x}_b}$ and $m^2_{x_a\bar{x}_b}$, which are given by the Hermitian block $m^2_{i\bar{\jmath}}$ of eq. (4.10) along the directions defined by F^i and X^i_a. The resulting expressions can be significantly simplified by making use of the stationarity condition, which holds at the vacuum, as well as the relations implied by gauge invariance, which hold at any point and can therefore also be differentiated. Most importantly, the dependence on the second derivatives of the superpotential can be completely eliminated. Defining the obvious notation $R_{v\bar{w}y\bar{z}} = R_{i\bar{\jmath}k\bar{l}}\, v^i \bar{w}^{\bar{\jmath}} y^k \bar{z}^{\bar{l}}$ and $Q_{av\bar{w}} = Q_{i\bar{\jmath}}\, v^i \bar{w}^{\bar{\jmath}}$ for any complex directions v^i, w^i, y^i and z^i, and recalling that $M^2_{ab} = M_a M_b\, \delta_{ab}$, one finds:

$$m^2_{f\bar{f}} = -\bigg[R_{f\bar{f}f\bar{f}} - 4g^2 \sum\nolimits_c \frac{Q_{cf\bar{f}}\, Q_{cf\bar{f}}}{M_c^2}\bigg] |F|^2\,, \tag{4.28}$$

$$m^2_{x_a\bar{x}_b} = \frac{1}{2} M^2_{ab} - \bigg[R_{f\bar{f}x_a\bar{x}_b} - 2g^2 \sum\nolimits_c \frac{Q_{cf\bar{f}}\, Q_{cx_a\bar{x}_b}}{M_c^2} - 2g^2 \frac{(Q_a \cdot Q_b)_{f\bar{f}}}{M_a M_b}\bigg] |F|^2\,, \tag{4.29}$$

$$m^2_{f\bar{x}_b} = -\bigg[R_{f\bar{f}f\bar{x}_b} - 4g^2 \sum\nolimits_c \frac{Q_{cf\bar{f}}\, Q_{cf\bar{x}_b}}{M_c^2}\bigg] |F|^2. \tag{4.30}$$

The entry Δ has instead a more complicated expression, and it is not possible to simplify it in any relevant way by using the stationarity and the gauge invariance conditions. Most importantly, the dependence on the third derivatives of the superpotential cannot be eliminated, and varying such derivatives allows to vary Δ over the entire complex plane. Therefore:

$$\Delta = \text{generic complex number that can be adjusted by tuning } W_{ijk}\,. \tag{4.31}$$

We may now ask what is the upper bound on the smallest eigenvalue of the above matrix $m^2_{\alpha\bar{\beta}}$ when $m^2_{f\bar{f}}$, $m^2_{f\bar{x}_b}$ and $m^2_{x_a\bar{x}_b}$ are held fixed and Δ is freely varied. As already explained, this would also represent an upper bound on the smallest eigenvalue $m^2_{\min}$ of the full mass matrix $m^2_{I\bar{J}}$. Unfortunately, this question is still quite complicated for generic theories with arbitrary gauge symmetries, where k can be arbitrarily large and it is thus difficult to study the full $(2+k)$-dimensional matrix. The importance of the Goldstone directions with respect to the Goldstino direction depends however crucially on the relative size of the vector masses M_a compared to the chiral supersymmetry breaking scale $\sqrt{|F|}$. When the M_a are much larger than $\sqrt{|F|}$, the situation simplifies substantially and the heavy vector multiplets can in fact be integrated out in a supersymmetric way to define an effective theory for the light chiral multiplets; the way in which this can be done has been described in detail in the previous chapter. In this situation the only dangerous light modes are those associated with f^I_A and f^I_B, and the largest value for the smallest mass is obtained by tuning Δ to zero. The detailed computation of the effects induced by heavy vector multiplets on the mass along the Goldstino direction will be explicitly presented in the next chapter. For the moment we can anticipate the main result: the upper bound $m^2_{\rm min}$ is given by (4.28), up to negligible effects of order $\mathcal{O}(|F|^4/M_a^2)$, and the square bracket in (4.28) can be interpreted as the effective Riemann curvature of the low energy theory along the Goldstino direction [61]. When the M_a are instead comparable-or-smaller than $\sqrt{|F|}$, the modes associated to x^I_{a-} are a priori as light and as dangerous as the modes associated to f^I_A and f^I_B, and the study of the bound become more complicated. It is this situation that we would like to study in some detail.

For the sake of clarity, we shall mostly restrict our study to the simplest case of theories with a single $U(1)$ gauge symmetry and $k=1$. In this case, it is possible to extract analytically the full information and derive a simple necessary and sufficient bound, which can be saturated by adjusting the superpotential. In more complicated theories with several gauge symmetries forming a more general group G, on the other hand, one may get some partial analytic information by studying smaller sub-blocks of dimension one, two and three, and derive simple necessary but not sufficient bounds, which can a priori not be saturated by adjusting the superpotential. In particular, one may look separately at all the possible directions in the generator space and figure out which one leads to the strongest bound. A natural naive guess for a special direction to look at is the direction $D^a X^i_a/|D|$ defined by the vector auxiliary fields D^a. As anticipated in Section 1.3.2, the relevance of this special direction is suggested by the fact that it appears together with F^i in the definition of the Goldstino fermion. When looking at this special direction, some partial and interesting simplifications do indeed occur in the expressions (4.29) and (4.30), but since we were not able to reach a really simple and useful result by pursuing this direction, we will not comment any further on this, and restrict from now on to the basic case involving only one symmetry generator.

4.3 Bound on the Lightest Scalar Mass

Let us now consider the case of theories with a single $U(1)$ gauge symmetry, where the index a takes a single value and can therefore be dropped. The matrix (4.25) is then 3-dimensional, and it turns out that it is possible to study the behavior of its eigenvalues in a fully analytic way. In order to illustrate the fact that the study of larger sub-blocks of the mass matrix leads to sharper bounds on the lightest eigenvalue, we shall however successively study sub-blocks of dimensions one, two and three.

There are three possible principal blocks of dimension one, which correspond to the diagonal elements, but only two of them are independent, namely:

$$m^2_{f\bar f}\,,\quad 2\,m^2_{x\bar x}\,. \tag{4.32}$$

Both of these values represent upper bounds on $m^2_{\rm min}$. Which one is the smallest and thus leads to the strongest bound depends however on the situation. We therefore conclude that a first bound that we can write is:

$$m^2_{\rm min}\le m^2_{(1)}\,,\quad m^2_{(1)}=\min\big\{m^2_{f\bar f},2\,m^2_{x\bar x}\big\}\,. \tag{4.33}$$

There are then three possible principal blocks of dimension two, but again only two of these are independent. The first possibility is the upper 2-dimensional block of (4.25), with two identical diagonal elements given by $m^2_{f\bar{f}}$ and off-diagonal element given by Δ. The two eigenvalues of such a matrix are $m^2_{f\bar{f}} \pm |\Delta|$. The maximal value for the smallest of these is achieved by choosing $\Delta = 0$ and is given by $m^2_{f\bar{f}}$. This sets an upper bound on $m^2_{\rm min}$, but this bound is already contained in the previously derived bound (4.33). The second possibility is the lower 2-dimensional block of (4.25), which is given by

$$\begin{pmatrix} m^2_{f\bar{f}} & \sqrt{2} i\, m^2_{f\bar{x}} \\ -\sqrt{2} i\, m^{2*}_{f\bar{x}} & 2\, m^2_{x\bar{x}} \end{pmatrix} . \tag{4.34}$$

The eigenvalues of this matrix are easily computed and are given by:

$$m^2_\pm = \frac{1}{2}\big(m^2_{f\bar{f}} + 2\, m^2_{x\bar{x}}\big) \pm \frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\, m^2_{x\bar{x}}\big)^2 + 8\, |m^2_{f\bar{x}}|^2} . \tag{4.35}$$

Both of these eigenvalues set upper bounds on $m^2_{\rm min}$. The smallest one that leads to the strongest bound is always the one with the negative sign choice. This leads to a new bound, which is always stronger-or-equal than the previous bound (4.33) and takes into account the non-trivial level-repulsion effect induced by the off-diagonal element $m^2_{f\bar{x}}$:

$$m^2_{\rm min} \le m^2_{(2)} \,, \;\; m^2_{(2)} = \frac{1}{2}\big(m^2_{f\bar{f}} + 2\, m^2_{x\bar{x}}\big) - \frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\, m^2_{x\bar{x}}\big)^2 + 8\, |m^2_{f\bar{x}}|^2} . \tag{4.36}$$

Finally, one may try to look at the full block of dimension three, which should in this case yield the full information. This is given by:

$$\begin{pmatrix} m^2_{f\bar{f}} & \Delta & -\sqrt{2} i\, m^{2*}_{f\bar{x}} \\ \Delta^* & m^2_{f\bar{f}} & \sqrt{2} i\, m^2_{f\bar{x}} \\ \sqrt{2} i\, m^2_{f\bar{x}} & -\sqrt{2} i\, m^{2*}_{f\bar{x}} & 2\, m^2_{x\bar{x}} \end{pmatrix} . \tag{4.37}$$

For generic Δ, the eigenvalues of this matrix are quite complicated, since they are determined by the roots of a cubic characteristic polynomial. However, their values for the optimal choice of Δ that maximizes the smallest of them can be determined analytically. To understand this, let us first recall that by the anti-crossing theorem of Wigner and von Neumann, one generically needs to tune two or three real parameters to force the eigenvalue of a real-symmetric or Hermitian matrix to cross. In our case, the matrix is Hermitian but due to its very special form it actually behaves like a real-symmetric one.[3] One can then verify that its eigenvalues always cross at isolated points in the Δ complex plane. Knowing this, it becomes clear that the highest value

[3]In fact we know that there actually exists a basis where the matrix simplifies from Hermitian to real-symmetric.

for the minimal eigenvalue is obtained at such a crossing point. But since at that point two eigenvalues become degenerate, the cubic characteristic polynomial simplifies and it should be possible to solve the problem analytically. One way to derive the desired result is to start from the characteristic equation written after decomposing the two complex entries Δ and $m^2_{f\bar{x}}$ in the form of a modulus times a phase:

$$\begin{aligned}&\big(\lambda - m^2_{f\bar{f}}\big)^2\big(\lambda - 2\,m^2_{x\bar{x}}\big) - 4\,|m^2_{f\bar{x}}|^2\big(\lambda - m^2_{f\bar{f}}\big)\\&-\;|\Delta|^2\big(\lambda - 2\,m^2_{x\bar{x}}\big) + 4\,|\Delta||m^2_{f\bar{x}}|^2\cos\big(\arg\Delta - 2\arg m^2_{f\bar{x}}\big) = 0\,.\end{aligned} \tag{4.38}$$

Form the form of this equation, it is clear that the optimal choice for the phase of Δ is the one minimizing the last term, in such a way that the cosine is equal to -1, that is:

$$\arg\Delta = 2\arg m^2_{f\bar{x}} + \pi\,. \tag{4.39}$$

Plugging back this expression into the characteristic equation (4.38), this simplifies to $\big(\lambda - m^2_{f\bar{f}} + |\Delta|\big)\big[\big(\lambda - 2\,m^2_{x\bar{x}}\big)\big(\lambda - m^2_{f\bar{f}} - |\Delta|\big) - 4\,|m^2_{f\bar{x}}|^2\big] = 0$. The three solutions of this cubic equation for λ are now easy to find analytically and they are given by $m^2_{f\bar{f}} - |\Delta|$ and $\frac{1}{2}\big(m^2_{f\bar{f}} + 2\,m^2_{x\bar{x}} + |\Delta|\big) \pm \frac{1}{2}\big[\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}} + |\Delta|\big)^2 + 16\,|m^2_{f\bar{x}}|^2\big]^{1/2}$. The optimal value for $|\Delta|$, which maximizes the minimal eigenvalue, is obtained when the first eigenvalue crosses the smallest of the other two, which is the one with the relative minus sign. This fixes:

$$|\Delta| = \frac{1}{2}\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}}\big) + \frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}}\big)^2 + 8\,|m^2_{f\bar{x}}|^2}\,. \tag{4.40}$$

At the optimal point defined by (4.39) and (4.40), the values of the two degenerate lowest eigenvalues and the highest eigenvalues are finally given by:

$$m^2_{\pm} = \frac{1}{2}\big(m^2_{f\bar{f}} + 2\,m^2_{x\bar{x}}\big) \pm \frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}}\big)^2 + 8\,|m^2_{f\bar{x}}|^2}\,. \tag{4.41}$$

Both of these eigenvalues give upper bounds on $m^2_{\rm min}$. The smallest one that leads to the strongest bound is, as before, the one with the negative sign choice. This leads to a new bound, which is however seen to be identical to the previous bound (4.36), showing that the potential level-repulsion effect that is induced by a generic off-diagonal element Δ can be trivialized by optimally choosing the value of this element through a tuning of the superpotential:

$$m^2_{\rm min} \le m^2_{(3)}\,,\;\; m^2_{(3)} = \frac{1}{2}\big(m^2_{f\bar{f}} + 2\,m^2_{x\bar{x}}\big) - \frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}}\big)^2 + 8\,|m^2_{f\bar{x}}|^2}\,. \tag{4.42}$$

Summarizing, we have managed to find explicit expressions for the upper bounds $m^2_{(1)}$, $m^2_{(2)}$, $m^2_{(3)}$ on the lightest mass that descend from blocks of dimension 1, 2, 3. As expected, these are increasingly strong and satisfy:

$$m^2_{(1)} \ge m^2_{(2)} \ge m^2_{(3)}\,. \tag{4.43}$$

These bounds hold however for a fixed theory at a fixed vacuum. In particular, they depend on the direction f^i and on the vacuum coordinates ϕ^i, which determine the direction x^i and the values of $R_{i\bar{j}k\bar{l}}$ and $Q_{i\bar{j}}$. We may then derive a more useful and universal bound by further optimizing the superpotential W to maximize the smallest mass. The strongest version of this fully optimized bound, which is our main result, then takes the form

$$m^2_{\rm min} \leq m^2\,, \tag{4.44}$$

where

$$m^2 = \max\bigg\{\,\frac{1}{2}\big(m^2_{f\bar{f}} + 2\,m^2_{x\bar{x}}\big) - \,\frac{1}{2}\sqrt{\big(m^2_{f\bar{f}} - 2\,m^2_{x\bar{x}}\big)^2 + 8\,|m^2_{f\bar{x}}|^2}\,\bigg\}\,. \tag{4.45}$$

More precisely, the optimization of W defining (4.45) can be performed as follows. At any given point one can adjust $n-1$ independent complex first derivatives W_i, $n(n-1)/2$ independent complex second derivatives W_{ij}, and $(n-1)n(n+1)/6$ independent complex third derivatives W_{ijk}, compatibly with gauge invariance. One may then tune the $n-1$ W_i to freely adjust the direction f^i and $\sqrt{|F|}$, $n-1$ of the W_{ij} to adjust the values of $n-1$ of the fields ϕ^i compatibly with the $n-1$ stationary conditions in the non-Goldstone directions, and finally 1 of the W_{ijk} to adjust the quantity Δ to its optimal value. In this optimized situation, however, there is still 1 combination of fields ϕ^i related to the vector mass $M^2 = 2\,g^2|X|^2$ that cannot be freely adjusted, because the stationarity condition (4.9) along the Goldstone direction does not depend on W_{ij} and W_{ijk}. As a result, (4.9) represents a relation between the scales $\sqrt{|F|}$ and M, for given gauge coupling g. One may however still imagine to tune the real gauge coupling g to achieve any desired value of $\sqrt{|F|}$ and M compatibly with this real stationarity condition. Notice finally that after the above optimization procedure we are left with $(n-1)(n-2)/2$ free complex W_{ij} and $(n-1)n(n+1)/6-1$ free complex W_{ijk}. This is more than enough to be able to decouple all the $n-2$ complex scalar fields that occur in addition to the Goldstino and the Goldstone partners. The simplest possibility is to take the left-over W_{ij} to be large and the left-over W_{ijk} to be moderate, so that all these extra scalars become very massive and do not induce any sizable negative level-repultion effect on the masses of the Goldstino and Goldstone partners. This shows that the bound (4.45) can indeed always be saturated by a last tuning of the superpotential. An explicit implementation of this procedure is illustrated with a numerical example in Appendix A.

