Integrated schedule planning with supply-demand interactions

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STRC 2011 **May 2011**

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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May 2011

Abstract

We present an integrated schedule planning model where the decisions of schedule design, fleeting and pricing are made simultaneously. Pricing is integrated through a logit demand model where itinerary choice is modeled by defining the utilities of the alternative itineraries. Utilities are explained with the fare price, departure time and number of stops. Spill and recapture effects are incorporated in the model to better represent the demand. For the recapture ratios we use a logit formulation similar to the demand model so that the ratios are determined by the model according to the utilities of the alternatives. Furthermore fare class segmentation is considered in such a way that the model decides the seats allocated to each fare class. To deal with the high complexity of the resulting mixed integer nonlinear problem, we propose a heuristic algorithm based on Lagrangian relaxation and sub-gradient optimization. The study is in the context of a project regarding the design of an innovative air transportation system called Clip-Air which has flexible transportation capacity. In order to quantify the potential advantages of this new system, models are extended to work with Clip-Air fleet and comparative analysis is carried out using a dataset for a major European company. It is observed that, the enhanced flexibility of Clip-Air allows to transport around 15% more passengers with the same overall fleet capacity.

Keywords

Fleet assignment, supply-demand interactions, integrated schedule planning, discrete choice modeling, itinerary choice

1 Introduction and Motivation

Transportation demand is constantly increasing in the last decades for both passenger and freight transportation. According to the statistics provided by the Association of European Airlines (AEA), air travel traffic has grown at an average rate over 5% per year over the last three decades and in 2012 passenger-km values is expected to be doubled compared to 1997. This increase results with disruptions in the operations. To give an example, 21.4% of flight departures in Europe were delayed by more than 15 minutes in 2008. It is estimated by US National Aviation System (NAS) that 92.5% of the delays are a result of scheduling more flights than the actual capacity. Given these trends in the air transportation, actions need to be taken both in supply operations and the demand management to have a demand responsive tranportation capacity for the sustainability of transportation.

The utilization of optimization techniques in airline scheduling process has improved the operations of airlines in the last decades. However to have a demand responsive supply capacity airline operators need new approaches to simplify their fleet management. Clip-Air, which is a new air transportation concept developed at EPFL, is designed to answer these needs providing flexibility in transportation capacity. Clip-Air simplifies the fleet management by allowing to decouple the carrying (wing) and the load (capsule) units. Capsules are modular detachable units such that the transportation capacity can be modified according to the demand. This modularity allows flexibility in fleeting as well as other operations including the crew scheduling and recovery operations. Maintenance requirements are also simplified due to the decoupling of wing, which needs the crucial maintenance steps, from the capsules. From a broader point of view, Clip-Air is designed for mixed passenger and freight transportation in a more efficient way and is expected to improve the integration of air transportation in multi-modal networks. Therefore Clip-Air is expected to improve airline operations from several aspects and in this study we develop models and algorithms to quantify the potential advantages of Clip-Air.

In this paper we present an integrated schedule design and fleet assignment model with supply-demand interactions. Supply and demand is related through a demand model where the attributes of itineraries define the utility of itinerary alternatives. As preliminary steps simple demand models are tested which were found to be very sensitive to the specification. Therefore a more reliable demand model is developed for itinerary choice using discrete choice methodology. Furthermore, fare class segmentation is included in the optimization model, which is inspired from the behavioral model, so that the model decides the configuration of the seats according to different demand elasticities of fare classes. Fleet assignment model also considers spill and recapture effects to better utilize the capacity which is also based on the demand model such that the redirection of passengers between itineraries is determined according to their utilities.

The rest of the paper is organized as follows: in section 2 we provide a literature review on fleet assignment models and air travel demand models as well as the initial attempts to integrate supply and demand decisions. Section 3 provides the integrated schedule model for standard fleet and the extension of the model with Clip-Air together with the demand model specification and the way spill and recapture effects are handled. In section 4 we present the results for the comparison between standard fleet and Clip-Air as well as the results with and without the integrated demand model. In section 5 we propose a heuristic method to deal with the high complexity of the resulting mixed integer nonlinear problem. Finally we conclude the paper and give future directions in section 6.

2 Related Literature

Integrated schedule design and fleet assignment models are studied in the literature with the purpose of increasing the revenue by making simultaneous decisions on the schedule and the fleet assignment. Schedule design is handled in different ways according to the flexibility allowed for the changes in the schedule. Desaulniers *et al.* (1997) and Rexing *et al.* (2000) study in an environment where the origin and destinations are known but the departure and arrival times can be shifted within a given time-windows. Lohatepanont and Barnhart (2004) work with sets of mandatory and optional flights where optional flights can be canceled to increase the profit.

In airline scheduling decisions, demand and price values are usually taken as inputs to the models. However, supply and demand depend on each other, that is decisions taken for supply influence the demand figures and vice versa. In the literature, choice models have been used to model the utility of each itinerary depending on specific attributes. Coldren *et al.* (2003) propose some logit models and Coldren and Koppelman (2005) extend the previous work with the introduction of GEV and nested logit models. Koppelman *et al.* (2008) model the time of day preferences under a multinomial logit setting in order to analyze the effect of schedule delay. Carrier (2008) and Wen and Lai (2010) propose some advanced demand modeling in which customer segmentation is modeled as a latent class. We refer to the work of Garrow (2010) for a comprehensive review of different specifications of choice behavior models for air travel demand.

