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Interaction of D-branes on orbifolds and massless particle emission

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Abstract

We discuss various D-brane configurations in 4-dimensional orbifold compactifications of type II superstring theory which are point-like 0-branes from the 4-dimensional space-time point of view. We analyze their interactions and compute the amplitude for the emission of a massless NSNS boson from them, in the case where the branes have a non vanishing relative velocity. In the large distance limit, we compare our computation to the expected field theory results, finding complete agreement.

Talk presented by Claudio A. Scrucca

1 Introduction and summary

We discuss various D-brane configurations in generic orbifold compactifications which are 0-branes from the 4-dimensional space-time point of view, but can have extension in the compact directions. More precisely, two cases turn out to be particularly interesting; the 0-brane of type IIA and the 3-brane of type IIB.

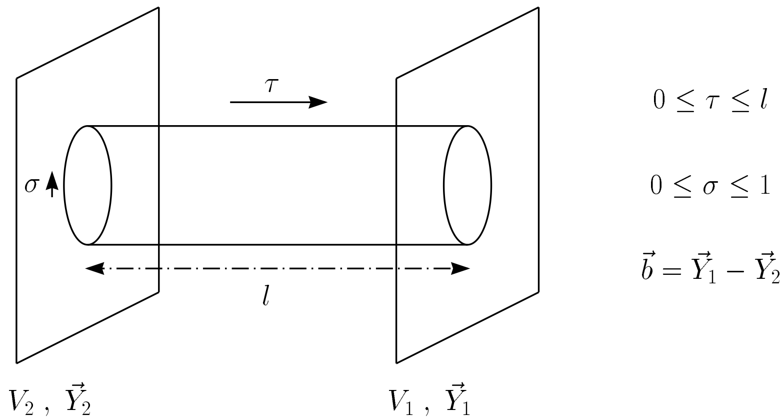
The dynamics of these D-branes is determined by a one loop amplitude which can be conveniently evaluated in the boundary state formalism [1, 2]. In particular, one can compute the force between two D-branes moving with constant velocity, extending Bachas' result [3] to compactifications breaking some supersymmetry [4].

Analyzing the large distance behavior of the interaction and its velocity dependence, it is possible to read the charges with respect to the massless fields, and relate the various D-brane configurations to known solutions of the 4-dimensional low energy effective supergravity.

Finally, we will discuss the emission of massless NSNS states from two interacting D-branes [5]. The correlators that are involved have twisted boundary conditions because of the non zero velocity of the branes, but they can be systematically computed in a natural way using again the boundary state formalism. We will then briefly outline the large distance behavior of the string amplitude and its field theory interpretation.

2 Interactions on orbifolds

Consider two D-branes moving with velocities $V_1 = \tanh v_1$, $V_2 = \tanh v_2$ (say along 1) and transverse positions $\vec{Y}_1$, $\vec{Y}_2$ (along 2,3).



The potential between these two D-branes is given by the cylinder vacuum amplitude and can be thought either as the Casimir energy stemming from open string vacuum fluctuations or as the interaction energy related to the exchange closed strings between the two branes. The amplitude in the closed string channel

$$\mathcal{A} = \int_0^\infty dl \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} | B, V_2, \vec{Y}_2 >_s$$

is just a tree level propagation between the two boundary states, which are defined to implement the boundary conditions defining the branes.

There are two sectors, RR and NSNS, corresponding to periodicity and antiperiodicity of the fermionic fields around the cylinder, and after the GSO projection there are four spin structures, R$\pm$ and NS$\pm$, corresponding to all the possible periodicities of the fermions on the covering torus.

In the static case, one has Neumann b.c. in time and Dirichlet b.c. in space. The velocity twists the 0-1 directions and gives them rotated b.c. The moving boundary state is most simply obtained by boosting the static one with a negative rapidity $v = v_1 - v_2$ [6].

$$|B, V, \vec{Y} >= e^{-ivJ^{01}} |B, \vec{Y} > \quad .$$

In the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute, and momentum or winding in the compact directions can be safely neglected since they correspond to massive subleading components.

The moving boundary states

$$|B, V_1, \vec{Y}_1 >= \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1} |B, V_1 > \otimes |k_B > , \tag{1}$$

$$|B, V_2, \vec{Y}_2 >= \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{Y}_2} |B, V_2 > \otimes |q_B > , \tag{2}$$

can carry only space-time momentum in the boosted combinations

$$k_B^\mu = (V_1\gamma_1 k^1, \gamma_1 k^1, \vec{k}_T) = (\sinh v_1 k^1, \cosh v_1 k^1, \vec{k}_T) \ ,$$
$$q_B^\mu = (V_2\gamma_2 q^1, \gamma_2 q^1, \vec{q}_T) = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T) \ .$$

Notice that because of their non zero velocity, the branes can also transfer energy, and not only momentum as in the static case.

