Effects of heavy modes on vacuum stability in supersymmetric theories

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Abstract

We study the effects induced by heavy fields on the masses of light fields in supersymmetric theories, under the assumption that the heavy mass scale is much higher than the supersymmetry breaking scale. We show that the square-masses of light scalar fields can get two different types of significant corrections when a heavy multiplet is integrated out. The first is an indirect level-repulsion effect, which may arise from heavy chiral multiplets and is always negative. The second is a direct coupling contribution, which may arise from heavy vector multiplets and can have any sign. We then apply these results to the sGoldstino mass and study the implications for the vacuum metastability condition. We find that the correction from heavy chiral multiplets is always negative and tends to compromise vacuum metastability, whereas the contribution from heavy vector multiplets is always positive and tends on the contrary to reinforce it. These two effects are controlled respectively by Yukawa couplings and gauge charges, which mix one heavy and two light fields respectively in the superpotential and the Kähler potential. Finally we also comment on similar effects induced in soft scalar masses when the heavy multiplets couple both to the visible and the hidden sector.

1 Introduction

In supersymmetric theories, vacua that preserve supersymmetry are automatically stable, whereas vacua that break supersymmetry are not guaranteed to be stable. In order to assess stability, one then has to study the mass matrix of scalar fluctuations around the vacuum and check that it is positive definite. It was however shown in [1, 2, 3] (see also [4] for a related analysis), by looking at the sGoldstino direction, that there exists a simple necessary condition for metastability depending on the sectional curvature of the scalar manifold along the supersymmetry breaking direction. Moreover, it has been further argued in [5, 6] that this condition becomes also sufficient if for a given Kähler potential K one allows the superpotential W to be adjusted. These results are quite helpful for discriminating between theories where metastable vacua may exist and theories where they cannot exist, by looking only at K and not at W. A comprehensive review of these results and some extensions of them within rigid supersymmetry can be found in [7].

In some cases, like for instance for the moduli sector of string models where supersymmetry is supposed to be spontaneously broken, one may be interested in studying the possibility that some of the fields are stabilized in a supersymmetry-breaking way with a small mass, whereas the remaining fields are stabilized in a supersymmetry-preserving way with a large mass. One may then study the low-energy dynamics and in particular the question of vacuum metastability within a supersymmetric effective theory obtained by integrating out the heavy multiplets. The way in which this can be done in a manifestly supersymmetric way is well known, see for instance [8, 9], and turns out to hold true also in the presence of gravity [10].[1] At leading order in the low-energy expansion in the number of derivatives, fermions and auxiliary fields, the basic recipe is that chiral and vector superfields can be integrated out by using an approximate equation of motion corresponding to imposing stationarity of W and K respectively. One may then ask the practical question of what is the effect of heavy modes on the light masses, and in particular whether the induced corrections tend to improve or to worsen the situation concerning metastability of the vacuum. More specifically, it would be very valuable to have some criterium to distinguish situations where the effect of heavy modes on the scalar square-masses are negative, and must therefore necessarily be computed to be able to assess vacuum stability, from situations where this effect is positive and can thus be safely ignored to check vacuum metastability. To derive such a criterium, we shall study in some detail the structure and the sign of the effect induced by heavy modes on the sGoldstino mass, which captures the crucial condition for achieving

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[1]See also the earlier work [11] where this question was raised and the works [12, 13] studying it in the case of effective theories describing string models with fluxes.

metastability. For simplicity we shall restrict to rigid supersymmetry, but the extension of supergravity is straightforward, since as explained in [10] the two steps of integrating out heavy multiplets and adding the coupling to gravity commute at leading order in the low-energy expansion.

In order to illustrate the basic point that we want to make, let us consider a generic theory involving both light and heavy modes l^i and h^α that interact among each other. For simplicity, we shall think of these as real scalar fields in a non-supersymmetric theory, but the results are clearly more general. In such a situation, one may define a low-energy effective theory for the light modes l^i by integrating out the heavy modes h^α. At lowest order in the low-energy expansion, this can be done by requiring stationarity of the potential energy V with respect to the heavy modes and solving the equation $V_\alpha = 0$. This determines $h^\alpha = h^\alpha(l)$. By differentiating the stationarity equation with respect to the light fields, one also deduces that $\partial_i h^\alpha = -V_{\text{inv}}^{\alpha\beta} V_{\beta i}$, where $V_{\text{inv}}^{\alpha\beta}$ denotes the inverse of $V_{\alpha\beta}$ as a matrix. The effective Lagrangian for the low-energy theory is then obtained by substituting back this solution into the original Lagrangian. For the wave-function factor and the potential, one easily obtains $g_{ij}^{\text{eff}}(l) = (g_{ij} + \partial_i h^\alpha g_{\alpha j} + \partial_j h^\beta g_{i\beta} + \partial_i h^\alpha \partial_j h^\beta g_{\alpha\beta})(l, h(l))$ and $V^{\text{eff}}(l) = V(l, h(l))$. The light masses may finally be derived by computing derivatives of V^{eff}. Using the chain rule, these can be related to derivatives of V. One finds $V_i^{\text{eff}} = V_i$ and $V_{ij}^{\text{eff}} = V_{ij} - V_{i\alpha} V_{\text{inv}}^{\alpha\beta} V_{\beta j}$, so that the light masses $m_{ij}^{2\text{eff}} = V_{ij}^{\text{eff}}$ are given by the following expression in terms of the light, heavy and mixing blocks $m_{ij}^2 = V_{ij}$, $M_{\alpha\beta}^2 = V_{\alpha\beta}$ and $\mu_{i\alpha}^2 = V_{i\alpha}$ of the full mass matrix:

$$m_{ij}^{2\text{eff}} = m_{ij}^2 - \mu_{i\alpha}^2 M^{-2\alpha\beta} \mu_{\beta j}^2 \,. \tag{1.1}$$

This expressions is easily seen to coincide with the mass matrix of light states obtained by diagonalizing the full mass matrix of the microscopic theory at leading order in an expansion in powers of the inverse heavy mass matrix. The formula (1.1) moreover shows that integrating out the heavy modes generically gives two types of effects on the masses of the light modes. The first is a direct effect hidden in the first term on the right hand side and is due to the fact that the light block of the mass matrix m_{ij}^2 gets influenced by the coupling to the heavy modes. It has a sign that depends on the form of the couplings between light and heavy modes. The second is an indirect effect described by the second term on the right-hand side and is due to the fact that the presence of an off-diagonal block in the mass matrix mixing light and heavy fields makes the true light mass matrix differ from the original light block. It has a sign that is manifestly always negative. In parallel with what happens to a quantum mechanical system with two separated sets of low and high energy levels, we see that there is a direct effect correcting significantly the light energy levels and negligibly the heavy ones, which is due to diagonal interactions and can have any

sign, and an indirect level-repulsion effect that further splits apart the two sets of levels, which is due to off-diagonal interactions and has a definite sign.

In this work, we shall consider $N = 1$ supersymmetric theories and compute the detailed form of the effective mass matrix for light scalar fields belonging to chiral multiplets in the two cases where the heavy modes that are integrated out are respectively chiral and vector multiplets. More precisely, we shall focus on the mass along the sGoldstino direction, to extract the metastability condition. It turns out that two radically different results occur in these two situations. In the case of heavy chiral multiplets only the indirect level-repulsion effect generically arises with a non-negligible size. The correction is always negative and thus dangerous, as suggested from the arguments in [14]. We will derive its general form and show that it is controlled by the mixed third derivatives of W. In the case of heavy vector multiplets, on the other hand, only the direct effect occurs. Moreover the correction turns out to be always positive and therefore harmless, as already argued in [3]. We will rederive more precisely its form, which is controlled by the mixed third derivatives of K.

