GOAL-BASED DAYLIGHTING DESIGN USING AN INTERACTIVE SIMULATION METHOD

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ABSTRACT

This paper proposes an interactive goal-based method for designing daylit buildings. The lighting simulation tool which supports this process is a hybrid global illumination rendering method which efficiently computes annual daylighting metrics. The goal-based method uses a knowledge base populated using a set of previously completed simulations that quantify the effects of various façade design modifications. The knowledge base guides a simple algorithm over an iterative design process. The current knowledge base includes information about window size, shape, location on the facade, and simple shading devices. Three case studies are given in which this iterative optimization method was applied; all resulted in improved daylighting performance.

INTRODUCTION

Successful daylighting schemes may enhance architectural design, provide health benefits to occupants, and reduce energy consumption for electric lighting [Edwards and Torcellini, 2002; Rashid and Zimring, 2008]. In recent years, simulation tools have gained popularity among both students and practitioners for evaluating daylighting performance [Reinhart and Fitz, 2006; Sarawgi, 2006]. However, such tools have yet to achieve total integration into the design process. Experience and heuristics continue to be used more often than simulation tools during the schematic design phase, the period in which most major design decisions relating to daylighting are made [Galasiu and Reinhart, 2008]. This design phase, however, should be the one where feedback is received and where the overall design proposal is first assessed (even roughly) against performance goals.

One reason why simulation tends to be more commonly utilized for analysis of a near-completed design than for early design exploration may be that most simulation tools generally do not provide the designer with the means to easily gauge his early design options against performance objectives, or with some kind of feedback about how he might change his design to meet these goals. As a result, the designer takes the risk of wasting time exploring options that do not improve performance. Furthermore, creating models and running simulations can become too time-consuming if getting a comprehensive understanding of the performance requires too extensive an analysis. This combination of unguided search with time-intensive simulations can make the whole process of integrating daylighting considerations early on too tedious and inefficient for the designer.

One method for improving efficiency in design exploration is the use of a knowledge-based or expert system. A knowledge-based system is one in which human expert knowledge about a specific domain is encoded in an algorithm or computer system [Luger, 2004]. In the daylighting domain, such a system would function as a virtual lighting consultant, guiding the designer towards design modifications which improve overall daylighting performance. Knowledge-based systems have already been successfully implemented for artificial lighting scenarios [Jung et al., 2003]. For daylighting, however, only qualitative systems are currently available [Paule and Scartezzini, 1997].

We propose a new method which combines a daylighting knowledge-based system with an efficient simulation engine, and which will ultimately be part of the LightSolve approach described in [Andersen et al, 2008]. The knowledge-base has been populated using a set of previously completed simulations chosen based on the Design of Experiments methodology [Montgomery, 2004]. It includes information about changes in illuminance levels due to various design modifications to facade elements. Windows and shading devices were chosen as design parameters because their placement on the façade is typically determined early in the design process and has a large influence on daylighting performance. The current knowledge-base has been populated using solar angle and weather data from Boston. MA.

The lighting simulation tool used to support the proposed design process is an interactive and physically-accurate hybrid global illumination rendering method, described below. This method has been validated against Radiance and allows for efficient image renderings and computation of annual daylighting metrics for custom 3D geometries.

This paper presents a series of three case studies for which this method was applied to improve daylighting performance. Each case study begins with the same initial design but has a different daylighting performance goal or set of goals. Using the knowledge-based method, we were able to improve performance in all three cases, including one case study which had two conflicting goals. For each case study, the design modifications suggested by the goal-based process is shown and the performance of the final design is discussed.

RENDERING METHOD

To build our rendering system, we use a hybrid global illumination method of patch-based radiosity [Goral et al. 1984] for the sky, and, for direct illumination by the sun, of indirect illumination and shadow volumes for pixel-based shadows [Crow 1977; Heidmann 1991]. Forward ray tracing is used to compute the direct illumination from the sun and sky. Light inter-reflection between surfaces is then computed by radiosity, and the per-patch direct illumination from the sun is replaced by per-pixel shadow volumes rendering. This hybrid method is efficient and allows interactive (~1 fps) recomputation of the lighting solution when the time or day changes and a real-time speed (>60 fps) when the user changes the camera to navigate through the This rendering implementation will be design. referred to as the LightSolve Viewer (LSV) throughout this paper.

