Magnetohydrodynamic properties of nominally axisymmetric systems with 3D helical core

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Abstract

Magnetohydrodynamic equilibrium states with a three-dimensional helical core are computed to model the MAST spherical tokamak and the RFX-mod reversed field pinch. The boundary is fixed as axisymmetric. The MAST equilibrium state has the appearance of an internal kink mode and is obtained under conditions of weak reversed central shear. The RFX-mod equilibrium state has seven-fold periodicity. An ideal magnetohydrodynamic stability analysis reveals that the reversal of the core magnetic shear can stabilise a periodicity-breaking mode that is dominantly $m/n = 1/8$ strongly coupled to a $m/n = 2/15$ component, as long as the central rotational transform does not exceed the value of 8.

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1.. Introduction

Tokamaks and Reversed Field Pinches (RFP) are designed nominally to be axisymmetric devices which may be weakly perturbed by the magnetic field ripple induced by the necessarily discrete nature of the toroidal coils. The experimental conditions in such systems, however, reveal that the internal magnetic field structure can be drastically modified acquiring three-dimensional (3D) properties. Standard Grad-Shafranov solvers are unable to cope with the breaking of axisymmetry. The most obvious example is the "snake" phenomenon that has been observed in the JET tokamak [1, 2]. But 3D internal structures could be the cornerstone of the continuous modes observed after the disappearance of sawteeth in the TCV tokamak [3, 4], the "long-lived modes" in the MAST spherical torus [5], the saturated internal kinks reported in the NSTX device [6] and the transition of mode behaviour as a function of plasma shaping in the DIII-D tokamak [7]. In RFX-mod, the Single Helical Axis (SHAx) reconstruction of the plasma constitutes the manifestation of a dominant single helicity mode of operation that results in a significant improvement of confinement properties [8].

The theoretical investigation of such internal helical states in tokamaks has relied mostly on analytic techniques of saturated internal kink modes [9, 10, 11] or large scale nonlinear magnetohydrodynamic (MHD) stability codes [12, 13]. Bifurcated equilibrium states have been also obtained that model ballooning-like features on low order rational surfaces through the second variation of the potential energy that reveal the formation of 3D structures that are interpreted as the incipent formation of magnetic islands [14]. MHD equilibria with imposed nested magnetic flux surfaces and an axisymmetric plasma boundary that can reproduce the SHAx state in RFX-mod have been successfully computed with the VMEC code [15, 8] and that can also predict helical structures similar to saturated internal kink modes [16] in tokamaks have been calculated with ANIMEC [17] (a variant of the VMEC code designed to obtain 3D anisotropic pressure equilibria).

We address in this work the generation of 3D equilibria that model the MAST spherical tokamak with core reversed shear and the examination of the ideal MHD stability properties of RFX-mod SHAx equilibrium states.

2.. MAST helical core equilibrium state

The ANIMEC code [17] is used to determine a bifurcated equilibrium state that models the MAST tokamak. The MAST boundary is axisymmetric and is obtained from a fit to the formula

$$R_b = R_0 + a\cos(u + \delta\sin u + \tau\sin 2u)$$

$$Z_b = Ea\sin u,$$

where the major radius is $R_0 = 0.9m$, the minor radius is $a = 0.54m$, the elongation is $E = 1.744$, the triangularity is $\delta = 0.3985$ and the quadrangularity is $\tau = 0.1908$. The variable u represents a poloidal angle.

The plasma mass and the toroidal current profiles are prescribed such that the resulting plasma pressure is relatively constant in the core with steeper gradients in the outer half of the plasma as displayed in Fig. 1 and the inverse rotational transform q-profile is weakly shear reversed in the centre of the plasma with a minimum value $q_{min} \simeq 1$ inside mid-radius. The toroidal plasma current and corresponding q-profile are plotted in Fig. 2.

