FLAVOUR PHYSICS FROM EXTRA DIMENSIONS

G. Martinelli[a], M. Salvatori[b], C. A. Scrucca[c], L. Silvestrini[a]
[a]INFN, Sez. di Roma, Dip. di Fisica, Univ. di Roma "La Sapienza",
Piazzale Aldo Moro 2, 00185 Rome, Italy
[b]Dep. de Física Teórica, C-XI, and Ist. de Física Teórica, C-XVI,
Univ. Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
[c]Physics Dep., Theory Div., CERN, 1211 Geneva 23, Switzerland

We discuss the possibility of introducing an $SU(2)$ global flavour symmetry in the context of flat extra dimensions. In particular we concentrate on the 5-dimensional case and we study how to obtain the flavour structure of the Standard Model quark sector compactifying the fifth dimension on the orbifold $S_1/\mathbf{Z_2}$ *à la* Scherk-Schwarz (SS). We show that in this case it is possible to justify the five orders of magnitude among the values of the quark masses with only one parameter: the SS flavour parameter. The non-local nature of the SS symmetry breaking mechanism allows to realize this without introducing new instabilities in the theory.

1 Introduction

In the last quarter of century the central problem in particle physics has been the mechanism for the breaking of the electroweak gauge symmetry and the consequent generation of masses for the gauge bosons and matter fermions. In the Standard Model (SM), the problem manifests itself in two different ways: on one hand in the instability of the weak interaction scale (the so-called hierarchy problem), on the other hand in the arbitrariness of the Yukawa couplings, which span at least five orders of magnitude, and the related problem of the strength of the CKM matrix elements. In the quest for a solution of these fundamental questions, a plethora of models have been built, starting from technicolour, softly broken global supersymmetry, supergravity, string theories, etc. None of the proposed solutions is fully satisfactory in this respect.

More recently, the idea came on the stage that it would be possible to overcome the hierarchy problem by implementing the breaking of the gauge symmetry via an alternative to the standard Higgs mechanism. Several models in this direction have been produced, in particular in the context of extra dimensions, and attempts have also been made to generate fermion masses without fundamental Yukawa couplings [1]. In particular, we will refer to the possibility of identifying the extra component of the gauge field (A_5) with the Higgs scalar field (ϕ): the 5-dimensional gauge invariance forbids any mass term for ϕ. In this context, using the Scherk-Schwarz (SS) [2] compactification on the orbifold [3] $S_1/\mathbf{Z}_2$, it is possible to regard the Vacuum Expectation Value (VEV) of A_5 as playing the rôle of the usual Higgs VEV. Effective couplings involving the Wilson line $\exp[ig\int A_5(y)dy]$ can play the rôle of Yukawa couplings for fermions that are mixtures of bulk and boundary fields [4,5,6]. The quadratic divergences that plague the SM are however absent since the renormalization of the A_5 mass term is protected by gauge invariance.

We will concentrate on the scenario described in ref. [6] (that we call SSS model) and we extend the strategy by introducing a flavour symmetry, broken *à la* SS, to generate the quark mass matrices. We show that by a wise choice of the flavour quantum numbers for bulk and brane fermion fields, it is possible to reproduce the observed values, and pattern, of the quark mass ratios and CKM couplings. Effectively, our model generates a 4-dimensional theory equipped with the Froggatt-Nielsen (FN) $U(1)$ couplings [7] without any new fundamental Higgs and the related instabilities in the renormalized theory.