To facilitate rendering with LSV, each surface in the mesh is labeled as opaque material, glass, or light sensor (either opaque or invisible). Most surfaces in the design, such as the walls, ceilings, and floors, are comprised of Lambertian (diffuse) opaque materials. Glass surfaces are used for windows that allow light from sun and sky to penetrate and illuminate the room. Sensors are used to measure the illuminance of the users' areas of interest. An opaque sensor is a special kind of opaque surface that also tracks its received light. In contrast, invisible sensors do not absorb or reflect light during the simulation and can be used to measure lighting on a hypothetical working plane, floating in the middle of the room.

This rendering system was validated through a set of qualitative and quantitative comparisons with Radiance [Ward 1994]. Radiance is the accepted industry standard for architectural lighting simulations and the most commonly used engine in the daylighting community [Reinhart and Fitz, 2006]. A pixel difference of less than 10% was found between LSV and Radiance for a variety of different scenes, camera positions, and daylighting conditions; more information about early validation results can be found in [Cutler et al 2008]. This analysis has since been extended by comparing data collected from area-based patch sensors in LSV with point sensors in Radiance. To do this, each patch sensor was sampled with a sufficient number of points that are fed to rtrace, the function in Radiance that computes irradiance. The irradiance values from the sampled points were averaged for each patch and compared with the results from LSV. Table 1 summarizes these comparisons for three different times of day on March 21 for an example model in Boston, MA. The values of the sensor patches with the lowest and highest relative difference from Radiance are indicated. Similar values were found for June 21 and December 21 (with an overall highest difference of 28%). Figure 1 shows renderings from both LSV and Radiance at the same time and day for visual comparison. This set of analyses brought confidence that our system provides reasonably accurate renderings, appropriate for use in daylighting design, although further improvement of the results' accuracy is underway.

Time	Sensor	LSV	Radiance	Relative Difference	
10 am	Best	46800	46890	0.19%	
	Worst	2672	2478	7.83%	
12 pm	Best	61240	61130	0.19%	
	Worst	3952	3711	6.47%	
2 pm	Best	4859	4851	0.17%	
	Worst	4288	4155	3.20%	

 Table 1 Example comparison of irradiance sensor data collected from LSV and Radiance: Values of sensors with lowest and highest relative difference at three times of day for March 21.



Figure 1 Sample renderings of a test scene using Radiance (left image) and LSV (middle image) at noon on March 21. The right image shows the same scene with the 16 area sensors distributed across the working plane.

Goal-Based Metric

A goal-based metric was created to allow the method to work towards user-defined performance goals. This metric is a numerical version of the graphical metric presented in [Kleindienst et al., 2008] and uses the same logic for climate and temporal simplifications. The metric assumes a user-defined sensor plane and a user-defined illuminance goal range. The rendering method divides the sensor plane into small patches, and calculates the illuminance for each patch. For a given moment of time, the goal-based metric represents the percentage of the area of the sensor plane on which the calculated illuminance is within the user's goal range. The user may indicate the time(s) of day (morning, mid-day, or afternoon) and/or the season(s) (winter, spring/fall, or summer) associated with each goal. The resultant metric would then represent the percentage of time and sensor plane area in which the calculated climate-base illuminance falls within the user-defined goal illuminance range.

To allow for climate-specific calculations, we calculate the illuminance of each sensor plane patch for each of four sky types, ranging from overcast to clear. A climate-based representative illuminance is calculated as a weighted average of illuminances from each sky type. To make whole-year calculations more efficient, the year is split into 56 periods and climate-based illuminance is calculated for each of them. This simplification has been validated in [Kleindienst et al., 2008].

