Yukawa couplings and flavour symmetries in minimal gauge-Higgs unification

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ABSTRACT

We discuss the possibility of introducing an $SU(2)$ global flavour symmetry in the context of flat extra dimensions. In particular we concentrate on the 5-dimensional case and we study how to obtain the flavour structure of the Standard Model quark sector compactifying the fifth dimension on the orbifold $S_1/\mathbf{Z_2}$ *à la* Scherk-Schwarz (SS). We show that in this case it is possible to justify the five orders of magnitude among the values of the quark masses with only one parameter: the SS flavour parameter. The non-local nature of the SS symmetry breaking mechanism allows to realize this without introducing new instabilities in the theory.

1. Introduction

Recently, the idea came on the stage that it might be possible to overcome the hierarchy problem by implementing the breaking of gauge symmetries via alternative mechanisms relying on the presence of one or more extra dimensions. In particular, it is has been known for a long time that realizing the Higgs field as the zero-mode of the internal component of a higher-dimensional gauge field leads to an effective potential with improved stability [1]. This idea has been recently reinvestigated from various points of view [2], and exploited to construct concrete higher-dimensional orbifold models with this type of gauge–Higgs unification [3,4]. A simple prototype of this kind of models is the minimal five-dimensional (5D) scenario described in [4], where the gauge symmetry is broken by the combination of a $\mathbf{Z}_2$ orbifold projection and a continuous SS twist along the extra compact dimension. The electroweak symmetry breaking is spontaneous and occurs through the Hosotani mechanism [5]. The order parameter is the Wilson loop $W = \exp\{ig \oint A_5(y)dy\}$ constructed out of the internal component A_5 of the gauge field along the internal circle S^1. The role of the Higgs field is played by the zero-mode of A_5, but the effective potential can depend only on the non-local gauge-invariant W and is therefore finite.

In this paper, we study the possibility of endowing the above-mentioned class of higher-dimensional models with a flavour symmetry of the Froggatt–Nielsen (FN) type [6]. This is done by introducing an extended flavour symmetry, which is then broken, as for the electroweak symmetry, by the combination of an orbifold projection and a SS twist. We focus on the model of ref. [4] and describe its minimal flavour extension. We show that by a wise choice of the flavour quantum numbers for bulk and brane fermion fields, it

[a]Talk given by L.S.

is possible to reproduce the observed pattern of the quark masses and CKM angles, and observe that a similar approach is possible also for lepton masses and PMNS angles. The resulting model generates a 4D effective theory with a stabilized electroweak scale and a $U(1)$ FN symmetry.

2. The Basic Construction

Our starting point is the model described in ref. [4]. The basic physical idea is to start with a suitable 5D gauge theory and compactify it in such a way as to produce a 4D effective theory where the internal component A_5 of the original gauge field acts as a Higgs field with a dynamics that is interestingly constrained by the remnant of the 5D gauge symmetry. As in the standard electroweak theory, the VEV of the Higgs field can induce a mass for the matter fermions. The relevant Yukawa couplings, however, originate in this case from the 5D gauge coupling. For bulk fermions, this implies that the Yukawa couplings are universal and their magnitude is simply the gauge coupling times a Clebsch–Gordan (CG) factor, depending only on the representation and forbidding any generation mixing. In order to achieve realistic Yukawa couplings, one is therefore led to consider the more general case of fermions that are a mixture of bulk and brane fields with wave functions depending non-trivially on the internal dimension. This situation is most easily realized, as suggested in ref. [4], by considering bulk and brane fields that mix through non-universal bilinear couplings localized at the fixed-points. The new eigenstates, resulting from the diagonalization of the quadratic Lagrangian for these fields, will then inherit non-vanishing and non-universal Yukawa couplings to the Higgs field. These can be pretty general, but their size is always at most of the order of the gauge coupling. This implies that the natural value of all the fermion masses induced in this way is of the order of m_W. This framework is very similar to the one occurring in models with flavour symmetries, the breaking of which is transmitted to the effective Yukawa couplings through a heavy fermion, and suggests that it should be possible to naturally generalize the model of ref. [4] to include a flavour symmetry.

A minimal prototype of such a model can be constructed as follows. The standard fermions are taken to live at the orbifold fixed-points, whereas the messenger fermions that activate the mechanism of symmetry breaking live in the bulk. A spontaneously broken Abelian flavour symmetry is then incorporated much in the same way as for the spontaneously broken electroweak symmetry, and both symmetry breakings are implemented at once by letting the orbifold projection and the SS twist act on both the electroweak and the flavour groups. The minimal choice of 5D flavour group allowing a Abelian group in the intermediate step and a full breaking in the final step is an $SU(2)_F$ group. For simplicity, we assume this to be a global symmetry, but the case of a local symmetry is similar. This flavour group is broken to a $U(1)_F$ subgroup that is identified with the one of ref. [6] through the orbifold projection, and finally to nothing through the SS twist.