4.4 Renormalizable Gauge Theories

Let us illustrate the implications of our result in the simplest case of renormalizable gauge theories with a single $U(1)$ gauge group, where the Kähler potential is quadratic and the Killing vector is linear:

$$K = \delta_{i\bar{j}}\Phi^i\bar{\Phi}^{\bar{j}}\,,\quad X^i = -iq_i\Phi^i\,. \tag{4.46}$$

In this situation, $Q_{i\bar{\jmath}} = q_i \delta_{ij}$. Moreover, one finds $K_i = \delta_{i\bar{\jmath}} \bar{\phi}^{\bar{\jmath}}$ and $K^i = \phi^i$. It then follows that $X^i = -i\, Q^i{}_j K^j$. Thanks to this last property, and calling $Q^{-1i}{}_j$ the inverse of $Q^i{}_j$ restricted to the subspace of non-vanishing charges, one may write:

$$D = g\, Q^{-1}_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}}\,, \tag{4.47}$$

$$M^2 = 2\, g^2 \delta_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}}\,. \tag{4.48}$$

In this simple situation, the scale of the D auxiliary field is related in a very simple and direct way to the mass scale M. Indeed, it follows from the above definitions that $D = \frac{1}{2} g^{-1} Q^{-1}_{x\bar{x}} M^2$. Moreover, the condition (4.9) holding at stationary points reads in this case $Q_{f\bar{f}} |F|^2 = \frac{1}{2} g^{-1} M^2 D$. Using the above relation for D, and assuming that $Q_{f\bar{f}} \neq 0$, this further implies that $|F|^2 = \frac{1}{4} g^{-2} Q^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} M^4$. From these relations, we see that stationary points are possible only if

$$Q^{-1}_{x\bar{x}} Q_{f\bar{f}} \geq 0\,. \tag{4.49}$$

Even though this constraint appears to be not too stringent, it can impose some non-trivial restrictions especially in the case of models with a small number of fields. For example, in the case of only two charged chiral fields, the previous relation implies that vacua which break supersymmetry by F and D terms exist only if the two fields have charges of the same sign.

The values of the overall $|F|$ and of $|D|$ are related to M and their ratio is fixed in terms of the values of $Q^i{}_j$ along the directions f^i and x^i:

$$|D| = \frac{1}{2} g^{-1} |Q^{-1}_{x\bar{x}}| M^2\,, \tag{4.50}$$

$$|F| = \frac{1}{2} g^{-1} \sqrt{Q^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1}} M^2\,. \tag{4.51}$$

$$\left|\frac{D}{F}\right| = \sqrt{Q^{-1}_{x\bar{x}} Q_{f\bar{f}}}\,. \tag{4.52}$$

When instead $Q_{f\bar{f}} = 0$, eq. (4.9) implies that $|D| = 0$, whereas $|F|$ and M can be arbitrary. This is the only situation where M can be adjusted independently of $|F|$.

Notice that we may write down the following simple bound on the relative importance of D-type and F-type supersymmetry breaking, in terms of the pair of charges $q_{\rm min}$ and $q_{\rm max}$ which possess the largest possible ratio with the constraint that they have the same sign [126]:

$$\left|\frac{D}{F}\right| \leq \sqrt{\left|\frac{q_{\rm max}}{q_{\rm min}}\right|}\,. \tag{4.53}$$

This bound can be saturated by choosing the directions f^i and x^i to be the eigenvectors of $Q^i{}_j$ corresponding to the eigenvalues $q_{\rm max}$ and $q_{\rm min}$.

The scalar masses (4.28), (4.29) and (4.30) undergo two relevant simplifications. The first is that all the curvature terms drop, since in this case the scalar manifold is flat. The second is that due to the relation (4.51) the supersymmetric term in $m^2_{x\bar{x}}$ is forced to be of the same order of magnitude as the non-supersymmetric terms. One then finds the following simple expressions:

$$m^2_{f\bar{f}} = \Big[Q^{-1}_{x\bar{x}}\, Q_{f\bar{f}}\Big] M^2, \tag{4.54}$$

$$m^2_{x\bar{x}} = \frac{1}{2}\Big[1 + Q^{-1}_{x\bar{x}}\, Q_{x\bar{x}} + Q^{-1}_{x\bar{x}}(Q_{f\bar{f}})^{-1} Q^2_{f\bar{f}}\Big] M^2, \tag{4.55}$$

$$m^2_{f\bar{x}} = \Big[Q^{-1}_{x\bar{x}}\, Q_{f\bar{x}}\Big] M^2. \tag{4.56}$$

We observe now that by the restriction (4.49) and some simple linear algebra, we can get some useful constraints on the various pieces of these masses. In particular, we have that $Q^{-1}_{x\bar{x}}\, Q_{f\bar{f}} \geq 0$ and $Q^{-1}_{x\bar{x}}(Q_{f\bar{f}})^{-1} Q^2_{f\bar{f}} \geq Q^{-1}_{x\bar{x}}\, Q_{f\bar{f}} \geq 0$, since $Q^2_{f\bar{f}} \geq (Q_{f\bar{f}})^2$. Moreover, $Q^{-1}_{x\bar{x}}\, Q_{x\bar{x}}$ has indefinite sign but becomes equal to 1 whenever x^i is an eigenvector of ${Q^i}_j$, and $Q^{-1}_{x\bar{x}}\, Q_{f\bar{x}}$ has indefinite sign but becomes equal to 0 whenever either f^i or x^i is an eigenvector of ${Q^i}_j$.

In this class of models, the masses $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$ and $m^2_{f\bar{x}}$ depend on the vacuum point only through the orientation of the direction x^i and the size of M. Moreover, by varying the vacuum point at fixed M one may achieve all the possible orientations for x^i, thanks to the simple linear form of X^i and quadratic form of K. The optimization of the superpotential defining the bound (4.45) then amounts in this case to optimizing the orientation of the directions f^i and x^i, with the only constraint that they should be orthogonal. There is then a natural guess for the optimal choice of f^i and x^i. This consists in choosing these two orthogonal directions to be the eigenvectors of ${Q^i}_j$ with largest and smallest eigenvalues with common sign, namely $q_{\rm max}$ and $q_{\rm min}$. With such a choice, $m^2_{f\bar{f}}$ is maximal, $m^2_{f\bar{x}}$ vanishes and $2\, m^2_{x\bar{x}}$ is larger than $m^2_{f\bar{f}}$. The precise values are

$$m^2_{f\bar{f}} \to \left|\frac{q_{\rm max}}{q_{\rm min}}\right| M^2\,,\quad m^2_{x\bar{x}} \to \left[1 + \frac{1}{2}\left|\frac{q_{\rm max}}{q_{\rm min}}\right|\right] M^2\,,\quad m^2_{f\bar{x}} \to 0\,. \tag{4.57}$$

With this choice, one gets that $m^2_{(1)}$, $m^2_{(2)}$ and $m^2_{(3)}$ all coincide with the maximal possible value of $m^2_{f\bar{f}}$. This value certainly represents the maximal possible value for $m^2_{(1)}$ taken on its own. But then it must necessarily represent also the maximal possible value for $m^2_{(2)}$ and $m^2_{(3)}$, because by construction one has $m^2_{(1)} \geq m^2_{(2)} \geq m^2_{(3)}$ for any choice of f^i and x^i. This proves that the above choice for f^i and x^i is indeed the optimal one, and the bound (4.45) thus reads in this case

$$m^2 = \left|\frac{q_{\rm max}}{q_{\rm min}}\right| M^2\,. \tag{4.58}$$

Notice finally that the optimal configuration corresponds in this case to the one that maximizes the size of the D auxiliary field relative to the F auxiliary fields:

$$\left|\frac{D}{F}\right| \to \sqrt{\left|\frac{q_{\rm max}}{q_{\rm min}}\right|}\,. \tag{4.59}$$

Summarizing, we see that in the case of a flat scalar manifold and a linear isometry, the lightest scalar field is identified with a partner of the Goldstino, and its square mass is positive. In this particular case, one would thus have obtained the same bound by looking only at the Goldstino partners and maximizing the smallest of their masses by making the effect of the gauging as large as possible. This is however an accidental feature of these models, which is due to the flatness and maximal symmetry of the space, as well as the fact that there is a single generator. In next section we will show that in the case of curved scalar manifolds, the situation is no-longer so trivial.

4.5 Non-linear Gauged Sigma Models

Let us next consider the more general case of effective theories with a non-trivial Kähler potential and a single $U(1)$ gauge symmetry generated by a Killing vector of unspecified form:

$$K = K(\Phi^i \bar{\Phi}^{\bar{\jmath}}) \,, \quad X^i = X^i(\Phi^i) \,. \tag{4.60}$$

This situation is of course much more complex than the simple particular case considered in previous section. Yet one may try to follow the same steps as before. A major difference is that since the Killing vector X^i is not linear and K is not quadratic, X^i and K^j are no longer linearly related through $Q^i{}_j$. One may however introduce the new quantity

$$\tilde{Q}^i{}_j = \frac{i X^i K_j}{K^m K_m} \,, \tag{4.61}$$

which allows to write the relation $X^i = -i\,\tilde{Q}^i{}_j K^j$. In the case of renormalizable gauge theories with a phase symmetry, $\tilde{Q}^i{}_j$ coincides with $Q^i{}_j$ and is constant, but in the more general situation considered here $\tilde{Q}^i{}_j$ differs from $Q^i{}_j$ and is not constant. With this notation, and calling $\tilde{Q}^{-1i}{}_j$ the inverse of $\tilde{Q}^i{}_j$ in the subspace where it does not vanish, one can then write:

$$D = g\,\tilde{Q}^{-1}_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}} \,, \tag{4.62}$$

$$M^2 = 2\,g^2 g_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}} \,. \tag{4.63}$$

In this more complicated case, the auxiliary field D is again related to the mass scale M, but in a more involved and implicit way. Indeed, from the above definitions one deduces that $D = \frac{1}{2} g^{-1} \tilde{Q}^{-1}_{x\bar{x}} M^2$. Moreover, the condition (4.9) implies that at a stationary point $Q_{f\bar{f}} |F|^2 = \frac{1}{2} g^{-1} M^2 D$. Using the above relation for D, and assuming that $Q_{f\bar{f}} \neq 0$, this further implies that $|F|^2 = \frac{1}{4} g^{-2} \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} M^4$. From these relations, we see that stationary points are possible only if

$$\tilde{Q}^{-1}_{x\bar{x}} Q_{f\bar{f}} \geq 0 \,. \tag{4.64}$$

The values of the overall $|F|$ and of $|D|$ are again related to M and their ratio takes as before a simple form, but now these relations depend not only on $Q^i{}_j$ but also on the new quantities $\tilde{Q}^i{}_j$, taken respectively along the directions f^i and x^i:

$$|D| = \frac{1}{2} g^{-1} \, |\tilde{Q}^{-1}_{x\bar{x}}| M^2 \,, \tag{4.65}$$

$$|F| = \frac{1}{2} g^{-1} \sqrt{\tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1}} M^2 \,. \tag{4.66}$$

$$\left|\frac{D}{F}\right| = \sqrt{\tilde{Q}^{-1}_{x\bar{x}} Q_{f\bar{f}}} \,. \tag{4.67}$$

When instead $Q_{f\bar{f}} = 0$, eq. (4.9) implies that $|D| = 0$, whereas $|F|$ and M can be arbitrary. As before, this is the only situation where M can be adjusted independently of $|F|$.

In this case, the relative importance of D-type and F-type supersymmetry breaking depends on the vacuum point not only through the direction x^i but also through $Q^i{}_j$ and $\tilde{Q}^i{}_j$. Finding an explicit and quantitative bound on their ratio is then more difficult (see for instance [127] for some attempts). Nevertheless, from the above relations one may still infer a simple although somewhat implicit bound that involves the maximal eigenvalue $Q_{\rm max}$ of $Q^i{}_j$ and the minimal eigenvalue $\tilde{Q}_{\rm min}$ of $\tilde{Q}^i{}_j$, with the constraint that these should have the same sign:

$$\left|\frac{D}{F}\right| \le \sqrt{\left|\frac{Q_{\rm max}}{\tilde{Q}_{\rm min}}\right|} \,. \tag{4.68}$$

In general, this bound can however not be saturated, because $Q^i{}_j$ and $\tilde{Q}^i{}_j$ are different matrices that cannot be diagonalized simultaneously, and it is therefore not possible to choose the orthogonal directions f^i and x^i in such a way to get simultaneously $Q_{f\bar{f}} = Q_{\rm max}$ and $\tilde{Q}_{x\bar{x}} = \tilde{Q}_{\rm min}$.

The masses (4.28), (4.29) and (4.30) can now be computed more explicitly. In this case there is an additional contribution coming from the curvature. As before, the relation (4.66) allows to rewrite the non-supersymmetric pieces in terms of the same scale as the supersymmetric piece. One then finds the following expressions:

$$m^2_{f\bar{f}} = \Big[-\frac{1}{4} g^{-2} M^2 R_{f\bar{f}f\bar{f}} \, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}} \, Q_{f\bar{f}} \Big] M^2, \tag{4.69}$$

$$m^2_{x\bar{x}} = \frac{1}{2}\Big[1 - \frac{1}{2} g^{-2} M^2 R_{f\bar{f}x\bar{x}} \, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}} \, Q_{x\bar{x}} + \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} Q^2_{f\bar{f}} \Big] M^2, \tag{4.70}$$

$$m^2_{f\bar{x}} = \Big[-\frac{1}{4} g^{-2} M^2 R_{f\bar{f}f\bar{x}} \, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}} \, Q_{f\bar{x}} \Big] M^2. \tag{4.71}$$

There are again various restrictions on the ingredients appearing in these expressions. Concerning the contractions of $Q_{i\bar{\jmath}}$ and $\tilde{Q}_{i\bar{\jmath}}$, the restriction (4.64) implies as before useful constraints. In particular, we have $\tilde{Q}^{-1}_{x\bar{x}} \, Q_{f\bar{f}} \ge 0$ and $\tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} Q^2_{f\bar{f}} \ge \tilde{Q}^{-1}_{x\bar{x}} \, Q_{f\bar{f}} \ge 0$. Moreover, $\tilde{Q}^{-1}_{x\bar{x}} \, Q_{x\bar{x}}$ is indefinite and deviates from 1 even when x is an eigenvector of $Q^i{}_j$, whereas $\tilde{Q}^{-1}_{x\bar{x}} \, Q_{f\bar{x}}$ has indefinite sign but becomes as before equal to 0 whenever either f^i or x^i is an eigenvector of $Q^i{}_j$. Concerning the contractions of $R_{i\bar{\jmath}k\bar{l}}$, on the other hand, there does not seem to exist any sharp inequality.

The masses $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$ and $m^2_{f\bar{x}}$ depend on the vacuum point not only through the orientation of the direction x^i and the size of M, but also through the values of $R_{i\bar{\jmath}k\bar{l}}$, $Q_{i\bar{\jmath}}$ and $\tilde{Q}_{i\bar{\jmath}}$, which are in general not constant. Moreover, it is no longer granted that by varying the vacuum point at fixed M one may achieve all the possible orientations for x^i. The optimization of the superpotential defining the bound (4.45) is then a complicated task, and does not simply amount to optimizing the orientation of the directions f^i and x^i. Moreover, even ignoring this difficulty, finding the optimal choice is more involved also because of the fact that generically it emerges from a competition between the terms that depend only on $Q_{i\bar{\jmath}}$ and $\tilde{Q}_{i\bar{\jmath}}$ and those that depend also on $R_{i\bar{\jmath}k\bar{l}}$, although there may be regimes where one or the other of these two contributions dominates. As a consequence of this, we were not able to find any general result for this type of models based on curved geometries. We will however study in some detail a few particular examples in the next section, based on simple geometries with covariantly constant curvature and simple isometries. The only few remarks that can be made in general concern the behavior of the various contractions that appear in the masses $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$ and $m^2_{f\bar{x}}$ when the directions f^i and x^i are varied. To get an idea of what may happen, we may treat f^i and x^i as arbitrary directions and enforce the constraints that $g_{i\bar{\jmath}} f^i \bar{f}^{\bar{\jmath}} = 1$, $g_{i\bar{\jmath}} x^i \bar{x}^{\bar{\jmath}} = 1$ and $g_{i\bar{\jmath}} f^i \bar{x}^{\bar{\jmath}} = 0$ through Lagrange multipliers. Proceeding in this way, one then finds the following results. When $Q_{f\bar{f}}$ is extremal $Q_{f\bar{x}} = 0$, when $\tilde{Q}^{-1}_{x\bar{x}}$ is extremal $\tilde{Q}^{-1}_{f\bar{x}} = 0$, when $Q_{f\bar{f}}\,\tilde{Q}^{-1}_{x\bar{x}}$ is extremal $Q_{f\bar{x}}\,\tilde{Q}^{-1}_{x\bar{x}} + Q_{f\bar{f}}\,\tilde{Q}^{-1}_{f\bar{x}} = 0$, and finally when $R_{f\bar{f}f\bar{f}}$ is extremal $R_{f\bar{f}f\bar{x}} = 0$.

Summarizing, we see that in the case of a curved scalar manifold and a generic isometry, the lightest scalar field is generically identified with a linear combination of Goldstino and Goldstone partners, and its square mass is not necessarily positive. In this case, one would thus have obtained a too optimistic bound by proceeding along the lines of [61] and looking only at the Goldstino partners and maximizing the smallest of their mass. Notice finally that the optimal situation does not necessarily correspond to the one that maximizes the effect of the gauging.