DAYLIGHTING KNOWLEDGE-BASE

The daylighting knowledge-base used for this study contains information about the relative effects of various design changes to the façade on the illuminance levels on a workplane. Information is available for whole-year performance, or for more specific periods of the day (morning, mid-day, afternoon), and for each season (winter, spring/fall, and summer). This database allows the system to suggest and perform design changes which will improve the design's performance as a response to the varying outside conditions and the façade properties, based on the user's specific goals.

Traditional knowledge-bases are often populated using information known to a human expert. However, because daylight is dynamic and performance relies on many variables (geometry, orientation, materials, and so on), it would be difficult or impossible for a human to accurately and quantitatively describe the effect of each design change at specific times. Instead, simulations are used to allow us to examine such highly specific situations.

The current knowledge-base was populated using a set of previously completed simulations for a test model. The LSV rendering method was used to perform the simulations. The full set of simulated

models was developed based on a two level, fullfactorial Design of Experiments scheme [Montgomery, 2004]. As daylight is highly dependent on latitude and climate, it is necessary to perform full sets of experiments for each location of interest. The current knowledge-base has been populated using solar angle and weather data from Boston, MA.

Simulated Test Model

The test model is a simulated virtual 3D model with a single height space which is 30ft by 30ft in area and 10ft in height (9.1m x 9.14 x 3.1m). The four facades are oriented towards the four cardinal directions. Interior materials are entirely diffuse with reflectances of 80%, 50%, and 20% for the ceiling, walls, and floor, respectively. The sensor plane on which illuminance is measured is located at a workplane height of 3ft (0.9m) from the floor. It is divided into five zones: four perimeter zones, each facing a cardinal direction, and a core zone which is 10ft by 10ft $(3.1m \times 3.1m)$ in area (Figure 2).

The test model dimensions were chosen based on the common daylighting rule-of-thumb that light from a window may penetrate up to 1.5 times the head height for shaded windows and up to 2.5 times the head height for unshaded windows [Reinhart, 2005]. In section, the zones boundaries correspond to depths of 1, 2, and 3 times the window head height for windows located at the maximum height. Such zones allow us to capture information about a daylit zone, a drop-off zone, and a deep zone. Zones further than the deep zone would likely see only negligible effects from changes on the façade in question.

Each individual experiment model has a single window located on one of the four facades. The glazing type is clear single-pane with a transmissivity of 85%. Each set of experiments examines the effects of varying the window and shading device location and dimensions on one of the four facades. All overhangs were dimensioned to block direct sunlight at solar noon during the months from the spring equinox to the fall equinox in Boston, MA. For this study, a full set of experiments was conducted for each facade.



Figure 2 Test model with five sensor plane zones indicated as a different shades of grey

Design Parameters

Five façade parameters were examined. These include window area, window height-to-width ratio, vertical location of the window on the façade, horizontal location of the window on the façade, and existence of a horizontal overhang. Due to the nature of the Design of Experiments method, two levels for each parameter were chosen. These are indicated in Table 2. In total, ten different design conditions were examined for each of the four facades.

	LEVELS		
Parameter	0	1	
Window Area	5% of wall area	10% of wall area	
Window	1:1.5 height-to-	1.5:1 height-to-	
Shape	width ratio	width ratio	
Vertical	3.5ft (1.1m)	3.5ft (1.1m) from	
Location	from floor	ceiling	
Horizontal	5ft (1.5m) from	5ft (1.5m) from	
Location	left edge of wall	right edge of wall	
Horizontal			
Overhang	None	Overhang	

Table 2 Values tested for each design parameter

Results

Main effects were obtained for all design conditions for each of the five zones during three periods of the day and during three seasons of the year. These effects were obtained for illuminance on the workplane and can be sorted to find the relative effect of each condition. Highly positive values indicate that the particular design condition will result in increased illuminance relative to other design conditions. Highly negative values indicate that the design condition will result in decreased illuminance relative to other design conditions.

To use these results for a user-defined geometry and target time period, the results corresponding to all the relevant zones and all the relevant seasons and times of day are averaged. For example, Table 3 shows the first eight highest ranked actions for increasing illuminance in a south zone during the morning and mid-day in fall, winter, and spring. Further details about the knowledge-base and its results can be found in [Lee and Andersen, 2009].