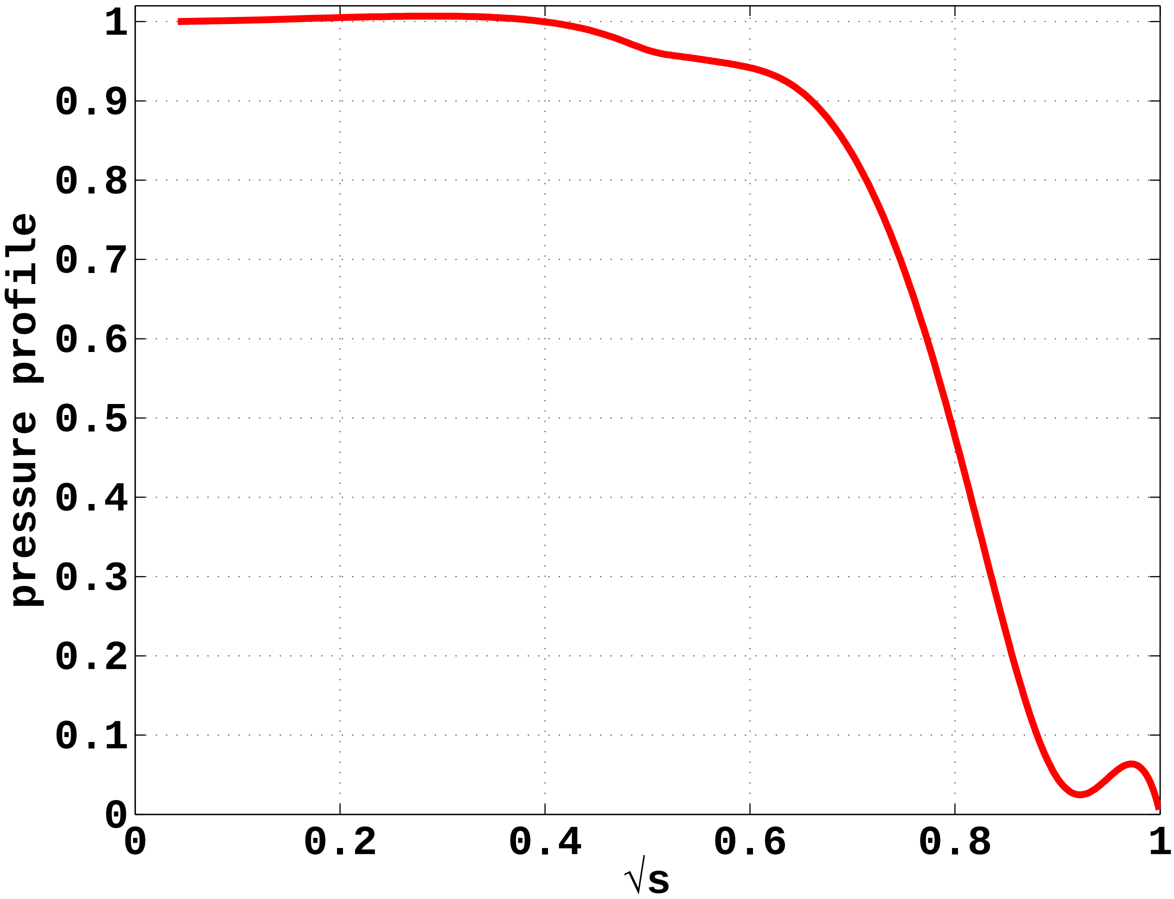

Figure 1. The pressure profile of a model MAST equilibrium as a function of the radial variable $\sqrt{s}$, where s is proportional to the toroidal magnetic flux normalised to it value at the plasma boundary (corresponding to $0.63Wb$ for the case considered). (colour online)

The mass profile chosen is described by a polynomial expansion in s, a radial variable proportional to the enclosed toroidal magnetic flux. The toroidal current profile is prescribed by a piecewise continuously differentiable function in s composed of a quadratic expression in the centre of the pasma, a linear term in the outer part of the plasma and these are connected with a cubic function [18, 19]. The ANIMEC code predicts two possible solutions to the MHD equilibrium equations [16]. The standard solution is axisymmetric. A helical core bifurcated solution can also be achieved when q_{min} is in the neighbourhood of unity and we provide an initial guess to the position of the magnetic