Integrating over the bosonic zero modes and taking into account momentum conservation ($k_B^\mu = q_B^\mu$), the amplitude factorizes into a bosonic and a fermionic partition functions:

$$\mathcal{A} = \frac{1}{\sinh v}\int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q_B^2}{2}} \sum_s Z_B Z_F^s = \frac{1}{\sinh v}\int_0^\infty \frac{dl}{2\pi l} e^{-\frac{b^2}{2l}} \sum_s Z_B Z_F^s \tag{3}$$

with

$$Z_{B,F} = < B, V_1 | e^{-lH} | B, V_2 >_{B,F}^s \ .$$

From now on, $X^\mu \equiv X^\mu_{osc}$; moreover, it will prove convenient to group the fields into pairs

$$\begin{aligned} X^\pm = X^0 \pm X^1 &\rightarrow \alpha_n, \beta_n = a_n^0 \pm a_n^1 \ , \\ X^i, X^{i*} = X^i \pm i X^{i+1} &\rightarrow \beta_n^i, \beta_n^{i*} = a_n^i \pm i a_n^{i+1} \ , \quad i = 2,4,6,8 \ , \\ \chi^{A,B} = \psi^0 \pm \psi^1 &\rightarrow \chi_n^{A,B} = \psi_n^0 \pm \psi_n^1 \ , \\ \chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} &\rightarrow \chi_n^i, \chi_n^{i*} = \psi_n^i \pm i\psi_n^{i+1} \ , \quad i = 2,4,6,8 \ , \end{aligned}$$

with the commutation relations $[\alpha_m, \beta_{-n}] = -2\delta_{mn}$, $[\beta_m^i, \beta_{-n}^{i*}] = 2\delta_{mn}$, $\{\chi_m^A, \chi_{-n}^B\} = -2\delta_{mn}$ and $\{\chi_m^i, \chi_n^{i*}\} = 2\delta_{mn}$. In this way, any rotation or boost will reduce to a simple phase transformation on the modes.

2.1 Orbifold construction

Let us briefly recall the orbifold construction. An orbifold compactification can be obtained by identifying points in the compact part of space-time which are connected by discrete rotations $g = e^{2\pi i \sum_a z_a J_{aa+1}}$ on some of the compact pairs X^a,χ^a, a=4,6,8. In order to preserve at least one supersymmetry, one has to impose the condition $\sum_a z_a = 0$.

We will consider three case: toroidal compactification on T_6 and orbifold compactification on $T_2 \otimes T_4/Z_2$ and T_6/Z_3. The construction is universal, and these three cases can be obtained by explicit choices for the angles z_a:

$$\begin{aligned} T_6/Z_3 \ (N=2 \text{ SUSY}): &\ \text{take } z_4, z_6 = \tfrac{1}{3}, \tfrac{2}{3} \ , \quad z_8 = -z_4 - z_6 \ , \\ T_2 \otimes T_4/Z_2 \ (N=4 \text{ SUSY}): &\ \text{take } z_4 = -z_6 = \tfrac{1}{2} \ , \quad z_8 = 0 \ , \\ T_6 \ (N=8 \text{ SUSY}): &\ \text{take } z_4 = z_6 = z_8 = 0 \ . \end{aligned}$$

The spectrum of the theory now contains additional twisted sectors, in which periodicity is achieved only up to an element of the quotient group Z_N. One can diagonalize the fields such that they satisfy the periodicity condition ($g_a = e^{2\pi i z_a}$)

$$X^a(\sigma+1) = g_a X^a(\sigma) \ , \quad X^{*a}(\sigma+1) = g_a^* X^{*a}(\sigma) \ ,$$

and similarly for fermions. This leads to fractional moding in the compact directions.

These twisted states exist at fixed points of the orbifold. They thus occur only for the 0-brane of type IIA, which corresponds to Dirichlet b.c. in all the compact directions and can thus be localized at a fixed point.

Finally, in all sectors, one has to project onto invariant states to get the physical spectrum of the theory which is invariant under orbifold rotations. In particular, the physical boundary state is given by the projection

$$|B_{phys}> = \frac{1}{N}(|B,1> + |B,g> + ... + |B,g^{N-1}>)$$

in terms of the twisted boundary states $|B,g^k> = g^k|B>$.

2.2 0-brane: untwisted sector

Consider first the static case. The b.c. are Neumann for time and Dirichlet for all other directions (i=2,4,6,8 and a=2,4,6).

