Multichannel Compressed Sensing via Source Separation for Hyperspectral Images

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Outline

- Problem Statement
- Signal Model
- Sampling and Transmission Mechanism
- Joint Recovery Methods
- Simulation Results

Compressed Sensing

- Idea: Merging the sampling and compression steps together
  ⇒ *Sampling under the Nyquist rate*

- Condition: Signal to be compressible ≡ sparse in some basis.

- **Problem statement**

  Recovering a sparse vector $x \in \mathbb{R}^N$ *(*$\|x\|_0 = K$*) from M linear, noisy measurements* $y = \Phi\Psi x + z$*, where, M<<N.*

  $z \in \mathbb{R}^M$: the noise vector

  $\Phi \in \mathbb{R}^{M \times N}$: The sensing dictionary, often a random matrix.

  $\Psi \in \mathbb{R}^{N \times N}$: The sparsifying O.N. basis.

Compressed Sensing

- **Recovery algorithms:** Convex relaxation: l1/TV-minimization, Greedy:COSAMP, ROMP, IHT,...
- **Guarantees:**
- **Definition.** Matrix A satisfies restricted isometry property (RIP) with constant δ_s if for all *s-sparse* vectors *x* we have:

$$(1-\delta_s)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta_s)\|x\|_2^2$$

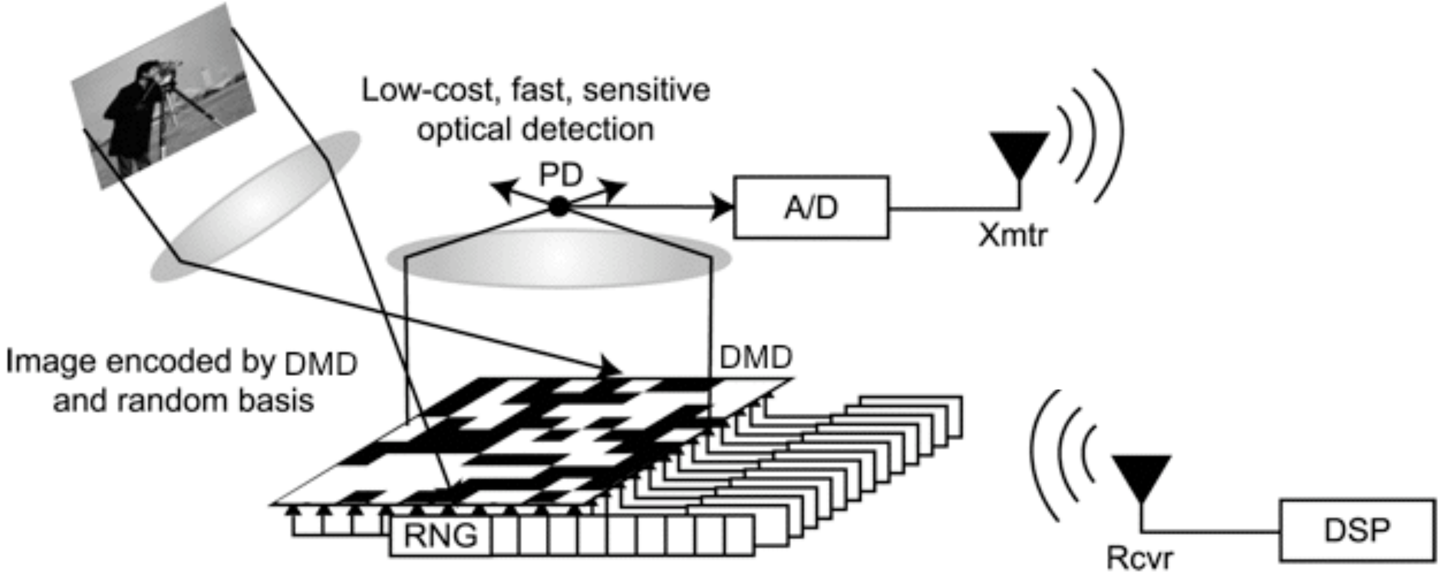
- Stable recovery of all *K*-sparse vectors, if $\Phi\Psi$ satisfies RIP with $\delta_{c.K} \leq C_0$.
- For all subgaussian Φ, if $M \gtrsim O\Big(K \log(N/K)\Big)$, $\Phi\Psi$ is RIP.
- Low cost sampling for complex recovery e.g., for IHT complexity of each iteration scales with *M*×*N*.