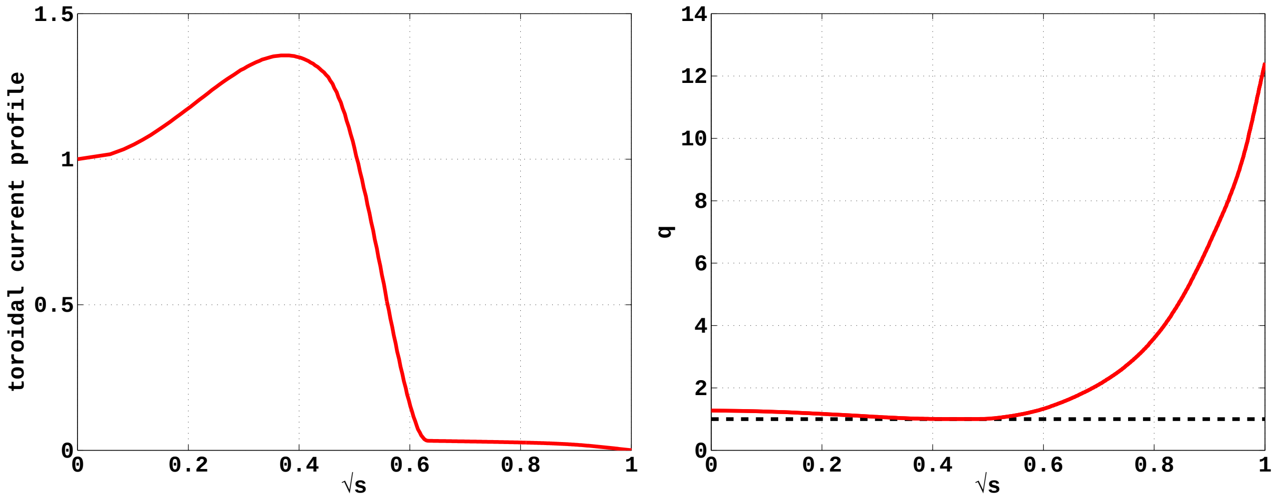

Figure 2. The toroidal current profile (left) and the corresponding inverse rotational transform q-profile (right) as a function of the radial variable $\sqrt{s}$ for a MAST equilibrium state. The dashed line identifies the value of $q = 1$. (colour online)

axis that has a helical distortion. We concentrate hereon on the helical branch solution.

The contours of constant pressure at four toroidal cross sections that span half the torus are presented in Fig. 3. The outer region of the plasma remains axisymmetric

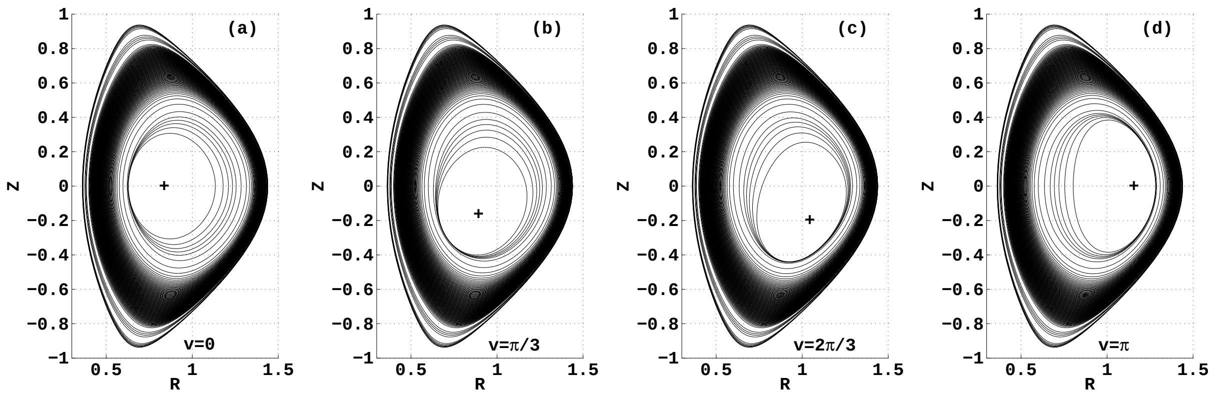

Figure 3. Contours of constant pressure at cross sections with toroidal angle (a) $v = 0$, (b) $v = \pi/3$, (c) $v = 2\pi/3$ and (d) $v = \pi$ that encompass half the torus for a MAST equilibrium with $q_{min} \sim 1$. The position of the magnetic axis at each cross section is marked with a "+" symbol.