For the bosons, the b.c. translate into the following equations

$$(\alpha_n + \tilde{\beta}_{-n})|B>_B = 0\ ,\quad (\beta_n + \tilde{\alpha}_{-n})|B>_B = 0\ ,$$
$$(\beta^i_n - \tilde{\beta}^i_{-n})|B>_B = 0\ ,\quad (\beta^{i*}_n - \tilde{\beta}^{i*}_{-n})|B>_B = 0\ ,$$

The boundary state which solves them is given by a Bogolubov transformation

$$|B>_B = \exp\left\{\frac{1}{2}\sum_{n=1}^{\infty}(\alpha_{-n}\tilde{\alpha}_{-n} + \beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})\right\}|0>\ .$$

For the fermions, one has integer or half-integer moding in the RR and NSNS sectors respectively. The b.c lead to

$$(\chi^A_n + i\eta\tilde{\chi}^B_{-n})|B,\eta>_F = 0\ ,\quad (\chi^B_n + i\eta\tilde{\chi}^A_{-n})|B,\eta>_F = 0\ ,$$
$$(\chi^i_n - i\eta\tilde{\chi}^i_{-n})|B,\eta>_F = 0\ ,\quad (\chi^{i*}_n - i\eta\tilde{\chi}^{i*}_{-n})|B,\eta>_F = 0\ .$$

Here $\eta = \pm 1$ has been introduced to deal later on with the GSO projection.

The corresponding boundary state can be factorized into zero mode and oscillator parts:

$$|B,\eta>_F = |B_o>_F \otimes |B_{osc}>_F\ .$$

The oscillator part is the same for both sectors, with appropriate moding

$$|B_{osc},\eta>_F = \exp\left\{\frac{i\eta}{2}\sum_{n>0}(\chi^A_{-n}\tilde{\chi}^A_{-n} + \chi^B_{-n}\tilde{\chi}^B_{-n} - \chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})\right\}|0>\ .$$

The zero mode part exists only in the RR sector, and is slightly more subtle to construct.

Since they satisfy a Clifford algebra, the zero modes are proportional to Γ-matrices $\psi^\mu_o = i/\sqrt{2}\Gamma^\mu$, $\tilde{\psi}^\mu_o = i/\sqrt{2}\tilde{\Gamma}^\mu$. One can then construct the creation-annihilation operators $a, a^* = 1/2(\Gamma^0 \pm \Gamma^1)$, $b^i, b^{i*} = 1/2(-i\Gamma^i \pm \Gamma^{i+1})$ and similarly for tilded operators, satisfying the usual algebra $\{a, a^*\} = \{b^i, b^{i*}\} = 1$.

The b.c. for the zero modes can then be rewritten as

$$(a + i\eta\tilde{a}^*)|B_o,\eta>_F = 0\ ,\quad (a^* + i\eta\tilde{a})|B_o,\eta>_F = 0\ ,$$
$$(b^i - i\eta\tilde{b}^i)|B_o,\eta>_F = 0\ ,\quad (b^{i*} - i\eta\tilde{b}^{i*})|B_o,\eta>_F = 0\ ,$$

Defining the spinor vacuum $|0> \otimes|\tilde{0}>$ such that $a|0> = \tilde{a}|\tilde{0}> = b^i|0> = \tilde{b}^{i*}|\tilde{0}> = 0$ the zero mode part of the boundary state can then be written as

$$|B_o,\eta>_{RR} = \exp\left\{-i\eta(a^*\tilde{a}^* - b^{i*}\tilde{b}^i)\right\}|0> \otimes|\tilde{0}>\ .$$

The complete boundary state is already invariant under orbifold rotations, for which

$$\beta^a_n \to g_a\beta^a_n\ ,\quad \chi^a_n \to g_a\chi^a_n\ ,\quad b^a \to g_a b^a\ ,$$
$$\beta^{a*}_n \to g^*_a\beta^{a*}_n\ ,\quad \chi^{a*}_n \to g^*_a\chi^{a*}_n\ ,\quad b^{a*} \to g^*_a b^{a*}\ . \tag{4}$$

This comes from the fact that the Z_N action rotates pairs of fields with the same b.c. and is thus irrelevant.

