## **EXPLORATION OF SIGNALING CYCLES USING DYNAMIC OPTIMIZATION**

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#### Context

### **Background and Scope**

Signal transduction pathways - enable cells to:

- sense changes in their environment
- integrate external or internal signals,

and respond to them by changes in transcriptional activity, metabolism, or other regulatory measures.

These pathways are very important for major developmental and genetic changes in every organism (cell growth, mitogenesis, differentiation, embryo development), and their malfunction is associated with diseases (cancer, asthma, diabetes..).

### **Analysis of MAPK Cascades**

$$\begin{split} \frac{\mathrm{d}x^*}{\mathrm{d}\tau} &= \rho^{S/X} \tilde{k}_X \{x:s^*\} + \rho^{P_X/X} (\tilde{d}_X^* \{x^*:p_X\} - \tilde{a}_X^* x^* p_X) \\ \frac{\mathrm{d}\{x:s^*\}}{\mathrm{d}\tau} &= \tilde{a}_X x s^* - (\tilde{d}_X + \tilde{k}_X) \{x:s^*\} \\ \frac{\mathrm{d}\{x^*:p_X\}}{\mathrm{d}\tau} &= \tilde{a}_X^* x^* p_X - (\tilde{d}_X^* + 1) \{x^*:p_X\} \\ 0 &= 1 - x - x^* - \rho^{S/X} \{x:s^*\} - \rho^{P_X/X} \{x^*:p_X\} \\ 0 &= 1 - s - s^* - \{x:s^*\} \\ 0 &= 1 - p_X - \{x^*:p_X\} \end{split}$$

Mass balance

+
Mass-Action Kinetics

=
Dimensionless
transient model

↓
Differential-algebraic
equation system

### Signaling Cascade

Simple biochemical network:

- enzyme-catalyzed chemical modification of a protein molecule and
- the reverse reaction catalyzed by a different enzyme (monocyclic cascades)

This basic unit is found repeatedly in multiple configuration, throughout a wide variety of biological pathways, including phosphorylation-dephosphorylation of mitogen-activated protein kinase (MAPK) signaling cascades.

**Optimal Design** 

Ultrasensitivity

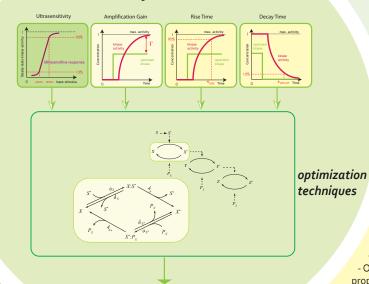
Dimensionless Michaelis-Menten

 $\tilde{K}_X = \frac{\tilde{d}_X + \tilde{k}_X}{\tilde{a}_X} \qquad \tilde{K}_{X^*} = \frac{\tilde{d}_{X^*} + 1}{\tilde{a}_{X^*}}$ 

for Maximal

# Methodology Dynamic Optimization Problem

### variation in functional behavior



## required parameter values

### **Key System Parameters**

 $\begin{array}{ll} \text{Species:} & x, x^*, s, s^*, \{x:s^*\}, p_X, \{x^*:p_X\} \\ & y, y^*, \{y:x^*\}, p_Y, \{y^*:p_Y\} \\ & z, z^*, \{z:y^*\}, p_Z, \{z^*:p_Z\} \end{array}$ 

 $\begin{array}{ll} \text{Parameters:} & \tilde{a}_X, \tilde{a}_{X^*}, \tilde{d}_X, \tilde{d}_{X^*}, \tilde{k}_X & \tilde{K}_X, \tilde{K}_{X^*} \\ & \tilde{a}_Y, \tilde{a}_{Y^*}, \tilde{d}_Y, \tilde{d}_{Y^*}, \tilde{k}_Y, \tilde{k}_{Y^*} & \tilde{K}_Y, \tilde{K}_{Y^*} \\ & \tilde{a}_Z, \tilde{a}_{Z^*}, \tilde{d}_Z, \tilde{d}_{Z^*}, \tilde{k}_Z, \tilde{k}_{Z^*} & \tilde{K}_Z, \tilde{K}_{Z^*} \end{array}$ 

Concentration ratios:  $\varrho^{S/X}, \varrho^{P_X/X} - \varrho^{X/Y}, \varrho^{P_Y/Y} - \varrho^{Y/Z}, \varrho^{P_Z/Z}$ 

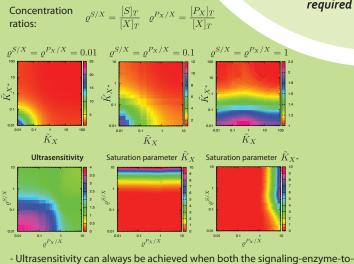
Acitivity coefficient:

 $\alpha_X = \tilde{k}_X \frac{\varrho^{S/X}}{\varrho^{P_X/X}}$ 

# Optimal Design for Minimal Signal Propagation Time

- Given amplification gain
- Given maximal ultrasensitivity
- Optimization of the sum of two signal propagation times

parameters for bi- and tricyclic cascade.



- Ultrasensitivity can always be achieved when both the signaling-enzyme-to -substrate and phosphatase-to-substrate ratios are small.
- However, this condition alone is not sufficient to allow ultrasensitivity. It is also required that both the activation and deactivation reactions operate at enzyme saturation.