CS Image Acquisition

- Rice Single-Pixel Camera

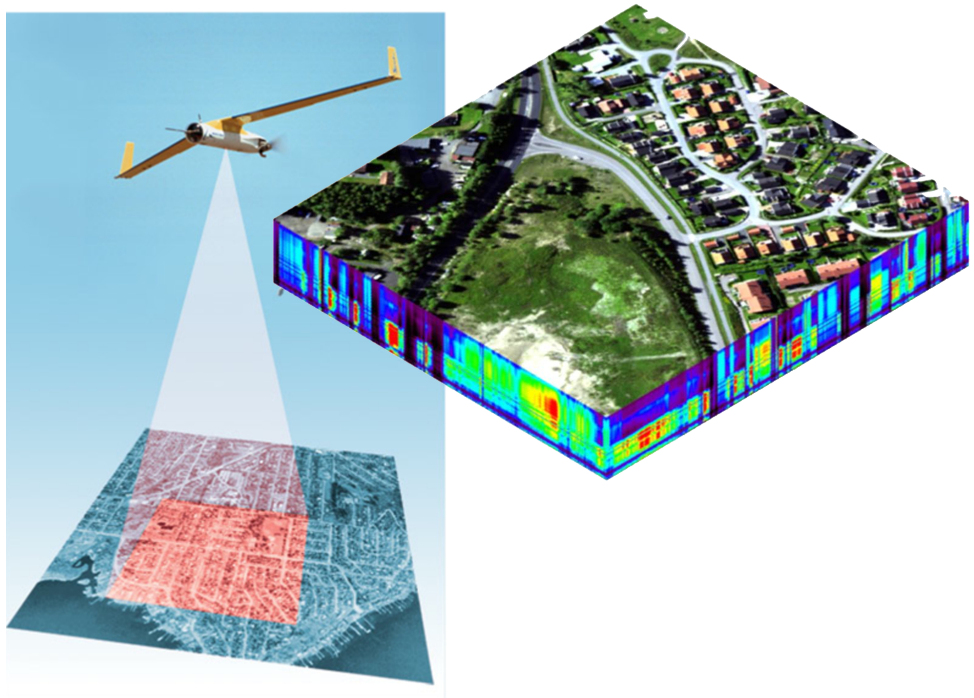
  http://dsp.rice.edu/cscamera

  Φ : uniformly at random selecting rows of Walsh-Hadamard matrix.



Hyperspectral Images

- HSI: A collection of hundreds of images acquired simultaneously in narrow and adjacent spectral bands/channels.



- Applications include agriculture, mineral exploration and environmental monitoring.

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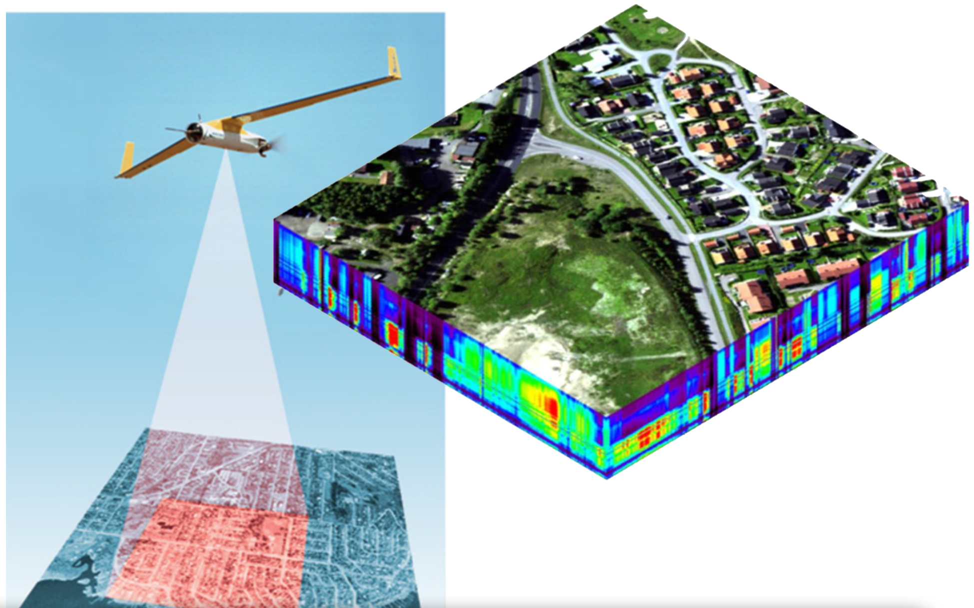