2 The Basic Idea

Our starting point is the SSS model described in ref. [6]. The basic physical idea is to break the electroweak symmetry by using a 5-dimensional theory equipped with a suitable gauge group, which is broken in 4 dimensions using as an effective Higgs the fifth component of the gauge field A_5. The symmetry breaking proceeds in two steps: i) the orbifold projection reduces the original gauge symmetry to the electroweak symmetry subgroup, with a massless "Higgs" boson, which transforms as a doublet under the weak isospin; ii) the "Higgs" subsequently acquires a mass via the Hosotani mechanism [8]. The renormalization of the "Higgs" mass is then protected by gauge invariance. The minimal gauge group that contains the electroweak group $SU(2)\times U(1)$, and has the same rank, is $SU(3)$, which was chosen in ref. [6]. The enlarged $SU(3)_W$ weak interaction gauge group is broken to $SU(2)\times U(1)$ via the $S_1/\mathbf{Z}_2$ orbifold projection from 5 to 4 dimensions, which we choose to act in the gauge space through the group element $P_W^{\mathcal{R}} = e^{i\pi T^8(\mathcal{R})}$, where $T^8(\mathcal{R})$ is the $SU(3)_W$ generator in the representation $\mathcal{R}$. The residual gauge symmetries are those associated with the generators T^a with $a = 1, 2, 3, 8$ that satisfy the condition: $[T^a, P_W^3] = 0$. Subsequently, the residual $SU(2)\times U(1)$ gauge symmetry is reduced to $U(1)$ *à la* SS. The SS gauge symmetry breaking is controlled by the parameter α: $T_W(\alpha) = e^{2\pi i\alpha T^7(\mathcal{R})}$. This choice satisfies the consistency condition $(T_W P_W)^2 = 1$. As for the standard symmetry breaking, the effective Higgs can induce a mass for the fermions of the theory. The Yukawa coupling is the gauge coupling, and thus induces a mass of the same order of magnitude for all fermions of the bulk, at the scale $m_W = \alpha/R$, where R is the compactification radius.

To the $SU(3)_W$ group, we add a $SU(2)_f$ flavour group which, for simplicity, we take as a global symmetry. The flavour group is broken by the $\mathbf{Z}_2$ orbifold projection to an effective $U(1)$ which mimics the $U(1)$ symmetry of ref. [7]. The orbifold projection is chosen to act in the flavour space through $P_f^{\mathcal{R}'} = e^{i\pi T^3(\mathcal{R}')}$, where $T^3(\mathcal{R}')$ is the $SU(2)_f$ generator acting in the representation $\mathcal{R}'$. The flavour symmetry is then reduced to the $U(1)$ group associated with T^3, which is the only generator satisfying the condition $[T^a, P_f^2] = 0$. This is the $U(1)$ that allows the FN mechanism. In this way, the flavour charge is quantized and it is represented by $q_f = T^3$. The residual $U(1)$ is finally broken through the SS mechanism. The SS flavour symmetry breaking is controlled by the parameter β: $T_f(\beta) = e^{2\pi i\beta T^7(\mathcal{R})}$. This choice satisfies the consistency condition $(T_f P_f)^2 = 1$. Concering the flavour group, we have chosen $SU(2)$ because in this case, after the SS compactification on orbifold $S_1/\mathbf{Z}_2$, it is possible to obtain a 4-dimensional effective theory without any residual flavour symmetry, as we expect from experimental evidence.

Standard Model fermions live on the two branes, depending on their chirality, whereas other messenger fermions, which activate the mechanism of symmetry breaking, live in the bulk. The SM fermions have to be charged under the residual $U(1)$ flavour symmetry on the brane, *i.e.* they have to be characterized by a $U(1)$ flavour charge Q_f. In the bulk, we introduce two pairs of fermion fields $(\psi^l, \tilde{\psi}^l)$, where $l = d, u$, with opposite $\mathbf{Z}_2$ parity, in such a way that we can write a mass term wich mixes them.

As long as the gauge and flavour symmetries are unbroken, it is not possible to generate any 4-dimensional effective mass term.

When we break only the brane residual gauge symmetry $SU(2) \times U(1)$ to $U(1)$ by the SS mechanism, we can obtain only flavour diagonal effective mass terms through diagrams of the type described in fig. 1. In this diagram, $\Psi(2, Q_f)$ and $\Psi(1, Q_f)$ are the $SU(2)$ doublet and singlet components of the bulk fields respectively and Q_f is the flavour charge. We emphasize that in this case all fields have to carry the same flavour quantum numbers. [a]

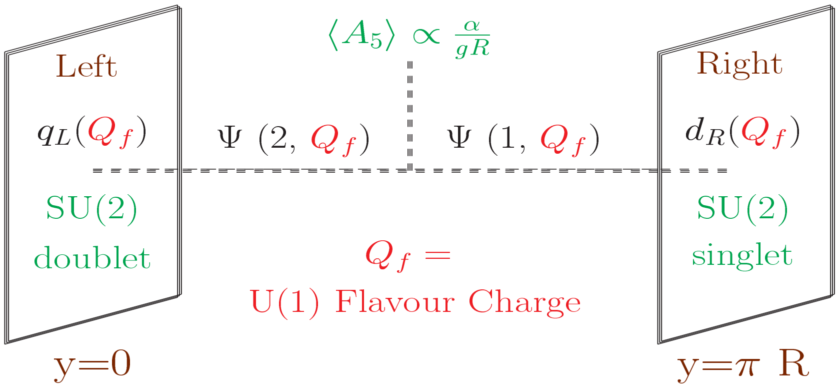

Figure 1: Possible process in the presence of gauge symmetry breaking only: all the fields carry the same flavour charge.