4.6 Explicit Examples with Constant Curvature

In this section, we study in some detail a few concrete examples to illustrate our general results. We focus on models with two fields and one gauge symmetry. In this situation, the Goldstino and Goldstone directions f^i and x^i are rigidly tied and can be parametrized with a single angle θ, which we shall define in such a way that the mass M is constant. Another simplification that occurs in the two-field case is that one simply has $Q^2_{f\bar{f}} = (Q_{f\bar{f}})^2 + |Q_{f\bar{x}}|^2$. We shall take $\theta \in [0, 2\pi]$, but in all the examples below the behaviors of the masses in the four quadrants are related by simple reflections.

4.6.1 Flat Kähler Potential and Linear Isometry

As a first simple example, let us discuss the case of quadratic Kähler potential and linear Killing vector, which corresponds to a flat scalar manifold with a phase isometry defined by positive charges:

$$K = \Phi^1\bar{\Phi}^1 + \Phi^2\bar{\Phi}^2\,, \quad X^i = -i\left(q_1\Phi^1,\, q_2\Phi^2\right). \tag{4.72}$$

In this case, we can parametrize the vacuum in the following way:

$$\Phi^i = \frac{1}{\sqrt{2}}\,g^{-1}M\left(q_1^{-1}\cos\theta,\, q_2^{-1}\sin\theta\right). \tag{4.73}$$

The Goldstone and Goldstino directions are then given by $x^i = -i\left(\cos\theta,\, \sin\theta\right)$ and $f^i = -i\left(\sin\theta, -\cos\theta\right)$, and the metric is clearly trivial: $g_{i\bar{j}} = \delta_{ij}$. The relations between $|D|$, $|F|$ and M^2 are in this case:

$$|D| = \frac{1}{2}\,g^{-1}\Big(q_1^{-1}\cos^2\theta + q_2^{-1}\sin^2\theta\Big)M^2\,, \tag{4.74}$$

$$|F| = \frac{1}{2}\,g^{-1}(q_1q_2)^{-1/2}M^2\,. \tag{4.75}$$

We then get:

$$\left|\frac{D}{F}\right| = \sqrt{\frac{q_2}{q_1}}\cos^2\theta + \sqrt{\frac{q_1}{q_2}}\sin^2\theta\,. \tag{4.76}$$

In this case $R_{i\bar{j}k\bar{l}}$ vanishes identically and we therefore get:

$$R_{f\bar{f}f\bar{f}} = 0\,, \quad R_{f\bar{f}x\bar{x}} = 0\,, \quad R_{f\bar{f}f\bar{x}} = 0\,. \tag{4.77}$$

The matrix elements of $Q_{i\bar{j}}$ are instead given simply by:

$$Q_{f\bar{f}} = q_2\cos^2\theta + q_1\sin^2\theta\,, \tag{4.78}$$

$$Q_{x\bar{x}} = q_1\cos^2\theta + q_2\sin^2\theta\,, \tag{4.79}$$

$$Q_{f\bar{x}} = (q_1 - q_2)\cos\theta\sin\theta\,. \tag{4.80}$$

The elements $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$, $m^2_{f\bar{x}}$ and the eigenvalues $m^2_{\pm}$ of the mass matrix are equal to M^2 times some functions of θ and q_1/q_2. The behavior of $m^2_{f\bar{f}}/M^2$ and m^2_{-}/M^2 as functions of θ is shown in Fig. 4.1 for some particular choice of q_1/q_2. More in general, one finds the following behavior. If $q_1 > q_2$, $m^2_{f\bar{f}}$ and m^2_{-} both reach their maxima for $\theta = \frac{\pi}{2}$, and at that point $m^2_{f\bar{f}}/M^2 = q_1/q_2$, $m^2_{x\bar{x}}/M^2 = \frac{1}{2}(2 + q_1/q_2)$ and $m^2_{f\bar{x}}/M^2 = 0$, so that $m^2_{-}/M^2 = q_1/q_2$. The optimal direction is therefore $\theta = \frac{\pi}{2}$, and the bound is $m^2/M^2 = q_1/q_2$. If instead $q_2 > q_1$, the situation is similar but with $q_1 \leftrightarrow q_2$ and $\theta \leftrightarrow \frac{\pi}{2} - \theta$.

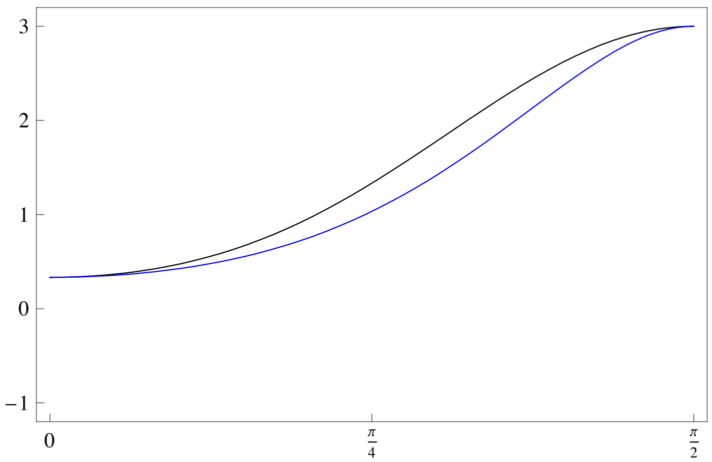

Figure 4.1: Plot of $m^2_{f\bar{f}}/M^2$ (upper curve) and m^2_{-}/M^2 (lower curve) as functions of θ for the model with quadratic Kähler potential and linear Killing vectors defined by (4.72), with $q_1/q_2 = 3$.

4.6.2 Logarithmic Kähler Potential and Shift-Isometry

As a second simple example, let us discus the case of logarithmic Kähler potential and constant Killing vector, which corresponds to a constantly and positively curved scalar manifold with a shift isometry defined by positive shifts:

$$K = -\Lambda_1^2 \log\left(\frac{\Phi^1 + \bar{\Phi}^1}{\Lambda_1}\right) - \Lambda_2^2 \log\left(\frac{\Phi^2 + \bar{\Phi}^2}{\Lambda_2}\right), \quad X^i = i\left(A_1,\, A_2\right). \tag{4.81}$$

The two scales Λ_1 and Λ_2 define the curvatures of the two field sectors, whereas the two scales A_1 and A_2 define the gauge shifts. It is then convenient to introduce the following dimensionless parameters:

$$\lambda_1 = \frac{g\Lambda_1}{M}, \quad \lambda_2 = \frac{g\Lambda_2}{M}, \quad a_1 = \frac{gA_1}{M}, \quad a_2 = \frac{gA_2}{M}. \tag{4.82}$$

In this case, we can parametrize the vacuum in the following way, by including absolute values to take into account that the fields are in this case restricted to have a positive real part:

$$\Phi^i = \frac{1}{\sqrt{2}}\, g^{-1} M \left(a_1 \lambda_1 |\sec\theta|,\, a_2 \lambda_2 |\csc\theta|\right). \tag{4.83}$$

The Goldstone and Goldstino directions are then given by $x^i = \sqrt{2} i\left(a_1,\, a_2\right)$ and $f^i = \sqrt{2} i\left(a_1|\tan\theta|, -a_2|\cot\theta|\right)$, whereas $g_{i\bar{\jmath}} = \frac{1}{2}\,\mathrm{diag}\big(a_1^{-2}\cos^2\theta,\, a_2^{-2}\sin^2\theta\big)$. The relation between $|D|$, $|F|$ and M^2 are in this case:

$$|D| = \frac{1}{\sqrt{2}}\, g^{-1}\big(\lambda_1|\cos\theta| + \lambda_2|\sin\theta|\big)\, M^2, \tag{4.84}$$

$$|F| = \frac{1}{\sqrt{2}}\, g^{-1}\sqrt{\lambda_1\lambda_2}\, |2\cos\theta\sin\theta|^{-1/2}\, M^2. \tag{4.85}$$

We then get:

$$\left|\frac{D}{F}\right| = \sqrt{|2\cos\theta\sin\theta|}\left(\sqrt{\frac{\lambda_1}{\lambda_2}}|\cos\theta| + \sqrt{\frac{\lambda_2}{\lambda_1}}|\sin\theta|\right). \tag{4.86}$$

The contractions of $R_{i\bar{j}k\bar{l}}$ are given by

$$R_{f\bar{f}f\bar{f}} = 2g^2M^{-2}\big(\lambda_2^{-2}\cos^4\theta + \lambda_1^{-2}\sin^4\theta\big)\,, \tag{4.87}$$
$$R_{f\bar{f}x\bar{x}} = 2g^2M^{-2}\big(\lambda_2^{-2} + \lambda_1^{-2}\big)\cos^2\theta\sin^2\theta\,, \tag{4.88}$$
$$R_{f\bar{f}f\bar{x}} = 2g^2M^{-2}\big(\lambda_1^{-2}\sin^2\theta - \lambda_2^{-2}\cos^2\theta\big)|\cos\theta\sin\theta|\,. \tag{4.89}$$

The matrix elements of $Q_{i\bar{j}}$ are instead found to be independent of the shifts a_i and dominated by the effect of the connection term in their definition, as a result of the fact that the Killing vectors are constant:

$$Q_{f\bar{f}} = \sqrt{2}\left(\lambda_2^{-1}|\cos\theta| + \lambda_1^{-1}|\sin\theta|\right)|\cos\theta\sin\theta|\,, \tag{4.90}$$
$$Q_{x\bar{x}} = \sqrt{2}\left(\lambda_1^{-1}|\cos^3\theta| + \lambda_2^{-1}|\sin^3\theta|\right), \tag{4.91}$$
$$Q_{f\bar{x}} = \sqrt{2}\left(\lambda_1^{-1}|\cos\theta| - \lambda_2^{-1}|\sin\theta|\right)|\cos\theta\sin\theta|\,. \tag{4.92}$$

The elements $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$, $m^2_{f\bar{x}}$ and the eigenvalues $m^2_\pm$ of the mass matrix are equal to M^2 times some functions of θ and λ_1/λ_2. The behavior of $m^2_{f\bar{f}}/M^2$ and m^2_-/M^2 as functions of θ is shown in Fig. 4.6.2 for some particular choice of λ_1/λ_2. More in general, one finds the following behavior. $m^2_{f\bar{f}}$ reaches its maximum for $\theta = \frac{\pi}{4}$ and at that point $m^2_{f\bar{f}}/M^2 = 1 + \frac{1}{4}(\lambda_1/\lambda_2 + \lambda_2/\lambda_1)$, $m^2_{x\bar{x}}/M^2 = 1 + \frac{1}{2}(\lambda_1/\lambda_2 + \lambda_2/\lambda_1)$ and $m^2_{f\bar{x}}/M^2 = -\frac{1}{4}(\lambda_1/\lambda_2 - \lambda_2/\lambda_1)$, so that m^2_-/M^2 is smaller-or-equal than $m^2_{f\bar{f}}/M^2$. The maximum of m^2_-/M^2 occurs instead for some $\theta \leq \frac{\pi}{4}$ if $\lambda_1 > \lambda_2$ and for some $\theta \geq \frac{\pi}{4}$ if $\lambda_1 < \lambda_2$, and takes a value that is smaller than $1 + \frac{1}{4}(\lambda_1/\lambda_2 + \lambda_2/\lambda_1)$. For $\lambda_1 \simeq \lambda_2$, the optimal direction is $\theta \simeq \frac{\pi}{4}$ and the bound is $m^2/M^2 \simeq \frac{3}{2}$, which is identical to the one that one would have obtained by looking just at the Goldstino direction. For $\lambda_1 \gg \lambda_2$, on the other hand, a numerical study shows that the optimal direction is $\theta \simeq 0.67$ and the bound is $m^2/M^2 \simeq 0.13\,\lambda_1/\lambda_2$, which is a factor 1.86 smaller than the one that one would have inferred by looking just at the Goldstino direction, although still positive. For $\lambda_1 \ll \lambda_2$, the situation is similar but with $\lambda_1 \leftrightarrow \lambda_2$ and $\theta \leftrightarrow \frac{\pi}{2} - \theta$.

4.6.3 Logarithmic Kähler Potential and Linear Isometry

As a third slightly more complicated and richer example, let us finally discus the case of logarithmic Kähler potential and linear Killing vector, which corresponds to a constantly and positively curved scalar manifold with a phase isometry defined by positive charges:

$$K = -\Lambda_1^2\log\left(1 - \frac{\Phi^1\bar{\Phi}^1}{\Lambda_1^2}\right) - \Lambda_2^2\log\left(1 - \frac{\Phi^2\bar{\Phi}^2}{\Lambda_2^2}\right), \quad X^i = -i\left(q_1\Phi^1, q_2\Phi^2\right). \tag{4.93}$$

The two scales Λ_1 and Λ_2 define as before the curvatures of the two field sectors. It turns out that by varying the overall scale of these curvatures with respect to the

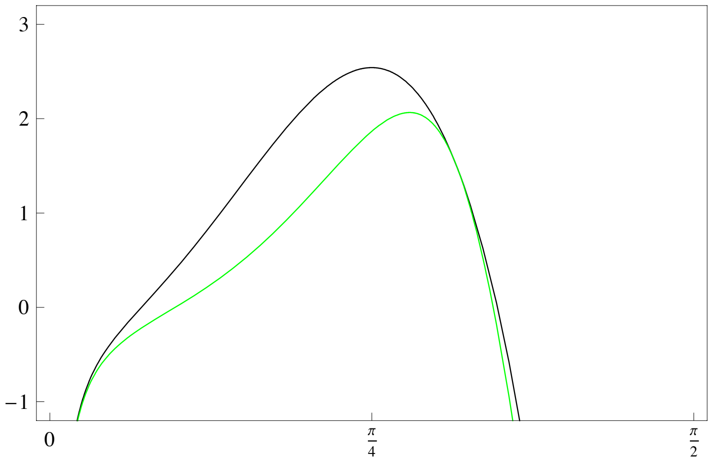

Figure 4.2: Plot of $m^2_{f\bar{f}}/M^2$ (upper curve) and m^2_{-}/M^2 (lower curve) as functions of θ for the model with logarithmic Kähler potential and constant Killing vectors defined by (4.81), with $\lambda_1/\lambda_2 = \frac{1}{6}$.

vector mass scale, this new model interpolates between the two previous ones. This can be seen as follows. The small curvature limit corresponds to take Λ_i large and Φ^i finite, so that Φ^i/Λ_i is close to 0. In this limit one can keep the same coordinates and just expand the logarithm in K. In this way one then recovers the model (4.72). The large curvature limit corresponds instead to take Λ_i small and Φ^i also small, so that Φ^i/Λ_i is close to 1. In this limit, it is convenient to change coordinates to describe the model in a more transparent way. The appropriate reparametrization turns out to be $\Phi^i/\Lambda_i \to (1 - \frac{1}{2}\Phi^i/\Lambda_i)/(1 + \frac{1}{2}\Phi^i/\Lambda_i)$. Discarding an irrelevant Kähler transformation, one then finds $K \to -\sum_i \Lambda_i^2 \log((\Phi^i + \bar{\Phi}^i)/\Lambda_i)$ and $X^i \to i\, q_i \Lambda_i\, (1 - \frac{1}{4}\Phi^{i2}/\Lambda_i^2)$. In these new coordinates, Φ^i/Λ_i is close to 0. In this limit one then manifestly recovers the model (4.81) with the same field parametrization and shifts given by $A_i = q_i \Lambda_i$. To parametrize the effects of the curvatures, we introduce as before the dimensionless parameters

$$\lambda_1 = \frac{g\Lambda_1}{M}\,, \quad \lambda_2 = \frac{g\Lambda_2}{M}\,. \tag{4.94}$$

It will also be useful to introduce the short-hand notation

$$u(\theta) = H\bigg(\frac{\cos\theta}{q_1\lambda_1}\bigg)\,, \quad v(\theta) = H\bigg(\frac{\sin\theta}{q_2\lambda_2}\bigg)\,. \tag{4.95}$$

where $H(x)$ is the following monotonically decreasing function:

$$H(x) = \frac{\sqrt{1+2\,x^2}-1}{x^2} \simeq \begin{cases} 1 & , \quad |x| \ll 1 \\ \sqrt{2}/|x|\,, & |x| \gg 1 \end{cases}\,. \tag{4.96}$$

In this case, we can parametrize the vacuum in the following way:

$$\Phi^i = \frac{1}{\sqrt{2}} g^{-1} M \left(q_1^{-1} u(\theta) \cos\theta,\, q_2^{-1} v(\theta) \sin\theta\right) . \tag{4.97}$$