As it is costly to acquire each pixel of HSI, it becomes very interesting to use CS approach!

environmental monitoring.

Observation Model

- HSI are represented by a matrix $X \in \mathbb{R}_{+}^{J \times N}$

  J: spectral bands/channels

  N: image resolution per channel

- **Signal Priors**

  1) HSI is generated from few source images based on a *linear mixture* model

  2) Each source image is *sparse* in wavelet basis

  3) Source images are sometimes disjoint.

  4) Mixture parameters are known.

Observation Model (Cont.)

- HSI can be decomposed as,

$$X = AS = A\Sigma\Psi^T.$$

$S \in [0,1]^{I \times N}$ is the "*source matrix*", rows collecting *I source images*, each representing the percentage of a given material in each pixel of the scene. Sometimes we can assume each pixel corresponds to only one material i.e., sources are disjoint, $S \in \{0,1\}^{I \times N}$

$A \in \mathbb{R}_+^{J \times I}$ is the "*mixing matrix*", whose columns are the *spectral reflectance* of the respective source images (rows of *S*).

$\Sigma \in \mathbb{R}^{I \times N}$ whose rows are sparse vectors, representing the Wavelet coefficients of the source images (Ψ the 2D Wavelet basis).

CS Acquisition for HSI

- How to recover HSI from CS measurements?

  A "*joint recovery scheme*" to exploit the correlations among channels rather than channel-by-channel individual recovery?

- Our contribution:

  Rephrasing the CS recovery of multichannel data as the "*compressive source separation*" problem by knowing the mixture parameters.

  Instead of recovering the whole data, first recover the few underlying sparse sources!

Sampling and Transmission

1. Taking *M* linear measurements per channel using a "*universal*" random compression matrix

$$Y = X\Phi^T \qquad Y \in \mathbb{R}^{J\times M}$$

2. Applying separation matrix $A^\dagger = (A^T A)^{-1} A^T$, and a thresholding operator $[.]_+$, to extract measurements of active sources

$$\overline{Y} = [A^\dagger Y]_+ = \Sigma_{\mathcal{I}^*}\Psi^T\Phi^T. \qquad \overline{Y} \in \mathbb{R}^{I\times M}$$

$\mathcal{I}^*$: indices of the active sources.

3. Transmit indices $\mathcal{I}^*$ plus CS measurements $\overline{Y}$ to the receiver.

- **Limitation.** 1) $A^\dagger$ may bring instability issues, at least $I \leq J$ is required, 2) Mixing matrix *A*, the spectral signature of the existing materials in HSI must be known.

Reconstruction Problem

- Noisy measurements at the receiver point (vector formulation),

$$\mathbf{Vec}(\widetilde{Y}) = \tilde{\mathbf{\Phi}}\,\mathbf{Vec}(\Sigma_{\mathcal{I}^*}) + \mathbf{Vec}(Z).$$

$\tilde{\mathbf{\Phi}} = \Phi\Psi \otimes \mathbf{Id}_{I^*}$, $\otimes$ the Kronecker product, $\mathbf{Id}_{I^*}$ identity matrix.

- **Source reconstruction problem (P_0)**

$$\begin{aligned} \widehat{\Sigma}_{\mathcal{I}*} &= \mathrm{argmin}_{\Sigma\in\mathcal{B}}\|\mathbf{Vec}(\Sigma)\|_0 \\ s.t. & \quad \|\mathbf{Vec}(\widetilde{Y}) - \tilde{\mathbf{\Phi}}\,\mathbf{Vec}(\Sigma)\|_2 \leq \epsilon \end{aligned}$$

where $\mathcal{B} := \{\Sigma \in \mathbb{R}^{I^*\times N} : (\Sigma\Psi^T)_{ij} \in \{0,1\}\ \&\ \mathrm{Offdiag}(\Sigma\Sigma^T) = 0\}.$

- This is an NP hard problem!