In the effective Lagrangian of our model, mass terms are generated by coupling fermions that correspond to different components of the same representation of $SU(2)_F$. These components have different $U(1)$ charges. The effective mass coupling is therefore proportional to n insertions of the raising/lowering operator $\sigma_\pm$, *i.e.* $(\lambda\sigma_\pm)^n$, which are necessary to connect different components of the same multiplet. λ is a symmetry-breaking parameter of the order of the Cabibbo angle. Just as with the FN mechanism, in the presence of a spontaneously broken Abelian flavour symmetry, the entries of the Yukawa couplings $Y^{u,d}_{i,j}$, with $i,j = 1,2,3$ indices in flavour space, can be expressed as powers of the symmetry breaking parameter:

$$Y^u_{ij} \sim \lambda^{q_i - u_j}\,, \qquad Y^d_{ij} \sim \lambda^{q_i - d_j}\,, \tag{1}$$

where q_i, u_i and d_i are the $U(1)$ flavour charges of doublets, up-type singlets and down-type singlets respectively. Our main improvement with respect to the FN model is that the 4-dimensional $U(1)$ flavour charge is naturally quantized and that the non-local nature (in the fifth dimension) of the SS flavour symmetry breaking protects the theory from new ultra-violet divergences.

3. Results and Conclusions

We now present the simplest prototype of model realizing the above ideas; we refer to [7] for a complete discussion about the construction of a more realistic model and its phenomenology. We concentrate on the following phenomenologically acceptable parametrization of the Yukawa couplings:

$$Y^d \sim \lambda \begin{pmatrix} \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda) & \mathcal{O}(1) \end{pmatrix}, \quad Y^u \sim \lambda \begin{pmatrix} \mathcal{O}(\lambda^6) & \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{pmatrix}. \tag{2}$$

We can then fix the $U(1)$ flavour charges of the SM fermions by comparing eqs. (1) with eqs. (2):

$$q_i = \left(-\frac{7}{2}, -\frac{5}{2}, -\frac{1}{2}\right), \qquad d_i = \left(\frac{5}{2}, \frac{3}{2}, \frac{1}{2}\right), \qquad u_i = \left(\frac{7}{2}, \frac{5}{2}, \frac{1}{2}\right). \tag{3}$$

Charges spanning values from $+7/2$ to $-7/2$ imply that the bulk fields must belong at least to the 8-dimensional representation of $SU(2)_F$ to have the possibility to couple to SM fermions at fixed points. The 8-dimensional representation corresponds to the simplest possible choice. Thus, in the bulk, besides 5-dimensional gauge fields, we introduce two pairs of fermion fields $(\psi^l, \tilde{\psi}^l)$ with $l = d, u$ belonging to a 8-dimensional representation of the flavour group $SU(2)_F$ and to the fundamental and 6-dimensional symmetric representation of $SU(3)_W$ respectively.

The basic rules to generate 4-dimensional effective mass (and kinetic) terms are the following: i) Write the most general Lagrangian compatible with gauge and global symmetries for the bulk fermions; ii) Write the most general effective Lagrangian for the fermions on the branes, including the couplings to the bulk fermions. The latter induce gauge-invariant non-local couplings between the fermions on the branes and the fifth component of the gauge field A_5, through Wilson lines. The possible terms are all those allowed by the residual symmetries on the branes. We will only consider renormalizable contributions to the Lagrangian, in the spirit of neglecting the effect of irrelevant operators at low energy. iii) Integrate out the heavy bulk degrees of freedom (bulk masses $\sim 1/R$). By integrating out the heavy bulk fermions, we are able to reproduce eqs. (2). Moreover, using the bulk mass parameters M_d and M_u, it is possible to justify the hierarchy between top and bottom quark masses: $m_b/m_t = e^{-(M_d - M_u)\pi R}$.

Therefore, introducing a new global flavour symmetry in 5 dimensions and using the SS compactification on the orbifold $S_1/\mathbf{Z}_2$, we are able to reproduce the qualitative flavour structure of the SM. In particular, we can explain the five orders of magnitude among the values of the quark masses using only the SS parameter. Effectively, our model generates a 4-dimensional theory equipped with a Froggatt-Nielsen $U(1)$ structure, without introducing a new fundamental Higgs and the related instabilities in the renormalized theory. Here we have presented the basic idea; we refer to ref. [7] for a detailed analysis of the main aspects of this prototype model, a complete discussion about the construction of a more realistic model, and the question of how to explain the ratio m_t/m_W, which we have not addressed here.

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