but the inner core acquires a 3D helical character similar to a saturated ideal MHD internal kink mode (the magnetic field lines do not break consistent with the condition imposed in the equilibrium calculation that the magnetic flux surfaces remain nested). The toroidal plasma current in the configuration examined is $340kA$, the vacuum toroidal magnetic field is approximately $0.39T$ at the centre of the cross section and the volume averaged $\langle\beta\rangle \simeq 6.2\%$ corresponding to $\beta_N \simeq 5$. Stable operation at this value of β_N has been reported on MAST experimental discharges [20]. The extent of the 3D helical core depends most sensitively to details of the q-profile, particularly the proximity of q_{min} to unity. Pressure profile variations at fixed $\langle\beta\rangle$ are not critically important. The helical distortion of the magnetic axis increases by less than 25% when we increment $\langle\beta\rangle$ from 1% to 6%.

3.. Equilibrium and ideal MHD stability of RFX-mod

MHD equilibrium states that model the SHAx regime on RFX-mod have been computed with the VMEC code in fixed boundary mode [8, 15]. The plasma boundary is circular, the pressure and inverse rotational transform profiles are prescribed as polynomial functions with respect to the normalised toroidal magnetic flux. Therefore, the q-profiles that are investigated only approach the field reversal point. This is quite adequate to investigate the physics of the core region which can develop a 3D internal structure which is roughly independent of the dynamics where the toroidal magnetic field reversal occurs. A new version of the VMEC code that employs the poloidal flux as the independent radial coordinate is under development and can treat problems specifically related to field reversal physics [8].

The ideal MHD stability properties of RFX-mod SHAx equilibrium models have been investigated with the TERPSICHORE code [21]. The equilibrium state develops a central helical distortion with a seven-fold toroidally periodic structure when the inverse rotational transform q decreases below 1/7 on axis. The toroidal magnetic flux contours computed in the coordinates of the VMEC code and transformed to the Boozer coordinate frame [22] are plotted on the top and middle rows of Fig. 4, respectively. The indentations of these flux contours in the core region are less pronounced at fixed Boozer toroidal angles ϕ compared with those at the corresponding planes of fixed geometric toroidal angle v used in the VMEC code representation. In RFP configurations, the main source of energy for MHD instability is the parallel current density. Thus contours of constant $\boldsymbol{j} \cdot \boldsymbol{B}/B^2$ covering half a toroidal field period are displayed in in the Boozer frame on the bottom row of Fig. 4. The Pfirsch-Schlüter contribution to $\boldsymbol{j} \cdot \boldsymbol{B}/B^2$ is weak despite finite pressure gradients, thus $\boldsymbol{j} \cdot \boldsymbol{B}/B^2$ is virtually constant on each flux surface. The blank contour near mid-radius identifies the transition position of the magnetic field structure from external axisymmetric to internal helical.

We explore the ideal MHD stability properties of the RFX-mod equilibrium state to mode structures that break the seven-fold periodicity of the system. In particular, we examine the mode family [23, 24] of the immediately contiguous side-band. This is labelled as the $N = 1$ family and includes the toroidal mode numbers $n = n_{eq} \pm 1$ with $n_{eq} = 7\ell$, where ℓ is an integer (positive or negative). Thus this mode family contains $n = ... - 1, 1, 6, 8, 13, 15, 20, ...$. We prescribe a conformal conducting wall that is 1.1 times the plasma radius which is close to that in the experiment. Three specific configurations are studied. These include a case with monotonic q-profile with vanishing central shear and on-axis $q = 1/8$, a core shear reversed configuration also with on-axis $q = 1/8$ and an equilibrium state with reversed central shear but on-axis $q < 1/8$. The monotonic q system is weakly unstable to a mode structure which dominantly couples a $m/n = 1/8$ with a $m/n = 2/15$ component. Here m is the poloidal mode number. When the central shear is reversed retaining on-axis $q = 1/8$, the mode is stabilised. However, when on-axis q decreases below $1/8$, the plasma becomes strongly unstable with respect to the coupled $(m/n = 1/8; 2/15)$ mode. The q-profiles and eigenvalues of the configurations analysed