The Goldstone and Goldstino directions then read $x^i = -i\left(u(\theta)\cos\theta,\, v(\theta)\sin\theta\right)$ and $f^i = -i\left(u(\theta)\sin\theta, -v(\theta)\cos\theta\right)$, and the metric is $g_{i\bar{\jmath}} = \mathrm{diag}\big(1/u^2(\theta),\, 1/v^2(\theta)\big)$. The relation between $|D|$, $|F|$ and M^2 are in this case:

$$|D| = \frac{1}{2} g^{-1}\left(q_1^{-1} u(\theta)\cos^2\theta + q_2^{-1} v(\theta)\sin^2\theta\right) M^2 \tag{4.98}$$

$$|F| = \frac{1}{2} g^{-1} \sqrt{\frac{q_1^{-1} u(\theta)\cos^2\theta + q_2^{-1} v(\theta)\sin^2\theta}{q_2\big[2/v(\theta)-1\big]\cos^2\theta + q_1\big[2/u(\theta)-1\big]\sin^2\theta}}\, M^2 . \tag{4.99}$$

We then get:

$$\left|\frac{D}{F}\right| = \sqrt{\sqrt{\frac{q_2}{q_1}} u(\theta)\cos^2\theta + \sqrt{\frac{q_1}{q_2}} v(\theta)\sin^2\theta} \times \sqrt{\sqrt{\frac{q_2}{q_1}}\big[2/v(\theta)-1\big]\cos^2\theta + \sqrt{\frac{q_1}{q_2}}\big[2/u(\theta)-1\big]\sin^2\theta} . \tag{4.100}$$

The contractions of $R_{i\bar{\jmath}k\bar{l}}$ are given by

$$R_{f\bar{f}f\bar{f}} = 2g^2 M^{-2}\left(\lambda_2^{-2}\cos^4\theta + \lambda_1^{-2}\sin^4\theta\right) , \tag{4.101}$$

$$R_{f\bar{f}x\bar{x}} = 2g^2 M^{-2}\left(\lambda_2^{-2} + \lambda_1^{-2}\right)\cos^2\theta\sin^2\theta , \tag{4.102}$$

$$R_{f\bar{f}f\bar{x}} = 2g^2 M^{-2}\left(\lambda_1^{-2}\sin^2\theta - \lambda_2^{-2}\cos^2\theta\right)\cos\theta\sin\theta . \tag{4.103}$$

The matrix elements of $Q_{i\bar{\jmath}}$ are instead found to be:

$$Q_{f\bar{f}} = q_2\big[2/v(\theta)-1\big]\cos^2\theta + q_1\big[2/u(\theta)-1\big]\sin^2\theta , \tag{4.104}$$

$$Q_{x\bar{x}} = q_1\big[2/u(\theta)-1\big]\cos^2\theta + q_2\big[2/v(\theta)-1\big]\sin^2\theta , \tag{4.105}$$

$$Q_{f\bar{x}} = \left(q_1\big[2/u(\theta)-1\big] - q_2\big[2/v(\theta)-1\big]\right)\cos\theta\sin\theta . \tag{4.106}$$

The elements $m^2_{f\bar{f}}$, $m^2_{x\bar{x}}$, $m^2_{f\bar{x}}$ and the eigenvalues $m^2_\pm$ of the mass matrix are equal to M^2 times some functions of θ, λ_1/λ_2, q_1/q_2 and $q_1 q_2 \lambda_1 \lambda_2$. The behavior of $m^2_{f\bar{f}}/M^2$ and m^2_-/M^2 as functions of θ is shown in Fig. 4.3 for some particular choice of λ_1/λ_2, q_1/q_2 and $q_1 q_2 \lambda_1 \lambda_2$. More in general, one finds the following behavior. $m^2_{f\bar{f}}$ and m^2_- reach maxima for two different values of θ, and the maximal value of m^2_- is always smaller than the maximal value of $m^2_{f\bar{f}}$. This shows once again that the bound that one would have inferred by looking only at the Goldstino direction is weaker than the bound m^2 that one obtains by taking into account also the Goldstone direction. One moreover verifies that in the limit $\lambda_i \gg 1$ one recovers the behavior of the model with quadratic K and linear X^i with charges q_i, whereas in the limit $\lambda_i \ll 1$ one reaches the behavior of the model with logarithmic K and constant X^i with shifts $A_i = q_i \Lambda_i$.

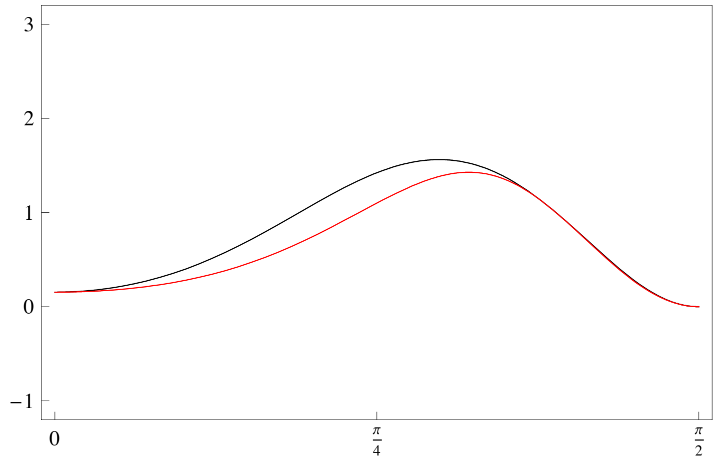

Figure 4.3: Plot of $m^2_{f\bar{f}}/M^2$ (upper curve) and m^2_-/M^2 (lower curve) as functions of θ for the model with logarithmic Kähler potential and linear Killing vectors defined by (4.93), with $\lambda_1/\lambda_2 = \frac{1}{6}$, $q_1/q_2 = 3$ and $q_1 q_2 \lambda_1 \lambda_2 = 1$.

4.7 Summary

In this chapter, we have shown that it is possible to derive an absolute upper bound on the mass of the lightest scalar field of a theory with spontaneously broken supersymmetry and local gauge symmetries. This can be obtained by focusing on the subset of scalar fields corresponding to the partners of the Goldstino fermion and the gauge vector bosons, for which the mass is constrained by symmetry arguments. The resulting bound has the property that it can be saturated by adjusting the superpotential. Requiring it to be positive is therefore a necessary and sufficient condition on the remaining functions specifying the kinetic terms for the existence of a metastable supersymmetry-breaking vacuum. We have shown that by including also the Goldstone partners one finds in general a stronger bound than by considering just the Goldstino partners, and we have illustrated this fact through several explicit examples.

The results we presented in this chapter have interesting implications on the conditions for the existence of metastable supersymmetry breaking vacua in generic supersymmetric theories with local gauge symmetries. Indeed, the region of parameter space where tachyons can be avoided is reduced when one considers not only the Goldstino partners but also the Goldstone partners, since there are points where the former have positive square mass while the latter or linear combinations of the two have negative square mass. We believe that there may in fact exist models where the upper bound derived from just the Goldstino partners is positive whereas the upper bound derived by including also the Goldstone partners is negative. In such a situation, one would then find an obstruction against the existence of metastable supersymmetry-breaking vacua that comes from the Goldstone partners rather than from the Goldstino partners.

In the light of this possibility, it would be interesting to apply the result that we have derived to reexamine the conditions for the existence of metastable supersymmetry-breaking vacua in theories where the gauging plays a crucial role. One class of models where this could perhaps uncover new instabilities is that of theories with extended supersymmetry, and more specifically those where the Goldstino partners do not seem to lead necessarily to tachyons. This is for instance the case of $N = 2$ theories with non-Abelian vector multiplets and/or charged hyper multiplets (see [128] for an extended discussion on this topic).

To conclude, we would like to comment on the generalization of our result to the case of supergravity theories. The only technical difficulty to extend our analysis to that case is the fact that the Goldstino direction f^i and the Goldstone directions x_a^i are no longer orthogonal, as a consequence of the additional gravitational term in the definition of the auxiliary fields. More precisely, one gets $g_{i\bar{\jmath}} f^i \bar{x}_a^{\bar{\jmath}} = i g^{-1} m_{3/2} D_a$. As a consequence, the set of vectors f^i and x_a^i can no longer be chosen to form an orthonormal set, although it still represents a complete set of dangerous directions. The restriction of the mass matrix to this subspace is then no longer given just by eq. (4.25) but by a more complex expression. As a result, the analysis becomes technically more complicated. But for the rest one can apply the same strategy we developed in this chapter for theories with rigid supersymmetry.

Chapter 5

Effects of Heavy Multiplets on Vacuum Stability

In this chapter we are going to combine the main results of the previous two chapters to study the effects induced by heavy supermultiplets on the masses of light scalar fields and in particular on the metastability conditions; this is done in the limit in which the heavy mass scale is much larger than the supersymmetry breaking scale and heavy multiplets can be integrate out supersymmetrically by following the procedure described in Chapter 3. We restrict to the case in which all the vector multiplets have large supersymmetric masses and the low-energy supersymmetric effective theory contains only light chiral multiplets. As we have seen at the end of Section 4.2, in this situation the Goldstino is the only dangerous direction in the scalar field space whereas the directions associated to Killing vectors are automatically safe.

We will show that the square-masses of light scalar fields can get two different types of significant corrections when a heavy multiplet is integrated out. The first is an indirect level-repulsion effect, which may arise from heavy chiral multiplets and is always negative. The second is a direct coupling contribution, which may arise from heavy vector multiplets and can have any sign. We then apply these results to the sGoldstino mass and study the implications for the vacuum metastability condition. We find that the correction from heavy chiral multiplets is always negative and tends to compromise vacuum metastability, whereas the contribution from heavy vector multiplets is always positive and tends on the contrary to reinforce it. These two effects are controlled respectively by Yukawa couplings and gauge charges, which mix one heavy and two light fields respectively in the superpotential and the Kähler potential. Finally we will also comment on similar effects induced in soft scalar masses when the heavy multiplets couple both to the visible and the hidden sector. This chapter is based on our paper [3].

5.1 General Considerations

In the following we are interested in studying the low-energy dynamics and in particular the question of vacuum metastability within the supersymmetric effective theory obtained by integrating out heavy fields in a manifestly supersymmetric way. In Chapter 3 we have shown that at leading order in the low-energy expansion in the number of derivatives, fermions and auxiliary fields, the basic recipe is that chiral and vector superfields can be integrated out by using approximate equations of motion corresponding to imposing stationarity of W and K respectively. In the forthcoming sections we will make more explicit the computation of the scalar masses in the low energy effective theory in the case in which only chiral multiplets are light; as anticipated, this analysis may play a relevant role in the study of the moduli sector of string models where supersymmetry is supposed to be spontaneously broken (see Section 2.3).

More specifically, we ask the practical question of what is the effect of heavy modes on the light masses, and in particular whether the induced corrections tend to improve or to worsen the situation concerning metastability of the vacuum. It would be very valuable to have some criterion to distinguish situations where the effect of heavy modes on the scalar square-masses are negative, and must therefore necessarily be computed to be able to assess vacuum stability, from situations where this effect is positive and can thus be safely ignored to check vacuum metastability. To derive such a criterion, we shall study in some detail the structure and the sign of the effect induced by heavy modes on the sGoldstino mass, which captures the crucial condition for achieving metastability. Most of the details are developed for simplicity in the rigid case; however, as we explained in Chapter 3 gravity does not introduce new complications and a similar analysis can be performed in the supergravity case. We comment on these aspects at the end of the chapter.

In order to illustrate the basic point that we want to make, let us consider a generic theory involving both light and heavy modes, indicated as ϕ^i and ϕ^α respectively, that interact among each other. For simplicity, we shall think of these as real scalar fields in a non-supersymmetric theory, but the results are clearly more general. In such a situation, one may define a low-energy effective theory for the light modes ϕ^i by integrating out the heavy modes ϕ^α. As we already discussed, at lowest order in the low-energy expansion, this can be done by requiring stationarity of the potential energy V with respect to the heavy modes and solving the equation $V_\alpha = 0$. This determines $\phi^\alpha = \phi^\alpha_0(\phi^i)$. By differentiating the stationarity equation with respect to the light fields, one also deduces that $\partial_i \phi^\alpha_0 = -V^{\alpha\beta}_{\rm inv} V_{\beta i}$, where $V^{\alpha\beta}_{\rm inv}$ denotes the inverse of $V_{\alpha\beta}$ as a matrix.[1]

[1]We use $(\ )_{\rm inv}$ instead of $(\ )^{-1}$ to indicate inverse matrices in order to clearly distinguish between two different kinds of inverse matrices which appear in the computations. More precisely, consider a general square matrix X; as a general rule we adopt the convention that the subscript "inv" defines the inverse of an arbitrary invertible sub-block of X, whereas the symbol "-1" is used to represent the inverse of the whole matrix X. For example, $X^{\alpha\beta}_{\rm inv}$ represents the inverse of the sub-block $X_{\alpha\beta}$ whereas $X^{-1\alpha\beta}$ represents the $(\alpha\beta)$ components of the inverse matrix X^{-1}. It is then obvious that in general

The effective Lagrangian for the low-energy theory is then obtained, as usual, by substituting back this solution into the original Lagrangian. For the wave-function factor and the potential, one easily obtains $g^{\rm eff}_{ij}(\phi^i) = (g_{ij} + \partial_i\phi_0^\alpha g_{\alpha j} + \partial_j\phi_0^\beta g_{i\beta} + \partial_i\phi_0^\alpha\partial_j\phi_0^\beta g_{\alpha\beta})(\phi^i,\phi_0^\alpha)$ and $V^{\rm eff}(\phi^i) = V(\phi^i,\phi_0^\alpha(\phi^i))$. The light masses may finally be derived by computing derivatives of $V^{\rm eff}$. Using the chain rule, these can be related to derivatives of V. One finds $V^{\rm eff}_i = V_i$ and $V^{\rm eff}_{ij} = V_{ij} - V_{i\alpha}V^{\alpha\beta}_{\rm inv}V_{\beta j}$, so that the light masses $m^{2\rm eff}_{ij} = V^{\rm eff}_{ij}$ are given by the following expression in terms of the light, heavy and mixing blocks $m^2_{ij} = V_{ij}$, $M^2_{\alpha\beta} = V_{\alpha\beta}$ and $\mu^2_{i\alpha} = V_{i\alpha}$ of the full mass matrix:

$$m^{2\rm eff}_{ij} = m^2_{ij} - \mu^2_{i\alpha}\, M^{-2\alpha\beta}\mu^2_{\beta j}\,. \tag{5.1}$$

This expression is easily seen to coincide with the mass matrix of light states obtained by diagonalizing the full mass matrix of the microscopic theory at leading order in an expansion in powers of the inverse heavy mass matrix. The formula (5.1) moreover shows that integrating out the heavy modes generically gives two types of effects on the masses of the light modes. The first is a direct effect hidden in the first term on the right hand side and is due to the fact that the light block of the mass matrix m^2_{ij} gets influenced by the coupling to the heavy modes. It has a sign that depends on the form of the couplings between light and heavy modes. The second is an indirect effect described by the second term on the right-hand side and is due to the fact that the presence of an off-diagonal block in the mass matrix mixing light and heavy fields makes the true light mass matrix differ from the original light block. It has a sign that is manifestly always negative. In parallel with what happens to a quantum mechanical system with two separated sets of low and high energy levels, we see that there is a direct effect correcting significantly the light energy levels and negligibly the heavy ones, which is due to diagonal interactions and can have any sign, and an indirect level-repulsion effect that further splits apart the two sets of levels, which is due to off-diagonal interactions and has a definite sign.

5.2 Effect of Heavy Chiral Multiplets

Let us now consider a situation where the chiral multiplets Φ^I split into a set of light multiplets Φ^i parametrizing the low-energy theory and a set of heavy multiplets Φ^α [2] with a large supersymmetric mass $W_{\alpha\beta}$ to be integrated out. For later convenience we recall the expressions of the scalar masses discussed in Section 1.3.1:

$$m^2_{0I\bar J} = \nabla_I W_K \nabla_{\bar J}\bar W^{K} - R_{I\bar J K\bar L}\, F^K \bar F^{\bar L}\,, \tag{5.2}$$

$$m^2_{0IJ} = -\nabla_I\nabla_J W_K\, F^K\,. \tag{5.3}$$

situations $X^{\alpha\beta}_{\rm inv}$ and $X^{-1\alpha\beta}$ do not coincide.

[2]Notice that here we come back to the notation of Chapter 3 in which capital indices indicate collectively light and heavy fields.