Supply-demand interactions are considered in fleeting model from different perspectives. Yan and Tseng (2002) study an integrated schedule design and fleet assignment model in which the set of flight legs is built considering the itineraries under a given expected demand for every origin-destination pair. In an itinerary-based setting, Barnhart *et al.* (2002) consider the spill and recapture effects separately for each fare class resulting from insufficient capacity. Similarly, Lohatepanont and Barnhart (2004) study the network effects including the demand adjustment in case of flight cancellations.

Advanced supply and demand interactions can be modeled by letting the model to optimize itinerary's attributes (e.g., the price). Talluri and van Ryzin (2004a) integrate discrete choice modeling into the single-leg, multiple-fare-class revenue management model that determines the subset of fare products to offer at each point in time. Authors, provide characterization of optimal policies under a general choice model of demand. To overcome the missing no-purchase information in airline booking data, they use expectation-maximization (EM) method. Schön (2006) develops a market-oriented integrated schedule design and fleet assignment model with integrated pricing decisions. It is assumed that customers can be segmented according to some characteristics and different fares can be charged for these segments. Schön (2008) gives several specifications for the inverse price-demand function described in Schön (2006) including discrete choice models of multinomial logit model as well as nested logit model where the explanatory variable is taken as the fare price. Budhiraja *et al.* (2006) also work on a similar topic where the change in unconstrained itinerary demand is incorporated into the model as a function of supply.

3 Integrated Schedule Planning

In this section we provide the integrated schedule planning model. We first give the specification for the demand model and explain how we deal with spill and recapture effects and fare class segmentation. We first provide the model for a standard fleet and then we provide the extensions for a fleet type composed of Clip-Air wings and capsules.

3.1 Demand model for itinerary choice

The reliability of the demand model and its complexity are related. For long term and aggregated decisions simplistic models can be sufficient and appropriate. For medium-term strategic decisions, such as scheduling and fleeting, more accurate models are needed. Their integration into scheduling and fleeting models is desirable but it comes at the cost of additional complexity resulting in unmanageable models for real-world instances.

For the different specifications of common demand functions we refer to the work of Talluri and van Ryzin (2004b) who give place to linear models as well as nonlinear specifications such as exponential and multinomial logit models.

We introduce a demand model based on discrete choice analysis. The choice of an itinerary is modeled by defining the utilities of the alternatives. To explain the utilities, we have used *fare*, *time of day*, and *level of service* as found to be important in the context of itinerary choice in the studies of Coldren *et al.* (2003), Coldren and Koppelman (2005) and Garrow (2010). Therefore utility for each itinerary $i \in I$ and for each fare class $h \in H$ (e.g., business and economy) is given by:

$$V_i^h = \beta_{fare}^h p_i^h + \beta_{time}^h time_i + \beta_{stops}^h nonstop_i,$$

where p_i^h is the fare price of itinerary i for class h, $time_i$ is a dummy variable for the time of the day which is 1 if departure time is between 07:00-11:00, and $nonstop_i$ is a dummy variable for number of stops which is 1 if i is a non-stop itinerary. The coefficients of these variables are estimated with a multinomial logit model which are specific to each fare class h, since price and time elasticities of business and economy demand are known to be different (Belobaba *et al.* (2009)).

Defining the utility of the itineraries, a portion of unconstrained (expected) demand is captured by each itinerary according to their comparative utilities. Since all the itineraries do not serve as an alternative to each other we need to define a market segment. In this study we assume that each origin and destination (OD) pair defines a market segment which is indexed by $s \in S$ and the corresponding set of itineraries is represented by I_s. Therefore the total expected demand for the OD pair s, D_s^h, is split to the itineraries according to the formula (1): each itinerary i in segment s attracts $\tilde{d}_i^h$ many class h passengers. We include a set of no-purchase itineraries $I_s^{'} \in I_s$ for each segment s which stands for the itineraries offered by other airlines.

$$\tilde{d}_i^h = D_s^h \frac{\exp{(V_i^h)}}{\sum_{j \in I_s} \exp{(V_j^h)}} \quad \forall s \in S, h \in H, i \in I_s \tag{1}$$

Similarly to what Schön (2008) proposes, we define a variable υ_s^h for the ease of notation which is given by:

$$\upsilon_s^h = \frac{1}{\sum_{j \in I_s} \exp{(V_j^h)}} \quad \forall s \in S, h \in H,$$

now equation (1) can be re-written as:

$$\tilde{d}_i^h = D_s^h \upsilon_s^h \exp{(V_i^h)} \quad \forall s \in S, h \in H, i \in I_s \tag{2}$$

Finally, we impose that all the unconstrained demand is covered either by some itineraries offered by the airline or lost in favor to the competitors. In other words, the choice probabilities must sum up to 1:

$$\sum_{i \in I_s} \upsilon_s^h \exp\left(V_i^h\right) = 1 \quad \forall s \in S, h \in H.$$

3.2 Spill and recapture effects

Although the purpose of the fleet assignment is to optimize the assignment of aircrafts to the flight legs, capacity restrictions and the uncertainties in demand may result in lost passengers or under utilized capacity. In case of capacity shortage some passengers, who can not fly on their desired itineraries, may accept to fly onto other available itineraries in the same market segment offered by the company. This effect is referred as spill and recapture effect. In this paper we model accurately the spill and recapture in order to better represent the demand. We assume that the spilled passengers are recaptured by the other itineraries with a recapture ratio based on a multinomial logit choice model similar to the demand model.