When we break both the brane residual gauge and flavour symmetries by the SS mechanism, we can generate effective 4-dimensional mass terms with a non-trivial structure in flavour space. In fact, in this case we are able to reproduce an effective FN mechanism.

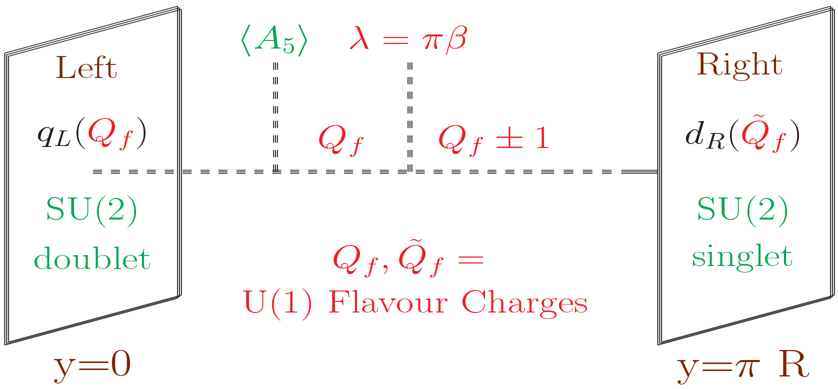

Figure 2: Possible process in the presence of both gauge and flavour symmetry breaking.

In the effective Lagrangian of our model, mass terms are generated by coupling fermions that correspond to different components of the same representation of $SU(2)_f$, as in fig. 2. These components have different $U(1)$ charges. In the limit in which the masses of the bulk fermions are much bigger than the electroweak scale and $\lambda = \pi\beta \ll 1$, the effective mass coupling is therefore proportional to n insertions of the $SU(2)_f$ raising/lowering operator $T_\pm$, *i.e.* $(\lambda T_\pm)^n$, which are necessary to connect different components of the same multiplet. λ is a parameter of

[a] The rôle played by the gauge fields has been extensively explained in ref. [6] and will not be repeated here.

the order of the Cabibbo angle. With suitable assignments of the $SU(2)_f$ quantum numbers, it is then possible to arrange the mass matrix in such a way to produce a pattern of matrix elements whose diagonalization reproduces the observed quark masses and mixing angles. Just as with the FN mechanism, in the presence of a spontaneously broken Abelian flavour symmetry, the entries of the Yukawa couplings $Y^{u,d}_{i,j}$, with $i, j = 1, 2, 3$ indices in flavour space, can be expressed as powers of the symmetry breaking parameter. The exponents can be written as the difference of flavour charges as follows:

$$Y^d \sim \begin{pmatrix} \lambda^{q_1-d_1} & \lambda^{q_1-d_2} & \lambda^{q_1-d_3} \\ \lambda^{q_2-d_1} & \lambda^{q_2-d_2} & \lambda^{q_2-d_3} \\ \lambda^{q_3-d_1} & \lambda^{q_3-d_2} & \lambda^{q_3-d_3} \end{pmatrix}, \quad Y^u \sim \begin{pmatrix} \lambda^{q_1-u_1} & \lambda^{q_1-u_2} & \lambda^{q_1-u_3} \\ \lambda^{q_2-u_1} & \lambda^{q_2-u_2} & \lambda^{q_2-u_3} \\ \lambda^{q_3-u_1} & \lambda^{q_3-u_2} & \lambda^{q_3-u_3} \end{pmatrix}, \tag{1}$$

where the q_i are the $U(1)$ flavour charges of left-handed doublets, whereas the d_i and u_i are the flavour charges of the right-handed singlets. Our main improvement with respect to the FN model is that the 4-dimensional $U(1)$ flavour charge is naturally quantized and that the non-local nature (in the fifth dimension) of the SS flavour symmetry breaking protects the theory from new ultra-violet divergences.