2 Models with chiral multiplets

Let us start by considering the simplest case of $N = 1$ theories with only chiral multiplets Φ^I. The most general two-derivative Lagrangian is specified in terms of a real Kähler potential K and a holomorphic superpotential W, and reads:

$$\mathcal{L} = \int d^4\theta \, K(\Phi, \bar{\Phi}) + \int d^2\theta \, W(\Phi) + \text{h.c.} \,. \tag{2.1}$$

In components, this gives $\mathcal{L} = T - V$ where

$$T = -g_{I\bar{J}} \, \partial_\mu \phi^I \partial^\mu \bar{\phi}^{\bar{J}} - i g_{I\bar{J}} \, \psi^I \big(\slashed{\partial} \bar{\psi}^{\bar{J}} + \Gamma^{\bar{J}}_{\bar{M}\bar{N}} \, \slashed{\partial} \bar{\phi}^{\bar{M}} \bar{\psi}^{\bar{N}} \big) \,, \tag{2.2}$$

$$V = g^{I\bar{J}} \, W_I \bar{W}_{\bar{J}} + \frac{1}{2} \nabla_I W_J \, \psi^I \psi^J + \text{h.c.} - \frac{1}{4} R_{I\bar{J}K\bar{L}} \, \psi^I \psi^K \bar{\psi}^{\bar{J}} \bar{\psi}^{\bar{L}} \,, \tag{2.3}$$

A vacuum is defined by constant values of the scalars ϕ^I and vanishing values of the fermions ψ^I, such that V is stationary. Supersymmetry is spontaneously broken whenever some of the auxiliary fields F^I have non-vanishing values. The form of these auxiliary fields is given by

$$F^I = -g^{I\bar{J}} \, \bar{W}_{\bar{J}} \,. \tag{2.4}$$

The stationarity condition implies moreover that

$$\nabla_I W_J \, F^J = 0 \,. \tag{2.5}$$

The masses for the scalar and fermion fields describing fluctuations around the vacuum are then found to be given by

$$m_{0I\bar{J}}^2 = \nabla_I W_K \nabla_{\bar{J}} \bar{W}^K - R_{I\bar{J}K\bar{L}} F^K \bar{F}^{\bar{L}} \,, \tag{2.6}$$

$$m_{0IJ}^2 = -\nabla_I \nabla_J W_K F^K \,, \tag{2.7}$$

and

$$m_{1/2IJ} = \nabla_I W_J \,. \tag{2.8}$$

We see from the above expressions that the supersymmetric part of the mass is controlled by the quadratic terms in the superpotential and given by W_{IJ}.

The direction F^I in field space is special. For fermions it defines the Goldstino $\eta = \bar{F}_I \psi^I$, which is massless and represents the Goldstone mode of broken supersymmetry: $m_\eta = 0$. For scalars it defines instead the sGoldstino $\varphi = \bar{F}_I \phi^I$, which describes two real scalar fields with masses that are entirely controlled by supersymmetry breaking effects. One may then look at the average of these two masses, which is defined as

$$m_\varphi^2 = \frac{m_{0I\bar{J}}^2 F^I \bar{F}^{\bar{J}}}{F^K \bar{F}_K} \,. \tag{2.9}$$

A simple computation shows that this is given by [1, 7]

$$m_\varphi^2 = R \, F^I \bar{F}_I \,, \tag{2.10}$$

where

$$R = -\frac{R_{I\bar{J}K\bar{L}} F^I \bar{F}^{\bar{J}} F^K \bar{F}^{\bar{L}}}{(F^M \bar{F}_M)^2} \,. \tag{2.11}$$

From this result it follows that a necessary condition for not having a tachyonic mode is that the holomorphic sectional curvature R be positive [1, 2]. This necessary condition becomes also sufficient if for a given K one allows W to be adjusted [5]. Indeed, at the stationary point one may tune W_I to maximize the average sGoldstino mass, W_{IJ} to make the other masses arbitrarily large, and W_{IJK} to set the splitting between the two sGoldstino masses to zero. Moreover, in such a situation one can prove that the two real sGoldstino modes become degenerate mass eigenstates [6].

2.1 Integrating out heavy chiral multiplets

Let us now consider a situation where the chiral multiplets Φ^I split into a set of light multiplets Φ^i parametrizing the low-energy theory and a set of heavy multiplets Φ^α with a large supersymmetric mass $W_{\alpha\beta}$ to be integrated out. In order to

distinguish light from heavy multiplets in a sensible way, we must assume that the supersymmetric mass mixing $W_{i\alpha}$ between them is not too large. In the following, we shall denote these heavy and mixing blocks of the supersymmetric mass matrix in the following way:

$$M_{\alpha\beta} = W_{\alpha\beta}\,, \quad \mu_{i\alpha} = W_{i\alpha}\,. \tag{2.12}$$

The most relevant interactions for our purposes will be the cubic terms in W, namely the Yukawa couplings

$$\lambda_{\alpha ij} = W_{\alpha ij}\,, \quad \lambda_{\alpha\beta j} = W_{\alpha\beta j}\,, \quad \lambda_{\alpha\beta\gamma} = W_{\alpha\beta\gamma}\,. \tag{2.13}$$

At leading order in the low-energy expansion in number of derivatives, fermions and auxiliary fields, the low-energy effective theory can be obtained in component fields by imposing stationarity of V with respect to each heavy field and substituting back the solution into the original Lagrangian. Equivalently, this effective theory can be derived directly in superfields, by demanding the stationarity of W with respect to each heavy chiral multiplet. For convenience, we shall assume without loss of generality normal coordinates in the microscopic theory around the point under consideration. This substantially simplifies the computations, although the effective theory does not automatically inherit normal coordinates, due to the corrections induced to the Kähler metric.

The corrections due to the supersymmetric mass mixing between heavy and light multiplets are encoded in the following small dimensionless matrix:

$$\epsilon_i^\alpha = -M^{-1\alpha\beta}\mu_{\beta i}\,. \tag{2.14}$$

It should be emphasized that it is always possible to perform a holomorphic field redefinition in such a way to diagonalize the supersymmetric mass matrix W_{IJ} at a given point in field space, thereby setting ϵ_i^α to zero. This means that all the effects depending on ϵ_i^α only serve to compensate a choice of light and heavy fields that does not exactly diagonalize the supersymmetric part of the mass matrix, and therefore do not represent genuine non-trivial corrections. Moreover, since ϵ_i^α must be small, these effects are anyhow quantitatively irrelevant. We may then set $\epsilon_i^\alpha = 0$ by suitably choosing the fields. We shall however keep $\epsilon_i^\alpha \neq 0$ during the computations to verify more explicitly the above claims and set $\epsilon_i^\alpha = 0$ only at the very end. We can anticipate that all the tensorial quantities characterizing the light fields will receive additional contributions coming from heavy indices converted to light indices through the matrix ϵ_i^α. This leads us to introduce already at this stage the following

deformed tensors:

$$g^\epsilon_{i\bar{j}} = g_{i\bar{j}} + \epsilon^\alpha_i g_{\alpha\bar{j}} + \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} g_{i\bar{\beta}} + \epsilon^\alpha_i \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} g_{\alpha\bar{\beta}}\,, \tag{2.15}$$

$$\lambda^\epsilon_{\alpha ij} = \lambda_{\alpha ij} + \epsilon^\beta_i \lambda_{\alpha\beta j} + \epsilon^\gamma_j \lambda_{\alpha i\gamma} + \epsilon^\beta_i \epsilon^\gamma_j \lambda_{\alpha\beta\gamma}\,, \tag{2.16}$$

$$\begin{aligned}
R^\epsilon_{i\bar{j}k\bar{l}} = &\, R_{i\bar{j}k\bar{l}} + \epsilon^\alpha_i R_{\alpha\bar{j}k\bar{l}} + \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} R_{i\bar{\beta}k\bar{l}} + \epsilon^\gamma_k R_{i\bar{j}\gamma\bar{l}} + \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{i\bar{j}k\bar{\delta}} + \epsilon^\alpha_i \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} R_{\alpha\bar{\beta}k\bar{l}} \\
&+ \epsilon^\alpha_i \epsilon^\gamma_k R_{\alpha\bar{j}\gamma\bar{l}} + \epsilon^\alpha_i \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{\alpha\bar{j}k\bar{\delta}} + \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \epsilon^\gamma_k R_{i\bar{\beta}\gamma\bar{l}} + \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{i\bar{\beta}k\bar{\delta}} + \epsilon^\gamma_k \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{i\bar{j}\gamma\bar{\delta}} \\
&+ \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \epsilon^\gamma_k \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{i\bar{\beta}\gamma\bar{\delta}} + \epsilon^\alpha_i \epsilon^\gamma_k \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{\alpha\bar{j}\gamma\bar{\delta}} + \epsilon^\alpha_i \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{\alpha\bar{\beta}k\bar{\delta}} + \epsilon^\alpha_i \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \epsilon^\gamma_k R_{\alpha\bar{\beta}\gamma\bar{l}} \\
&+ \epsilon^\alpha_i \bar{\epsilon}^{\bar{\beta}}_{\bar{j}} \epsilon^\gamma_k \bar{\epsilon}^{\bar{\delta}}_{\bar{l}} R_{\alpha\bar{\beta}\gamma\bar{\delta}}\,.
\end{aligned} \tag{2.17}$$

Finally, we shall define the following quantity for later use, which characterizes the heavy block $W_{\alpha I} g^{I\bar{J}} \bar{W}_{\bar{J}\bar{\beta}}$ of the square of the supersymmetric mass matrix:

$$|M^\epsilon|^2_{\alpha\bar{\beta}} = M_{\alpha\gamma} \big(g^{\gamma\bar{\delta}} + \epsilon^\gamma_i g^{i\bar{\delta}} + \bar{\epsilon}^{\bar{\delta}}_{\bar{j}} g^{\gamma\bar{j}} + \epsilon^\gamma_i \bar{\epsilon}^{\bar{\delta}}_{\bar{j}} g^{i\bar{j}} \big) \bar{M}_{\bar{\delta}\bar{\beta}}\,. \tag{2.18}$$