In the model, $t_{i,j}^h$ is the decision variable for the number of class h passengers redirected from itinerary i to j for the same segment s. We define a recapture ratio $b_{i,j}^h$ which represents the ratio of $t_{i,j}^h$ spilled passengers from itinerary i being recaptured by itinerary j. We may lose passengers toward the no-purchase itineraries but no spill exist from them.

The recapture ratio is defined by the multinomial logit as:

$$b_{i,j}^h = \frac{\exp\left(V_j^h\right)}{\sum_{k \in I_s \setminus i} \exp\left(V_k^h\right)} \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}), j \in I_s, \tag{3}$$

With the use of variable υ_s^h we can rewrite the equation 3 as:

$$b_{i,j}^h = \frac{\exp\left(V_j^h\right)}{\frac{1}{\upsilon_s^h} - \exp\left(V_i^h\right)} \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}), j \in I_s.$$

Table 1: ORY-NCE itineraries

OD	fare	nonstop	time
ORY-NCE$_1$	220	1	1
ORY-NCE$_2$	218	1	0
ORY-NCE$_3$	214	1	0
ORY-NCE$^{'}$	250	1	1

We illustrate the concept using three itineraries belonging to the same segment ORY-NCE. We also include the no-purchase option (ORY-NCE$'$). The resulting recapture ratios can be seen in Table 2 with the corresponding information on the itineraries given in Table 1. Fare values for the itineraries are determined by the model except the no-purchase itinerary which has a fixed price.

For example, in case of capacity shortage for itinerary 1, at most 40.1% and 50.3% of passengers in excess will be recaptured by the second and third itineraries respectively. 9.6% will be lost to the outside itineraries. Recapture ratio is higher for ORY-NCE$_3$ compared to ORY-NCE$_2$ since it has a lower fare price. Similar analysis can be done for the remaining recapture ratios.

Table 2: Recapture ratios for ORY-NCE

	ORY-NCE$_1$	ORY-NCE$_2$	ORY-NCE$_3$	ORY-NCE$'$
ORY-NCE$_1$	0	0.401	0.503	0.096
ORY-NCE$_2$	0.417	0	0.490	0.093
ORY-NCE$_3$	0.463	0.434	0	0.103

3.3 Fare class segmentation

The demand model presented in section 3.1 is specific to each fare class to model different demand elasticities. In this study we extend the segmentation of fare classes to the fleeting decisions. The configuration of the seats for each fare class is determined by the model according to the profitability of the itineraries. Let π_f^h be the variable that represents the number of seats allocated for class h passengers on flight f.

According to the statistics provided in the *Analysis of European Air Transport Industry* (DG-TREN (2002)), the percentage of business class tickets sold in major Western European markets is 21% in 2002. Therefore, we make the assumption that the percentage of business seats allocated for a flight can vary between 10% and 30% although we allow the model to determine the number seats for each class. Similarly, we assume that 20% of the total forecasted demand is for business tickets.

3.4 Integrated schedule design, fleet assignment and demand model

Our integrated schedule design and fleet assignment model is an extension of the model proposed by Barnhart *et al.* (2002) and Lohatepanont and Barnhart (2004). Similarly to Schön (2008), we integrate discrete choice demand models into a fleeting and scheduling model with the additional definition of variable spill function which allows a more realistic representation of this effect. The model, which considers a single airline, is provided in Figure 1 for a standard fleet.

Let F be the set of flight legs, there are two subsets of flights one being mandatory flights (F_M), which should be flown, and the other being the optional flights (F_O) which can be canceled in terms of optimization purposes. A represents the set of airports and K is for the set of fleet. The schedule is represented by time-space network such that $N(k, a, t)$ is the set of nodes in the time-line network for plane type k, airport a and time $t \in T$. $In(k, a, t)$ and $Out(k, a, t)$ are the sets of inbound and outbound flight legs for node (k, a, t).

Objective (4) is to maximize the profit which is calculated with revenue for business and economy demand, that takes into account to lost revenue due to spill, minus operating costs. Operating cost for flight f when using fleet type k is represented by $C_{k,f}$ which is associated with a binary variable of $x_{k,f}$ that is one if a plane of type k is assigned to flight f.