In order to distinguish light from heavy multiplets in a sensible way, we must assume that the supersymmetric mass mixing $W_{i\alpha}$ between them is not too large. We denote the heavy, light and mixing blocks of the whole supersymmetric mass matrix $\mathcal{M}_{1/2}$ in the following way:

$$\mathcal{M}_{1/2} = \left(\begin{array}{c|c} M & \mu \\ \hline \mu^\dagger & m \end{array}\right) , \tag{5.4}$$

where

$$M_{\alpha\beta} = W_{\alpha\beta} \,, \qquad \mu_{\alpha i} = W_{\alpha i} \,, \qquad m_{ij} = W_{ij} \,. \tag{5.5}$$

The most relevant interactions for our purposes will be the cubic terms in W, namely the Yukawa couplings (3.27)

$$\lambda_{\alpha ij} = W_{\alpha ij} \,, \qquad \lambda_{\alpha\beta j} = W_{\alpha\beta j} \,, \qquad \lambda_{\alpha\beta\gamma} = W_{\alpha\beta\gamma} \,. \tag{5.6}$$

At leading order in the low-energy expansion in number of derivatives, fermions and auxiliary fields, the low-energy effective theory can be obtained in component fields by imposing stationarity of V with respect to each heavy field and substituting back the solution into the original Lagrangian. Equivalently, this effective theory can be derived directly in superfields, by demanding the stationarity of W with respect to each heavy chiral multiplet. For convenience, we shall assume without loss of generality normal coordinates in the microscopic theory around the point under consideration. This substantially simplifies the computations, although the effective theory does not automatically inherit normal coordinates, due to the corrections induced to the Kähler metric.

The holomorphic coordinate transformations to go in normal coordinates is defined by asking that the Kähler connection Γ is locally vanishing at an arbitrary point of the scalar manifold. We get (see for example [129]):

$$\Phi^{I'} = \Lambda^{I'}{}_{I}\Big(\Phi^I + \frac{1}{2}\,\Gamma^I{}_{JK}|\Phi^J\Phi^K + \frac{1}{6}\,g^{I\bar{L}}\partial_P\Gamma_{\bar{L}JK}|\Phi^J\Phi^K\Phi^P + \cdots\Big) , \tag{5.7}$$

where $\Lambda^{I'}{}_{I}$ is defined in such a way that:

$$K_{I'\bar{J}'}| = \delta_{I'\bar{J}'} = \Lambda_{I'}^{\;I}\Lambda_{\bar{J}'}^{\;\bar{J}}K_{I\bar{J}}| \,. \tag{5.8}$$

In the new coordinate system, the Käheler potential can locally be approximated as:

$$K(\Phi,\bar{\Phi}) = K| + F(\Phi) + \bar{F}(\bar{\Phi}) + \delta_{I\bar{J}}\,\Phi^I\bar{\Phi}^J + \frac{1}{4}R_{I\bar{J}K\bar{L}}|\Phi^I\bar{\Phi}^{\bar{J}}\Phi^K\bar{\Phi}^{\bar{L}} + \mathcal{O}(\Phi^5) \,, \tag{5.9}$$

where F is an irrelevant holomorphic function. The Kähler metric is then:

$$g_{I\bar{J}}(\Phi,\bar{\Phi}) = \delta_{I\bar{J}} + R_{I\bar{J}K\bar{L}}|\Phi^K\bar{\Phi}^{\bar{L}} + \mathcal{O}(\Phi^3) \,. \tag{5.10}$$

We stopped at fourth order in the expansion of K since higher terms do not contribute in the computation of sGoldstinos mass.

The corrections due to the supersymmetric mass mixing between heavy and light multiplets are encoded in the following small dimensionless matrix:

$$\epsilon_i^\alpha = -M^{-1\alpha\beta}\mu_{\beta i}\,. \tag{5.11}$$

It should be emphasized that it is always possible to perform a holomorphic field redefinition in such a way to diagonalize the supersymmetric mass matrix W_{IJ} at a given point in field space, thereby setting ϵ_i^α to zero. This means that all the effects depending on ϵ_i^α only serve to compensate a choice of light and heavy fields that does not exactly diagonalize the supersymmetric part of the mass matrix, and therefore do not represent genuine non-trivial corrections. Moreover, since ϵ_i^α must be small, these effects are anyhow quantitatively irrelevant. We may then set $\epsilon_i^\alpha = 0$ by suitably choosing the fields. We shall however keep $\epsilon_i^\alpha \neq 0$ during the computations to verify more explicitly the above claims and set $\epsilon_i^\alpha = 0$ only at the very end. We can anticipate that all the tensorial quantities characterizing the light fields will receive additional contributions coming from heavy indices converted to light indices through the matrix ϵ_i^α. This leads us to introduce already at this stage the following deformed tensors:

$$g^\epsilon_{i\bar\jmath} = g_{i\bar\jmath} + \epsilon_i^\alpha g_{\alpha\bar\jmath} + \bar\epsilon_{\bar\jmath}^{\bar\beta} g_{i\bar\beta} + \epsilon_i^\alpha \bar\epsilon_{\bar\jmath}^{\bar\beta} g_{\alpha\bar\beta}\,, \tag{5.12}$$

$$\lambda^\epsilon_{\alpha ij} = \lambda_{\alpha ij} + \epsilon_i^\beta \lambda_{\alpha\beta j} + \epsilon_j^\gamma \lambda_{\alpha i\gamma} + \epsilon_i^\beta \epsilon_j^\gamma \lambda_{\alpha\beta\gamma}\,, \tag{5.13}$$

$$\begin{aligned} R^\epsilon_{i\bar\jmath k\bar l} = {} & R_{i\bar\jmath k\bar l} + \epsilon_i^\alpha R_{\alpha\bar\jmath k\bar l} + \bar\epsilon_{\bar\jmath}^{\bar\beta} R_{i\bar\beta k\bar l} + \epsilon_k^\gamma R_{i\bar\jmath\gamma\bar l} + \bar\epsilon_{\bar l}^{\bar\delta} R_{i\bar\jmath k\bar\delta} + \epsilon_i^\alpha \bar\epsilon_{\bar\jmath}^{\bar\beta} R_{\alpha\bar\beta k\bar l} \\ & + \epsilon_i^\alpha \epsilon_k^\gamma R_{\alpha\bar\jmath\gamma\bar l} + \epsilon_i^\alpha \bar\epsilon_{\bar l}^{\bar\delta} R_{\alpha\bar\jmath k\bar\delta} + \bar\epsilon_{\bar\jmath}^{\bar\beta} \epsilon_k^\gamma R_{i\bar\beta\gamma\bar l} + \bar\epsilon_{\bar\jmath}^{\bar\beta} \bar\epsilon_{\bar l}^{\bar\delta} R_{i\bar\beta k\bar\delta} + \epsilon_k^\gamma \bar\epsilon_{\bar l}^{\bar\delta} R_{i\bar\jmath\gamma\bar\delta} \\ & + \bar\epsilon_{\bar\jmath}^{\bar\beta} \epsilon_k^\gamma \bar\epsilon_{\bar l}^{\bar\delta} R_{i\bar\beta\gamma\bar\delta} + \epsilon_i^\alpha \epsilon_k^\gamma \bar\epsilon_{\bar l}^{\bar\delta} R_{\alpha\bar\jmath\gamma\bar\delta} + \epsilon_i^\alpha \bar\epsilon_{\bar\jmath}^{\bar\beta} \bar\epsilon_{\bar l}^{\bar\delta} R_{\alpha\bar\beta k\bar\delta} + \epsilon_i^\alpha \bar\epsilon_{\bar\jmath}^{\bar\beta} \epsilon_k^\gamma R_{\alpha\bar\beta\gamma\bar l} \\ & + \epsilon_i^\alpha \bar\epsilon_{\bar\jmath}^{\bar\beta} \epsilon_k^\gamma \bar\epsilon_{\bar l}^{\bar\delta} R_{\alpha\bar\beta\gamma\bar\delta}\,. \end{aligned} \tag{5.14}$$

Finally, we shall define the following quantity for later use, which characterizes the heavy block $W_{\alpha I}\, g^{I\bar J}\, \bar W_{\bar J\bar\beta}$ of the square of the supersymmetric mass matrix:

$$|M^\epsilon|^2_{\alpha\bar\beta} = M_{\alpha\gamma}\big(g^{\gamma\bar\delta} + \epsilon_i^\gamma g^{i\bar\delta} + \bar\epsilon_{\bar\jmath}^{\bar\delta} g^{\gamma\bar\jmath} + \epsilon_i^\gamma \bar\epsilon_{\bar\jmath}^{\bar\delta} g^{i\bar\jmath}\big)\bar M_{\bar\delta\bar\beta}\,. \tag{5.15}$$

In the following, we shall compute within the component approach the average sGoldstino mass in the low-energy effective theory, defined at a stationary point as [3]

$$m_\varphi^{2\text{eff}} = \frac{m^{2\text{eff}}_{0i\bar\jmath} F^{i\text{eff}} \bar F^{\bar\jmath\text{eff}}}{F^{k\text{eff}} \bar F_k^{\text{eff}}}\,. \tag{5.16}$$

We shall then reproduce the same result within the superfield approach by first computing the Riemann tensor $R^{\text{eff}}_{i\bar\jmath k\bar l}$ of the effective theory at a generic point and then

[3]For convenience in this chapter we indicate the mass in the Goldstino direction by m_φ^2, avoiding the heavier notation $m^2_{f\bar f}$ which would be confusing when generalized to the effective theory. For the same reason we will also avoid the symbol $R_{f\bar f f\bar f}$ introduced in the previous chapter to denote the contraction of the Riemann tensor with the Goldstino direction.

applying the standard expression for the sGoldstino mass at a stationary point (4.28), correctly reinterpreted in the new notation and applied to the effective theory where no vector multiplets are present. The relevant formula is:

$$m_{\varphi}^{2\text{eff}} = R^{\text{eff}} F^{i\text{eff}} \bar{F}_i^{\text{eff}} \,, \tag{5.17}$$

where we defined the effective sectional curvature [4]

$$R^{\text{eff}} = -\frac{R^{\text{eff}}_{i\bar{\jmath}k\bar{l}} F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}} F^{k\text{eff}} \bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}} \bar{F}_m^{\text{eff}})^2} \,. \tag{5.18}$$

5.2.1 Component Approach

Consider first the component approach. For simplicity we shall focus on the bosonic fields and discard fermions, since we are interested in computing effective scalar masses; in the low-energy expansion the values of the heavy scalar fields are defined by

$$\phi^\alpha = \phi_0^\alpha(\phi^i, \bar{\phi}^{\bar{\imath}}) \qquad \text{solution of} \qquad V_\alpha(\phi^i, \bar{\phi}^{\bar{\imath}}, \phi_0^\alpha, \bar{\phi}_0^{\bar{\alpha}}) = 0 \,. \tag{5.19}$$

At leading order in the number of auxiliary fields, this stationarity condition implies that $W_{\alpha I}\bar{W}^I = 0$ and gives the following values for the heavy auxiliary fields:

$$F^\alpha = \epsilon_i^\alpha F^i \,. \tag{5.20}$$

The effective theory for the light fields is then obtained by substituting these expressions for ϕ^α and F^α into the original Lagrangian.

To derive the effective theory, we will need to compute the derivatives of the heavy fields ϕ_0^α and $\bar{\phi}_0^{\bar{\alpha}}$ with respect to the light fields ϕ^i. These can be deduced, as in the non-supersymmetric case, by differentiating the stationarity conditions $V_\alpha = 0$ with respect to the light fields. One finds:

$$\frac{\partial \phi^\alpha}{\partial \phi^i} = -M_0^{-2\alpha\bar{\beta}} \mu^2_{0\bar{\beta}i} - M_0^{-2\alpha\beta} \mu^2_{0\beta i} \,, \tag{5.21}$$

$$\frac{\partial \bar{\phi}^{\bar{\alpha}}}{\partial \phi^i} = -M_0^{-2\bar{\alpha}\beta} \mu^2_{0\beta i} - M_0^{-2\bar{\alpha}\bar{\beta}} \mu^2_{0\bar{\beta}i} \,. \tag{5.22}$$

where we defined the heavy and off-diagonal blocks of the complete scalar mass matrix $\mathcal{M}_0^2$ of the microscopic theory:

$$\mathcal{M}_0^2 = \left(\begin{array}{c|c} M_0^2 & \mu_0^2 \\ \hline \mu_0^{\dagger\,2} & m_0^2 \end{array} \right) , \tag{5.23}$$

[4]Notice the minus sign introduced to match the usual definition of sectional curvature for real manifolds. See App. B of [130].

where

$$M_0^2 = \left(\begin{array}{c|c} V_{\alpha\bar\beta} & V_{\alpha\beta} \\ \hline V_{\bar\alpha\bar\beta} & V_{\bar\alpha\beta} \end{array}\right), \qquad \mu_0^2 = \left(\begin{array}{c|c} V_{\alpha\bar\jmath} & V_{\alpha j} \\ \hline V_{\bar\alpha\bar\jmath} & V_{\bar\alpha j} \end{array}\right), \qquad m_0^2 = \left(\begin{array}{c|c} V_{i\bar\jmath} & V_{ij} \\ \hline V_{\bar\imath\bar\jmath} & V_{\bar\imath j} \end{array}\right), \quad (5.24)$$

Notice that μ_0 and M_0 differ from μ and M, since the former refer to the full mass matrix (5.2)-(5.3) whereas the latter parametrize only its supersymmetric part, namely W_{IJ}. More precisely:

$$(M_0^2)_{\alpha\bar\beta} = V_{\alpha\bar\beta}\,, \qquad (\mu_0^2)_{\alpha\bar\jmath} = V_{\alpha\bar\jmath}\,, \qquad (m_0^2)_{i\bar\jmath} = V_{i\bar\jmath}\,, \quad (5.25)$$

coincide with M^2, μ^2 and m^2 in (5.5) only in the supersymmetric limit. At quadratic order in the auxiliary fields one finds:

$$M_0^{-2\alpha\bar\beta} = V_{\text{inv}}^{\alpha\bar\beta} + V_{\text{inv}}^{\alpha\bar\gamma}\, V_{\bar\gamma\bar\delta}\, V_{\text{inv}}^{\bar\delta\sigma} V_{\sigma\tau}\, V_{\text{inv}}^{\tau\bar\beta}\,, \quad (5.26)$$

$$M_0^{-2\alpha\beta} = -V_{\text{inv}}^{\alpha\bar\gamma}\, V_{\bar\gamma\bar\delta}\, V_{\text{inv}}^{\bar\delta\beta}\,. \quad (5.27)$$

It is convenient for the forthcoming analysis to roughly estimate the leading powers of auxiliary fields F in each term of previous expressions. We have that $V_{\text{inv}}^{\alpha\bar\beta} \sim a\,F^0 + b\,F^2 + \mathcal{O}(F^3)$ whereas $V_{\alpha\beta} \sim c\,F$ and this implies, at quadratic order in F, that $M_0^{-2\alpha\bar\beta}$ contains only terms which are quadratic or constant in F whereas $M_0^{-2\alpha\beta}$ contains only linear terms.

The effective Kähler metric of the light fields can be determined by looking at the scalar kinetic terms and substituting the values of the heavy scalar fields. One may in this case work at leading order in the auxiliary fields, since these terms already involve two derivatives. Focusing also on the leading order in the light masses and the heavy-light mass mixing, the relations (5.21) and (5.22) then simplify to $\partial_i\phi^\alpha = \epsilon_i^\alpha + \mathcal{O}(F)$ and $\partial_i\bar\phi^{\bar\alpha} = \mathcal{O}(F)$. Using these expressions, which actually turn out to be correct even at order ϵ^2, one finds that the kinetic term can be rewritten in the standard supersymmetric form with an effective Kähler metric given by

$$g_{i\bar\jmath}^{\text{eff}} = g_{i\bar\jmath}^{\epsilon}\,. \quad (5.28)$$

The effective mass matrix of the light scalar fields can on the other hand be determined by using the supersymmetric generalization of the expression (5.1), which can be derived by using the same logic. More precisely the effective masses are obtained by taking holomorphic and anti-holomorphic derivatives of $V_i^{\text{eff}} = V_i + V_\alpha \frac{\partial\phi_0^\alpha}{\partial\phi^i} + V_{\bar\alpha}\frac{\partial\bar\phi_0^{\bar\alpha}}{\partial\phi^i} = V_i$. One then finds:

$$\begin{aligned} m_{0i\bar\jmath}^{2\text{eff}} &= m_{0i\bar\jmath}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\beta}\mu_{0\beta\bar\jmath}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\bar\beta}\mu_{0\bar\beta\bar\jmath}^2 \\ &\quad - \mu_{0i\alpha}^2 M_0^{-2\alpha\beta}\mu_{0\beta\bar\jmath}^2 - \mu_{0i\alpha}^2 M^{-2\alpha\bar\beta}\mu_{0\bar\beta\bar\jmath}^2\,, \end{aligned} \quad (5.29)$$

$$\begin{aligned} m_{0ij}^{2\text{eff}} &= m_{0ij}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\bar\beta}\mu_{0\bar\beta j}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\beta}\mu_{0\beta j}^2 \\ &\quad - \mu_{0i\alpha}^2 M_0^{-2\alpha\bar\beta}\mu_{0\bar\beta j}^2 - \mu_{0i\alpha}^2 M_0^{-2\alpha\beta}\mu_{0\beta j}^2\,. \end{aligned} \quad (5.30)$$

As in the general non-supersymmetric case, the result corresponds to a perturbative diagonalization of the full scalar mass matrix $\mathcal{M}_0^2$, at leading order in the inverse mass matrix of the heavy scalars M_0^{-2}:

$$m_0^{2\text{eff}} = m_0^2 - \mu_0^{\dagger\,2}\, M_0^{-2}\, \mu_0^2\,. \tag{5.31}$$

Let us now focus on the Hermitian block $m_{0i\bar{\jmath}}^{2\text{eff}}$. Using eqs. (5.26) and (5.27) in the formula (5.29), and restricting to terms that are at most quadratic in the auxiliary fields as demanded by supersymmetry at the two-derivative level, we see that there are three kinds of effects coming from the four correction terms. The first type involves second derivatives of W and no auxiliary fields, and comes only from the first correction term. The second type involves the Riemann tensor and two auxiliary fields, and comes again only from the first correction term. The third type involves third derivatives of W and two auxiliary fields, and comes from all four correction terms. All together, these three effects give a negative level-repulsion correction with respect to $m_{0i\bar{\jmath}}^2$.