In the following, we shall compute within the component approach the average sGoldstino mass in the low-energy effective theory, defined at a stationary point as

$$m^{2\mathrm{eff}}_\varphi = \frac{m^{2\mathrm{eff}}_{0i\bar{j}} F^{i\mathrm{eff}} \bar{F}^{\bar{j}\mathrm{eff}}}{F^{k\mathrm{eff}} \bar{F}^{\mathrm{eff}}_k}\,. \tag{2.19}$$

We shall then reproduce the same result within the superfield approach by first computing the Riemann tensor $R^{\mathrm{eff}}_{i\bar{j}k\bar{l}}$ of the effective theory at a generic point and then applying the standard expression for the sGoldstino mass at a stationary point within the effective theory, namely

$$m^{2\mathrm{eff}}_\varphi = R^{\mathrm{eff}} F^{i\mathrm{eff}} \bar{F}^{\mathrm{eff}}_i\,, \tag{2.20}$$

in terms of an effective sectional curvature

$$R^{\mathrm{eff}} = -\frac{R^{\mathrm{eff}}_{i\bar{j}k\bar{l}} F^{i\mathrm{eff}} \bar{F}^{\bar{j}\mathrm{eff}} F^{k\mathrm{eff}} \bar{F}^{\bar{l}\mathrm{eff}}}{(F^{m\mathrm{eff}} \bar{F}^{\mathrm{eff}}_m)^2}\,. \tag{2.21}$$

2.2 Component approach

Consider first the component approach. For simplicity we shall focus on the bosonic fields and discard fermions, since we are interested in computing effective scalar masses. At leading order in the low-energy expansion, the values of the heavy scalar fields are defined by

$$\phi^\alpha = \phi^\alpha(\phi^i, \bar{\phi}^{\bar{i}}) \ \text{ solution of } \ V_\alpha(\phi^i, \bar{\phi}^{\bar{i}}, \phi^\alpha, \bar{\phi}^{\bar{\alpha}}) = 0\,. \tag{2.22}$$

At leading order in the number of auxiliary fields, this stationarity condition implies that $W_{\alpha I} \bar{W}^I = 0$ and gives the following values for the heavy auxiliary fields:

$$F^\alpha = \epsilon^\alpha_i F^i\,. \tag{2.23}$$

The effective theory for the light fields is then obtained by substituting these expressions for ϕ^α and F^α into the original Lagrangian.

To derive the effective theory, we will need to compute the derivatives of the heavy fields ϕ^α and $\bar{\phi}^{\bar\alpha}$ with respect to the light fields ϕ^i. These can be deduced by differentiating the stationarity conditions with respect to the light fields. One finds:

$$\frac{\partial \phi^\alpha}{\partial \phi^i} = -M_0^{-2\alpha\bar\beta}\mu_{0\bar\beta i}^2 - M_0^{-2\alpha\beta}\mu_{0\beta i}^2\,, \tag{2.24}$$

$$\frac{\partial \bar{\phi}^{\bar\alpha}}{\partial \phi^i} = -M_0^{-2\bar\alpha\beta}\mu_{0\beta i}^2 - M_0^{-2\bar\alpha\bar\beta}\mu_{0\bar\beta i}^2\,. \tag{2.25}$$

Here M_0^2 and μ_0^2 represent the heavy and off-diagonal blocks of the complete scalar mass matrix of the microscopic theory. Notice that μ_0 and M_0 differ from μ and M, since the former refer to the full mass matrix whereas the latter parametrize only its supersymmetric part. At quadratic order in the auxiliary fields one finds:

$$M_0^{-2\alpha\bar\beta} = V_{\text{inv}}^{\alpha\bar\beta} + V_{\text{inv}}^{\alpha\bar\gamma}V_{\bar\gamma\delta}V_{\text{inv}}^{\bar\delta\sigma}V_{\sigma\tau}V_{\text{inv}}^{\tau\bar\beta}\,, \tag{2.26}$$

$$M_0^{-2\alpha\beta} = -V_{\text{inv}}^{\alpha\bar\gamma}V_{\bar\gamma\delta}V_{\text{inv}}^{\bar\delta\beta}\,. \tag{2.27}$$

The effective Kähler metric of the light fields can be determined by looking at the scalar kinetic terms and substituting the values of the heavy scalar fields. One may in this case work at leading order in the auxiliary fields, since these terms already involve two derivatives. Focusing also on the leading order in the light masses and the heavy-light mass mixing, the relations (2.24) and (2.25) then simplify to $\partial_i \phi^\alpha = \epsilon_i^\alpha$ and $\partial_i \bar{\phi}^{\bar\alpha} = 0$. Using these expressions, which actually turn out to be correct even at order ϵ^2, one finds that the kinetic term can be rewritten in the standard supersymmetric form with an effective Kähler metric given by

$$g_{i\bar\jmath}^{\text{eff}} = g_{i\bar\jmath}^\epsilon\,. \tag{2.28}$$

The effective mass matrix of the light scalar fields can on the other hand be determined by using the supersymmetric generalization of the expression (1.1), which can be derived by using the same logic. As in the general non-supersymmetric case, the result corresponds to a perturbative diagonalization of the full scalar mass matrix, at leading order in the inverse mass matrix of the heavy scalars. Denoting with m_0^2 the light block of the scalar mass matrix, one finds:

$$\begin{aligned} m_{0i\bar\jmath}^{2\text{eff}} &= m_{0i\bar\jmath}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\beta}\mu_{0\beta\bar\jmath}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\bar\beta}\mu_{0\bar\beta\bar\jmath}^2 \\ &\quad - \mu_{0i\alpha}^2 M_0^{-2\alpha\beta}\mu_{0\beta\bar\jmath}^2 - \mu_{0i\alpha}^2 M^{-2\alpha\bar\beta}\mu_{0\bar\beta\bar\jmath}^2\,, \end{aligned} \tag{2.29}$$

$$\begin{aligned} m_{0ij}^{2\text{eff}} &= m_{0ij}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\bar\beta}\mu_{0\bar\beta j}^2 - \mu_{0i\bar\alpha}^2 M_0^{-2\bar\alpha\beta}\mu_{0\beta j}^2 \\ &\quad - \mu_{0i\alpha}^2 M_0^{-2\alpha\bar\beta}\mu_{0\bar\beta j}^2 - \mu_{0i\alpha}^2 M_0^{-2\alpha\beta}\mu_{0\beta j}^2\,. \end{aligned} \tag{2.30}$$

Let us now focus on the Hermitian block $m_{0i\bar{j}}^{2\text{eff}}$. Using eqs. (2.26) and (2.27) in the formula (2.29), and restricting to terms that are at most quadratic in the auxiliary fields as demanded by supersymmetry at the two-derivative level, we see that there are three kinds of effects coming from the four correction terms. The first type involves second derivatives of W and no auxiliary fields, and comes only from the first correction term. The second type involves the Riemann tensor and two auxiliary fields, and comes again only from the first correction term. The third type involves third derivatives of W and two auxiliary fields, and comes from all four correction terms. All together, these three effects give a negative level-repulsion correction with respect to $m_{0i\bar{j}}^2$.

Let us now compute more specifically the average sGoldstino mass $m_\varphi^{2\text{eff}}$ defined by eq. (2.19) at a stationary point of the effective theory and compare it to its analogue m_φ^2 defined by eq. (2.9) in the microscopic theory. Recall that we are using normal coordinates, so that $g_{i\bar{j}} = \delta_{i\bar{j}}$ and $g_{i\bar{j}}^{\text{eff}} = \delta_{i\bar{j}} + \epsilon_i^\alpha \bar{\epsilon}_{\bar{j}}^{\bar{\alpha}}$. The first thing we need to make more explicit are the effective auxiliary fields. To do so we start by deriving W^{eff} by substituting the solution (2.22) into in W. Taking a derivative we then find that $W_i^{\text{eff}} = W_i + \epsilon_i^\alpha W_\alpha$. But using the stationarity condition $W_{\alpha I}\bar{W}_{\bar{I}} = 0$ of the heavy scalars we see that $W_\alpha = \bar{\epsilon}_{\bar{i}}^{\bar{\alpha}} W_i$, so that $W_i^{\text{eff}} = (\delta_{i\bar{j}} + \epsilon_i^\alpha \bar{\epsilon}_{\bar{j}}^{\bar{\alpha}})W_j = g_{i\bar{j}}^{\text{eff}} W_j$. The auxiliary fields in the effective theory thus coincide with the light components of the auxiliary fields in the microscopic theory: $F^{i\text{eff}} = -g^{\text{eff}i\bar{j}}\bar{W}_{\bar{j}}^{\text{eff}} = -\bar{W}_{\bar{i}} = F^i$. Recalling (2.23) one also finds that $g_{i\bar{j}}^{\text{eff}} F^{i\text{eff}}\bar{F}^{\bar{j}\text{eff}} = F^I \bar{F}^{\bar{I}}$. In summary, we get:

$$F^{i\text{eff}} = F^i \,, \quad F^{i\text{eff}}\bar{F}_i^{\text{eff}} = F^I \bar{F}_I \,. \tag{2.31}$$