Constraints (5) ensure the coverage of mandatory flights which must be served according to the schedule development. Therefore every mandatory flight should be assigned exactly one type of fleet. Constraints (6) are for the optional flights that have the possibility to be canceled. Constraints (7) are for the flow conservation of fleet, where y_{k,a,t^-} and y_{k,a,t^+} are the variables representing the number of type k planes at airport a just before and just after time t. Constraints (8) limit the usage of fleet by the available amount which is R_k for fleet type k. In this study it is assumed that the network configuration at the beginning of the period (which is one day) is the same as the end of the period in terms of the number of planes at each airport. Constraints (9) ensure this circular schedule property.

Constraints (10) maintain the capacity restriction for business and economy demand. The assigned number of seats for a flight should satisfy the demand for the corresponding itineraries considering the spill effects. Similarly when a flight is canceled, all the related itineraries should not realize any demand. $\delta_{i,f}$ is a binary parameter which is 1 if itinerary i uses flight f and enables us to write the capacity constraints over the itineraries rather than the flights. Since we let the configuration of business and economy seats to be determined by the model we need to make sure that the total does not exceed the capacity (11) where Q_k is the available seat capacity of plane type k. Constraints (12) are for business and economy demand conservation for each itinerary saying that total redirected passengers from itinerary i to all other itineraries including the no-purchase options should not exceed its realized demand.

Existence of the demand model induces additional constraints as mentioned previously in sections 3.1 and 3.2. For the sake of completeness we again provide the explanation for the related constraints. Constraints (13) give the demand split between the itineraries in the same segment according to their utilities using logit formulation for each fare class. According to the utilities we may lose passengers to the outside options. Constraints (14) ensure that the probability of being assigned to one of the itineraries sums up to 1. Constraints (15) provide the spill ratios between itinerary i and j. Basically the redirected passengers are accommodated to the remaining options with the same demand model excluding the desired (original) itinerary. Since

$$
\begin{aligned}
& Max \sum_{s\in S}\sum_{h\in H}\sum_{i\in(I_s\setminus I'_s)} (d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I'_s)\\ i\neq j}} t_{j,i}^h b_{j,i}^h) p_i^h \\
& \qquad - \sum_{\substack{k\in K\\ f\in F}} C_{k,f} x_{k,f} && (4)\\
& s.t. \sum_{k\in K} x_{k,f} = 1 && \forall f \in F^M \quad (5)\\
& \sum_{k\in K} x_{k,f} \leq 1 && \forall f \in F^O \quad (6)\\
& y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} \\
& \qquad = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} && \forall [k,a,t] \in N \quad (7)\\
& \sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \leq R_k && \forall k \in K \quad (8)\\
& y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} && \forall k \in K, a \in A \quad (9)\\
& \sum_{s\in S}\sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} \delta_{i,f} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I'_s)\\ i\neq j}} \delta_{i,f} t_{j,i}^h b_{j,i}^h \\
& \qquad \leq \sum_{k\in K} \pi_{k,f}^h && \forall h \in H, f \in F \quad (10)\\
& \sum_{h\in H} \pi_{k,f}^h = Q_k x_{k,f} && \forall f \in F, k \in K \quad (11)\\
& \sum_{\substack{j\in I_s\\ i\neq j}} t_{i,j}^h \leq d_i^h && \forall s \in S, h \in H, i \in (I_s \setminus I'_s) \quad (12)\\
& \tilde{d}_i^h = D_s^h \upsilon_s^h \exp(V_i^h) && \forall s \in S, h \in H, i \in I_s \quad (13)\\
& \sum_{i\in I_s} \upsilon_s^h \exp(V_i^h) = 1 && \forall s \in S, h \in H \quad (14)\\
& b_{i,j}^h = \frac{\exp(V_j^h)}{\frac{1}{\upsilon_s^h} - \exp(V_i^h)} && \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \quad (15)\\
& x_{k,f} \in \{0,1\} && \forall k \in K, f \in F \quad (16)\\
& y_{k,a,t} \geq 0 && \forall [k,a,t] \in N \quad (17)\\
& \pi_{k,f}^h \geq 0 && \forall h \in H, k \in K, f \in F \quad (18)\\
& d_i^h \leq \tilde{d}_i^h \leq D_i^h && \forall h \in H, i \in I \quad (19)\\
& 0 \leq p_i^h \leq UB_i^h && \forall h \in H, i \in I \quad (20)\\
& t_{i,j}^h \geq 0 && \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \quad (21)\\
& b_{i,j}^h \geq 0 && \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \quad (22)\\
& \upsilon_s^h \geq 0 && \forall s \in S, h \in H \quad (23)
\end{aligned}
$$

Figure 1: Integrated schedule planning model

no-purchase options are outside our network we can not redirect passengers from them. Instead we just lose passengers who are attracted by those options.

Constraints (16)-(23) specify the decision variables. Demand value provided by the logit model, $\tilde{d}_i^h$, serves as an upper bound for the actual number of transported passengers that is represented by d_i^h for each itinerary and fare class. Furthermore, the price of each itinerary is limited by a specified upper bound UB_i since logit formulation considers only the difference between the utilities. This upper bound is assumed to be the average price in the market plus one standard deviation.

