# Effect of uncertainties on the real-time operation of a lowland water system in The Netherlands

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**Abstract** Due to the limited pumping capacity in lowland water systems, reduction of system failure requires anticipation of extreme precipitation events. This can be done by Model Predictive Control that optimizes an objective function over a certain time horizon, for which the system behaviour is calculated by a model and a prediction of the inputs to the system. The forecast inputs usually contain large uncertainties. Because the pump constraints make the optimization problem non-certainty equivalent, uncertainties need to be considered to adequately control the water system. In this paper, the way uncertainties influence the control decision is investigated. An information-control horizon and an information-prediction horizon are introduced as time-limits for the sensitivity to future input information and the value of predictions. These horizons need to be considered in the design of a controller. Multiple Model Predictive Control is suggested to deal with the uncertainties in a risk based way.

**Key words** Boezem canals; decision support system; model predictive control; multiple model optimization; Netherlands; polder; prediction horizon; real time operation; risk; uncertainty

## INTRODUCTION

The storage canals (or "Boezem" canals) in lowland water systems in The Netherlands have the function to convey water, pumped from low-lying polders and draining from the higher-lying areas, to the boundaries of the canal system. From there, the main pumping stations discharge the water to the sea or rivers. In this way, the Boezem canals facilitate the drainage in the polders, where several economic interests are dependent on adequately maintained water levels (Schuurmans *et al*., 2002).

Apart from the transport, temporary storage is also an important function, as pumping capacity is limited and can sometimes be exceeded by the inflow into the storage canal system. This inflow into the Boezem canals is necessary to maintain the water levels in the polders. If available storage in the Boezem canals is insufficient, drainage of the polders cannot be secured, which may result in flooding problems there. Also the Boezem canals themselves may flood. In such cases, we speak of a failure of the Boezem system.

In this paper, we focus on the control of the Boezem canals using the main pumping stations, considering the polders as autonomously controlled systems, as is still common in present day water management. For long-term optimal policies, decisions should minimize risk, defined as the expected value of future damage. This requires explicit consideration of uncertainties.

## Objective of operation

Damage occurs if the water system will fail during extreme events. Making optimal use of the possibilities to control the Boezem system will reduce both failure frequency and impacts.

Avoiding system failure is not the only objective. In fact, failure in a Boezem system should not be viewed as a single Boolean event, but rather as a continuous damage function of the water level deviations from a target level. Low water levels are associated with long-term effects such as acceleration of land subsidence, decay of foundations and instability of embankments. High water levels can cause flooding, risk of dike breach and the necessity to impose a pump stop for the polders, causing flooding problems there. The challenge for the operators is to secure the evacuation of water, discharged from the surrounding land, while balancing the short- and long-term costs associated with high and low water levels.

Usually a target water level has been set by the water board with democratically elected representatives of the various interest sectors. These include owners of agricultural land, house owners and inhabitants. Because zero deviation of the target level throughout the system is never attainable, the objective should also quantify the "cost" associated with the deviations, as a function of their direction, magnitude, location and duration. This makes it possible to balance the deviations in a way that minimizes costs or damage. Apart from the water-level related variables, the variables associated with control actions, like the pump flow, could also be part of the objective function. In this way, the operation costs of the pumps can be incorporated into the objective. In the most cases, these are relatively low compared to the water level related costs, making minimization of operating costs a secondary objective that becomes relevant for the decisions only in non-critical situations.

The objective function that was assumed is quadratic for the deviation from target level and the control flow and linear (by summation over the time steps) for the duration. The spatial variability has not been included, but this is possible by introducing extra state variables for water levels in the different areas. This objective function can either be used as a performance indicator to evaluate control rules, or directly in a control policy, based on real time optimization. In the last case optimization should take place over a certain time horizon, to balance current and future costs and to allow anticipation of future events if necessary. The objective is minimizing the cost function $J$ over this time horizon:

$$\min_u J = \sum_{i=0}^{n} e^T(k+i\,|\,k)\cdot Q\cdot e(k+i\,|\,k) + u^T(k+i\,|\,k)\cdot R\cdot u(k+i\,|\,k) \tag{1}$$

in which $e$ is the state vector (water level deviation from target level), $k$ is the actual time step, $n$ is the number of time steps within the prediction horizon, $i$ is the counter for these time steps, $u$ is the control action vector (pump flow), $Q$ and $R$ are the weight matrices for the penalty on states and control actions, respectively. In the example case, these vectors and matrices are scalars.