For a boost of rapidity v, the transformations on the modes are

$$\begin{aligned}&\alpha_n \to e^{-v}\alpha_n\ ,\quad \chi^A_n \to e^{-v}\chi^A_n\ ,\quad a \to e^{-v}a\ ,\\ &\beta_n \to e^{v}\beta_n\ ,\quad \chi^B_n \to e^{v}\chi^B_n\ ,\quad a^* \to e^{v}a^*\ .\end{aligned} \tag{5}$$

The spinor vacuum is no longer invariant, but transforms as $|0>\otimes|\tilde{0}>\to e^{-v}|0>\otimes|\tilde{0}>$. Finally, the complete boosted boundary state is

$$\begin{aligned}&|B,V>_B = \exp\left\{\frac{1}{2}\sum_{n>0}(e^{-2v}\alpha_{-n}\tilde{\alpha}_{-n}+e^{2v}\beta_{-n}\tilde{\beta}_{-n}+\beta^i_{-n}\tilde{\beta}^{i*}_{-n}+\beta^{i*}_{-n}\tilde{\beta}^{i}_{-n})\right\}|0>\ ,\\ &|B_{osc},V,\eta>_F = \exp\left\{\frac{i\eta}{2}\sum_{n>0}(e^{-2v}\chi^A_{-n}\tilde{\chi}^A_{-n}+e^{2v}\chi^B_{-n}\tilde{\chi}^B_{-n}-\chi^i_{-n}\tilde{\chi}^{i*}_{-n}-\chi^{i*}_{-n}\tilde{\chi}^{i}_{-n})\right\}|0>\ ,\\ &|B_o,V,\eta>_{RR} = e^{-v}\exp\left\{-i\eta(e^{2v}a^*\tilde{a}^*-b^{i*}\tilde{b}^i)\right\}|0>\otimes|\tilde{0}>\ .\end{aligned} \tag{6}$$

In both sectors, the fermion number operator reverses the sign of the parameter η, that is $(-1)^F|B,V,\eta>=-|B,V,-\eta>$, and the GSO-projected boundary state is

$$|B,V>=\frac{1}{2}(|B,V,+>-|B,V,->)\ .$$

The partition function can then be computed carrying out some simple oscillator algebra; the ghosts cancel one untwisted pair, say 2-3, and the result is the product of the contributions of the 0-1 pair and the 3 compact pairs.

For the bosons, one finds ($q=e^{-2\pi l}$)

$$\begin{aligned}&<B,V_1|e^{-lH}|B,V_2>^{(0,1)}_B=\prod_{n=1}^{\infty}\frac{1}{(1-e^{-2v}q^{2n})(1-e^{2v}q^{2n})}\ ,\\ &<B,V_1|e^{-lH}|B,V_2>^{(a,a+1)}_B=\prod_{n=1}^{\infty}\frac{1}{(1-q^{2n})^2}\ .\end{aligned}$$

The total bosonic partition function is thus (zero-point energy $q^{-\frac{2}{3}}$)

$$Z_B=16\pi^3 i\sinh v q^{\frac{1}{3}}f(q^2)^4\frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)^3}\ . \tag{7}$$

For the fermions, the 0-1 pair gives

$$<B,V_1,\eta|e^{-lH}|B,V_2,\eta'>^{s(0,1)}_F=Z^s_o(\eta\eta')\prod_{n>0}(1+\eta\eta' e^{-2v}q^{2n})(1+\eta\eta' e^{2v}q^{2n})\ ,$$

with $\eta\eta'=\pm1$ and the zero mode contributions

$$Z^R_o(+)=2\cosh v\ ,\quad Z^R_o(-)=2\sinh v\ ,\quad Z^{NS}_o(\pm)=1\ .$$

Each compact pair gives instead

$$<B,V_1,\eta|e^{-lH}|B,V_2,\eta'>^{s(a,a+1)}_F=Z^s_o(\eta\eta')\prod_{n>0}(1+\eta\eta' q^{2n})^2\ ,$$

with

$$Z_o^R(+) = 2\ ,\quad Z_o^R(-) = 0\ ,\quad Z_o^{NS}(\pm) = 1\ .$$

After the GSO projection, only the three even spin structures R+ and NS± contribute, and (zero-point energy $q^{-\frac{1}{3}}$ for NSNS and $q^{\frac{2}{3}}$ for RR)

$$\begin{aligned} Z_F &= q^{-\frac{1}{3}} f(q^2)^{-4} \left\{ \vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)^3 - \vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)^3 + \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)^3 \right\} \\ &\sim V^4\ , \end{aligned} \tag{8}$$

corresponding to the usual cancellation of the force between two BPS states [7, 3]. Thus, the untwisted sector for the 0-brane gives the same result as the uncompactified theory for every compactification scheme.

2.3 0-brane: twisted sector

Consider now the twisted sector, which has to be included when the 0-brane is at an orbifold fixed point. In this case, the boundary state is similar to the one of the untwisted sector, with fractional moding in the compact directions.