3.5 Model extension: Clip-Air

Clip-Air changes the concept of fleet by decoupling of wings and capsules as mentioned previously. This new concept necessitates the modification of fleet assignment problem. The operating cost of a flight is separated between wing and capsules. C_f^w represents the cost for wing and $C_{k,f}$ is the cost of flying with k capsules for flight f. The total operating cost in the objective is then given by:

$$\sum_{f \in F} C_f^w x_f^w + \sum_{k \in K} C_{k,f} x_{k,f},$$

where x_f^w is a binary variable which is 1 if a wing is assigned to flight f. As the cost of assigning 1 to 3 capsules to a flight is non-linear we provide a linear specification by introducing variable $x_{k,f}$ which is 1 if k (1,2, or 3) capsules assigned to flight f. This allows to compute operational costs in a preprocessing phase.

For the flight coverages the constraints are replaced by (24) which says that each mandatory flight should be assigned at least one capsule. Constraints (25) ensure that if there is no wing assigned to a flight there can not be any capsules assigned to that flight and similarly no flight can be realized without any wing.

$$\sum_{k \in K} x_{k,f} = 1 \quad \forall f \in F^M \tag{24}$$

$$\sum_{k \in K} x_{k,f} \leq x_f^w \quad \forall f \in F \tag{25}$$

Constraints related to the fleet assignment including flow conservation, fleet availability, circular property of the schedule are adjusted accordingly for both wings and capsules. Constraints for the demand model and spill effects are the same as in the model of standard fleet.

4 Results

We work with a dataset from a major European airline company. Data provides information on the sets of aircrafts, airports, flights and itineraries with average prices and unconstrained demand. The model is implemented in AMPL and BONMIN is used as a solver which can deal with mixed integer nonlinear problems.

4.1 Cost figures for Clip-Air

As a preliminary analysis we provide the assumptions regarding the configuration of Clip-Air in comparison with Airbus A320 as seen in Table 3. We use weight differences to adjust the related operating costs for wings and capsules. From the presented values it is observed that when Clip-Air is flying with one capsule it is 63% heavier than one A320 plane. However if Clip-Air flies more than one capsule it becomes advantages over A320 such that it has 1% and 23% less weight when flying with two and three capsules respectively. Therefore these weight differences are applied to the fuel cost, airport and air navigation charges which represents the 16% and 10% of the total operating costs according to the study of Smith (2004).

Table 3: Clip-Air configuration

		Clip-Air	**A320**
Maximum Capacity		3x145 (435 seats)	150 seats
Engines		**3** engines	**2** engines
Maximum	1 (plane/capsule)	126t	77.5t
Aircraft Weight	2 (planes/capsules)	153t	2x77.5t (155t)
	3 (planes/capsules)	180t	3x77.5t (232t)

Since we are able to separate wings and capsules Clip-Air flies with one set of flight crews regardless of the number of capsules used for the flight. It is given by the study of Aigrain and Dethier (2011) that flight crew constitutes 60% of the total crew cost for A320. Therefore Clip-Air decreases the crew costs by 30% and 40% when flying with two and three capsules respectively. Remaining cost values are assumed to be the same as A320 for the utilization of each capsule.

4.2 Parameters for the demand model

Table 4 presents the demand parameters used as input to the integrated schedule planning model. The parameters are estimated by maximum likelihood estimation for the two fare classes. However since we are using a booking data we do not have information regarding the non-chosen alternatives. To be able to deal with this lack of variability we adjusted the parameters to reflect more realistic elasticities.

The parameters suggest that economy passengers are more sensitive to fare price. On the other hand business passengers are more sensitive to level of service and prefer morning flights being in line with intuition.

Table 4: Parameters used for the demand model

	Business demand	**Economy demand**
β_{fare}	-0.025	-0.050
β_{time}	0.323	0.139
$\beta_{nonstop}$	1.150	0.900

4.3 Results with a small data instance

The information regarding the small data instance is provided in Table 5. Given the airports, there are 4 different OD pairs: ORY-LYS, ORY-NCE, LYS-ORY and NCE-ORY.

Table 5: Small data instance

Airports	3 (ORY, LYS, NCE)
Flights	9
Passengers	800
Capsule capacity	50
Standard fleet types	A318 (123), ERJ145 (50)
Total fleet size (seats)	400
Fare classes	Business, economy

Comparison between standard fleet and Clip-Air

Clip-Air offers potential improvements in fleeting operations. In order to quantify these advantages compared to a fleet composed of standard planes we have run models for both cases and performed a comparative analysis.

Table 6 reports on the comparison between a standard fleet and a Clip-Air fleet for the small instance. Clip-Air is able to transport more passengers (+17%) using less seat capacity. Overall profit is also increased with Clip-Air although cost figures for Clip-Air need further validation.

The effect of the embedded demand model

In order to understand the effect of the demand model, we built a model, called *fixed demand model*, where price and demand values for the itineraries are given as input data. For this

Table 6: Results for the small data instance

	Standard Fleet	**Clip-Air**
Operating cost	65,635	52,924
Revenue	118,494	143,193
Profit	**52,859**	**81,269**
Transported pax.	**532**	**621**
	124 B, 408 E	132 B, 489 E
Flight count	8	8
Average pax/flight	**66**	**78**
Total Flight Hours (min)	590	590
Used fleet	2 A318	4 wings
	3 ERJ145	7 capsules
Used capacity (seats)	396	350
Running time(min)	0.5	3.5

analysis we limit the study to economy class only.