To proceed, we also need to compute more explicitly the mass-matrix blocks (2.26) and (2.27) entering in the expression (2.29) for the effective mass matrix $m_{0i\bar{j}}^{2\text{eff}}$. In normal coordinates, these quantities depend on $|M^\epsilon|_{\alpha\bar{\beta}}^2 = M_{\alpha\gamma}(g^{\gamma\bar{\delta}} + \epsilon_i^\gamma \bar{\epsilon}_{\bar{i}}^{\bar{\delta}})\bar{M}_{\bar{\delta}\beta}$, and at quadratic order in the auxiliary fields one finds that

$$V_{\alpha\bar{\beta}} = |M^\epsilon|_{\alpha\bar{\beta}}^2 - R_{\alpha\bar{\beta}K\bar{L}}F^K \bar{F}^{\bar{L}} \,, \quad V_{\alpha\beta} = -\lambda_{\alpha\beta K}F^K \,, \tag{2.32}$$

$$V_{\text{inv}}^{\alpha\bar{\beta}} = |M^\epsilon|^{-2\alpha\bar{\beta}} + |M^\epsilon|^{-2\alpha\bar{\delta}}|M^\epsilon|^{-2\bar{\beta}\gamma}R_{\gamma\bar{\delta}K\bar{L}}F^K \bar{F}^{\bar{L}} \,. \tag{2.33}$$

We are now in position to evaluate the average sGoldstino mass in the effective theory by computing the four correction terms in eq. (2.29). As explained after eqs. (2.29) and (2.30), these give rise to three types of effects. But when looking along the sGoldstino direction, some simplifications occur, due to the fact that only supersymmetry-breaking effects matter. The first type of effect cancels the corresponding leading part of $m_{0i\bar{j}}^2$. The second type of effect combines with the corresponding subleading term in $m_{0i\bar{j}}^2$ to reconstruct the average sGoldstino mass of the microscopic theory. The third type of effect gives instead a genuine correction. The precise evaluation of these effects can be simplified by noticing that at

8

a stationary point $W_{IJ}\bar{W}_{\bar{J}} = 0$, which implies that at leading order in the auxiliary fields $V_{\alpha\bar{i}}W_i = -V_{\alpha\bar{\beta}}W_\beta$. After a straightforward computation one finds that $m_\varphi^{2\text{eff}} = -\big(R_{I\bar{J}K\bar{L}} + \lambda_{\alpha IK}|M^\epsilon|^{-2\alpha\bar{\beta}}\bar{\lambda}_{\bar{\beta}\bar{J}\bar{L}}\big)F^I\bar{F}^{\bar{J}}F^K\bar{F}^{\bar{L}}/F^M\bar{F}_M$. Recalling then that $F^\alpha = \epsilon_i^\alpha F^{i\text{eff}}$ and $F^I\bar{F}_I = F^{i\text{eff}}\bar{F}_i^{\text{eff}}$, one may finally rewrite the above result as

$$m_\varphi^{2\text{eff}} = \big(R^\epsilon - \lambda_\alpha^\epsilon|M^\epsilon|^{-2\alpha\bar{\beta}}\bar{\lambda}_{\bar{\beta}}^\epsilon\big)F^{i\text{eff}}\bar{F}_i^{\text{eff}} \,, \tag{2.34}$$

with

$$R^\epsilon = -\frac{R_{i\bar{j}k\bar{l}}^\epsilon F^{i\text{eff}}\bar{F}^{\bar{j}\text{eff}}F^{k\text{eff}}\bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}}\bar{F}_m^{\text{eff}})^2} \,, \tag{2.35}$$

$$\lambda_\alpha^\epsilon = \frac{\lambda_{\alpha ij}^\epsilon F^{i\text{eff}}F^{j\text{eff}}}{F^{k\text{eff}}\bar{F}_k^{\text{eff}}} \,. \tag{2.36}$$

The first term in the result (2.34) corresponds to m_φ^2, whereas the second term describes a negative level-repulsion effect controlled by the Yukawa couplings $\lambda_{\alpha ij}$ mixing one heavy and two light fields. As anticipated, the dependence on ϵ amounts to a transformation of all the tensorial quantities accounting for the need to disentangle light from heavy eigenmodes of the supersymmetric mass matrix, and can thus be dropped by setting ϵ to zero.

2.3 Superfield approach

The above results can also be derived by integrating out the heavy fields directly at the superfield level, and then computing the sGoldstino mass in the resulting effective theory by applying eqs. (2.20) and (2.21). To do this, one derives the effective Kähler potential and superpotential by solving the following approximate superfield equations of motion:

$$\Phi^\alpha = \Phi^\alpha(\Phi^i) \text{ solution of } W_\alpha(\Phi^i, \Phi^\alpha) = 0 \,. \tag{2.37}$$

The bosonic components of this superfield equations of motion coincide, at leading order in the number of fermions and auxiliary fields, with the equations of motion (2.22)–(2.23) that we have used in the component approach.

To proceed, we will need to compute the first and second derivatives of the heavy scalar fields with respect to the light scalar fields. These can be derived by differentiating eq. (2.37), and one finds the following results:

$$\frac{\partial\phi^\alpha}{\partial\phi^i} = \epsilon_i^\alpha \,, \quad \frac{\partial^2\phi^\alpha}{\partial\phi^i\partial\phi^j} = -M^{-1\alpha\beta}\lambda_{\beta ij}^\epsilon \,. \tag{2.38}$$

The effective geometry can be derived by taking derivatives with respect to the light fields of the effective Kähler potential K^{eff}, where the heavy fields have been

substituted by the solution (2.37) in terms of light fields. We focus again on a given point in the light field space, around which we choose normal coordinates, but this point no longer needs to be a stationary point. Then, using the chain rule and eqs. (2.38), one easily computes $K^{\text{eff}}_{i\bar{j}} = \delta_{i\bar{j}} + \epsilon_i^\alpha \bar{\epsilon}_{\bar{j}}^{\bar{\alpha}}$, $K^{\text{eff}}_{\bar{i}jk} = -M^{-1\alpha\beta} \bar{\epsilon}_{\bar{i}}^{\bar{\alpha}} \lambda^\epsilon_{\beta jk}$ and $K^{\text{eff}}_{i\bar{j}k\bar{l}} = R^\epsilon_{i\bar{j}k\bar{l}} + \lambda^\epsilon_{\alpha ik} |M|^{-2\alpha\bar{\beta}} \bar{\lambda}^\epsilon_{\bar{\beta}\bar{j}\bar{l}}$. This finally implies that the effective metric is given by $g^{\text{eff}}_{i\bar{j}} = g^\epsilon_{i\bar{j}}$, the effective Christoffel symbol by $\Gamma^{\text{eff}}_{\bar{i}jk} = -M^{-1\alpha\beta} \bar{\epsilon}_{\bar{i}}^{\bar{\alpha}} \lambda^\epsilon_{\beta jk}$ and finally the effective Riemann tensor by the following expression:

$$R^{\text{eff}}_{i\bar{j}k\bar{l}} = R^\epsilon_{i\bar{j}k\bar{l}} + \lambda^\epsilon_{\alpha ik} |M^\epsilon|^{-2\alpha\bar{\beta}} \bar{\lambda}^\epsilon_{\bar{\beta}\bar{j}\bar{l}} \tag{2.39}$$

Plugging this expression into eqs. (2.20) and (2.21), we then reproduce the form of the result (2.34).[2]

3 Models with chiral and vector multiplets

Let us now consider the case of $N = 1$ theories with chiral multiplets Φ^I and vector multiplets V^a. The most general two-derivative Lagrangian is in this case specified by a real Kähler potential K, a holomorphic superpotential W, a holomorphic gauge kinetic function f_{ab} and some holomorphic Killing vectors X_a^I:[3]

$$\mathcal{L} = \int d^4\theta \, K(\Phi, \bar{\Phi}, V) + \int d^2\theta \left[W(\Phi) + \frac{1}{4} f_{ab}(\Phi) \, W^{a\alpha} W^b_\alpha \right] + \text{h.c.} . \tag{3.1}$$

The gauge transformations of the chiral multiplets are defined by the Killing vectors X_a^I whereas those of the vector superfields depend only on the structure constants $f_{ab}{}^c$ of the gauge group. Gauge invariance of the Lagrangian imposes that the variation of the non-holomorphic terms should be at most a Kähler transformation of the form $\Lambda^a f_a + \bar{\Lambda}^a \bar{f}_a$, where the f_a are some holomorphic functions, whereas the holomorphic terms should be strictly invariant. This implies the following conditions:

$$X_a^I K_I - \frac{i}{2} K_a = f_a \, , \tag{3.2}$$

$$X_a^I W_I = 0 \, , \tag{3.3}$$

$$X_a^I f_{bcI} = -2 f_{a(b}{}^d f_{c)d} \, . \tag{3.4}$$

These equations show that $-\frac{1}{2} K_a$ can be identified with the real Killing potential for the Killing vector X_a^I. They also imply that $K_{aI} = 2i\bar{X}_{aI}$ and $K_{ab} = 4g_{I\bar{J}} X_{(a}^I \bar{X}_{b)}^{\bar{J}}$.