## Formulation of the model predictive controller

The objective function in equation (1) can be solved repeatedly over a receding horizon, using a Model Predictive Control algorithm (MPC). This technique from control theory uses a model of the water system to predict future states (Camacho & Bordons, 1999; van Overloop, 2006). For this prediction, it makes use of actual information about the measured water levels and measured and predicted precipitation. Every time step, it minimizes a quadratic cost function of states and control actions over a fixed time horizon. In this optimization, an optimal sequence of control actions is found, of which only the first one is executed. After every time step, the optimization is repeated with updated information.

If the canals of the Boezem system are well interconnected and water level gradients are relatively small, the system of canals can be modelled as a single reservoir. This could later be extended to several reservoirs, if necessary. The internal model that is used in this paper reads as follows:

$$e(k+1) = e(k) - \frac{T_c}{A_s} Q_c(k) + \frac{T_c}{A_s} Q_d(k) \qquad (2)$$

in which $T_c$ is the control time step, $Q_c$ is the controlled pump flow, $Q_d$ is the uncontrolled inflow into the system (denoted as $u$ in equation 1), and $A_s$ is the storage area of the reservoir (canal system). This model is used to predict future water levels over the time horizon, based on the results for $Q_d$ of a rainfall–runoff model and optimizing the control sequence $Q_c$.

Apart from the objective function in equation (1) and the internal model in equation (2), also non-negativity constraints and constraints on the maximum pump flow are part of the optimization. A more detailed description of the formulation of this model predictive controller can be found in van Overloop (2006).

## Influence of uncertainties in input information

Optimal operation, given a certain rainfall event, is only possible with perfect information and foresight. In practice, however, there are large uncertainties concerning water system states, behaviour, and past and future rainfall. As a result of this, control actions are never optimal in hindsight. Reduction of uncertainties leads to decisions closer to the optimal ones and therefore has a certain value, which can be balanced against the costs of more accurate information or extra measurements to reduce uncertainty. What this value is depends on the extent to which the decisions are affected.

Because uncertainties usually increase with the lead time of predictions, one would expect that the most problematic uncertainties exist close to the end of the time horizon. However, depending on the problem and formulation, the influence of the prediction on the current decision usually also decreases with lead time. This means that also the optimality of the control will be less sensitive to this information and thus to the uncertainties therein.

To get insight into the uncertainties in the forecast, use of probabilistic forecasts (e.g. ensemble forecasts) is proposed. From a probabilistic, Bayesian point of view, such a forecast can be seen as the prior climatic distribution, conditioned on actual information, to become a posterior distribution (Krzysztofowicz, 1999, 2001; Murphy & Winkler, 1987). The forecast is thus based on the multi year average and spread of rainfall in a certain season, combined with information about actual weather patterns. With increasing lead time, this conditional, posterior distribution approaches the climatic distribution. If we use a probabilistic forecast in real time control, we can define two horizons relevant to the problem:

(a) The information-prediction horizon (TIp).
(b) The information-control horizon (TIc).

Horizon (a) can be defined as the time span from the actual moment until the moment where the conditional distribution of future events, conditional to all actual information, becomes the same as the marginal (climatic) distribution of these events. The length of this horizon depends on the forecast system and the statistical properties of the input forecasted. This may also depend on the season.

Horizon (b) is defined as the time span from the actual moment until the moment from which information does not influence the control actions anymore. This can be the case because the control action is at one of its constraints, or if optimality requires postponing actions.

Note that in control theory there is also a definition of the control (Tc) and prediction (Tp) horizons. These horizons define, respectively, the number of control moves to be calculated and number of prediction time steps over which the future state is calculated. These are parameters of the controller set-up. Rules to choose these parameters are usually based on characteristic time-constants of the system (Camacho & Bordons, 1999). In contrast to these definitions, we define horizons TIp and TIc, depending on predictability of inputs and sensitivity of the control action, respectively.