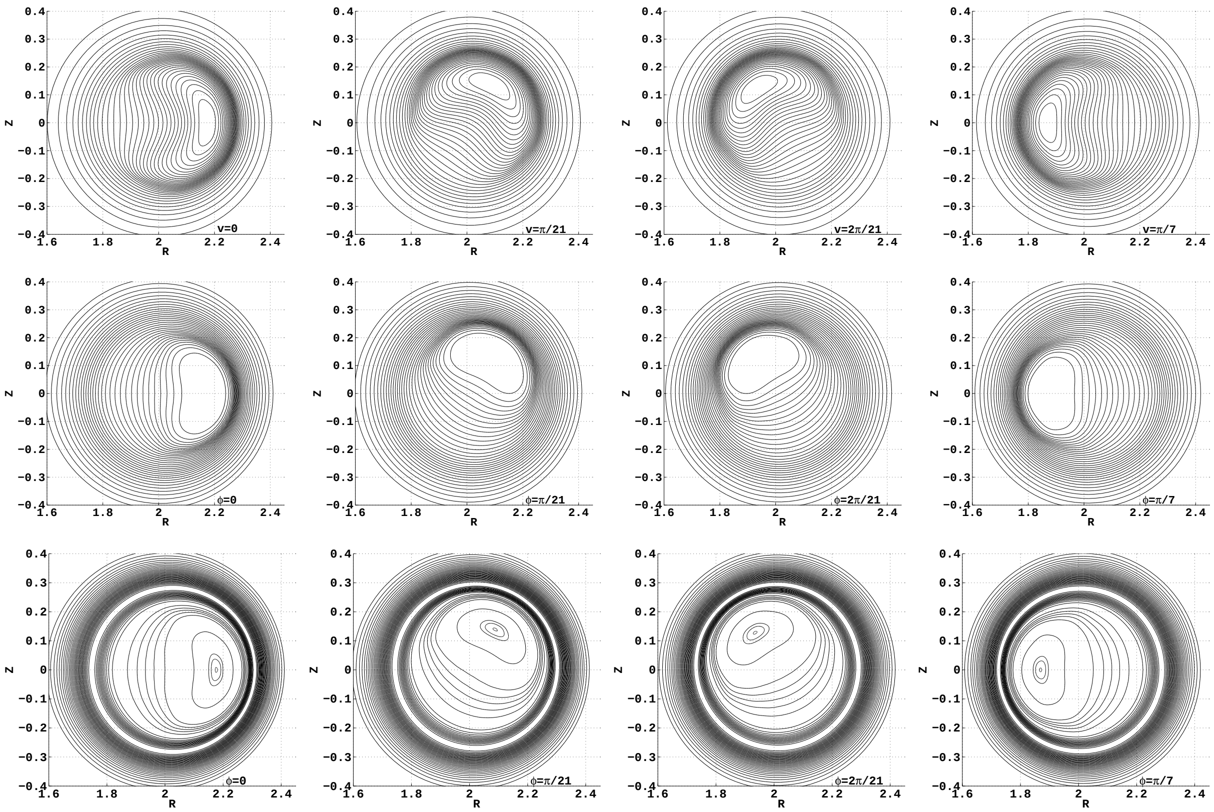

Figure 4. Contours of constant toroidal flux Φ on geometric cross sections with VMEC toroidal angle $v = 0$, $v = \pi/21$, $v = 2\pi/21$ and $v = \pi/7$ (upper row), the corresponding Φ contours in Boozer coordinates with toroidal angle $\phi = 0$, $\phi = \pi/21$, $\phi = 2\pi/21$ and $\phi = \pi$ (middle row), and the contours of constant parallel current density factor $\boldsymbol{j} \cdot \boldsymbol{B}/B^2$ at the same Boozer frame toroidal angle (bottom row) covering half of a period in an RFX-mod SHAx equilibrium model.