Table 3 Example knowledge-base: First eight actions to increase illuminance in south zone in morning and mid-day, from fall through spring)

Rank	ACTION
1	Increase area of south windows
2	Move east windows towards south
3	Move south windows higher on façade
4	Remove any overhangs on south windows
5	Move east windows higher on façade
6	Move south windows towards east
7	Increase area of east windows
8	Move west windows towards south

GOAL-BASED DESIGN PROCESS

The following sections describe the key calculations and methods used for the proposed interactive goalbased design process, which begins with the user's initial input and continues by iterating through design changes until the user's goals have been met or until the user stops the process. The process begins with the creation of one or more specific, customized knowledge-bases for the user's particular design. The process also relies on pre-computed algorithms used to determine which design change should occur at each iteration, and on a search method used to determine the magnitude of each design change.

User Input

Initial user input must include a 3D model with materials and one or more performance goals. Performance goals consist of an illuminance range (minimum and maximum allowed), a goal time period (whole-year or specific seasons and/or times of day), and one or more goal sensor plane(s). The sensor plane(s) can be modeled as a horizontal 2D plane within the 3D model.

The current version of the method allows for 3D models to be created in Google SketchUp. Reflectances of opaque materials and transmissivity of glazing must be specified.

Customized Knowledge-Base

Based on the user's initial input, a customized knowledge-base is created for the specific model location, orientation, geometry, and goal times. The goal times are given directly by the user, and only information for those relevent times will be used. Relevant zones are determined based on the the location of sensor planes within the user's initial input geometry. Sensor planes located within a distance of 1 time the ceiling height are considered peripheral to the closest wall. Sensor planes between 1 and 2 times the ceiling height from a given wall is considered a core zone. Sensor planes located deeper than 2 times the ceiling height from a wall is considered a deep zone. For sensor planes encompassing multiple zones, the average values of relevant zones will be used.

For designs with multiple goals, multiple knowledgebases have to be created. Because problems with multiple goals are more difficult to resolve, further intelligence is given to the knowledge-base. Each wall is divided into sections based on the orthogonal projections of the sensor planes. The distance from each sensor plane to each orthogonal wall projection is also determined. For each sensor, the wall sections are ordered in terms of likelihood to affect that sensor based on the sensor's distance and angle from each wall section. Individual knowledge-bases are then created for each wall section corresponding to each sensor plane. Case study 3 provides an example of this process.

Determining a Design Change

Before each design iteration, the goal-based illuminance metric is calculated. This value represents the percentage of area and time that the sensor plane is within the goal range. Additionally, the percentages above and below the goal range are calculated. The customized knowledge-base recommends a design change based on the greater percentage away from the goal range. For example, if 10% of the plane is above the goal range and 25% of the plane is below the goal range, the knowledge-base will suggest a change which is likely to increase illuminance over the sensor plane.

For situations in which there are multiple sensor planes, the multiple customized knowledge-bases are used and the recommended design change will be that which attempts to improve the worse performing sensor plane. If two sensor planes are performing within 5% of each other, the two corresponding knowledge-bases will be compared and the design change chosen will be that which is most likely to improve both planes simultaneously. Typically, this means that if the initial performance of one sensor plane is worse than that of another, the façade section(s) nearest that sensor plane is likely to be changed the most.

Determining the Magnitude of a Change

Once the knowledge-base has chosen a design change, the user will be asked to input any constraints on the new design. For example, if a window is to be moved higher on the façade, the user will be asked for the highest head height that he will allow.

The optimal magnitude for the particular design change is determined by sampling three points at equal intervals between the current design and the design at which the constraint is met. Models which correspond to each sample point are automatically created and simulated using the LSV engine. The performance (percentage within goal range) at each point is determined, and a polynomial function is fit to these data. For cases with multiple performance goals, the performance of each sensor plane is averaged. The maximum value of the polynomial is determined using the first and second derivative values. The design is then changed to that value which results in the highest performance for that design change. If no improvement is seen, the previous design is retained.