Regarding the relation between these four horizons, the following observations can be made:

- If the system controlled has delays in its behaviour, Tp should be much larger than the delay time.
- Tp should also be much larger than Tc + delay time. This is necessary to evaluate the full effects of each calculated control action.
- If TIc > TIp, extending the TIp by more advanced predictions based on more information helps to improve control. Tp and Tc should be chosen larger than TIp. At the end of Tp, the cost-to-go function can be used that is based on a steady-state optimization (Faber & Stedinger, 2001; Kelman *et al.*, 1990; Loucks *et al.*, 2005; Negenborn *et al.*, 2005).
- If TIp > TIc, we can know something about the future after TIc, but it has no influence on the decision now. The information about cost-to-go functions after TIc is not relevant, because the possible control sequences that lead to minimum total cost do not diverge yet. Tp does not need to be longer than TIc.
- In general, it can be stated that extending Tc and Tp beyond TIc is not necessary, but has no negative influence on the control, except for the computational cost.
- Extending Tp and Tc beyond TIp is not necessary either, and can have a negative influence, if the forecast system is biased. In that case, the climatic distribution would provide a better estimate than the forecast.

The next sections describe the impact of uncertainties on control actions for the Delfland Boezem canal system, which is controlled with the help of MPC.

## The Delfland Decision Support System

Delfland is the water board responsible for the southwestern part of the province of South Holland. With 1.4 million inhabitants on 41 000 hectares, it is one of the most densely populated water boards. The area is characterized by a high concentration of economic value. The water system has important functions for drainage, water supply for agriculture, navigation and recreation. The eastern part consists mainly of polder areas, while the western part also has higher-lying areas, draining to the Boezem canals without pumping stations. In the western part, especially, greenhouses cover a large area, resulting in a very fast runoff process. The total area of the Boezem canal system is 730 hectares (about 1.8% of the total area). The water level in these canals is usually kept close to the target level of –0.42 m below sea level. The tolerable range for extreme conditions is between –0.60 and –0.30 m below sea level. If large amounts of precipitation are expected, the water level can be temporarily lowered to the lower margin. The total capacity of the polder pumping stations discharging on the canals is around 50 $m^3$/s, but due to the fast runoff from the higher-lying areas, the total inflow can easily exceed 100 $m^3$/s during heavy rainstorms. The five main pumping stations discharge the water from the Boezem canals to the North Sea and the "Nieuwe Waterweg", a canalized tidal river connecting the port of Rotterdam to the sea. The total capacity of the main pumping stations depends on the tide of the outside water and can vary between 50 and 70 $m^3$/s. This capacity can not be reached in practice, however, because of limitations on the transport capacity of the canal system. High pump flows should be avoided where possible to avoid problems with high flow velocities and steep water level gradients.

Until recently, the main pumping stations were manually controlled by operators of the water board. To assist them in the difficult task, the Dutch engineering consultant Nelen & Schuurmans BV has designed and implemented a decision support system (DSS), in close cooperation with the operators of the water board (Schuurmans *et al.*, 2003).

The DSS is based on the Model Predictive Controller as described previously. Every 15 minutes, the optimal control sequence is calculated for 24 hours (the length of Tc and Tp) in 15 minute time steps. The calculation is based on the latest water level measurements and inflow predictions, based on a rainfall–runoff model fed by precipitation measurements and forecasts.

In current practice, the decision about the timing and magnitude of the pump flow is still taken by the operators, aided by this DSS. The DSS can also be switched to a mode in which it functions as a control system, automatically operating the pumping stations.

## Sensitivity for uncertainties for the Delfland DSS

For the case of the model predictive controller used in the DSS for Delfland, the sensitivity of the control action was tested by providing different forecast errors and measuring performance by evaluating the objective function over a closed loop simulation period. The sensitivity can be determined in two steps:

(a) Test whether the control action is sensitive to information after a certain time step.

(b) If so, test what the resulting reduction in optimality is, by evaluating the closed loop value of the objective function.

Because positive and negative deviations from the target level are punished equally strongly in the objective function, anticipatory pumping is only used if pump constraints are likely to be violated and positive deviations are expected somewhere in the future. In this case, the pumping necessary to counter the positive deviations is postponed as much as possible to avoid long periods with low water levels. Under normal conditions, in which the pump constraints are not relevant, the actual water level and the actual flow, therefore, are the only variables influencing control actions. In this case, the MPC controller behaves similar to a feedforward controller.

In cases where constraints become relevant (when the expected inflow exceeds the maximum outflow), anticipatory pumping becomes necessary. In these cases, there will be a period of time in the future in which the pumps are planned to operate at their full capacity. If the inflow peak is close enough to the actual moment, the pump flow is already at full capacity and will not be sensitive to increase of the forecasted flow. If the event is farther away or smaller, anticipation will be postponed, so in that case the action will be insensitive to changes in the future. In between these two, there is a relatively small range of situations in which the optimal action consists of anticipation of future constraints, but not at full pump capacity. In that case, the control action is sensitive to any changes in forecast, as long as they occur within the period of projected full pump capacity use. This period can be bounded by physical or operational constraints on the water level, but in the theoretical case that the expected inflow is very close to the maximum pump flow, can be unbounded. In this case, TIc goes to infinity. Summarizing, the controller can be in three situations:

(a) no anticipation necessary,
(b) sensitive to prediction,
(c) pump at full capacity.