Let us now compute more specifically the average sGoldstino mass $m_\varphi^{2\text{eff}}$ defined by eq. (5.16) at a stationary point of the effective theory and compare it to its analogue $m_\varphi^2 = m_{0i\bar{\jmath}}^2 f^i \bar{f}^{\bar{\jmath}}$ in the microscopic theory. Recall that we are using normal coordinates, so that $g_{i\bar{\jmath}} = \delta_{i\bar{\jmath}}$ and $g_{i\bar{\jmath}}^{\text{eff}} = \delta_{i\bar{\jmath}} + \epsilon_i^\alpha \bar{\epsilon}_{\bar{\jmath}}^{\bar{\alpha}}$. The first thing we need to make more explicit are the effective auxiliary fields. To do so we start by deriving W^{eff} by substituting the solution (5.19) into in W. Taking a derivative we then find that $W_i^{\text{eff}} = W_i + \epsilon_i^\alpha W_\alpha$. But using the stationarity condition $W_{\alpha I}\bar{W}_{\bar{I}} = 0$ of the heavy scalars we see that $W_\alpha = \bar{\epsilon}_{\bar{\imath}}^{\bar{\alpha}} W_i$, so that $W_i^{\text{eff}} = (\delta_{i\bar{\jmath}} + \epsilon_i^\alpha \bar{\epsilon}_{\bar{\jmath}}^{\bar{\alpha}})W_j = g_{i\bar{\jmath}}^{\text{eff}} W_j$. The auxiliary fields in the effective theory thus coincide with the light components of the auxiliary fields in the microscopic theory: $F^{i\text{eff}} = -g^{\text{eff}\,i\bar{\jmath}}\bar{W}_{\bar{\jmath}}^{\text{eff}} = -\bar{W}_{\bar{\imath}} = F^i$. Recalling (5.20) one also finds that $g_{i\bar{\jmath}}^{\text{eff}} F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}} = F^I \bar{F}^{\bar{I}}$. In summary, we get:

$$F^{i\text{eff}} = F^i\,, \quad F^{i\text{eff}} \bar{F}_i^{\text{eff}} = F^I \bar{F}_I\,. \tag{5.32}$$

To proceed, we also need to compute more explicitly the mass-matrix blocks (5.26) and (5.27) entering in the expression (5.29) for the effective mass matrix $m_{0i\bar{\jmath}}^{2\text{eff}}$. In normal coordinates, these quantities depend on $|M^\epsilon|_{\alpha\bar{\beta}}^2 = M_{\alpha\gamma}\big(\, g^{\gamma\bar{\delta}} + \epsilon_i^\gamma \bar{\epsilon}_{\bar{\imath}}^{\bar{\delta}}\,\big) \bar{M}_{\bar{\delta}\bar{\beta}}$ (see expression (5.15)), and at quadratic order in the auxiliary fields one finds that

$$V_{\alpha\bar{\beta}} \;=\; |M^\epsilon|_{\alpha\bar{\beta}}^2 - R_{\alpha\bar{\beta}K\bar{L}} F^K \bar{F}^{\bar{L}}\,, \tag{5.33}$$

$$V_{\alpha\beta} \;=\; -\lambda_{\alpha\beta K} F^K\,, \tag{5.34}$$

$$V_{\text{inv}}^{\alpha\bar{\beta}} \;=\; |M^\epsilon|^{-2\alpha\bar{\beta}} + |M^\epsilon|^{-2\alpha\bar{\delta}} |M^\epsilon|^{-2\bar{\beta}\gamma} R_{\gamma\bar{\delta}K\bar{L}} F^K \bar{F}^{\bar{L}}\,. \tag{5.35}$$

We are now in position to evaluate the average sGoldstino mass in the effective theory by computing the four correction terms in eq. (5.29). As explained after eqs. (5.29) and (5.30), these give rise to three types of effects. But when looking along the sGoldstino direction, some simplifications occur, due to the fact that only supersymmetry-breaking effects matter. The first type of effect cancels the corresponding leading part of $m^2_{0i\bar{\jmath}}$. The second type of effect combines with the corresponding subleading term in $m^2_{0i\bar{\jmath}}$ to reconstruct the average sGoldstino mass of the microscopic theory. The third type of effect gives instead a genuine correction. The precise evaluation of these effects can be simplified by noticing that at a stationary point $W_{IJ}\bar{W}_{\bar{J}} = 0$, which implies that at leading order in the auxiliary fields $V_{\alpha\bar{\imath}}W_i = -V_{\alpha\bar{\beta}}W_\beta$. After a straightforward computation one finds that $m_\varphi^{2\text{eff}} = -\big(R_{I\bar{J}K\bar{L}} + \lambda_{\alpha IK}|M^\epsilon|^{-2\alpha\bar{\beta}}\bar{\lambda}_{\bar{\beta}\bar{J}\bar{L}}\big)F^I\bar{F}^{\bar{J}}F^K\bar{F}^{\bar{L}}/F^M\bar{F}_M$. Recalling then that $F^\alpha = \epsilon_i^\alpha F^{i\text{eff}}$ and $F^I\bar{F}_I = F^{i\text{eff}}\bar{F}_i^{\text{eff}}$, one may finally rewrite the above result as

$$m_\varphi^{2\text{eff}} = \big(R^\epsilon - \lambda_\alpha^\epsilon|M^\epsilon|^{-2\alpha\bar{\beta}}\bar{\lambda}_{\bar{\beta}}^\epsilon\big)F^{i\text{eff}}\bar{F}_i^{\text{eff}}\,, \tag{5.36}$$

with

$$R^\epsilon = -\frac{R^\epsilon_{i\bar{\jmath}k\bar{l}}F^{i\text{eff}}\bar{F}^{\bar{\jmath}\text{eff}}F^{k\text{eff}}\bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}}\bar{F}_m^{\text{eff}})^2}\,, \tag{5.37}$$

$$\lambda_\alpha^\epsilon = \frac{\lambda^\epsilon_{\alpha ij}F^{i\text{eff}}F^{j\text{eff}}}{F^{k\text{eff}}\bar{F}_k^{\text{eff}}}\,. \tag{5.38}$$

The first term in the result (5.36) corresponds to m_φ^2, whereas the second term describes a negative level-repulsion effect controlled by the Yukawa couplings $\lambda_{\alpha ij}$ mixing one heavy and two light fields. As anticipated, the dependence on ϵ amounts to a transformation of all the tensorial quantities accounting for the need to disentangle light from heavy eigenmodes of the supersymmetric mass matrix, and can thus be dropped by setting ϵ to zero.

5.2.2 Superfield Approach

The above results can also be derived by integrating out the heavy fields directly at the superfield level, and then computing the sGoldstino mass in the resulting effective theory by applying eqs. (5.17) and (5.18). To do this, one derives the effective Kähler potential and superpotential by solving the following approximate superfield equations of motion:

$$\Phi^\alpha = \Phi^\alpha(\Phi^i)\ \text{ solution of }\ W_\alpha(\Phi^i,\Phi^\alpha) = 0\,. \tag{5.39}$$

As discussed in Chapter 3, the bosonic components of this superfield equations of motion coincide, at leading order in the number of fermions and auxiliary fields, with the equations of motion (5.19)–(5.20) that we have used in the component approach.

To proceed, we will need to compute the first and second derivatives of the heavy scalar fields with respect to the light scalar fields. These can be derived by differentiating eq. (5.39), and one finds the following results:

$$\frac{\partial\phi^\alpha}{\partial\phi^i} = \epsilon_i^\alpha\,,\quad \frac{\partial^2\phi^\alpha}{\partial\phi^i\partial\phi^j} = -M^{-1\alpha\beta}\lambda^\epsilon_{\beta ij}\,. \tag{5.40}$$

The effective geometry can be derived by taking derivatives with respect to the light fields of the effective Kähler potential K^{eff}, where the heavy fields have been substituted by the solution (5.39) in terms of light fields. We focus again on a given point in the light field space, around which we choose normal coordinates, but this point no longer needs to be a stationary point. Then, using the chain rule and eqs. (5.40), one easily computes $K^{\rm eff}_{i\bar{\jmath}} = \delta_{i\bar{\jmath}}+\epsilon^{\alpha}_{i}\bar{\epsilon}^{\bar{\alpha}}_{\bar{\jmath}}$, $K^{\rm eff}_{\bar{\imath}jk} = -M^{-1\alpha\beta}\bar{\epsilon}^{\bar{\alpha}}_{\bar{\imath}}\lambda^{\epsilon}_{\beta jk}$ and $K^{\rm eff}_{i\bar{\jmath}k\bar{l}} = R^{\epsilon}_{i\bar{\jmath}k\bar{l}}+\lambda^{\epsilon}_{\alpha ik}|M|^{-2\alpha\bar{\beta}}\bar{\lambda}^{\epsilon}_{\bar{\beta}\bar{\jmath}\bar{l}}$. This finally implies that the effective metric is given by $g^{\rm eff}_{i\bar{\jmath}} = g^{\epsilon}_{i\bar{\jmath}}$, the effective Christoffel symbol by $\Gamma^{\rm eff}_{\bar{\imath}jk} = -M^{-1\alpha\beta}\,\bar{\epsilon}^{\bar{\alpha}}_{\bar{\imath}}\,\lambda^{\epsilon}_{\beta jk}$ and finally the effective Riemann tensor by the following expression:

$$R^{\rm eff}_{i\bar{\jmath}k\bar{l}} = R^{\epsilon}_{i\bar{\jmath}k\bar{l}} + \lambda^{\epsilon}_{\alpha ik}|M^{\epsilon}|^{-2\alpha\bar{\beta}}\bar{\lambda}^{\epsilon}_{\bar{\beta}\,\bar{\jmath}\,\bar{l}}\,. \tag{5.41}$$

Plugging this expression into eqs. (5.17) and (5.18), we then reproduce the form of the result (5.36).[5]

5.3 Effect of Heavy Vector Multiplets

Let us now suppose that all the vector multiplets have a large supersymmetric mass, much larger than the splittings induced by supersymmetry breaking. We may then integrate out in a supersymmetric way the modes associated with these heavy vector multiplets, paying attention to the fact that in order to become massive they absorb the modes of some chiral multiplets.

For later convenience we recall the expressions of the scalar masses discussed in Section 1.3.2:

$$\begin{aligned} m^2_{0I\bar{J}} &= \nabla_I W_K \nabla_{\bar{J}} \bar{W}^K - R_{I\bar{J}K\bar{L}}\, F^K \bar{F}^{\bar{L}} + h^{ab}\bar{X}_{aI}X_{b\bar{J}} + h^{ab}h_{acI}h_{bd\bar{J}}\, D^b D^c \\ &\quad + \big(i\nabla_I X_{a\bar{J}} - ih^{bc}h_{abI}X_{c\bar{J}} + ih^{bc}h_{ab\bar{J}}\bar{X}_{cI}\big)D^a\,, \end{aligned} \tag{5.42}$$

$$\begin{aligned} m^2_{0IJ} &= -\nabla_I\nabla_J W_K\, F^K - h^{ab}\bar{X}_{aI}\bar{X}_{bJ} - \frac{1}{2}\big(\nabla_I h_{abJ} - 2h^{cd}h_{acI}h_{bdJ}\big)\, D^a D^b \\ &\quad + 2i\, h^{bc}h_{ab(I}\bar{X}_{cJ)}D^a\,, \end{aligned} \tag{5.43}$$

and the relation between F and D auxiliary fields valid at the vacuum (1.53):

$$i\nabla_I X_{a\bar{J}}\, F^I \bar{F}^{\bar{J}} - g_{I\bar{J}}X^I_{(a}\bar{X}^{\bar{J}}_{b)}\, D^b + \frac{1}{2}f_{ab}{}^{d}\theta_{dc}\, D^b D^c = 0\,. \tag{5.44}$$

The relevant scales in this case are the supersymmetric mass matrix $2g_{I\bar{J}}X^I_{(a}\bar{X}^{\bar{J}}_{b)} = \frac{1}{2}K_{ab}$ of the heavy vector multiplets and the quantity $iX_{aI} = \frac{1}{2}K_{aI}$ controlling the supersymmetric mixing between vector multiplets and chiral multiplets:

$$M^2_{ab} = \frac{1}{2}K_{ab}\,, \quad \nu_{aI} = \frac{1}{2}K_{aI}\,. \tag{5.45}$$

[5]Note that the results derived in this subsection are evaluated at values of the heavy scalar fields solving $W_\alpha = 0$, whereas the results of previous subsection were evaluated at values of the heavy scalar fields solving $V_\alpha = 0$. However it turns out that the difference between these two values is subleading in the counting of auxiliary fields and can therefore be discarded.

The couplings that are expected to be relevant are instead given by the cubic couplings in K, namely the generalized charges

$$Q_{aI\bar{J}} = -\frac{1}{2}K_{aI\bar{J}}\,, \quad Q_{abI} = -\frac{1}{2}K_{abI}\,, \quad Q_{abc} = -\frac{1}{2}K_{abc}\,. \tag{5.46}$$

At leading order in the expansion in number of derivatives, fermions and auxiliary fields, the low-energy effective theory for the light chiral multiplets can again be obtained in two different but equivalent ways. One may proceed in components and integrate out the heavy modes associated to the vector multiplets and the chiral multiplets that they absorb, by requiring stationarity of V with respect to them. One may however also proceed in superfields and integrate out the heavy vector superfields by requiring stationarity of K with respect to them. For convenience, we shall as before assume without loss of generality normal coordinates in the microscopic theory around the point under consideration.

In analogy with what happens in the case of only chiral multiplets, we expect that the corrections due to the supersymmetric mixing between heavy and light multiplets should be encoded in following parameter of dimension one:

$$\delta^a_I = -M^{-2ab}\nu_{bI}\,. \tag{5.47}$$

In this case, such a parameter cannot be set to zero by a simple holomorphic field redefinition, because it corresponds to the non-holomorphic mixing between the heavy gauge fields and the corresponding real would-be Goldstone modes. However, it can be set to zero by making a suitable choice of gauge. With any different choice of gauge, δ^a_I would be non-zero and the terms depending on it in the effective theory would take into account the mixing between light and heavy fields. By doing the computation in such a gauge one would presumably end up getting deformed versions of all the tensorial quantities for light fields, involving additional contributions where heavy indices are converted to light indices by δ^a_I. We shall however refrain from keeping a general $\delta^a_I \neq 0$ and set $\delta^a_I = 0$ from the beginning by choosing the unitary gauge.

To perform the splitting between light and heavy fields and the gauge fixing more precisely, we may start by splitting the chiral multiplets Φ^I into those that are orthogonal and those that are parallel to the Killing vectors X^I_a evaluated at the point under consideration. This decomposition can be done more explicitly with the help of the parallel projector $P^I{}_J = 2X^I_a M^{-2ab}\bar{X}_{bJ}$. We shall denote these two sets of fields respectively with Φ^i and Φ^a. The orthogonal components Φ^i define the light chiral multiplets of the low-energy effective theory. The parallel components Φ^a are instead either heavy or eliminable through the gauge fixing.

In the following, we shall follow the same logic as in the previous section and first compute within the component approach the average sGoldstino mass in the low-energy effective theory, defined at a stationary point as

$$m^{2\text{eff}}_{\varphi} = \frac{m^{2\text{eff}}_{0i\bar{\jmath}}F^{i\text{eff}}\bar{F}^{\bar{\jmath}\text{eff}}}{F^{k\text{eff}}\bar{F}^{\text{eff}}_{k}}\,. \tag{5.48}$$

As we discussed in the previous chapter, in the limit in which the vector masses are much larger than the supersymmetry breaking scale, the danger associated to the Killing directions disappear, and the Goldstino direction is the only relevant direction we should consider to discuss metastability.