3 A Prototype Model

Here we present the simplest prototype of model realizing the above ideas; we refer to ref. [9] for a complete discussion about the constrution of a more realistic model and its phenomenology. We concentrate on the following phenomenologically acceptable parametrization of the Yukawa couplings:

$$Y^d = \begin{pmatrix} \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda) & \mathcal{O}(1) \end{pmatrix}, \quad Y^u = \begin{pmatrix} \mathcal{O}(\lambda^8) & \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda^7) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{pmatrix}. \tag{2}$$

We can then fix the $U(1)$ flavour charges of the SM fermions by comparing eqs. (1) with eqs. (2). Our choice is presented in table 1. Charges ranging from $+4$ to -4 imply that the bulk fields

Field	q_f	Field	q_f
Q_{1L}	- 4	d_{1R}	1
Q_{2L}	- 3	d_{2R}	0
Q_{3L}	-1	d_{3R}	-1
Q^c_{1R}	4	$-u^c_{1L}$	- 4
Q^c_{2R}	3	$-u^c_{2L}$	- 1
Q^c_{3R}	1	$-u^c_{3L}$	1

Table 1: Flavour charge of SM fermions.

must belong at least to the 9-dimensional representation of $SU(2)_f$ to have the possibility to couple to SM fermions at fixed points. The 9-dimensional representation corresponds to the simplest possible choice. Thus, in the bulk, besides 5-dimensional gauge fields, we introduce two pairs of fermion fields $(\psi^l, \tilde{\psi}^l)$ with $l = d, u$ belonging to a 9-dimensional representation of the flavour group $SU(2)_f$ and to the fundamental and 6-dimensional symmetric representation of $SU(3)_W$ respectively. The orbifold projection is chosen in such a way that the bulk fermions have to satisfy the following conditions:

$$\begin{aligned} \psi^l(x, -y) &= [\gamma_5 \otimes P^{\mathbf{3}}_W \otimes P^{\mathbf{9}}_f]\, \psi^l(x, y) \\ \tilde{\psi}^l(x, -y) &= -[\gamma_5 \otimes P^{\mathbf{6}}_W \otimes P^{\mathbf{9}}_f]\, \tilde{\psi}^l(x, y)\ , \end{aligned} \tag{3}$$

where P_W^3 and P_W^6 are the orbifold projections on the fundamental and symmetric $SU(3)_W$ representations respectively. As the fields ψ^l, $\tilde{\psi}^l$ have different parities under the orbifold projection, we can introduce a gauge-invariant bulk mass term as $\bar{\psi}^l M_l^C \tilde{\psi}^l$. It is natural to think that these bulk mass parameters are all of the order of magnitude of the only scale of the theory, namely the compactification scale $\sim 1/R$. To simplify the notation, we embed the SM fermions living on the brane in a 9-dimensional representation of $SU(2)_f$. Let us call Q, Q^c, u, u^c, d, d^c and $\tilde{Q}$, $\tilde{Q}^c$, $\tilde{u}$, $\tilde{u^c}$, $\tilde{d}$, $\tilde{d^c}$ the fields that have the appropriate quantum numbers to couple to the non-zero components of the bulk fields at the fixed points, ψ^u and ψ^d or $\tilde{\psi}^u$ and $\tilde{\psi}^d$, respectively. More explicitly, for the case of $SU(2)$ doublets, for example, we can write:

$$Q = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_{3L} \\ 0 \\ Q_{2L} \\ 0 \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_{1L} \end{pmatrix} \quad Q^c = \begin{pmatrix} 0 \\ Q^c_{2R} \\ 0 \\ Q^c_{3R} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \tilde{Q}^c = \begin{pmatrix} Q^c_{1R} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