We provide the information for the itineraries in Table 7. To remind that $nonstop$ variable is 1 for the non-stop itineraries and $time$ variable is 1 for the itineraries departing between 07:00-11:00. For each of the OD markets we introduce a no-purchase option as provided in Table 8. In case of capacity restrictions or when it is more profitable to fly with less passengers, part of the passengers may be lost to these itineraries offered by other airlines in the market. The prices of the no-purchase itineraries are fixed for the two models since we do not have control over these.

Table 7: Information for the itineraries

	origin	destination	expected demand	nonstop	time
1	ORY	LYS	132	1	1
2	ORY	LYS	133	1	0
3	ORY	NCE	68	1	1
4	NCE	ORY	56	1	1
5	ORY	NCE	79	1	0
6	NCE	ORY	63	1	0
7	ORY	NCE	80	1	0
8	LYS	ORY	108	1	1
9	LYS	ORY	81	1	0

Table 8: No-purchase itineraries

	origin	destination	fixed price	nonstop	time
10	ORY	LYS	185	1	1
11	LYS	ORY	185	1	1
12	ORY	NCE	250	1	1
13	NCE	ORY	250	1	1

Table 9 reports on the comparison between the fixed demand model and the integrated demand model. The integrated model is able to take advantage of the low price elasticity of passengers and make profit by increasing prices. The resulting prices can be seen in Table 10. The capacity allocated to itineraries may change due to the existence of the demand model. For example, for itineraries 8 and 9 the allocated capacity is different for the two models. Logit model determines the demand values by evaluating the differences between the utilities of the itineraries in the same market. Itinerary 8, which is a morning itinerary, is more desirable compared to itinerary 9. Therefore more passengers can be transported in itinerary 8 without decreasing the price that much. As a result, model decides to allocate more capacity on itinerary 8 and increases the price of itinerary 9 to meet a lower capacity level.

When we look at the running times, integration of demand model increases the computation time as expected.

Table 9: Results with and without the demand model

	Fixed demand model	**Integrated model**
Operating cost	65,635	65,635
Revenue	97,252	102,497
Profit	**31,617**	**36,862**
Transported pax.	**546**	**531**
Flight count	8	8
Average pax/flight	**68**	**66**
Total Flight Hours (min)	590	590
Used fleet	1 A318	2 A318
	3 ERJ145	3 ERJ145
Used capacity (seats)	273	396
Running time (min)	0.12	0.28

Table 10: Resulting demand and price values

			Fixed demand model		**Integrated model**	
	origin	destination	realized demand	fixed price	realized demand	realized price
1	ORY	LYS	123	162	123	179
2	ORY	LYS	50	162	50	194
3	ORY	NCE	50	200	50	220
4	NCE	ORY	50	212	50	230
5	ORY	NCE	50	200	50	218
6	NCE	ORY	50	212	50	228
7	ORY	NCE	0	200	0	214
8	LYS	ORY	50	162	108	159
9	LYS	ORY	123	162	50	172

We also report the resulting recapture ratios for the two models in Tables 11 and 12. It is observed that integrated model may decide lose more passengers to no-purchase options (itineraries 10, 11, 12 and 13) so that the price can be increased further by decreasing the demand to fit the available capacity.

Table 11: Resulting recapture ratios for the fixed demand model

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0.733	0	0	0	0	0	0	0	0.267	0	0	0
2	0.760	0	0	0	0	0	0	0	0	0.240	0	0	0
3	0	0	0	0	0.429	0	0.429	0	0	0	0	0.142	0
4	0	0	0	0	0	0.625	0	0	0	0	0	0	0.375
5	0	0	0.464	0	0	0	0.403	0	0	0	0	0.133	0
6	0	0	0	0.657	0	0	0	0	0	0	0	0	0.343
7	0	0	0.464	0	0.403	0	0	0	0	0	0	0.133	0
8	0	0	0	0	0	0	0	0	0.733	0	0.267	0	0
9	0	0	0	0	0	0	0	0.760	0	0	0.240	0	0

Table 12: Resulting recapture ratios for the integrated model

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0.352	0	0	0	0	0	0	0	0.648	0	0	0
2	0.572	0	0	0	0	0	0	0	0	0.428	0	0	0
3	0	0	0	0	0.401	0	0.503	0	0	0	0	0.096	0
4	0	0	0	0	0	0.725	0	0	0	0	0	0	0.275
5	0	0	0.417	0	0	0	0.490	0	0	0	0	0.093	0
6	0	0	0	0.731	0	0	0	0	0	0	0	0	0.269
7	0	0	0.463	0	0.434	0	0	0	0	0	0	0.103	0
8	0	0	0	0	0	0	0	0	0.631	0	0.369	0	0
9	0	0	0	0	0	0	0	0.783	0	0	0.217	0	0

4.4 Results with a larger data instance

We generated a relatively larger data instance compared to the instance provided in section 4.3. A summary for the data instance properties is given in Table 13. The network of airports is the same with more flights and therefore more passengers. The fleet consists of more plane types. There are 6 0D pairs: ORY-LYS, ORY-NCE, LYS-ORY, NCE-ORY, NCE-LYS and LYS-NCE.