CASE STUDIES

Initial Model

A series of case studies was performed using a single initial design with increasingly difficult goals. To demonstrate the flexibility of the method, the initial design was deliberately designed with a different form than the test model which populated the knowledge-base. The initial model is an L-shaped



Figure 3 Initial model for all case studies: Southand West-facing L-shaped space

space with 30ft (9.14m) of South-facing façade and 20ft (6.10m) of West-facing façade. The depth of the space is 10ft (3.05m) from each façade (Figure 3). The South façade has three windows and the West façade has two windows. Each window is 4ft (1.23m) in height and 2.5ft (0.76m) in width. Materials used in this model are the same as those used in the knowledge-base test model. The goal time period for all case studies was school schedule (morning through mid-day, fall through spring). For each case study, the first six design changes are presented.

Case Study #1: Single Minimum Illuminance Goal

The first case study has a goal of a minimum of 400 lux over a sensor plane that covers the whole space at workplane height. No maximum illuminance limit is set.

The customized knowledge-base for this case study used data from morning and mid-day, fall/spring and winter, for the south, west, core, and east zones. Only the south and west facades were considered.

The performance of the initial model was 71% within goal range for the times considered. Because the goal for this case study had no maximum value, the knowledge-base only suggested design changes to increase illuminance in the space. The first six suggested design changes were (in order): move the south facing windows higher on façade, increase the area of the south windows, move the south windows right (towards east), make the west windows more narrow, make the south windows wider, and increase



Figure 4 Case Study #1 design after 6 steps (Goal: 400 lx minimum, no maximum)



Figure 5 Case Study #1 performance improvement over 6 steps (20% total)

the area of the west windows. At each step, an arbitrary design constraint was specified. All design changes resulted in improved performance. The final design had a performance of 91% within the goal range, a 20% improvement from the original. The final design is shown in Figure 4. The progression of performance improvement over the six design changes made is shown in Figure 5.

Case Study #2: Single Illuminance Range Goal

The second case study uses the same sensor plane as the first case study. The goal range is 200 lux minimum and 800 lux maximum. The customized knowledge-base for this case study was the same as the previous example.

Due to the low values and narrow range of illuminance in the goal range, the performance of the initial model for this case study was only 38% in range. Because this case study goal range had both a maximum and a minimum, the knowledge-base suggested a design change at each iteration based on whether a greater percentage of the sensor plane was above the goal range or below it. The first three suggested design changes were: move the south windows down, add overhangs to south windows, and decrease the area of south windows. These actions all acted to decrease the illuminance in the space. The final three design changes alternated between attempting to increase or decrease illuminance. These changes were: move the south windows towards east, add overhangs to the west windows, make the west windows narrower. All



Figure 6 Case Study #2 design after 6 steps (Goal: 200 lx minimum, 800 lx maximum)



Figure 7 Case Study #2 performance improvement over 6 steps (32% total)

actions succeeded in improving performance. The final design had a performance of 70% within the goal range, a 32% improvement from the original. The final design is shown in Figure 6. The progression of performance improvement over the six design changes made is shown in Figure 7.

Case Study #3: Multiple Illuminance Range Goals

The third case study has two sensor planes with different illuminance goals. The first sensor plane is located along the South façade and has a goal of 400 lux minimum (no maximum). The second sensor plane is located in the North-West corner and has a goal range of 200 lux minimum and 800 lux maximum.



Figure 8 Case Study #3 sensor plane locations and facade sections

Because this case study has two goals, multiple knowledge-bases are necessary. The façades are each divided into two sections, based on the orthogonal projections of the sensor planes. The facade sections are designated S1, S2, W1, and W2, as indicated in Figure 6. Based on proximity and angle from each façade section, it was determined that the façade sections most likely to affect sensor #1 were S1, S2, and W2 (in order) and those most likely to affect sensor #2 were W1, S2, and W2 (in order). For each pair of façade section and sensor, a customized knowledge-base was created. For example, for sensor #2, the W1 facade used information about the west zone, while the S2 facade used information about the core zone.