The complete Lagrangian for matter fields is then given by the following expression:

$$\begin{aligned}
\mathcal{L} &= \sum_{l=u,d} \Big[i\overline{\psi}^l \gamma^M D_M \psi^l + i\overline{\tilde{\psi}}^l \gamma^M D_M \tilde{\psi}^l - M_l^C [\overline{\psi}^l \tilde{\psi}^l + \overline{\tilde{\psi}}^l \psi^l] \Big] \\
&+ \delta(y) \sum_{i=1,2,3} \Big[i\overline{Q}_i \gamma^\mu D^4_\mu Q_i + i\overline{\tilde{Q}}_i \gamma^\mu D^4_\mu \tilde{Q}_i \\
&+ \frac{\epsilon^d_{1i}}{\sqrt{\pi R}} [\overline{Q}_i \psi^d_i + \overline{\tilde{Q}}_i \tilde{\psi}^d_i + \text{h.c.}] + \frac{\epsilon^u_{1i}}{\sqrt{\pi R}} [\overline{Q}^c_i \psi^u_i + \overline{\tilde{Q}}^c_i \tilde{\psi}^u_i + \text{h.c.}] \Big] \\
&+ \delta(y - \pi R) \sum_{i=1,2,3} \Big[i\overline{u}^c_i \gamma^\mu D^4_\mu u^c_i + i\overline{\tilde{u}}^c_i \gamma^\mu D^4_\mu \tilde{u}^c_i + i\overline{\tilde{d}}_i \gamma^\mu D^4_\mu \tilde{d}_i + i\overline{d}_i \gamma^\mu D^4_\mu d_i \\
&+ \frac{\epsilon^d_{2i}}{\sqrt{\pi R}} [\overline{d}_i \psi^d_i + \overline{\tilde{d}}_i \tilde{\psi}^d_i + \text{h.c.}] + \frac{\epsilon^u_{2i}}{\sqrt{\pi R}} [\overline{u}^c_i \psi^u_i + \overline{\tilde{u}}^c_i \tilde{\psi}^u_i + \text{h.c.}] \Big] .
\end{aligned} \tag{5}$$

D_M and D^4_M denote the 5- and 4-dimensional covariant derivatives respectively, ϵ^u_{1i}, ϵ^u_{2i}, ϵ^d_{1i}, ϵ^d_{2i} are dimensionless mixing parameters, and $i = 1, 2, 3$ is a family index.

4 Results and Conclusions

The basic rules to generate 4-dimensional effective mass (and kinetic) terms are the following:

- Write the most general Lagrangian compatible with gauge and global symmetries for the bulk fermions;
- Write the most general effective Lagrangian for the fermions on the branes, including the couplings to the bulk fermions. The latter induce gauge-invariant non-local couplings between the fermions on the branes and the fifth component of the gauge field A_5, through Wilson lines [4]. The possible terms are all those allowed by the residual symmetries on the branes. We only consider renormalizable contributions to the Lagrangian, in the spirit of neglecting the effect of irrelevant operators at low energy.
- Integrate out the heavy bulk degrees of freedom (bulk masses $\sim 1/R$).

By integrating out the heavy bulk fermions, in low energy limit $p^2_{\rm 4d} \ll M^2_{\rm bulk}$ and for $\lambda = \pi\beta \ll 1$, we are able to reproduce eqs. (2). In particular, when we diagonalize them we obtain the following results:

$$\begin{array}{l} m_u/m_t \sim \mathcal{O}(\lambda^8)\ ,\quad m_c/m_t \sim \mathcal{O}(\lambda^4)\ , \\ m_d/m_b \sim \mathcal{O}(\lambda^5)\ ,\quad m_s/m_b \sim \mathcal{O}(\lambda^3) \end{array} \qquad \text{and} \qquad V_{CKM} = \left(\begin{array}{ccc} \mathcal{O}(1) & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{array} \right)\ . \qquad (6)$$

Moreover, using the bulk mass parameters M_d and M_u, it's possible to justify the hierarchy between top and bottom quarks: $m_b/m_t = e^{-(M_d - M_u)\pi R}$.

Therefore, introducing a new global flavour symmetry in 5 dimensions and using the SS compactification on the orbifold $S_1/\mathbf{Z}_2$, we are able to reproduce the qualitative flavour structure of the SM. In particular, we can explain the five orders of magnitude among the values of the quark masses with only one parameter: the SS parameter $\lambda = \pi\beta$. Effectively, our model generates a 4-dimensional theory equipped with a Froggatt-Nielsen $U(1)$ structure, without introducing a new fundamental Higgs and the related instabilities in the renormalized theory. Here we have presented the basic idea; we refer to ref.[9] for a detailed analysis of the main aspects of this prototype model, a complete discussion about the construction of a more realistic model, and the question of how to explain the ratio m_t/m_W, which we have not addressed here.

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