Table 13: Large data instance

Airports	3 (ORY, LYS, NCE)
Flights	18
Passengers	1096
Capsule capacity	50
Standard fleet types	A318(123), A319(79), BAE300(100), ERJ135(37), ERJ145 (50)
Total fleet size (seats)	600
Fare classes	Business, economy

Comparison between standard fleet and Clip-Air

For the large data instance, the running time considerably increases as seen in Table 14. Therefore we report solutions with 3.2% and 1.5% optimality gap for standard fleet and Clip-Air respectively. Total transported number of passengers is higher (+10%) for Clip-Air as observed previously although less capacity (-32%) is used. Profit is higher for Clip-Air, since operating costs are significantly decreased and more revenue is realized with more transported passengers.

Table 14: Results for the large data instance

	Standard Fleet	**Clip-Air**
Operating cost	128,080	89,512
Revenue	188,405	198,905
Profit	**60,325**	**109,393**
Transported pax.	**828**	**909**
	183 B, 645 E	191 B, 718 E
Flight count	16	16
Average pax/flight	**52**	**57**
Total Flight Hours (min)	1200	1200
Used fleet	2 A318, 2 A319	5 wings
	1 ERJ135, 3 ERJ145	8 capsules
Used capacity (seats)	**591**	**400**
Running time (min)	2090	1470
Optimality gap	3.2%	1.5%

5 Heuristic approach

The resulting mixed integer nonlinear problem is highly complex and running times increase dramatically when we move to large data instances as observed in section 4.4. Therefore we propose a heuristic method based on Lagrangian relaxation combined with sub-gradient optimization and a Lagrangian heuristic. In this paper we present the method using the model for the standard fleet.

5.1 Lagrangian relaxation

When we relax the constraints (10) in the objective function introducing the Lagrangian multipliers λ_f^h one for each flight f and fare class h, objective function (4) is re-written as:

$$\begin{aligned} z(\lambda) = Max \sum_{s\in S}\sum_{h\in H}\sum_{i\in(I_s\setminus I'_s)} & \left(d_i^h - \sum_{\substack{j\in(I_s\setminus I'_s)\\ i\neq j}} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I'_s)\\ i\neq j}} t_{j,i}^h b_{j,i}^h \right) p_i^h \\ & - \sum_{k\in K}\sum_{f\in F} C_{k,f} x_{k,f} \\ & + \sum_{h\in H}\sum_{f\in F} \lambda_f^h \Big(\sum_{k\in K} \pi_{k,f}^h - \sum_{s\in S}\sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} d_i^h + \sum_{\substack{j\in I_s\\ i\neq j}} \delta_{i,f} t_{i,j}^h - \sum_{\substack{j\in(I_s\setminus I'_s)\\ i\neq j}} \delta_{i,f} t_{j,i}^h b_{j,i}^h\Big), \end{aligned} \tag{26}$$

which is subject to constraints (5)-(9) and (11)-(23).

When we sum the first term in the objective (26) over the set of flights F and multiply it with

$\delta_{i,f}$ we have an equivalent formulation. After arranging the terms we can write the objective function as:

$$z(\lambda) = Max \sum_{h \in H} \sum_{f \in F} \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f}(p_i^h - \lambda_f^h) \left(d_i^h - \sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j}^h + \sum_{\substack{j \in (I_s \setminus I'_s) \\ i \neq j}} t_{j,i}^h b_{j,i}^h \right)$$
$$+ \sum_{k \in K} \sum_{f \in F} \left(\sum_{h \in H} \lambda_f^h \pi_{k,f}^h - C_{k,f} x_{k,f} \right), \quad (27)$$

which is subject to constraints (5)-(9) and (11)-(23).

The model now can be decomposed into two subproblems. The first is a revenue maximization model with fare prices modified by the Lagrangian multipliers. The objective function is given by:

$$z_{REV}(\lambda) = Max \sum_{h \in H} \sum_{f \in F} \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f}(p_i^h - \lambda_f^h) \left(d_i^h - \sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j}^h + \sum_{\substack{j \in (I_s \setminus I'_s) \\ i \neq j}} t_{j,i}^h b_{j,i}^h \right), \quad (28)$$

which is subject to constraints (12)-(15) and (19)-(23).

The second subproblem is a fleet assignment model with class-fleet seat prizes. The objective function is given by:

$$z_{FAM}(\lambda) = Min \sum_{k \in K} \sum_{f \in F} \left(C_{k,f} x_{k,f} - \sum_{h \in H} \lambda_f^h \pi_{k,f}^h \right), \quad (29)$$

which is subject to constraints (5)-(9), (11) and (16)-(18).