Reconstruction Algorithm

- We propose the following method to approximate solution of $\mathbf{P_0}$:

1. **l1 relaxation,** solving with polynomial-time complexity

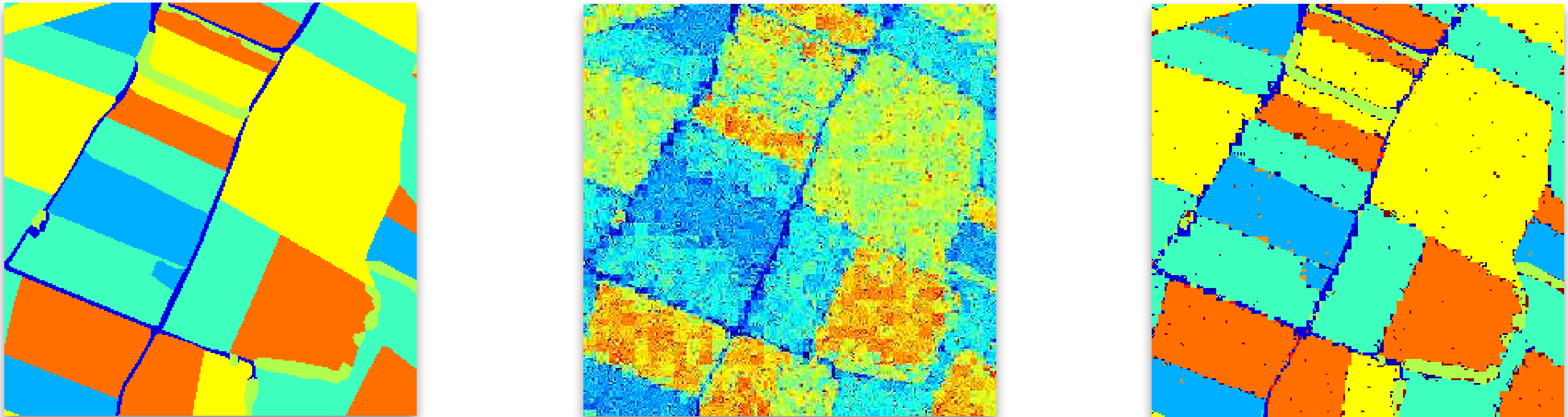
$$\begin{aligned} \widehat{\Sigma}_{\mathcal{I}*} &= \operatorname{argmin} \|\mathbf{Vec}(\Sigma)\|_1 \\ s.t. &\quad \|\mathbf{Vec}(\widetilde{Y}) - \tilde{\mathbf{\Phi}}\, \mathbf{Vec}(\Sigma)\|_2 \leq \epsilon \end{aligned}$$

2. Refinement: Projecting the solution onto the set $\mathcal{B}$ through a "*thresholding*" step, to match the priors.

3. Once the algorithm determines the sources, the whole HSI cube can be recovered through the mixing model.

Simulation Results I

Setup. Synthesized HSI (N=256×256, J=128) with I^*=6 distinct sources.
Experiment 1. Set M=6400, transmitting $M \times I^*$ measurements, about 0.45% of the original HSI.