Figure 9 Case Study #3 design after 6 steps (Sensor 1 Goal: 400 lx min, no max; Sensor 2 Goal: 200 lx min, 800 lx max)

For this case study, the average performance of both planes are presented. The performance of the initial model was 70% within the goal range (81% in range for sensor #1, and 59% in range for sensor #2). The suggested design changes were based on which sensor was performing worse at each step. Because sensor #2 is initially the worse performer, the first steps aim to improve performance over that sensor. The first two steps thus involve W1, the façade section most likely to affect this sensor. Later steps aim to increase illuminance in the sensor #2 region, thus the S1 façade section is changed. By the final step, performance is roughly equal over both sensors, so the S2 façade area is changed in an attempt to improve performance over both sensors. The first six steps suggested in this case study were: make the W1 window wider, add an overhang to the W1 window, move the S1 window higher, increase the area of the S1 window, make the W1 window smaller, move the S2 window higher. The final design is shown in Figure 9. The progression of performance improvement over the six design changes made is shown in Figure 10.

In this case study, we note that the average performance consistently improves after each step, while the individual performance of a single sensor may decrease after a single step. This occurs because a single step may favor one sensor plane over another. However, over several steps, we see that both performances improve. After six steps, the resultant design has an average performance of 82%, which is a 12% improvement from the original. The final performance of sensor #1 is 87% (6% improvement) and the final performance of sensor #2 is 72% (19% improvement). We note that sensor #2 has a larger improvement due to the formulation of the algorithm, which always attempts to improve the worse performer at each step.

CONCLUSIONS

This study has demonstrated the potential of a new method for guided goal-based design exploration using a daylighting knowledge-base combined with an efficient simulation engine. The knowledge-base is customized to a user's specific design and goals,



Figure 10 Case Study #3 performance improvement over 6 steps (12% total for the average of both sensor planes)

and has been created to improve the efficiency of search by suggesting design changes which are most likely to improve performance. In each of the three case studies presented, we saw that the knowledgebase always proposed changes which resulted in improved performance. The three case studies also demonstrate that this method has the potential to improve daylighting performance for general "nonbox" geometries and for varying goal types and complexities.

As a design tool, it is clear that this method has potential to educate students and new designers in the process of performance-based daylighting design. By exploring designs with such a tool, users will not only improve the performance of their original design ideas, but they will also quickly gain knowledge about how various design changes affect performance. As this method allows the user to input an arbitrary geometry and as it respects the user's initial design, this method also could be potentially valuable as a design exploration tool for early design stages in a professional setting. Informal interviews with architecture students have confirmed a great interest in performance-based feedback within a daylighting simulation tool. To continue this research, it will be necessary to study the use of the tool during an actual design process and to obtain feedback from users regarding their experiences and preferences.

There are still numerous limitations to the current method, including the limited number of design changes available to the knowledge-base and the collapsing of information about performance during specific times or in specific areas of a sensor plane. In future work, the knowledge-base will be expanded to include information about vertical shading devices, louvers, window distribution on the façade, interior reflectances, and glazing types. Additionally, more intelligence will be added to the algorithm regarding the original input geometry and the performance during specific times or over certain areas.

The "step-by-step" or univariate search method is another likely limitation. This approach was adopted because it easily allows for user interactivity, as the user provides constraints at each step. However, in general, univariate search algorithms are slow and may not result in optimal final solutions. Therefore, future work will involve a validation of the search method with traditional optimization methods and an investigation of other potential algorithms which would still allow for user interactivity.

The current method and case studies have only involved performance goals for illuminance on horizontal planes. In reality, users typically have additional goals regarding glare, solar thermal gains, or illuminance on vertical planes. To provide a complete method for daylighting design, new metrics, chosen based on user feedback, must be added to the knowledge-base. The addition of new metrics will also necessitate the investigation of other possible methods for search. In particular, illuminance and glare goals are often conflicting. These more complex scenarios will require more sophisticated algorithms.

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