In the Z_3 case, each pair of compact bosons gives

$$< B, V_1|e^{-lH}|B, V_2 >_B^{(a,a+1)} = \prod_{n=1}^{\infty} \frac{1}{(1-q^{2(n-\frac{1}{3})})(1-q^{2(n-\frac{2}{3})})}\ .$$

For a pair of compact fermions (no zero modes)

$$\begin{aligned} < B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_R^{s(a,a+1)} &= \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{3})})(1+\eta\eta' q^{2(n-\frac{2}{3})})\ , \\ < B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_{NS}^{s(a,a+1)} &= \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{6})})(1+\eta\eta' q^{2(n-\frac{5}{6})})\ . \end{aligned}$$

The total partition functions after the GSO projection are (the zero-point energies add to zero)

$$Z_B = 2i\sinh v f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1(-\frac{2}{3}il|2il)^3}\ , \tag{9}$$

$$\begin{aligned} Z_F &= f(q^2)^{-4} \left\{ \vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(-\frac{2}{3}il|2il)^3 - \vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(-\frac{2}{3}il|2il)^3 - \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(-\frac{2}{3}il|2il)^3 \right\} \\ &\sim V^2\ . \end{aligned} \tag{10}$$

In the Z_2 case, the analysis is similar and the results are

$$Z_B = 2i\sinh v q^{-\frac{1}{6}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1(0|2il)\vartheta_1(-il|2il)^2}\ , \tag{11}$$

$$\begin{aligned} Z_F &= q^{\frac{1}{6}} f(q^2)^{-4} \Big\{ \vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)\vartheta_2(-il|2il)^2 \\ &\qquad - \vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)\vartheta_3(-il|2il)^2 - \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)\vartheta_4(-il|2il)^2 \Big\} \\ &\sim V^2\ . \end{aligned} \tag{12}$$

2.4 3-brane

Let us now consider a particular 3-brane configuration. In the static case, we take Neumann b.c. for time, Dirichlet b.c. for space and mixed b.c. for each pair of compact directions, say Neumann for the a directions and Dirichlet for the a+1 directions.

The new b.c. for the compact directions are

$$
\begin{aligned}
&(\beta_n^a + \tilde{\beta}_{-n}^{a*})|B>_B = 0\ , \quad (\beta_n^{a*} + \tilde{\beta}_{-n}^{a})|B>_B = 0\ , \\
&(\chi_n^a + i\eta\tilde{\chi}_{-n}^{a*})|B_{osc},\eta>_F = 0\ , \quad (\chi_n^{a*} + i\eta\tilde{\chi}_{-n}^{a})|B_{osc},\eta>_F = 0\ , \\
&(b^a + i\eta\tilde{b}^{a*})|B_o,\eta>_F = 0\ , \quad (b^{a*} + i\eta\tilde{b}^{a})|B_o,\eta>_F = 0\ .
\end{aligned}
$$

Defining a new spinor vacuum $|0>\otimes|\tilde{0}>$ such that $b^a|0>=\tilde{b}^a|\tilde{0}>=0$ the compact part of the boundary state is

$$
\begin{aligned}
|B>_B &= \exp\left\{-\frac{1}{2}\sum_{n>0}(\beta_{-n}^a\tilde{\beta}_{-n}^a + \beta_{-n}^{a*}\tilde{\beta}_{-n}^{a*})\right\}|0>\ , \\
|B_{osc},\eta>_F &= \exp\left\{\frac{i\eta}{2}\sum_{n>0}(\chi_{-n}^a\tilde{\chi}_{-n}^a + \chi_{-n}^{a*}\tilde{\chi}_{-n}^{a*})\right\}|0>\ , \\
|B_o,\eta>_{RR} &= \exp\left\{-i\eta b^{a*}\tilde{b}^{a*}\right\}|0>\otimes|\tilde{0}>\ \ .
\end{aligned}
$$

In this case, the boundary state is not invariant under orbifold rotations, under which the modes of the fields transform as in eq. (4) and the spinor vacuum as $|0>\otimes|\tilde{0}>\rightarrow g_a|0>\otimes|\tilde{0}>$. This was expected since a Z_N rotation now mixes two directions with different b.c, and thus the corresponding closed string state does not need to be invariant under Z_N rotations.