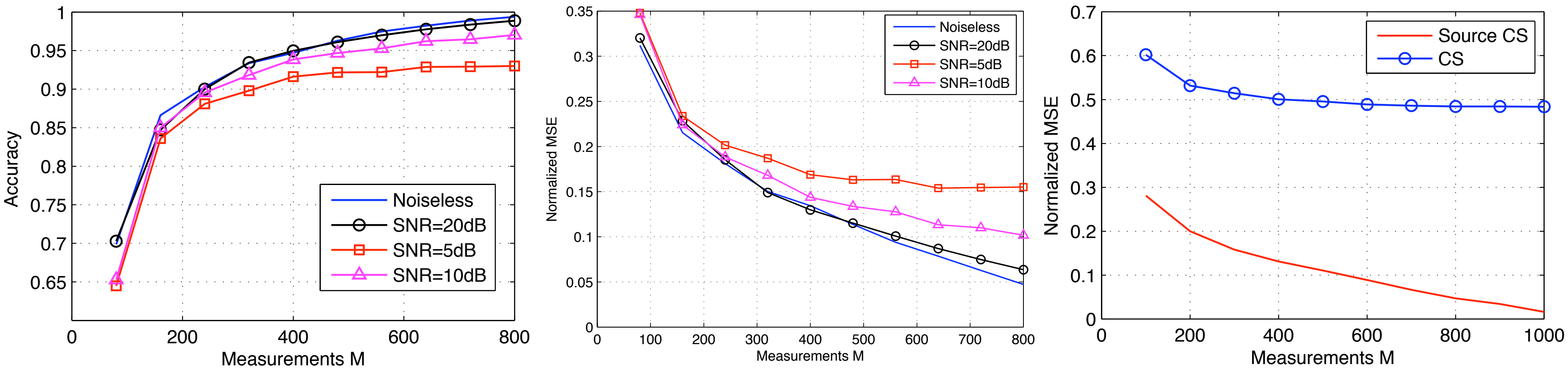


(a) (b) (c)

Reconstruction of HSI using our recovery scheme, demonstrated for a slice/channel j = 90. (a) Original data, (b) Reconstruction without thresholding step (Normalized MSE=0.23) and (c) Reconstruction with thresholding step (Normalized MSE=0.19).

Simulation Results II

- **Experiment 2.** Average performance evaluation of our method: Experiments on a 64×64×64 cubic image, extracted from the original HSI. Plots averaged over 20 independent realizations of Φ and Z.



(a) (b) (c)

- (a) Source separation *accuracy* and, (b) HSI reconstruction error of our recovery scheme, for different SNR and compression sizes *M*. (c) Reconstruction error of the HSI using classical CS recovery (applied separately on each channel) vs. our source separation based recovery scheme (Source CS).

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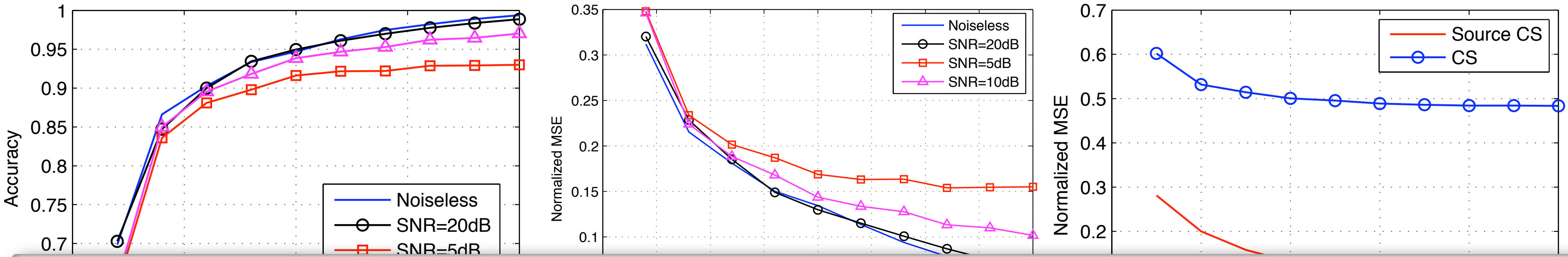