5.2 Solving the Lagrangian dual via sub-gradient optimization

We apply sub-gradient optimization to solve the Lagrangian dual $z_D = \min_{\lambda \geq 0} \max z(\lambda)$. The gradient for fare class h and flight f is defined as:

$$G_f^h = \sum_{k \in K} \pi_{k,f}^h - \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} (\delta_{i,f} d_i^h - \sum_{\substack{j \in I_s \\ i \neq j}} \delta_{i,f} t_{i,j}^h + \sum_{\substack{j \in (I_s \setminus I'_s) \\ i \neq j}} \delta_{i,f} t_{j,i}^h b_{j,i}^h)$$

The step size for fare class h and flight f is defined as:

$$T_f^h = \frac{\eta(z(\lambda) - Z_{UB})}{\sum_{h \in H} \sum_{f \in F} (G_f^h)^2},$$

where η is a scale parameter initialized at 0.5, Z_{UB} and Z_{LB} are upper and lower bounds, respectively. We update the Lagrangian multipliers using the gradient and the step size by:

$$\lambda_f^h = \max(0, \lambda_f^h - T_f^h G_f^h).$$

5.3 Lagrangian heuristic

At each iteration of the solution of the Lagrangian dual z_D, the optimal solution of $z(\lambda)$ may violate the capacity constraints (10) for some $f \in F$ and $h \in H$. Therefore we need to obtain a primal feasible solution which serves as a lower bound. To achieve that we devise a simple revenue maximization heuristic that uses the optimal solution to $z_{FAM}(\lambda) = \{\bar{x}, \bar{y}, \bar{\pi}\}$ to fix the fleet assignment and class capacity variables to the values, i.e., $x = \bar{x}$ and $\pi = \bar{\pi}$. Since fleet assignment part is fixed the constraints (5)-(9) and (11) are dropped. Therefore the model turns into a revenue optimization problem which is solved in the same way as $z_{REV}(\lambda)$.

5.4 Overall algorithm

Having provided the necessary steps, we can give the pseudo-code of the Lagrangian relaxation procedure.

no improvement() function checks if the upper bound is improved in the last 4 iterations in order to reduce the scale if there is no improvement. *Lagrangian heuristic()*, *compute sub-gradient()* and *compute step()* functions are explained in the previous sections.

5.5 Results on the performance of the heuristic

At this stage of the study the heuristic is under implementation. Preliminary computational study is presented at STRC 2011 and available upon request to authors.

Algorithm 1 Lagrangian procedure

Require: z_{LB}, $\bar{k}$, ϵ
  $\lambda^0 := 0$, $k := 0$, $z_{UB} := \infty$, $\eta := 0.5$
  repeat
    $\{\bar{d}, \bar{t}, \bar{b}\} :=$ solve $z_{REV}(\lambda^k)$
    $\{\bar{x}, \bar{y}, \bar{\pi}\} :=$ solve $z_{FAM}(\lambda^k)$
    $z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)$
    $z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$
    if no improvement(z_{UB}) **then**
      $\eta := \eta/2$
    end if
    $lb :=$ Lagrangian heuristic $(\{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})$
    $z_{LB} := \max(z_{LB}, lb)$
    $G :=$ compute sub-gradient$(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})$
    $T :=$ compute step$(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})$
    $\lambda^{k+1} := \max(0, \lambda^k - TG)$
  until $||TG||^2 \leq \epsilon$ **or** $k \geq \bar{k}$

6 Conclusions and Future Research

In this study we integrated a demand model into a schedule planning model where the demand model is specified as a logit model. Spill and recapture effects are considered in the model to better represent the reality by redirecting passengers to other itineraries in the same market in case of capacity restrictions. The recapture ratio is formulated in a similar way to the demand model. Furthermore both the demand model and the scheduling model is built considering fare class segmentation and the allocation of the seats to each fare class is determined by the integrated model.

The resulting mixed integer nonlinear problem is highly complex as seen from the examples provided. We propose a heuristic method based on Lagrangian relaxation and sub-gradient optimization. It allows us to decompose the problem into revenue maximization and fleet assignment subproblems. The implementation of the heuristic approach is under progress and analysis of the performance of the heuristic is one of the next steps of the study.

Since the study is motivated by the design of a flexible transportation system called Clip-Air we provide comparative results between standard fleet and Clip-Air. It is observed that the number of transported passengers are increased with Clip-Air although it uses less transportation capacity. It is also observed that there is a potential increase in the profit resulting from the decreased operating costs and increased transported passengers. The cost figures of Clip-Air are based on strong assumptions at this stage of the study. However we believe that this potential will persist when we obtain better estimates for the costs.

As further steps, the integrated model will be studied in order to increase the stability. This needs further investigation of the effects of the embedded logit formulation which generates both the demand values and the recapture ratios. Furthermore, for the performance analysis of Clip-Air we need to come up with a comprehensive scenario analysis.

7 Acknowledgments

The authors would like to thank EPFL Middle East for funding this research. Special thanks go to Claudio Leonardi for the information he provided regarding the design of Clip-Air. Authors would like to thank to Alexandre De Tenorio who worked on a preliminary comparative analysis of the performance of Clip-Air as his Master Thesis. Nancy Moret deserves an acknowledgment for initializing the integration of demand modeling into scheduling models in the context of her semester project.

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