The compact part of the twisted boundary state is finally found to be

$$
\begin{aligned}
|B,V,g_a>_B &= \exp\left\{-\frac{1}{2}\sum_{n>0}(g_a^2\beta_{-n}^a\tilde{\beta}_{-n}^a + g_a^{*2}\beta_{-n}^{a*}\tilde{\beta}_{-n}^{a*})\right\}|0>\ , \\
|B_{osc},V,g_a,\eta>_F &= \exp\left\{\frac{i\eta}{2}\sum_{n>0}(g_a^2\chi_{-n}^a\tilde{\chi}_{-n}^a + g_a^{*2}\chi_{-n}^{a*}\tilde{\chi}_{-n}^{a*})\right\}|0>\ , \\
|B_o,V,g_a,\eta>_{RR} &= g_a\exp\left\{-i\eta g_a^{*2}b^{a*}\tilde{b}^{a*}\right\}|0>\otimes|\tilde{0}>\ \ .
\end{aligned}
\tag{13}
$$

Each pair of compact bosons gives now a contribution to the partition function which depends on the orbifold relative angle ($(g_a^* g_a')^2 = e^{2\pi i w_a}$)

$$
< B,V_1,g_a|e^{-lH}|B,V_2,g_a'>_B^{(a,a+1)} = \prod_{n=1}^{\infty}\left|\frac{1}{1+\eta\eta' e^{2\pi i w_a}q^{2n}}\right|^2\ .
$$

For fermions one obtains

$$
< B,V_1,g_a,\eta|e^{-lH}|B,V_2,g_a',\eta'>_F^{s(a,a+1)} = Z_o^s(\eta\eta')\prod_{n>0}\left|1+\eta\eta' e^{2\pi i w_a}q^{2n}\right|^2\ ,
$$

where

$$
Z_o^R(+) = 2\cos\pi w_a\ ,\quad Z_o^R(-) = 2i\sin\pi w_a\ ,\quad Z_o^{NS}(\pm) = 1\ .
$$

After the GSO projection, the total partition functions for a given relative angle w_a are

$$
Z_B = 16i\sinh vq^{\frac{1}{3}}f(q^2)^4\frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)}\prod_a\frac{\sin\pi w_a}{\vartheta_1(w_a|2il)}\ , \tag{14}
$$

$$
\begin{aligned}
Z_F = q^{-\frac{1}{3}} f(q^2)^{-4} \Big\{ & \vartheta_2(i\frac{v}{\pi}|2il) \prod_a \vartheta_2(w_a|2il) \\
& -\vartheta_3(i\frac{v}{\pi}|2il) \prod_a \vartheta_3(w_a|2il) + \vartheta_4(i\frac{v}{\pi}|2il) \prod_a \vartheta_4(w_a|2il) \Big\} \\
\sim & \left\{ \begin{array}{ll} V^4 \ , & w_a = 0 \\ V^2 \ , & w_a \neq 0 \end{array} \right. .
\end{aligned}
\tag{15}
$$

Recall that to obtain the invariant amplitude, one has to average over all possible angles w_a.

Finally, for this 3-brane configuration there is no twisted sector, as already explained.

2.5 Large distance limit and field theory interpretation

In the large distance limit $l \to \infty$, explicit results with their exact dependence on the rapidity can be obtained and compared to a field theory computation. The behaviors that one finds are the following:

0-brane

a) Untwisted sector

$$\mathcal{A} \sim 4\cosh v - \cosh 2v - 3 \sim V^4 \ . \tag{16}$$

b) Twisted sector

$$\mathcal{A} \sim \cosh v - 1 \sim V^2 \ . \tag{17}$$

3-brane

$$
\begin{aligned}
\mathcal{A}(w_a) &\sim 4 \prod_a \cos \pi w_a \cosh v - \cosh 2v - \sum_a \cos 2\pi w_a \ , \\
\mathcal{A} &\sim \left\{ \begin{array}{ll} \cosh v - \cosh 2v \sim V^2 \ , & T_6/Z_3 \\ 4\cosh v - \cosh 2v - 3 \sim V^4 \ , & T_2 \otimes T_4/Z_2 \ , \ T_6 \end{array} \right. .
\end{aligned}
\tag{18}
$$

In the low energy effective supergravity field theories, the possible contributions to the scattering amplitude in the eikonal approximation come from vector exchange in the RR sector and dilaton and graviton exchange in the NSNS sector. The respective contributions have a peculiar dependence on the rapidity reflecting the tensorial nature and are:

$$\mathcal{A}^{NS}_{\phi} \sim -a^2 \ , \ \ \mathcal{A}^{R}_{V_\mu} \sim e^2 \cosh v \ , \ \ \mathcal{A}^{NS}_{g_{\mu\nu}} \sim -M^2 \cosh 2v \ . \tag{19}$$

Thus, the interpretation of the behaviors found in the various sectors and for the various brane configurations we have considered, is the following:

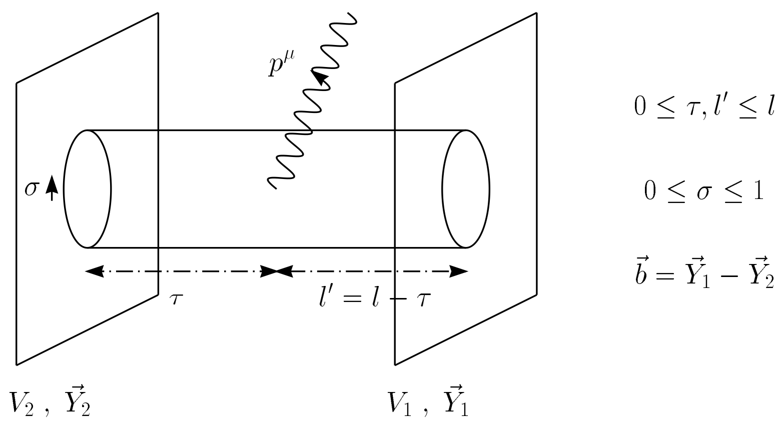
$$
\begin{aligned}
4\cosh v - \cosh 2v - 3 \quad &\Leftrightarrow \quad N = 8 \text{ Grav. multiplet} \ , \\
\cosh v - \cosh 2v \quad &\Leftrightarrow \quad N = 2 \text{ Grav. multiplet} \ , \\
\cosh v - 1 \quad &\Leftrightarrow \quad \text{Vec. multiplet} \ .
\end{aligned}
$$

The patterns of cancellation suggest that all the D-brane configurations that we have considered correspond to extremal p-brane solutions of the low energy supergravity, possibly coupling to the additional twisted vector multiplets; the 3-brane configuration on the Z_3 orbifold seems to be an exception since it does not couple to the scalars, and should thus correspond to a Reissner-Nordström extremal black hole.

Finally, notice that V^2 terms in the effective action give a non flat metric to the moduli space. Since in the dual open string channel a constant velocity V corresponds by T-duality to a constant electric field E, V^2 terms correspond to a renormalization of the Maxwell term E^2. It is well known that this can not happen for maximally supersymmetric theories; the V^2 behavior is thus forbidden for $N = 8$ compactifications, but generically allowed for compactifications breaking some supersymmetry, $N < 8$. Our results are compatible with this and show that V^2 terms do indeed appear in some cases.

3 Emission of massless NSNS bosons

Consider two moving D-branes in interaction emitting a massless NSNS boson.



The amplitude is computed inserting the usual vertex operator ($z = \sigma + i\tau$)

$$V(z,\bar{z}) = G_{ij}(\partial X^i - \frac{1}{2}p\cdot\psi\psi^i)(\bar{\partial}X^j + \frac{1}{2}p\cdot\bar{\psi}\bar{\psi}^j)e^{ip\cdot X}$$

between the two boundary states

$$\begin{aligned}\mathcal{A} &= \int_0^\infty dl \int_0^l d\tau \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} V(z,\bar{z}) | B, V_2, \vec{Y}_2 >_s \\ &= \int_0^\infty d\tau \int_0^\infty dl' \sum_s < V(z,\bar{z}) >_s \quad .\end{aligned}$$

We have chosen a purely space-like polarization tensor as allowed by gauge invariance.

As before, we split the bosons into zero mode and oscillators to be treated separately (again $X^\mu \equiv X^\mu_{osc}$). As usual, the zero mode part ensures momentum conservation ($p^\mu = k^\mu_B - q^\mu_B$) and gives the kinematics. The energies and longitudinal momenta are completely fixed by the momentum of the outgoing particle ($\cos\theta = p^1/p$, $p = p^0$),

$$\begin{aligned}k^0_B &= V_1 k^1_B\ ,\ \ k^1_B = \frac{p}{V_1 - V_2}(1 - V_2\cos\theta)\ , \\ q^0_B &= V_2 q^1_B\ ,\ \ q^1_B = \frac{p}{V_1 - V_2}(1 - V_1\cos\theta)\ .\end{aligned}$$

The zero mode contribution is found to have the simple structure ($v = v_1 - v_2$)

$$< e^{ip\cdot X} >_o = \frac{1}{\sinh v}\int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'}\ .$$

Further zero mode insertions give just additional momentum factors

$$\partial X^i_o \ \Rightarrow \ -\frac{1}{2}k^i_B \ , \ \ \bar{\partial} X^j_o \ \Rightarrow \ \frac{1}{2}k^j_B \ , \ \ \partial X^i_o \bar{\partial} X^j_o \ \Rightarrow \ -\frac{1}{4}k^i_B k^j_B \ .$$