Situation (b) is always a transition from situation (a) to (c). In a deterministic optimization with a perfect prediction, the time period of this transition is limited and very much determined by the objective function.

In practice, however, predictions are uncertain to a considerable extent. Apart from the small errors in prediction, that may or may not influence the actions depending on the situation, larger uncertainties in the prediction may also exist. These errors may also influence the situation (no anticipation, sensitive, full capacity) of the controller.

It is only if all possible paths of the forecast lead either to situation (a), or all lead to situation (c), that the controller is insensitive to the prediction. If this is not the case, the consequences and probabilities of all inflow scenarios have some impact on the decision.

## Risk-based operation

If uncertainties in the forecast information influence the control decisions, what would be the best decision, given the probability distribution of the inputs?

If a water system is certainty-equivalent, the optimal operation can easily be found by optimizing the control actions, given the best deterministic forecast, i.e. the expected value of the input (equation (3)). However, the necessary conditions for certainty equivalence require that (Philbrick & Kitanidis, 1999):

(a) The objective function is quadratic.
(b) System dynamics are linear.
(c) There are no inequality constraints.
(d) Uncertain inputs are normally distributed and independent.

For the Delfland case, the last two conditions are not fulfilled, making it a non-certainty equivalent problem. This means that:

$$u_{opt,deterministic} = \arg\min_{u} J(x,u,E(Q_d)) \tag{3}$$

$$u_{opt,stochastic} = \arg\min_{u} \underset{Q_d}{E}(J(x,u,Q_d)) \tag{4}$$

$$u_{opt,stochastic} \neq u_{opt,deterministic} \tag{5}$$

in which $x$ is the state (in this case $e$), $u$ is the control vector (in this case $Q_c$), $Q_d$ is the uncertain inflow, $J$ is the objective function (see equation (1)) and $E$ is the expectation operator.

In this case, optimal operation, given the uncertainties and the available information, is only possible by using a stochastic approach (equation (4)) to the optimization problem, routing the uncertainties to the objective function, which will then express risk instead of costs (Weijs *et al.*, 2006). By minimizing this objective function, expected damage is minimized. This can be approximated by using different inflow scenarios as a discrete representation of the uncertain inflow in a multiple model configuration of MPC (van Overloop, 2006; van Overloop *et al.*, 2006).

The use of an ensemble inflow prediction avoids the necessity to parameterize the stochastic inflow process, leaving the full stochastic properties of the physical model output intact.

In this research, the ensemble inflow forecast has been derived from a Monte Carlo simulation with the rainfall–runoff model, reflecting the parameter uncertainties. If ensemble weather predictions become available at a suitable timescale, it is possible to include these in the generation of the ensemble forecast.

## CONCLUSIONS AND RECOMMENDATIONS

Because of inflows exceeding the pump capacity, anticipation is necessary for the control of water levels in Dutch lowland water systems. A decision support system, based on Model Predictive Control assists operators in this task. The uncertainties in the information used by this controller have a negative influence on the performance. Since the problem is not certainty equivalent, use of multiple models is proposed to deal with the uncertainties, using risk minimization as the objective.

Two time scales are introduced, that have to be considered in the choice of the optimization horizon for the controller. The Information Control Horizon is a time-scale, dependent on the sensitivity of the actual decision for future events, depending on the water system characteristics, constraints and the objective function. The Information Prediction Horizon is a property of the predictability of the inputs and the forecast system used. These time scales are used in guidelines for the design of this type of Model Predictive Controllers and for assessing the impact of uncertainties.

To derive the first horizon, simulations can be done by the controller confronted with several forecasts. It might also be possible to derive analytical expressions for the horizon in certain types of problems. To derive the second horizon, time series of past forecasts and actual inflows need to be analysed. For real time control systems with relatively short time steps, in particular, multiple year data sets become very large and are usually not stored. The Delfland DSS has been coupled to a long-term database, storing this information since 2004. This growing data set might become very valuable for improving the forecasts and making the analyses proposed in this paper.

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