---

[2]Note that the results derived in this subsection are evaluated at values of the heavy scalar fields solving $W_\alpha = 0$, whereas the results of previous section were evaluated at values of the heavy scalar fields solving $V_\alpha = 0$. However it turns out that the difference between these two values is subleading in the counting of auxiliary fields and can therefore be discarded.

[3]We omit for simplicity the possibility of Fayet-Iliopoulos terms for Abelian factors.

Finally, the equivariance condition on the Killing vectors guarantees that the Killing potentials can be chosen to transform in the adjoint representation, so that

$$g_{I\bar{J}}X_{[a}^I\bar{X}_{b]}^{\bar{J}} = \frac{i}{4}f_{ab}{}^c K_c\,. \tag{3.5}$$

In components and in the Wess-Zumino gauge, one finds $\mathcal{L} = T - V$ where:

$$
\begin{aligned}
T = {}& -g_{I\bar{J}}\,D_\mu\phi^I D^\mu\bar{\phi}^{\bar{J}} - \frac{1}{4}h_{ab}\,F_{\mu\nu}^a F^{b\mu\nu} + \frac{1}{4}\theta_{ab}\,F_{\mu\nu}^a \tilde{F}^{b\mu\nu} \\
& - ig_{I\bar{J}}\,\psi^I\big(\slashed{D}\bar{\psi}^{\bar{J}} + \Gamma_{\bar{M}\bar{N}}^{\bar{J}}\,\slashed{D}\bar{\phi}^{\bar{M}}\bar{\psi}^{\bar{N}}\big) - \frac{i}{2}h_{ab}\,\lambda^a \slashed{D}\bar{\lambda}^b + \text{h.c.} \\
& + \frac{1}{\sqrt{2}}h_{abI}\,\lambda^a\sigma^{\mu\nu}\psi^I F_{\mu\nu}^b + \text{h.c.}\,, \tag{3.6}
\end{aligned}
$$

$$
\begin{aligned}
V = {}& g^{I\bar{J}}W_I\bar{W}_{\bar{J}} + \frac{1}{8}\,h^{ab}K_a K_b \\
& + \frac{1}{2}\Big[\nabla_I W_J\,\psi^I\psi^J - g^{I\bar{J}}h_{abI}\bar{W}_{\bar{J}}\,\lambda^a\lambda^b + \sqrt{8}\big(g_{I\bar{J}}\bar{X}_a^{\bar{J}} + \frac{i}{4}h^{bc}h_{abI}K_c\big)\psi^I\lambda^a\Big] + \text{h.c.} \\
& - \frac{1}{4}R_{I\bar{J}K\bar{L}}\,\psi^I\psi^K\bar{\psi}^{\bar{J}}\bar{\psi}^{\bar{L}} + \frac{1}{4}g^{I\bar{J}}h_{abI}h_{cd\bar{J}}\,\lambda^a\lambda^b\bar{\lambda}^c\bar{\lambda}^d + \frac{1}{2}h^{cd}h_{acI}h_{bd\bar{J}}\,\psi^I\lambda^a\bar{\psi}^{\bar{J}}\bar{\lambda}^b \\
& - \frac{1}{4}\Big[\nabla_I h_{abJ}\,\psi^I\psi^J\lambda^a\lambda^b + h^{cd}h_{acI}h_{bdJ}\psi^I\lambda^a\psi^J\lambda^b\Big] + \text{h.c.}\,. \tag{3.7}
\end{aligned}
$$

In these expressions D_μ is the covariant derivative acting as $D_\mu\phi^I = \partial_\mu\phi^I + A_\mu^a X_a^I$, $D_\mu\psi^I = \partial_\mu\psi^I + A_\mu^a\partial_J X_a^I\,\psi^J$ and $D_\mu\lambda^a = \partial_\mu\lambda^a + f_{bc}{}^a A_\mu^b\lambda^c$, whereas $F_{\mu\nu}^a$ is the field-strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{bc}{}^a A_\mu^b A_\nu^c$ and h_{ab} and θ_{ab} denote the real and imaginary parts of f_{ab}.

A vacuum is defined by constant values of the scalars ϕ^I and vanishing values of the fermions ψ^I, λ^a and the vectors A_μ^a, such that V is stationary. Supersymmetry is spontaneously broken whenever some of the auxiliary fields F^I, D^a have non-vanishing values. The form of these auxiliary fields is given by

$$F^I = -\bar{W}^I\,, \tag{3.8}$$

$$D^a = -\frac{1}{2}h^{ab}K_b\,. \tag{3.9}$$

The stationarity condition implies that

$$\nabla_I W_J\,F^J + \frac{1}{2}h_{abI}D^a D^b + i\bar{X}_{aI}D^a = 0\,. \tag{3.10}$$

Moreover, by contracting this relation with the Killing vectors X_a^I and taking the imaginary part, and using (3.3) and its derivative as well as (3.5), one finds the following relation between the values of F^I and D^a:

$$i\nabla_I X_{a\bar{J}}\,F^I\bar{F}^{\bar{J}} - g_{I\bar{J}}X_{(a}^I\bar{X}_{b)}^{\bar{J}}\,D^b + \frac{1}{2}f_{ab}{}^d\theta_{dc}\,D^b D^c = 0\,. \tag{3.11}$$

11

The masses of the scalar, fermion and vector fields describing fluctuations around the vacuum are found to be given by

$$m^2_{0I\bar{J}} = \nabla_I W_K \nabla_{\bar{J}} \bar{W}^K - R_{I\bar{J}K\bar{L}} F^K \bar{F}^L + h^{ab} \bar{X}_{aI} X_{b\bar{J}} + h^{ab} h_{acI} h_{bd\bar{J}} D^b D^c$$
$$+ \left(q_{aI\bar{J}} - i h^{bc} h_{abI} X_{c\bar{J}} + i h^{bc} h_{ab\bar{J}} \bar{X}_{cI} \right) D^a , \tag{3.12}$$

$$m^2_{0IJ} = -\nabla_I \nabla_J W_K F^K - h^{ab} \bar{X}_{aI} \bar{X}_{bJ} - \frac{1}{2} \left(\nabla_I h_{abJ} - 2 h^{cd} h_{acI} h_{bdJ} \right) D^a D^b$$
$$+ 2i\, h^{bc} h_{ab(I} \bar{X}_{cJ)} D^a , \tag{3.13}$$

then

$$m_{1/2IJ} = \nabla_I W_J , \tag{3.14}$$

$$m_{1/2ab} = h_{abI} F^I , \tag{3.15}$$

$$m_{1/2Ia} = \sqrt{2}\, \bar{X}_{aI} - \frac{i}{\sqrt{2}} h_{abI} D^b , \tag{3.16}$$

and finally

$$m^2_{1ab} = 2\, g_{I\bar{J}} X^I_{(a} \bar{X}^{\bar{J}}_{b)} . \tag{3.17}$$

We see that the supersymmetric parts of the mass matrices are given by W_{IJ} for the chiral multiplets and by $2 g_{I\bar{J}} X^I_{(a} \bar{X}^{\bar{J}}_{b)}$ for the vector multiplets.