We will finally reproduce the same result within the superfield approach by first computing the Riemann tensor of the effective theory at a generic point and then plugging it in the expression for the sGoldstino mass at a stationary point within the effective theory, which is given by

$$m_\varphi^{2\text{eff}} = R^{\text{eff}} F^{i\text{eff}} \bar{F}_i^{\text{eff}} \,, \tag{5.49}$$

in terms of an effective sectional curvature

$$R^{\text{eff}} = -\frac{R^{\text{eff}}_{i\bar{\jmath}k\bar{l}} F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}} F^{k\text{eff}} \bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}} \bar{F}^{\text{eff}}_m)^2} \,. \tag{5.50}$$

5.3.1 Component Approach

Let us first consider the component approach, where it is convenient to choose the Wess-Zumino gauge for the extra gauge symmetries implied by supersymmetry. For simplicity we shall as before focus on bosonic fields and discard fermions since we are interested in scalar masses. The relevant bosonic heavy modes coming from V^a and Φ^a are the following. In the vector multiplets V^a, the gauge fields A_μ^a contain heavy physical modes and should of course be considered. In the chiral multiplets Φ^a, on the other hand, the modes $\sigma^a = \text{Re}(\phi^a)$ correspond to the would-be Goldstone modes and can be eliminated by choosing the unitary gauge for the standard gauge symmetries, where the corresponding degrees of freedom are the longitudinal polarizations of the gauge bosons, whereas the modes $\rho^a = \text{Im}(\phi^a)$ are physical and, as we already discussed, they have a mass comparable to that of the vector fields, so that they must be considered. At leading order in the low-energy expansion, the heavy bosonic fields A_μ^a and ρ^a can then be integrated out by using the following approximate equations of motion:

$$\rho^a = \rho_0^a(\phi^i, \bar{\phi}^{\bar{\imath}}) \qquad \text{solution of} \qquad V_a(\phi^i, \bar{\phi}^{\bar{\imath}}, \rho_0^a) = 0 \,, \tag{5.51}$$

$$A_\mu^a = 0 \,. \tag{5.52}$$

Concerning the auxiliary fields, notice that those coming from the parallel chiral multiplets automatically vanish, as a consequence of the gauge invariance of the superpotential

$$W_I X_a^I = 0 \,, \tag{5.53}$$

whereas those of the vector multiplets are given by eq. (5.44), which corresponds to the equation of motion of ρ^a and reduces approximately to $Q_{aI\bar{J}} F^I \bar{F}^{\bar{J}} - \frac{1}{2} M_{ab}^2 D^b = 0$.

At leading order in the low-energy expansion one then finds:

$$F^a = 0\,, \tag{5.54}$$

$$D^a = 2\,M^{-2ab} Q_{bi\bar{\jmath}}\,F^i \bar{F}^{\bar{\jmath}}\,. \tag{5.55}$$

The effective theory for the light fields is finally obtained by substituting these expressions into the Lagrangian.

To derive the effective theory, one needs in principle to compute the derivatives of ρ_0^a with respect to ϕ^i. This can be deduced by taking a derivative of the stationarity condition for ρ^a with respect to ϕ^i. One then finds a result that is inversely proportional to the mass matrix of ρ^a, which is approximately equal to that of the vectors, and directly proportional to the mass mixing between ρ^a and ϕ^i. This mixing can be computed explicitly and after using the relation (5.53) ensuring the gauge invariance of W, as well as its first and second derivatives, one verifies that it contains only terms that are quadratic in the auxiliary fields or linear in the auxiliary fields but further suppressed by the ratio between light chiral masses and heavy vector mass, which must all be neglected. As a result, one finds:

$$\frac{\partial \rho^a}{\partial \phi^i} = 0\,. \tag{5.56}$$

The effective Kähler metric of the light fields is not affected. Indeed, neither A_μ^a nor ρ^a give any effect in the kinetic terms, as a consequence of eqs. (5.52) and (5.56). One thus simply finds:

$$g^{\rm eff}_{i\bar{\jmath}} = g_{i\bar{\jmath}}\,. \tag{5.57}$$

The effective scalar mass matrices can be computed by taking into account both the direct effect of the heavy modes on the microscopic mass evaluated in the light scalar directions ϕ^i and the indirect level-repulsion effect coming from the mass mixing with the heavy scalar directions ρ^a. It turns however out that the level-repulsion effect is negligible, for essentially the same reasons as those leading to eq. (5.56). We thus finally get:

$$m^{2\rm eff}_{0i\bar{\jmath}} = m^2_{0i\bar{\jmath}}\,, \tag{5.58}$$

$$m^{2\rm eff}_{0ij} = m^2_{0ij}\,. \tag{5.59}$$

There is nevertheless a direct effect in the Hermitian block $m^{2\rm eff}_{0i\bar{\jmath}}$, which consists of two significant contributions in $m^2_{0i\bar{\jmath}}$ coming from the couplings to heavy fields. The first contribution comes from plugging back the small but non-vanishing value of D^a into the last term of (5.42); notice that the terms proportional to the first derivatives of the gauge kinetic function vanish as a consequence of the orthogonality of F^I and X_a^I. This effect is easily evaluated by using eq. (5.55), and one finds $Q_{ai\bar{\jmath}}D^a = 2\,Q_{ai\bar{\jmath}}M^{-2ab}Q_{bk\bar{l}}F^k\bar{F}^{\bar{l}}$. The second contribution arises instead from the part of the first term in (5.42) that corresponds to values for the summed index K that run over the parallel chiral modes that are integrated out. It can be evaluated by using the projected metric $P^{I\bar{J}} = 2X_a^I M^{-2ab}\bar{X}_b^{\bar{J}}$, and reads $\nabla_i W_a \nabla_{\bar{\jmath}} \bar{W}^a = \nabla_i W_K P^{K\bar{L}} \nabla_{\bar{\jmath}} \bar{W}_{\bar{L}} = 2X_a^K \nabla_i W_K M^{-2ab} X_b^{\bar{L}} \nabla_{\bar{\jmath}} \bar{W}_{\bar{L}}$. But taking a derivative of the gauge invariance condition for the superpotential eq. (5.53) one deduces that $X_a^K \nabla_i W_K = -iQ_{ai\bar{K}}\bar{F}^{\bar{K}} = -iQ_{ai\bar{k}}\bar{F}^{\bar{k}}$, and finally $\nabla_i W_a \nabla_{\bar{\jmath}} \bar{W}^a = 2\,Q_{ai\bar{l}} M^{-2ab} Q_{bk\bar{\jmath}} F^k \bar{F}^{\bar{l}}$. These two contributions represent a direct correction to all the masses, which may be either positive or negative depending on the value of the charges along the direction that is considered.

Let us now evaluate more precisely the average sGoldstino mass defined by eq. (5.48) at a stationary point of the effective theory and compare it to its analogue defined by eqs. (5.49) and (5.50) in the microscopic theory. Along the supersymmetry breaking direction $F^{i\text{eff}} = F^i$ the two direct corrections discussed above give identical contributions that sum up and one easily finds:

$$m_\varphi^{2\text{eff}} = \left(R + 4\,Q_a M^{-2ab} Q_b\right) F^{i\text{eff}} \bar{F}_i^{\text{eff}}\,, \tag{5.60}$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}} F^{k\text{eff}} \bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}} \bar{F}_m^{\text{eff}})^2}\,, \tag{5.61}$$

$$Q_a = \frac{Q_{ai\bar{\jmath}}\, F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}}}{F^{k\text{eff}} \bar{F}_k^{\text{eff}}}\,. \tag{5.62}$$

The first term in the result (5.60) corresponds to m_φ^2, whereas the second term describes a positive direct effect controlled by the charges $Q_{ai\bar{\jmath}}$ mixing one heavy and two light fields. The absence of any indirect level-repulsion effect is due to the absence of genuine heavy chiral multiplets mixing to the light chiral multiplets. The above result reproduces through a more precise derivation the result advocated in [61].

5.3.2 Superfield Approach

It is straightforward to show that the above results can also be obtained by integrating out the heavy vector multiplets at the level of superfields. The only complication is that one should switch from the unitary plus Wess-Zumino gauge used in the component formulation, which fix respectively the standard and the extra gauge symmetries, to a supersymmetric unitary gauge to be used in the superfield formulation, which fixes at once all the multiplet of gauge symmetries. More precisely, we shall gauge fix all the parallel chiral multiplets Φ^a to constant values coinciding with their values at the stationary point. The superfields V^a become however general vector superfields in this gauge, and compared to the Wess-Zumino gauge that was chosen in the component approach, the modes that were described by the real scalar fields ρ^a in the Φ^a have now been transfered to the real scalar fields C^a in the general V^a. In this supersymmetric gauge, all the heavy degrees of freedom are thus contained in V^a, and can be integrated out by using the following approximate superfield equations of motion:

$$V^a = V^a(\Phi^i, \bar{\Phi}^{\bar{\imath}}) \text{ solution of } K_a(\Phi^i, \bar{\Phi}^{\bar{\imath}}, V^a) = 0\,. \tag{5.63}$$

The bosonic components of this superfield equations of motion map to the equations of motion (5.51)–(5.55) that we have used in the component approach, modulo the different gauge choice.

To proceed, we will need to compute the first and second derivatives of the lowest component of the heavy vector superfields with respect to the light scalar superfields. These can be derived by differentiating eq. (5.63), and at the point under consideration where $K_{ai} = 0$ one finds the following results:

$$\frac{\partial c^a}{\partial \phi^i} = 0\,, \quad \frac{\partial^2 c^a}{\partial \phi^i \partial \bar{\phi}^{\bar{\jmath}}} = M^{-2ab} Q_{bi\bar{\jmath}}\,. \tag{5.64}$$

The effective geometry can be derived by taking derivatives with respect to the light fields of the effective Kähler potential $K^{\rm eff}$, where the heavy fields have been substituted by the solution (5.63) in terms of light fields. We focus again on a given point in the light field space and use normal coordinates. Then, using the chain rule and eq. (5.64), and noticing that $K_{aij} = 0$ and $K_{ai\bar{\jmath}} = -2Q_{ai\bar{\jmath}}$, one easily computes $K^{\rm eff}_{i\bar{\jmath}} = \delta_{i\bar{\jmath}}$, $K^{\rm eff}_{\bar{\imath}jk} = 0$ and $K^{\rm eff}_{i\bar{\jmath}k\bar{l}} = K_{i\bar{\jmath}k\bar{l}} - 2\,Q_{ai\bar{\jmath}} M^{-2ab} Q_{bk\bar{l}} - 2\,Q_{ai\bar{l}} M^{-2ab} Q_{bk\bar{\jmath}}$. This finally implies that the effective metric is given by $g^{\rm eff}_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, the effective Christoffel symbol by $\Gamma^{\rm eff}_{\bar{\imath}jk} = 0$ and the effective Riemann tensor by the following expression:

$$R^{\rm eff}_{i\bar{\jmath}k\bar{l}} = R_{i\bar{\jmath}k\bar{l}} - 2\,Q_{ai\bar{\jmath}} M^{-2ab} Q_{bk\bar{l}} - 2\,Q_{ai\bar{l}} M^{-2ab} Q_{bk\bar{\jmath}}\,. \tag{5.65}$$

Plugging this expression into eqs. (5.49) and (5.50), we then reproduce the form of the result (5.60).

5.4 Summary

Summarizing, we have shown that integrating out heavy chiral multiplets Φ^α and vector multiplets V^a with large and approximately supersymmetric mass matrices $M^{2\alpha\bar{\beta}}$ and M^{2ab} induces corrections to the square masses of light scalars ϕ^i that are due respectively to an indirect level-repulsion effect and a direct coupling effect. The crucial dimensionless couplings that are involved in these effects are respectively the Yukawa couplings $\lambda_{\alpha ij} = W_{\alpha ij}$ and the generalized gauge charges $Q_{ai\bar{\jmath}} = -\frac{1}{2} K_{ai\bar{\jmath}}$, which corresponds to cubic couplings mixing one heavy and two light multiplets respectively in W and K. In particular, by looking along the chiral projection of the supersymmetry breaking direction, which is defined by the chiral auxiliary fields F^i, we showed that the averaged sGoldstino mass in the effective theory takes the form:

$$m^{2\rm eff}_\varphi = \big(R - \lambda_\alpha |M|^{-2\alpha\bar{\beta}} \bar{\lambda}_{\bar{\beta}} + 4\,Q_a M^{-2ab} Q_b\big)\, M_S^4\,. \tag{5.66}$$

The first term is what one would find by just restricting to the light fields. It is controlled by the sectional curvature R along the F-direction, and can have any sign. The second term is the correction induced by heavy chiral multiplets. It is controlled by the Yukawa couplings λ_α along the F-direction and is always negative. The third term is the correction induced by heavy vector multiplets. It is controlled by the gauge charges Q_a along the F-direction and is always positive. Finally M_S is the scale of supersymmetry breaking, which in our situation is set by the F auxiliary fields since the D auxiliary fields are suppressed.

The result (5.66) has been derived in rigid supersymmetry, in the limit where the supersymmetry breaking scale is much lower than the mass scale M of the heavy modes that are integrated out. Its generalization to gravity can however be derived in a straightforward way by using the results discussed in Chapter 3, where it has been shown that whenever the gravitino mass $m_{3/2}$ is also much smaller than the heavy mass scale M, one may first integrate out the fields in the rigid limit and then switch on the coupling to gravity. More precisely, the only modification induced by gravity in (5.66) is the addition of the correction $2m_{3/2}^2$; this can be see from the expression of the scalar masses eq. (1.100) by projecting along the Goldstino direction. One reconstructs in this way the supergravity result of [108]:

$$\Delta m_{\varphi}^{2\mathrm{eff}} = 2\, m_{3/2}^2 \,. \tag{5.67}$$

The origin of the difference in sign in the corrections induced by heavy chiral and vector multiplets is transparent in the component approach, where the first is due to an indirect level-repulsion effect whereas the second is due to a direct coupling effect. In the superfield approach, the two computations look instead very symmetric and the difference in sign is at first sight surprising. A closer inspection shows however that there too it can be understood quite robustly. For this we observe that for heavy chiral multiplets the stationarity condition $W_\alpha = 0$, the auxiliary fields $\bar{F}_\alpha = -W_\alpha$ and the relevant cubic couplings $\lambda_{\alpha ij} = W_{\alpha ij}$ are all controlled by the superpotential W, whereas for heavy vector multiplets the stationarity condition $K_a = 0$, the auxiliary fields $D_a = -\frac{1}{2}K_a$ and the relevant cubic couplings $Q_{ai\bar{j}} = -\frac{1}{2}K_{ai\bar{j}}$ are all controlled by the Kähler potential K. There is then a perfect symmetry between the two dynamics, which exchanges the roles of K and W. When one looks at the effects of these heavy dynamics onto the supersymmetry-breaking part of the masses of light scalar fields, this symmetry is however broken, because supersymmetry-breaking contributions to scalar masses arise only from K and not from W. This is what causes the difference in sign between the two effects.[6]

As anticipated in Section 2.3, the result that we have obtained may have interesting applications in the context of string models, where the situation in which some of the multiplets are stabilized in a supersymmetric way at a high energy scale naturally occurs and the question of their effect on the dynamics of the light multiplets, which are supposed to break supersymmetry, acquires a crucial importance. In such a situation one has in principle to honestly integrate out the heavy fields to properly describe the dynamics of the light fields. But it is in general cumbersome to do so, and this raises the question of whether or when one may get a qualitatively reliable indication on the

[6] A similar phenomenon has also been encountered in different context in [126].

light field dynamics by just freezing the heavy fields and truncating the theory. Some particular situations where one can safely do this truncation and get the right effective theory have been identified in [100, 101, 106]. Here we have shown more specifically and more systematically what kind of dangers may arise from the heavy fields concerning the masses of the light fields, which are the crucial issue for metastability of the vacuum.

A concrete example is that of string models where large classical effects related to background fluxes stabilize some moduli in a supersymmetric way with a large mass and small quantum effects related to gauge interactions stabilize some other moduli in a non-supersymmetric way with a small mass [96, 87]. The dynamics of these heavy and light modes, schematically denoted by H and L, is then described by $K = K_L(L, \bar{L}) + K_H(H, \bar{H}) + K_Q(L, \bar{L}, H, \bar{H})$ and $W = 0 + W_H(H) + W_Q(L, H)$. For gauge interactions with a field-dependent gauge kinetic function $H_{ab} \propto L$, the quantum effects have the following structure. The correction K_Q consists of both perturbative and non-perturbative effects suppressed by inverse powers and exponentials of $L + \bar{L}$, and can usually be neglected, since it represents a small correction to the kinetic terms of L. The correction W_Q consists instead only of non-perturbative effects suppressed by exponentials of L, and must be kept, since it represents the dominant source of potential for L.[7] In this situation, freezing the heavy moduli H to constant values is a priori not justified [88, 97, 98], but turns out a posteriori to give a sensible approximation to the effective theory for the light moduli L thanks to the smallness of the quantum corrections mixing L and H [101]. Applying our general result, we may now establish more quantitatively the importance of the corrections induced by integrating out the heavy modes on the light masses, and in particular the sGoldstino mass m. The relevant Yukawa coupling λ between one H and two L fields will involve the same exponential suppression factor as W_Q. The dangerous indirect level-repulsion effect on m^2 will then be suppressed by the square of this exponential factor. On the other hand, the direct effect induced on m^2 from the mixing K_Q involves both power and exponentially suppressed corrections. Given then that in these models there is a unique ultraviolet mass scale around M_P, the indirect effect is a priori smaller than the direct effect, and in all the situations where the direct effect is neglected also the indirect effect must be discarded. There is thus no problem in the limit of small quantum effects.