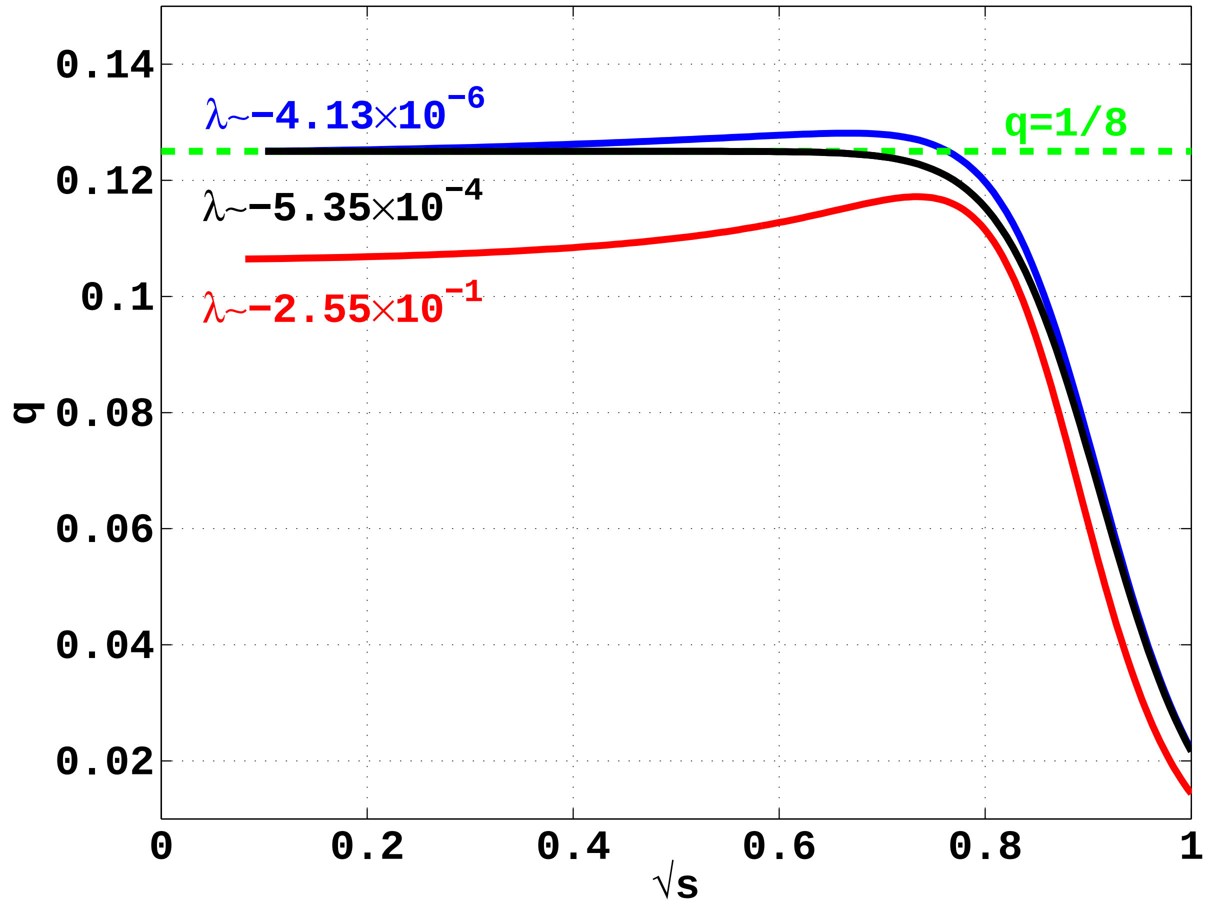

Figure 5. The inverse rotational transform q-profiles as a function of the radial variable $\sqrt{s}$ (s is proportional to the normalised toroidal magnetic flux) for equilibrium states with reversed core shear and on-axis $q = 1/8$ (top curve), monotonic q with vanishing core shear (middle curve) and low-q reversed core shear (bottom curve). The corresponding ideal MHD eigenvalues λ due to a dominant $(m/n = 1/8; 2/15)$ mode for each case are shown. The dashed curve identifies the $q = 1/8$ rational value. (colour online)