The directions F^I and D^a in field space are special. For fermions they define the Goldstino $\eta = \bar{F}_I \psi^I + \frac{i}{\sqrt{2}} D_a \lambda^a$, which is massless and represents the Goldstone mode of broken supersymmetry: $m_\eta = 0$. For scalars they define instead the projected sGoldstino $\varphi = \bar{F}_I \phi^I$, which describes two real scalar fields with masses that are entirely controlled by supersymmetry breaking effects. One may then consider the average of these two masses, which is as before given by

$$m^2_\varphi = \frac{m^2_{0I\bar{J}} F^I \bar{F}^{\bar{J}}}{F^K \bar{F}_K} . \tag{3.18}$$

A straightforward computation shows that the result is in this case given by [3, 7]

$$m^2_\varphi = R\, F^I \bar{F}_I + S\, D^a D_a + \frac{1}{4} T \frac{(D^a D_a)^2}{F^I \bar{F}_I} + M^2 \frac{D^a D_a}{F^I \bar{F}_I} , \tag{3.19}$$

where

$$R = -\frac{R_{I\bar{J}K\bar{L}} F^I \bar{F}^{\bar{J}} F^K \bar{F}^{\bar{L}}}{(F^M \bar{F}_M)^2} , \tag{3.20}$$

$$S = \frac{h_{acI} h^{cd} h_{db\bar{J}} F^I \bar{F}^{\bar{J}} D^a D^b}{(F^K \bar{F}_K)(D^c D_c)} , \tag{3.21}$$

$$T = \frac{h_{abI} h_{cb}{}^I D^a D^b D^c D^d}{(D^e D_e)^2} , \tag{3.22}$$

$$M^2 = \frac{2 X^I_a \bar{X}_{bI} D^a D^b}{D^c D_c} . \tag{3.23}$$

The directions X_a^I in field space are also special. In the supersymmetric limit they define the chiral multiplets Φ^a that are eaten up by the massless vector multiplets to produce massive vector multiplets. When supersymmetry is broken, things get a bit more complicated but the fermionic and bosonic components of Φ^a still get a mass comparable to that of the components of V^a.

From the above result it follows that a necessary condition for not having a tachyonic mode is that the holomorphic sectional curvature R be larger than some negative value depending on the gauge sector data [3]. In this case it is less clear whether this necessary condition becomes also sufficient if for a given K one allows W to be adjusted. Indeed, gauge-invariance forbids any tuning of W_I, W_{IJ} and W_{IJK} along the directions X_a^I. The corresponding modes thus represent a priori a left-over danger of instability [7]. This danger does however disappear in the limit we are considering here where the vector masses are much larger than the supersymmetry breaking scale, since these modes then become very heavy.

3.1   Integrating out heavy vector multiplets

Let us now suppose that all the vector multiplets have a large supersymmetric mass, much larger than the splittings induced by supersymmetry breaking. We may then integrate out in a supersymmetric way the modes associated with the heavy vector multiplets, paying attention to the fact that in order to become massive they absorb the modes of some chiral multiplets. The relevant scales in this case are the supersymmetric mass matrix $2g_{I\bar{J}}X_{(a}^I\bar{X}_{b)}^{\bar{J}} = \frac{1}{2}K_{ab}$ of the heavy vector multiplets and the quantity $iX_{aI} = \frac{1}{2}K_{aI}$ controlling the supersymmetric mixing between vector multiplets and chiral multiplets:

$$M_{ab}^2 = \frac{1}{2}K_{ab}\,, \ \ \nu_{aI} = \frac{1}{2}K_{aI}\,. \tag{3.24}$$

The couplings that are expected to be relevant are instead given by the cubic couplings in K, namely the generalized charges

$$q_{aI\bar{J}} = -\frac{1}{2}K_{aI\bar{J}}\,, \ \ q_{abI} = -\frac{1}{2}K_{abI}\,, \ \ q_{abc} = -\frac{1}{2}K_{abc}\,. \tag{3.25}$$

At leading order in the expansion in number of derivatives, fermions and auxiliary fields, the low-energy effective theory for the light chiral multiplets can again be obtained in two different but equivalent ways. One may proceed in components and integrate out the heavy modes associated to the vector multiplets and the chiral multiplets that they absorb, by requiring stationarity of V with respect to them. One may however also proceed in superfields and integrate out the heavy vector superfields by requiring stationarity of K with respect to them. For convenience,

we shall as before assume without loss of generality normal coordinates in the microscopic theory around the point under consideration.

In analogy with what happens in the case of only chiral multiplets, we expect that the corrections due to the supersymmetric mixing between heavy and light multiplets should be encoded in following parameter of dimension one:

$$\delta_I^a = -M^{-2ab}\nu_{bI} \,. \tag{3.26}$$

In this case, such a parameter cannot be set to zero by a simple holomorphic field redefinition, because it corresponds to the non-holomorphic mixing between the heavy gauge fields and the corresponding real would-be Goldstone modes. However, it can be set to zero by making a suitable choice of gauge. With any difference choice of gauge, δ_I^a would be non-zero and the terms depending on it in the effective theory would take into account the mixing between light and heavy fields. By doing the computation in such a gauge one would presumably end up getting deformed versions of all the tensorial quantities for light fields, involving additional contributions where heavy indices are converted to light indices by δ_I^a. We shall however refrain from keeping a general $\delta_I^a \neq 0$ and set $\delta_I^a = 0$ from the beginning by choosing the unitary gauge.

To perform the splitting between light and heavy fields and the gauge fixing more precisely, we may start by splitting the chiral multiplets Φ^I into those that are orthogonal and those that are parallel to the Killing vectors X_a^I evaluated at the point under consideration. This decomposition can be done more explicitly with the help of the parallel projector $P^I{}_J = 2X_a^I M^{-2ab}\bar{X}_{bJ}$. We shall denote these two sets of fields respectively with Φ^i and Φ^a. The orthogonal components Φ^i define the light chiral multiplets of the low-energy effective theory. The parallel components Φ^a are instead either heavy or eliminable through the gauge fixing.

In the following, we shall follow the same logic as in the previous section and first compute within the component approach the average sGoldstino mass in the low-energy effective theory, defined at a stationary point as

$$m_\varphi^{2\text{eff}} = \frac{m_{0i\bar{j}}^{2\text{eff}} F^{i\text{eff}}\bar{F}^{\bar{j}\text{eff}}}{F^{k\text{eff}}\bar{F}_k^{\text{eff}}} \,. \tag{3.27}$$

We will then reproduce the same result within the superfield approach by first computing the Riemann tensor of the effective theory at a generic point and then plugging it in the expression for the sGoldstino mass at a stationary point within the effective theory, which is given by

$$m_\varphi^{2\text{eff}} = R^{\text{eff}} F^{i\text{eff}}\bar{F}_i^{\text{eff}} \,, \tag{3.28}$$

in terms of an effective sectional curvature

$$R^{\text{eff}} = -\frac{R_{i\bar{j}k\bar{l}}^{\text{eff}} F^{i\text{eff}}\bar{F}^{\bar{j}\text{eff}} F^{k\text{eff}}\bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}}\bar{F}_m^{\text{eff}})^2} \,. \tag{3.29}$$

14

3.2 Component approach

Let us first consider the component approach, where it is convenient to choose the Wess-Zumino gauge for the extra gauge symmetries implied by supersymmetry. For simplicity we shall as before focus on bosonic fields and discard fermions since we are interested in scalar masses. The relevant bosonic heavy modes coming from V^a and Φ^a are the following. In the vector multiplets V^a, the gauge fields A_μ^a contain heavy physical modes and should of course be considered. In the chiral multiplets Φ^a, on the other hand, the modes $\sigma^a = \mathrm{Re}(\phi^a)$ correspond to the would-be Goldstone modes and can be eliminated by choosing the unitary gauge for the standard gauge symmetries, where the corresponding degrees of freedom are the longitudinal polarizations of the gauge bosons, whereas the modes $\rho^a = \mathrm{Im}(\phi^a)$ are physical and easily seen to have a mass comparable to that of the vector fields, so that they must be considered. At leading order in the low-energy expansion, the heavy bosonic fields A_μ^a and ρ^a can then be integrated out by using the following approximate equations of motion:

$$\rho^a = \rho^a(\phi^i, \bar{\phi}^{\bar{\imath}}) \ \text{ solution of } \ V_a(\phi^i, \bar{\phi}^{\bar{\imath}}, \rho^a) = 0 \,, \tag{3.30}$$

$$A_\mu^a = 0 \,. \tag{3.31}$$

Concerning the auxiliary fields, notice that those coming from the parallel chiral multiplets automatically vanish, as a consequence of the gauge invariance of the superpotential (3.3), whereas those of the vector multiplets are given by eq. (3.11), which corresponds to the equation of motion of ρ^a and reduces approximately to $q_{aI\bar{J}}F^I\bar{F}^{\bar{J}} - \frac{1}{2}M_{ab}^2 D^b = 0$. At leading order in the low-energy expansion one then finds:

$$F^a = 0 \,, \tag{3.32}$$

$$D^a = 2\,M^{-2ab} q_{bi\bar{\jmath}}F^i\bar{F}^{\bar{\jmath}} \,. \tag{3.33}$$

The effective theory for the light fields is finally obtained by substituting these expressions into the Lagrangian.