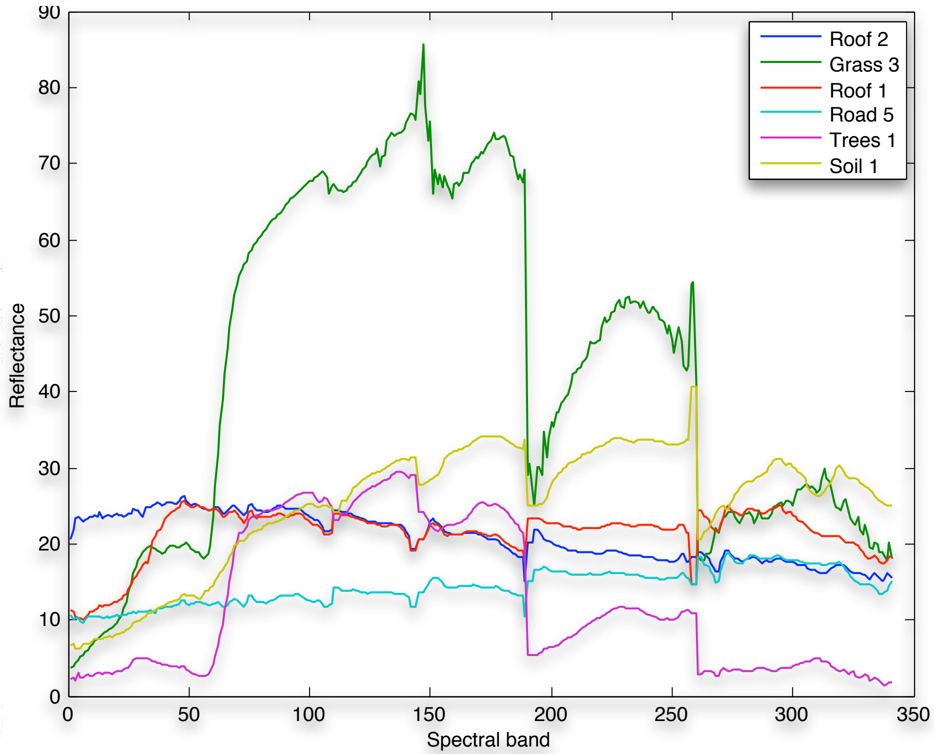
Remark: Individual channel-by-channel reconstruction requires $M \times J$ measurements, whereas our method (Source CS) outperforms by only $M \times I^*$ measurements, which significantly saves the battery life of the transmitter (satellite) and complexity of the decoding algorithm.

our source separation based recovery scheme (Source CS).

Real data

- URBAN data set

  HSI of size N=300×300, J=341, I = 6



Recovery Method II

1. Source images here are no more disjoint, but
   $\sum_i S^i(n) = 1, \quad 1 \le n \le N$ (another sort of sparsity)

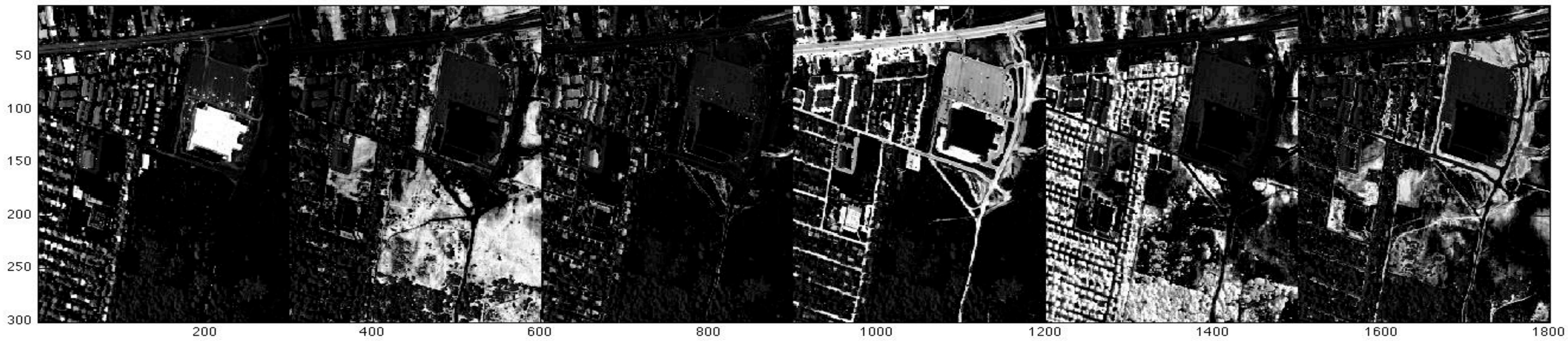
2. Sources have sparse gradient variation (TV norm)

- Recovery by the following convex-minimization:

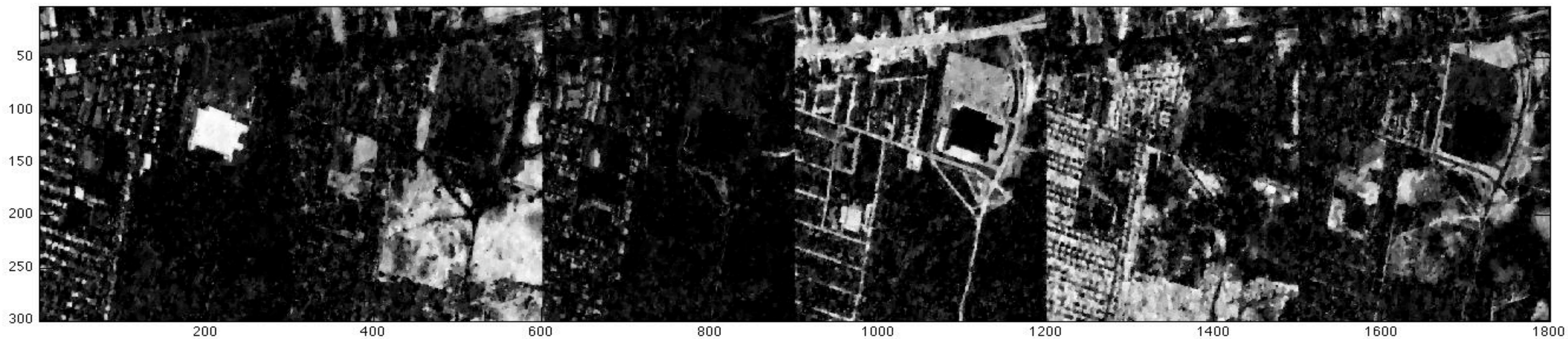
$$\begin{aligned} \arg\min_{S} \quad & \sum_i \|S^i\|_{TV} \\ \text{subject to} \quad & \left\| \widetilde{Y} - S\Phi^T \right\|_F \le \epsilon, \\ & S \ge 0, \\ & \sum_i S^i(n) = 1, \quad 1 \le n \le N, \end{aligned}$$

Simulation Results III

- CS acquisition with *M=N/8,* transmitting 0.21% of the whole cube!!



Ground truth

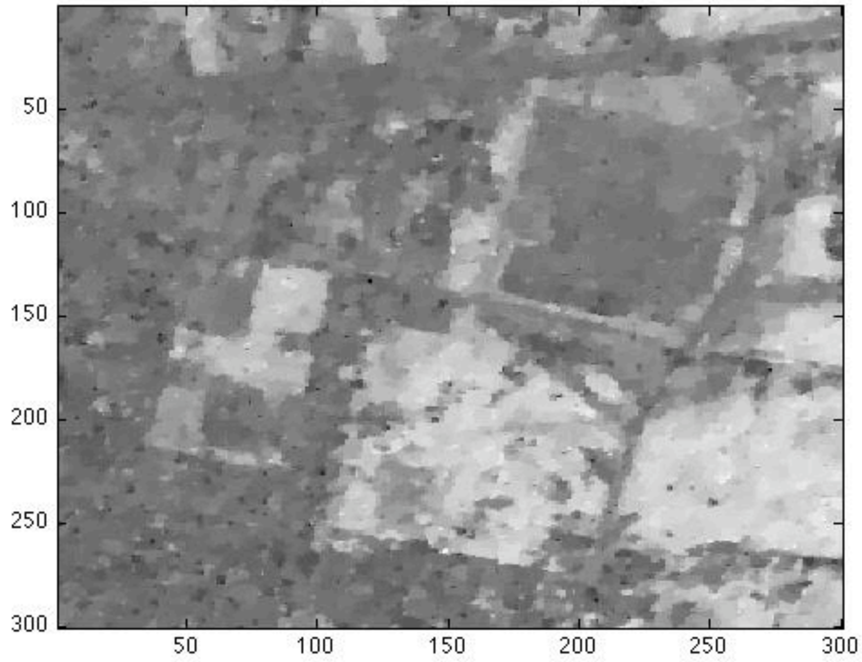


Source reconstruction by our method

Simulation Results IV

- HSI reconstruction from CS measurements *M=N/8* (demonstrated for spectral band *j*=230)

Source-separation based vs. Classical TVDN



Questions?!

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