are plotted in Fig. 5. Stability is considered to ensue when the eigenvalue $\lambda > -1 \times 10^{-4}$. The perturbed displacement vector in TERPSICHORE is denoted by $\boldsymbol{\xi}$ and its radial component is $\xi^s \equiv \boldsymbol{\xi} \cdot \boldsymbol{\nabla} s$. The Fourier amplitudes of ξ^s (denoted as ξ^s_{mn}) are displayed in Fig. 6 for the dominant $m/n = 1/8$ and $m/n = 2/15$ mode components for the core shear reversed and for the monotonic weak central shear examples (with on-axis $q = 1/8$). The shear reversal in the plasma bulk causes the mode structure amplitude to become

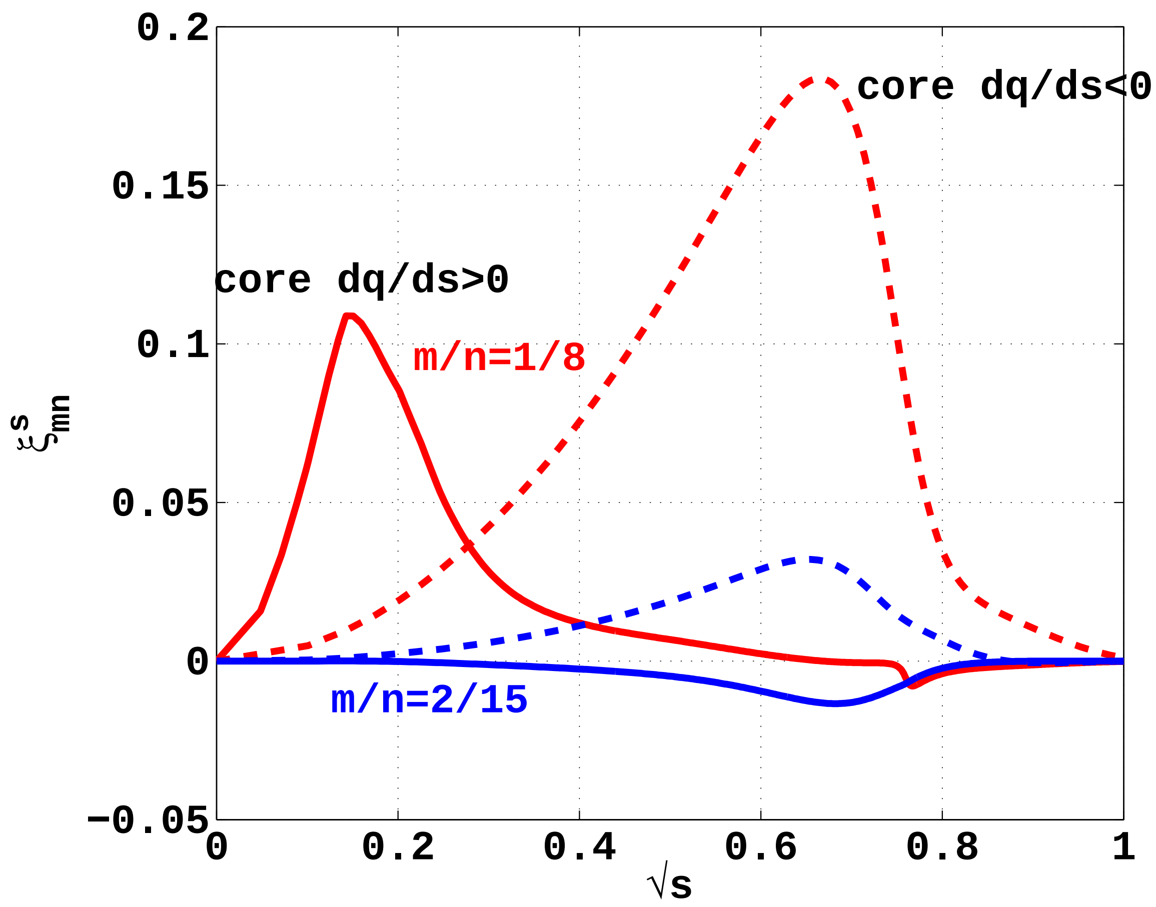

Figure 6. Profiles of the Fourier amplitudes of the radial component of the displacement vector as a function of $\sqrt{s}$. The solid (dashed) curves correspond to the two dominant terms of the mode structure for the reversed core (monotonic vanishing) shear case with on-axis $q = 1/8$ of the main periodicity-breaking instability. The two curves with largest amplitude represent the $m/n = 1/8$ Fourier component. (colour online)

smaller in magnitude and to shift towards the centre of the plasma, which constitute the ingredients for mode stabilisation.

4.. Conclusions and discussion

We have computed model MAST tokamak equilibrium states with internal 3D helical structures that are reminiscent of $m/n = 1/1$ saturated internal kink modes when the inverse rotational transform has weak core reversed magnetic shear and $q_{min} \sim 1$. The plasma pressure profile is very flat in the central region of the plasma, chosen consistent with the experimental observations associated with the "long-lived" mode conditions [5]. We have previously reported similar equilibrium solutions for TCV [16] and ITER [19]. In comparison, we find that for the tighter aspect systems like MAST, it is generally easier to calculate equilibria with helical internal distributions. In particular, the radial position of q_{min} can be more centrally located than for TCV [16] or ITER [19].

The MHD equilibria computed to model RFX-mod SHAx conditions are obtained with reversed core shear ($q'(s) > 0$ with prime $'$ indicating a derivative with respect to s) and with almost flat monotonic central shear ($q'(s) \leq 0$ everywhere). The large current in the RFP devices implies that the system is close to force-free even with finite plasma pressure. The parallel current density is thus not significantly affected by the Pfirsch-Schlüter currents and is almost constant on the flux surfaces. We have found that $\boldsymbol{j} \cdot \boldsymbol{B}/B^2$ constitutes an excellent diagnostic to separate the internal helical core from the external axisymmetric mantle. The configurations display seven-fold periodicity. The ideal MHD stability analysis with TERPSICHORE has concentrated on instabilities that break this periodicity. We have investigated the family of modes that straddle the main equilibrium toroidal component ($n = n_{eq} \pm 1$ with n_{eq} multiples of 7). We find that a $m/n = 1/8$ mode component coupled with a $m/n = 2/15$ term constitute the dominant features of the instability structure. The configuration with monotonic vanishing central shear is weakly unstable to this class of mode. With core reversed shear, the mode is stabilised. However, if the central q-value is decreased below $1/8$, the plasma becomes strongly unstable regardless of the sign of the central shear.

The RFX-mod experiment shows that a SHAx-like state exists for a relatively long time periodically interrupted by relaxation phenomena [25]. We conjecture that these relaxations may be the result of an evolution of the q-profile that can trigger an ideal MHD instability either by losing the reversed core shear or due to a drop of central q below 1/8. This suggests that local current drive could be employed to prevent the evolution of central q and therefore control the plasma to maintain the favourable confinement properties of the single helicity mode of operation in RFX-mod.

The MHD stability properties of tokamak systems with helical core are much more delicate to evaluate because any instability mode structure is also in principle part of the equilibrium spectrum.

Acknowledgments

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Reply to the comments of the 2nd referee on

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We thank the referee for the suggestions which we hope we have satisfactorily addressed and believe that they have contributed to enhance the quality of this paper.

Specifically, we have made the following alterations:

(i) We have modified Figure 3 to identify the position of the magnetic axis on the four toroidal cross sections plotted.

(ii) The β_N value does correspond to that obtained in the MAST device. A citation is provided at the end of Sect. 2 on p. 6.

(iii) The sensitivity to profiles and $\langle\beta\rangle$ is discussed at the end of Sect. 2 on p. 6.

(iv) We have corrected the typos that the referee found on pages 6 and 7 of the original manuscript.

All changes/additions in the revised manuscript appear in red.