To derive the effective theory, one needs in principle to compute the derivatives of ρ^a with respect to ϕ^i. This can be deduced by taking a derivative of the stationarity condition for ρ^a with respect to ϕ^i. One then finds a result that is inversely proportional to the mass matrix of ρ^a, which is approximately equal to that of the vectors, and directly proportional to the mass mixing between ρ^a and ϕ^i. This mixing can be computed explicitly and after using the relation (3.3) ensuring the gauge invariance of W, as well as its first and second derivatives, one verifies that it contains only terms that are quadratic in the auxiliary fields or linear in the auxiliary fields but further suppressed by the ratio between light chiral masses and heavy vector mass,

which must all be neglected. As a result, one finds:

$$\frac{\partial \rho^a}{\partial \phi^i} = 0 \,. \tag{3.34}$$

The effective Kähler metric of the light fields is not affected. Indeed, neither A_μ^a nor ρ^a give any effect in the kinetic terms, as a consequence of eqs. (3.31) and (3.34). One thus simply finds:

$$g_{i\bar{\jmath}}^{\text{eff}} = g_{i\bar{\jmath}} \,. \tag{3.35}$$

The effective scalar mass matrices can be computed by taking into account both the direct effect of the heavy modes on the microscopic mass evaluated in the light scalar directions ϕ^i and the indirect level-repulsion effect coming from the mass mixing with the heavy scalar directions ρ^a. It turns however out that the level-repulsion effect is negligible, for essentially the same reasons as those leading to eq. (3.34). We thus finally get:

$$m_{0i\bar{\jmath}}^{2\text{eff}} = m_{0i\bar{\jmath}}^2 \,, \tag{3.36}$$

$$m_{0ij}^{2\text{eff}} = m_{0ij}^2 \,. \tag{3.37}$$

There is nevertheless a direct effect in the Hermitian block $m_{0i\bar{\jmath}}^{2\text{eff}}$, which consists of two significant contributions in $m_{0i\bar{\jmath}}^2$ coming from the couplings to heavy fields. The first contribution comes from plugging back the small but non-vanishing value of D^a into the last term of (3.12). It is easily evaluated by using eq. (3.33), and one finds $q_{ai\bar{\jmath}}D^a = 2\,q_{ai\bar{\jmath}}M^{-2ab}q_{bk\bar{l}}F^k\bar{F}^{\bar{l}}$. The second contribution arises instead from the part of the first term in (3.12) that corresponds to values for the summed index K that run over the parallel chiral modes that are integrated out. It can be evaluated by using the projected metric $P^{I\bar{J}} = 2X_a^I M^{-2ab}\bar{X}_b^{\bar{J}}$, and reads $\nabla_i W_a \nabla_{\bar{\jmath}}\bar{W}^a = \nabla_i W_K P^{K\bar{L}}\nabla_{\bar{\jmath}}\bar{W}_{\bar{L}} = 2X_a^K \nabla_i W_K M^{-2ab}\bar{X}_b^{\bar{L}}\nabla_{\bar{\jmath}}\bar{W}_{\bar{L}}$. But taking a derivative of eq. (3.3) one deduces that $X_a^K \nabla_i W_K = -iq_{ai\bar{K}}\bar{F}^{\bar{K}} = -iq_{ai\bar{k}}\bar{F}^{\bar{k}}$, and finally $\nabla_i W_a \nabla_{\bar{\jmath}}\bar{W}^a = 2\,q_{ai\bar{l}}M^{-2ab}q_{bk\bar{\jmath}}F^k\bar{F}^{\bar{l}}$. These two contributions represent a direct correction to all the masses, which may be either positive or negative depending on the value of charges along the direction that is considered.

Let us now evaluate more precisely the average sGoldstino mass defined by eq. (3.27) at a stationary point of the effective theory and compare it to its analogue defined by eqs. (3.28) and (3.29) in the microscopic theory. Along the supersymmetry breaking direction $F^{i\text{eff}} = F^i$ the two direct corrections discussed above give identical contributions that sum up and one easily finds:

$$m_\varphi^{2\text{eff}} = \left(R + 4\,q_a M^{-2ab}q_b\right) F^{i\text{eff}}\bar{F}_i^{\text{eff}} \,, \tag{3.38}$$

where

$$R = -\frac{R_{i\bar{j}k\bar{l}} F^{i\text{eff}} \bar{F}^{\bar{j}\text{eff}} F^{k\text{eff}} \bar{F}^{\bar{l}\text{eff}}}{(F^{m\text{eff}} \bar{F}^{\text{eff}}_m)^2} \,, \tag{3.39}$$

$$q_a = \frac{q_{ai\bar{j}} F^{i\text{eff}} \bar{F}^{\bar{j}\text{eff}}}{F^{k\text{eff}} \bar{F}^{\text{eff}}_k} \,. \tag{3.40}$$

The first term in the result (3.38) corresponds to m_φ^2, whereas the second term describes a positive direct effect controlled by the charges $q_{ai\bar{j}}$ mixing one heavy and two light fields. The absence of any indirect level-repulsion effect is due to the absence of genuine heavy chiral multiplets mixing to the light chiral multiplets.

3.3   Superfield approach

It is straightforward to show that the above results can also be obtained by integrating out the heavy vector multiplets at the level of superfields. The only complication is that one should switch from the unitary plus Wess-Zumino gauge used in the component formulation, which fix respectively the standard and the extra gauge symmetries, to a supersymmetric unitary gauge to be used in the superfield formulation, which fixes at once all the multiplet of gauge symmetries. More precisely, we shall gauge fix all the parallel chiral multiplets Φ^a to constant values coinciding with their values at the stationary point. The superfields V^a become however general vector superfields in this gauge, and compared to the Wess-Zumino gauge that was chosen in the component approach, the modes that were described by the real scalar fields ρ^a in the Φ^a have now been transfered to the real scalar fields c^a in the general V^a. In this supersymmetric gauge, all the heavy degrees of freedom are thus contained in V^a, and can be integrated out by using the following approximate superfield equations of motion:

$$V^a = V^a(\Phi^i, \bar{\Phi}^{\bar{i}}) \text{ solution of } K_a(\Phi^i, \bar{\Phi}^{\bar{i}}, V^a) = 0 \,. \tag{3.41}$$

The bosonic components of this superfield equations of motion map to the equations of motion (3.30)–(3.33) that we have used in the component approach, modulo the different gauge choice.

To proceed, we will need to compute the first and second derivatives of the lowest component of the heavy vector superfields with respect to the light scalar superfields. These can be derived by differentiating eq. (3.41), and at the point under consideration where $K_{ai} = 0$ one finds the following results:

$$\frac{\partial c^a}{\partial \phi^i} = 0 \,, \quad \frac{\partial^2 c^a}{\partial \phi^i \partial \bar{\phi}^{\bar{j}}} = M^{-2ab} q_{bi\bar{j}} \,. \tag{3.42}$$

The effective geometry can be derived by taking derivatives with respect to the light fields of the effective Kähler potential K^{eff}, where the heavy fields have been

substituted by the solution (3.41) in terms of light fields. We focus again on a given point in the light field space and use normal coordinates. Then, using the chain rule and eq. (3.42), and noticing that $K_{aij} = 0$ and $K_{ai\bar{\jmath}} = -2q_{ai\bar{\jmath}}$, one easily computes $K_{i\bar{\jmath}}^{\text{eff}} = \delta_{i\bar{\jmath}}$, $K_{\bar{\imath}jk}^{\text{eff}} = 0$ and $K_{i\bar{\jmath}k\bar{l}}^{\text{eff}} = K_{i\bar{\jmath}k\bar{l}} - 2\,q_{ai\bar{\jmath}}M^{-2ab}q_{bk\bar{l}} - 2\,q_{ai\bar{l}}M^{-2ab}q_{bk\bar{\jmath}}$. This finally implies that the effective metric is given by $g_{i\bar{\jmath}}^{\text{eff}} = g_{i\bar{\jmath}}$, the effective Christoffel symbol by $\Gamma_{\bar{\imath}jk}^{\text{eff}} = 0$ and the effective Riemann tensor by the following expression:

$$R_{i\bar{\jmath}k\bar{l}}^{\text{eff}} = R_{i\bar{\jmath}k\bar{l}} - 2\,q_{ai\bar{\jmath}}M^{-2ab}q_{bk\bar{l}} - 2\,q_{ai\bar{l}}M^{-2ab}q_{bk\bar{\jmath}}\,. \tag{3.43}$$

Plugging this expression into eqs. (3.28) and (3.29), we then reproduce the form of the result (3.38).