One may finally wonder whether integrating out heavy chiral and vector multiplets has similar effects on soft masses in scenarios where both the visible and the hidden sectors couple to them. In fact, these effects are easily computed, since they are also encoded in the effective Riemann tensor, but with two visible-sector and two hidden-sector indices: $m^{2\text{eff}}_{u\bar{v}} = -R^{\text{eff}}_{u\bar{v}i\bar{\jmath}} F^{i\text{eff}} \bar{F}^{\bar{\jmath}\text{eff}}$. Applying the results (5.41) and (5.65) one would then find

$$m^{2\text{eff}}_{u\bar{v}} = -\big(R_{u\bar{v}i\bar{\jmath}} + \lambda_{\alpha ui} |M|^{-2\alpha\beta} \bar{\lambda}_{\bar{\beta}\bar{v}\bar{\jmath}} - 2\, Q_{au\bar{v}} M^{-2ab} Q_{bi\bar{\jmath}} - 2\, Q_{au\bar{\jmath}} M^{-2ab} Q_{bi\bar{v}}\big) F^i \bar{F}^{\bar{\jmath}} \,. \quad (5.68)$$

The first term is the usual expression for the soft masses,[8] the second term represents

[7] See [83, 82, 131] for a more detailed discussion of these effects for gaugino condensation.

[8] We have already discussed the general expression for the soft masses in the supergravity case

the correction induced by heavy chiral multiplets, and the third and fourth terms describe the corrections induced by heavy vector multiplets. The various couplings controlling these effects are however not always allowed by the Standard Model gauge symmetry $G_{\rm SM}$. If the heavy states are neutral, only $Q_{au\bar{v}}$ and $Q_{ai\bar{\jmath}}$ can be non-zero. The only effect then comes from the third term, with an arbitrary sign. This is the standard effect induced by a neutral heavy vector multiplet.[9] If on the other hand the heavy states are charged, only $\lambda_{\alpha ui}$ and $Q_{au\bar{\jmath}}$ can be non-zero. The only effects then come from the second and the fourth terms, which are respectively negative and positive. However a charged chiral multiplet cannot have a supersymmetric mass term, because $G_{\rm SM}$ does not allow holomorphic invariants, whereas a charged vector multiplet can, since non-holomorphic invariants exist; so actually only the fourth term is relevant. This is a less-standard but already-known effect that can be induced by charged vector multiplets.[10] In addition to these effects, there is as usual a separate gravitational effect, which for generic cosmological constant $V = M_S^4 - 3m_{3/2}^2 M_P^2$, and ignoring D-type effects, is given by: (see for examle [70, 71])

$$\Delta m^{2\rm eff}_{u\bar{v}} = g_{u\bar{v}} \left(m_{3/2}^2 + V M_P^{-2} \right) . \tag{5.69}$$

We clearly see that eqs. (5.68) and (5.69) for the soft scalar masses correspond to eqs. (5.66) and (5.67) for the average sGoldstino mass.

with vanishing cosmological constant (see eq. (2.16)); the rigid limit is obtained as usual by setting $m_{3/2}^2 = 0$.

[9] See for example [92].

[10] This kind of effect is relevant in Grand Unified Theories, where charged massive vector fields occur after the gauge symmetry is broken down from $G_{\rm GUT}$ to $G_{\rm SM}$, and induces important corrections to soft masses. This phenomenon and its phenomenological implications were studied in detail in [90, 91].

Conclusions

The main topics discussed in this thesis are low-energy supersymmetric effective theories and metastability conditions in generic non-linear σ-models with chiral and vector multiplets. This work has been motivated by some fundamental problems which occur when studying supersymmetry breaking in string-inspired models, where the hidden sector naturally includes the moduli sector of string compactification.

Effective field theories represent a very useful tool when dealing with the proliferation of moduli fields; this is because, in many scenarios, some of the moduli are stabilized in a supersymmetric way at the string scale and only induce mild indirect effects on the low-energy dynamics. In this work we have studied the conditions under which it is possible to consistently integrate out heavy multiplets directly at the superfield level in both globally and locally supersymmetric models. We have found that in the general case, the low-energy derivative expansion only preserves supersymmetry provided that all the auxiliary fields and fermion bilinears are small compared to the supersymmetric mass scale of the heavy multiplets. Our main result however concerns the supergravity case where we have proven that one also has to require a small gravitino mass or, equivalently, a small cosmological constant to guarantee the validity of the two-derivative expansion also in the gravitational sector. We have then shown that, once these conditions are satisfied, heavy multiplets can be integrated out by solving the associated (algebraic) superfield equations of motion obtained by discarding supercovariant derivatives; this translates into the fact that for heavy chiral superfields one has to impose the stationarity of the superpotential whereas for heavy vector superfields one has to impose the stationarity of the Kähler potential. Our most important conclusion is that the same procedure holds true both for rigid supersymmetry and supergravity, meaning that the process of integrating out heavy multiplets commutes at leading order in the low-energy expansion with switching on gravitational interactions. We have also discussed the conditions under which the integration procedure trivializes in the sense that heavy multiplets can be frozen to their expectation values; it turns out that this happens when the superpotential and the Kähler potential are separable respectively in the chiral and vector multiplet cases.

The second problem motivating our study is the necessity of defining some general criteria in order to efficiently discriminate among the plethora of different compactification scenarios arising in String Theory. In general the strongest constraint that one can imagine to impose is that any realistic model has to admit a sufficiently long-lived metastable vacuum, which breaks supersymmetry with a small value of the cosmological constant and provides sufficiently large masses for scalar fluctuations. A basic observation is the fact that in this class of models the Kähler potential is in general fixed by the compactification details whereas the form of the superpotential is more difficult to be determined. One may then take the point of view that K is given and W is arbitrary. In this work we have proposed a strategy to derive an absolute upper bound on the mass of the lightest scalar which is only sensitive to the geometry of the Kähler target manifold and can be saturated by properly tuning the superpotential. The bound is obtained by looking at all the directions in the scalar field space which cannot obtain an arbitrarily large supersymmetric mass; these directions do in general include not only the Goldstino direction, associated to supersymmetry breaking, but also the Goldstone directions, associated to gauge symmetry breaking. We have studied more explicitly this bound in theories with rigid supersymmetry and one Abelian gauge symmetry. For renormalizable gauge theories, the bound is saturated by orienting the Goldstino along the direction of maximal charge and the Goldstone partner along the direction of minimal charge; in this situation, the two degenerate sGoldstini are the lightest scalars and their masses assume the maximal value that can be achieved by optimizing the superpotential. In more general situations with a non-trivial Kähler potential, the lightest scalar is a linear combination of the two sGoldstini and the Goldstone partner. In that case, there is no simple way to derive general results and we have performed a case-by-case analysis studying some simple geometries with covariantly constant curvature. The main result is that the upper bound obtained by only considering the Goldstino direction in general turns out to be too optimistic and tends to overestimate the mass of the lightest scalar. This study extends some previous analysis in which only the supersymmetry breaking direction was assumed to be potentially dangerous for metastability and suggests that also the directions associated to gauge symmetry breaking may play a significant role.

We have finally computed the effects induced by the supersymmetric integration of heavy multiplets on the masses of light ones. In particular we have computed the corrections to the masses of the Goldstino partners in the limit in which all the vector multiplets are heavy and only the SUSY breaking direction is potentially dangerous for metastability. In this case, we have shown that there exist two kinds of effects: the first one is induced by heavy chiral multiplets and is a negative-definite level-repulsion effect controlled by Yukawa couplings mixing one heavy and two light fields in the superpotential; the second one comes from heavy vector multiplets, is controlled by the charges mixing one heavy and two light fields in the Kähler potential and is positive definite. Contrary to the former, this last contribution tends in general to reduce the effective curvature of the scalar manifold and to improve the metastability condition.

We conclude by discussing some future directions. Concerning the problem of metastability, we remark that the aim of our analysis was to illustrate with some simple examples the basic strategy one should follow to define the strongest upper bound on the mass of the lightest scalar. In particular, we wanted to clarify in some simple situations the role of the Goldstone partners in defining a necessary and sufficient condition for metastability. Even if the strategy we presented can be applied without conceptual complications to more general situations, our analysis does not pretend to be conclusive. In particular, the study which deserves more attention is, in our opinion, the case of renormalizable non-Abelian gauge theories; in our work we have not been able to find any simple general result, but we have given some indications for possible simplifications. Finally, a more detailed analysis is also necessary for the extension to supergravity, which is particularly interesting for applications to string-inspired models.

Acknowledgements

I would like to address special thanks to Prof. Claudio Scrucca for the opportunity to work under his supervision. I'm really grateful to him for his constant and precious contribution to my research activity and for his scientific and human support.

I would like to express my gratitude to Prof. Riccardo Rattazzi for his useful comments on our papers and for his illuminating (and unconventional) lessons.

I want to thank Marta Gómez-Reino for the fruitful collaboration in the first part of my doctorate.

I am grateful to my colleague and friend Christopher Andrey for all the scientific and non-scientific discussions, for the continuous support in learning French and for the moral support during the writing period.

I am also grateful to all my colleagues and friends of the seventh floor of the Cubotron who shared this important experience with me: Claude Becker, Jan Mrazek, Duccio Pappadopulo, Andrea Thamm, Paolo Torrielli, Alessandro Vichi and Daniel Zenhaeusern. I would also like to thank Jean-Claude Jacot for the useful discussions.

I would like to acknowledge the École Polytechnique Fédérale de Lausanne and the Institut de Théorie des Phénomènes Physiques and I am particularly grateful to Mme Porta, Leonor Camporeale, Tanya Castellino and Corinne Weibel for their fundamental administrative support and for their kindness.

Vorrei ringraziare tutti coloro che in questi 4 anni sono stati la mia "famiglia" qui a Losanna e hanno contribuito con il loro affetto a farmi sentire davvero a casa. Un ringraziamento speciale va a Luca Fontanesi perché con lui ho condiviso tutte le gioie e le fatiche di questa esperienza e perché è presente in tutti i ricordi più piacevoli che ho di questi anni. Grazie anche agli altri miei coinquilini e amici Elsa Lacroix e Floriano Traversi che mi hanno supportato (e sopportato) nei momenti più duri. Grazie agli amici di vecchia data Emanuele Facchinetti, Melanie Monks, Guillaume e Delphine Tarel per l'affetto dimostrato. Grazie a Gabriele Grosso e Francesco Manni che hanno reso questi ultimi due anni davvero indimenticabili. Grazie anche ai nuovi amici Federico Tortelli e Simone Ragusa per il loro supporto durante questo ultimo anno. Sono inoltre grato a tutti quelli che con la loro simpatia hanno allietato le mie giornate e dei quali conservo degli ottimi ricordi. Mi riferisco agli amici, conoscenti, compagni di sci e di conferenze, couchsurfers, frequentatori della Ruche, del Sat e dello

Zelig e a tutti quelli che hanno contribuito a rendere Av. de Tivoli 4 il posto più cool di Losanna. In tutto direi che sono circa un centinaio e sarebbe impossibile ringraziarli tutti singolarmente qui. Grazie mille a tutti!

Vorrei altresì ringraziare la mia famiglia, a cominciare dai miei genitori Lino e Barberina perché mi guidano e mi sostengono da sempre e non li ho mai sentiti lontani neppure per un giorno in questi quattro anni. Grazie anche alle mie care nonne Agnese e Luisa perché, nonostante tutti i consigli e le indicazioni, la mia cucina non sarà mai nemmeno lontanamente paragonabile alle specialità che mi preparano loro. Grazie ai miei cugini Marta e Davide che mi sono vicini da sempre. Grazie a Patrizio e Matilde per il loro affetto e grazie soprattutto alle mie sorelle Eleonora, Francesca e Valentina a cui dedico questo lavoro, perché il legame che ci unisce ha radici profonde e voglio loro un gran bene.

Per concludere, rivolgo un pensiero a Valfabbrica, il mio paese, perché posso anche girare mezzo mondo ma la mia vera casa resta sempre lei e quattro anni di lontananza mi hanno insegnato ad amarla ed apprezzarla ancora di più.

Lausanne, October 24th, 2011

Appendix A

Numerical Optimization of the Local Superpotential

In this appendix we illustrate the procedure discussed in Section 4.3 to locally reconstruct the optimized superpotential which saturates the bound on the mass of the light scalar. We do it for the specific example discussed in subsection 4.6.2 of the two-field model with logarithmic Kähler potential and gauged shift symmetry. The procedure must be implemented numerically, since neither the maximization equation for m^2_- nor the equations defining the optimal tuning of the superpotential parameters can be solved analytically. Following the approach presented in Chapter 4, we assume that the scalar manifold geometry and the gauged isometry are fixed, whereas the superpotential and the vacuum point can be freely chosen to saturate the bound, compatibly with gauge invariance and the stationarity conditions. To be concrete, we arbitrarily assign to the dimensionless parameters controlling the Kähler potential and the Killing vectors the following values:

$$\lambda_1 = 1\,, \quad \lambda_2 = 6\,, \quad a_1 = 1\,, \quad a_2 = 1\,. \tag{A.1}$$

The ratio λ_1/λ_2 has been chosen to be the same as in the plot of Fig. 4.6.2 whereas the values of a_1 and a_2 do not play any relevant role.

As we previously discussed we can tune the gauge coupling g in order to have the freedom to freely assign arbitrary values to the supersymmetry-breaking scale $\sqrt{|F|}$ and the vector mass M; in our case we chose the values:

$$M = 1, \quad \sqrt{|F|} = 2\,. \tag{A.2}$$

We then consider a superpotential which is a polynomial function of the gauge invariant combination of Φ^1 and Φ^2, which we call S:

$$W = \sum_n c_n S^n, \quad \text{with} \quad S = A_2\Phi^1 - A_1\Phi^2\,. \tag{A.3}$$

For simplicity we assume the coefficients c_n to be real; the minimal number of parameters necessary for the optimization of the superpotential will be discussed in a moment.

We parametrize the vacuum point using expression (4.83), which ensures that it is varied over a surface of constant vector mass M. The angular variable θ parametrizing the vacuum point controls also the directions f^i and x^i, which turn out to be rigidly tied in this simple example with two fields. We have seen that in this simple model, the value of θ that maximizes $m^2_{f\bar{f}}$ is $\theta = \pi/4$. In our case however we are interested in calculating the value of θ that maximizes m^2_-; as discussed this value depends only on M and on the ratio λ_1/λ_2 and it is numerically found to be:

$$\theta = 0.877765\,, \tag{A.4}$$

to which corresponds the maximal value $m^2_- = 2.06559$. The tuning of the gauge coupling can be done by imposing the stationarity condition along the Goldstone direction which give $g^2 = \frac{1}{4}\, M^4/|F|^2\, \lambda_1\lambda_2 \csc\theta \sec\theta$ (see eq.(4.85)). One finds:

$$g = 0.4367442\,. \tag{A.5}$$

We finally need to tune three parameters a_n in the superpotential to satisfy the three equations we are left with, namely: the stationarity condition along a direction orthogonal to x^i (we can chose f^i), the equation fixing the norm of the vector F^i and the equation (4.40) associated to the tuning of Δ. The numerical solution of this system of equations is found to be:

$$c_1 = 0.1578073\,, \qquad c_2 = 0.0165247\,, \qquad c_3 = -7.43647 \cdot 10^{-7}\,, \tag{A.6}$$

and we can finally set to zero the remaining coefficients: $c_{n>3} = 0$.

By tuning the parameters of the superpotential to these values, one can reconstruct a scalar potential that admits a metastable stationary point at the selected vacuum (corresponding to the preferred value of θ in eq. (A.4) and the fixed values (A.1) and (A.2)):

$$\Phi^1 = \bar{\Phi}^1 = 2.53422\,, \quad \Phi^2 = \bar{\Phi}^2 = 12.6272\,. \tag{A.7}$$

The scalar mass matrix has the expected form (4.37) with the following numerical values:

$$m^2_{\alpha\beta} = \begin{pmatrix} 2.36 & 0.29 & -1.27 \\ 0.29 & 2.36 & -1.27 \\ -1.27 & -1.27 & 7.57 \end{pmatrix}\,. \tag{A.8}$$

By computing the spectrum of this matrix, one can verify that the lightest eigenvalues are degenerate and correspond to the optimal value:

$$m^2_- = 2.06559\,. \tag{A.9}$$

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Curriculum Vitæ

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Typeset using $\LaTeX\,2_\varepsilon$ for Macintosh.
Printed by Reprographie EPFL, October 2011, Lausanne.