4   Conclusion

Summarizing, we have shown that integrating out heavy chiral multiplets Φ^{α} and vector multiplets V^a with large and approximately supersymmetric mass matrices $M^{2\alpha\bar{\beta}}$ and M^{2ab} induces corrections to the square masses of light scalars ϕ^i that are due respectively to an indirect level-repulsion effect and a direct coupling effect. The crucial dimensionless couplings that are involved in these effects are respectively the Yukawa couplings $\lambda_{\alpha ij} = W_{\alpha ij}$ and the generalized gauge charges $q_{ai\bar{\jmath}} = -\frac{1}{2}K_{ai\bar{\jmath}}$, which corresponds to cubic couplings mixing one heavy and two light multiplets respectively in W and K. In particular, by looking along the chiral projection of the supersymmetry breaking direction, which is defined by the chiral auxiliary fields F^i, we showed that the averaged sGoldstino mass in the effective theory takes the form:

$$m_{\varphi}^{2\text{eff}} = \left(R - \lambda_{\alpha}|M|^{-2\alpha\bar{\beta}}\bar{\lambda}_{\bar{\beta}} + 4\,q_a M^{-2ab}q_b\right)M_{\text{S}}^4\,. \tag{4.1}$$

The first term is what one would find by just restricting to the light fields. It is controlled by the sectional curvature R along the F-direction, and can have any sign. The second term is the correction induced by heavy chiral multiplets. It is controlled by the Yukawa couplings λ_{α} along the F-direction and is always negative. The third term is the correction induced by heavy vector multiplets. It is controlled by the gauge charges q_a along the F-direction and is always positive. Finally M_S is the scale of supersymmetry breaking, which in our situation is set by the F auxiliary fields since the D auxiliary fields are suppressed.

The result (4.1) has been derived in rigid supersymmetry, in the limit where the supersymmetry breaking scale is much lower than the mass scale M of the heavy modes that are integrated out. Its generalization to gravity can however be derived in a straightforward way by using the results of [10], where it was shown

that whenever the gravitino mass $m_{3/2}$ is also much smaller than the heavy mass scale M, one may first integrate out the fields in the rigid limit and then switch on the coupling to gravity. More precisely, the only modification induced by gravity in (4.1) is the addition of the correction $2m_{3/2}^2$, which reconstructs the supergravity result of [1] for the sGoldstino mass in the theory truncated to light modes:

$$\Delta m_\varphi^{2\text{eff}} = 2\, m_{3/2}^2\,. \tag{4.2}$$

The origin of the difference in sign in the corrections induced by heavy chiral and vector multiplets is transparent in the component approach, where the first is due to an indirect level-repulsion effect whereas the second is due to a direct coupling effect. In the superfield approach, the two computations look instead very symmetric and the difference in sign is at first sight surprising. A closer inspection shows however that there too it can be understood quite robustly. For this we observe that for heavy chiral multiplets the stationarity condition $W_\alpha = 0$, the auxiliary fields $\bar{F}_\alpha = -W_\alpha$ and the relevant cubic couplings $\lambda_{\alpha ij} = W_{\alpha ij}$ are all controlled by the superpotential W, whereas for heavy vector multiplets the stationarity condition $K_a = 0$, the auxiliary fields $D_a = -\frac{1}{2}K_a$ and the relevant cubic couplings $q_{ai\bar{j}} = -\frac{1}{2}K_{ai\bar{j}}$ are all controlled by the Kähler potential K. There is then a perfect symmetry between the two dynamics, which exchanges the roles of K and W. When one looks at the effects of these heavy dynamics onto the supersymmetry-breaking part of the masses of light scalar fields, this symmetry is however broken, because supersymmetry-breaking contributions to scalar masses arise only from K and not from W. This is what causes the difference in sign between the two effects.[4]

The result that we have obtained may have interesting applications in the context of string models, where the situation in which some of the multiplets are stabilized in a supersymmetric way at a high energy scale naturally occurs and the question of their effect on the dynamics of the light multiplets, which are supposed to break supersymmetry, acquires a crucial importance. In such a situation one has in principle to honestly integrate out the heavy fields to properly describe the dynamics of the light fields. But it is in general cumbersome to do so, and this raises the question of whether or when one may get a qualitatively reliable indication on the light field dynamics by just freezing the heavy fields and truncating the theory. Some particular situations where one can safely do this truncation and get the right effective theory have been identified in [16, 17, 18]. Here we have shown more specifically and more systematically what kind of dangers may arise from the heavy fields concerning the masses of the light fields, which are the crucial issue for metastability of the vacuum.

A concrete example is that of string models where large classical effects related to background fluxes stabilize some moduli in a supersymmetric way with a large

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[4] A similar phenomenon has also been encountered in different context in [15].

mass and small quantum effects related to gauge interactions stabilize some other moduli in a non-supersymmetric way with a small mass [19, 20]. The dynamics of these heavy and light modes, schematically denoted by H and L, is then described by $K = K_L(L, \bar{L}) + K_H(H, \bar{H}) + K_Q(L, \bar{L}, H, \bar{H})$ and $W = 0 + W_H(H) + W_Q(L, H)$. For gauge interactions with a field-dependent gauge kinetic function $f \propto L$, the quantum effects have the following structure. The correction K_Q consists of both perturbative and non-perturbative effects suppressed by inverse powers and exponentials of $L + \bar{L}$, and can usually be neglected, since it represents a small correction to the kinetic terms of L. The correction W_Q consists instead only of non-perturbative effects suppressed by exponentials of L, and must be kept, since it represents the dominant source of potential for L.[5] In this situation, freezing the heavy moduli H to constant values is a priori not justified [24, 25, 26], but turns out a posteriori to give a sensible approximation to the effective theory for the light moduli L thanks to the smallness of the quantum corrections mixing L and H [17]. Applying our general result, we may now establish more quantitatively the importance of the corrections induced by integrating out the heavy modes on the light masses, and in particular the sGoldstino mass m. The relevant Yukawa coupling λ between one H and to L fields will involve the same exponential suppression factor as W_Q. The dangerous indirect level-repulsion effect on m^2 will then be suppressed by the square of this exponential factor. On the other hand, the direct effect induced on m^2 from the mixing K_Q involves both power and exponentially suppressed corrections. Given then that in these models there is a unique ultraviolet mass scale around M_{Pl}, the indirect effect is a priori smaller than the direct effect, and in all the situations where the direct effect is neglected also the indirect effect must be discarded. There is thus no problem in the limit of small quantum effects.

One may finally wonder whether integrating out heavy chiral and vector multiplets has similar effects on soft masses in scenarios where both the visible and the hidden sectors couple to them. In fact, these effects are easily computed, since they are also encoded in the effective Riemann tensor, but with two visible-sector and two hidden-sector indices: $m_{u\bar{v}}^{2\mathrm{eff}} = -R_{u\bar{v}i\bar{j}}^{\mathrm{eff}} F^{i\mathrm{eff}} \bar{F}^{\bar{j}\mathrm{eff}}$. Applying the results (2.39) and (3.43) one would then find

$$m_{u\bar{v}}^{2\mathrm{eff}} = -\big(R_{u\bar{v}i\bar{j}} + \lambda_{\alpha ui}|M|^{-2\alpha\beta}\bar{\lambda}_{\bar{\beta}\bar{v}\bar{j}} - 2\,q_{au\bar{v}}M^{-2ab}q_{bi\bar{j}} - 2\,q_{au\bar{j}}M^{-2ab}q_{bi\bar{v}}\big)F^i \bar{F}^{\bar{j}}. \quad (4.3)$$

The first term is the usual expression for the soft masses, the second term represents the correction induced by heavy chiral multiplets, and the third and fourth terms describe the corrections induced by heavy vector multiplets. The various couplings controlling these effects are however not always allowed by the Standard Model gauge symmetry G_{SM}. If the heavy states are neutral, only $q_{au\bar{v}}$ and $q_{ai\bar{j}}$ can be

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[5]See [21, 22, 23] for a more detailed discussion of these effects for gaugino condensation.

non-zero. The only effect then comes from the third term, with an arbitrary sign. This is the standard effect induced by a neutral heavy vector multiplet.[6] If on the other hand the heavy states are charged, only $\lambda_{\alpha u i}$ and $q_{a u \bar{\jmath}}$ can be non-zero. The only effects then come from the second and the fourth terms, which are respectively negative and positive. However a charged chiral multiplet cannot have a supersymmetric mass term, because G_{SM} does not allow holomorphic invariants, whereas a charged vector multiplet can, since non-holomorphic invariants exist; so actually only the fourth term is relevant. This is a less-standard but already-known effect that can be induced by charged vector multiplets.[7] In addition to these effects, there is as usual a separate gravitational effect, which for generic cosmological constant $V = M_{\mathrm{S}}^4 - 3m_{3/2}^2 M_{\mathrm{Pl}}^2$ is given by:

$$\Delta m_{u\bar{v}}^{2\mathrm{eff}} = g_{u\bar{v}} \left(m_{3/2}^2 + V M_{\mathrm{Pl}}^{-2} \right). \tag{4.4}$$

We clearly see that eqs. (4.3) and (4.4) for the soft scalar masses correspond to eqs. (4.1) and (4.2) for the average sGoldstino mass.

Acknowledgements

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[6]See for example [9].

[7]This kind of effect is relevant in Grand Unified Theories, where charged massive vector fields occur after the gauge symmetry is broken down from G_{GUT} to G_{SM}, and induces important corrections to soft masses. This phenomenon and its phenomenological implications were studied in detail in [27, 28].

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