Finally, the amplitude can be rewritten (from now on $q^\mu \equiv q^\mu_B$ and $k^\mu \equiv k^\mu_B$) as

$$\mathcal{A} = \frac{1}{\sinh v}\int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} > \sum_s Z_B Z_F^s \mathcal{M}_s \ , \tag{20}$$

with

$$\begin{aligned}\mathcal{M}^s = G_{ij} \Big\{ & < \partial X^i \bar{\partial} X^j > - < \partial X^i p\cdot X >< \bar{\partial} X^j p\cdot X > \\ & + \frac{1}{4}\Big(< p\cdot\psi p\cdot\bar{\psi} >_s < \psi^i \bar{\psi}^j >_s - < p\cdot\psi\psi^i >_s < p\cdot\bar{\psi}\bar{\psi}^j >_s \\ & \qquad + < p\cdot\bar{\psi}\psi^i >_s < p\cdot\psi\bar{\psi}^j >_s \Big) \\ & + \frac{i}{2}\Big(< \partial X^i p\cdot X >< p\cdot\bar{\psi}\bar{\psi}^j >_s - < \bar{\partial} X^j p\cdot X >< p\cdot\psi\psi^i >_s \Big) \\ & - \frac{1}{2}k^i \Big(i < \bar{\partial} X^j p\cdot X > + \frac{1}{2} < p\cdot\bar{\psi}\bar{\psi}^j >_s \Big) + \frac{1}{2}k^j \Big(i < \partial X^i p\cdot X > - \frac{1}{2} < p\cdot\psi\psi^i >_s \Big) \\ & - \frac{1}{4}k^i k^j \Big\} \ . \end{aligned} \tag{21}$$

Obviously, the partition function factorizes, leaving connected correlators. In the odd spin structure, appropriate zero mode insertion is understood in order for these expressions to make sense.

3.1 Correlators

The boundary state formalism provides a systematic way of computing correlators with non trivial b.c., such as those needed here, through the definitions

$$< X^\mu X^\nu >= \frac{< B_1, V_1 | e^{-lH} X^\mu X^\nu | B_2, V_2 >_B}{< B_1, V_1 | e^{-lH} | B_2, V_2 >_B} \ , \tag{22}$$

$$< \psi^\mu \psi^\nu >_s = \frac{< B_1, V_1, \eta | e^{-lH} \psi^\mu \psi^\nu | B_2, V_2, \eta' >^s_F}{< B_1, V_1, \eta | e^{-lH} | B_2, V_2, \eta' >^s_F} \ . \tag{23}$$

For the bosons, one obtains an infinite series of logarithms corresponding to the propagation of all the string states with growing mass ($q = e^{-2\pi\tau}$):

$$\begin{aligned} & < X^0(z)\bar{X}^0(\bar{z}) >=< X^1(z)\bar{X}^1(\bar{z}) >= \\ & \quad = \frac{1}{4\pi}\sum_{n=0}^\infty \Big\{ \cosh 2[(v_1 - v_2)n - v_2] \ln(1 - q^{2n} e^{-4\pi\tau}) \\ & \qquad\qquad - \cosh 2[(v_2 - v_1)n - v_1] \ln(1 - q^{2n} e^{-4\pi l'}) \Big\} \ , \\ & < X^0(z)\bar{X}^1(\bar{z}) >=< X^1(z)\bar{X}^0(\bar{z}) >= \\ & \quad = -\frac{1}{4\pi}\sum_{n=0}^\infty \Big\{ \sinh 2[(v_1 - v_2)n - v_2] \ln(1 - q^{2n} e^{-4\pi\tau}) \\ & \qquad\qquad + \sinh 2[(v_2 - v_1)n - v_1] \ln(1 - q^{2n} e^{-4\pi l'}) \Big\} \ . \end{aligned}$$

For the fermions in the NS$\pm$ sectors, one has poles instead of logarithms, with a similar structure

$$\begin{aligned}
&< \psi^0(z)\bar{\psi}^0(\bar{z}) >_{NS\pm} = < \psi^1(z)\bar{\psi}^1(\bar{z}) >_{NS\pm} = \\
&= -i\sum_{n=0}^{\infty}(\mp)^n \left\{\cosh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\
&\qquad\qquad \left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} , \\
&< \psi^0(z)\bar{\psi}^1(\bar{z}) >_{NS\pm} = < \psi^1(z)\bar{\psi}^0(\bar{z}) >_{NS\pm} = \\
&= i\sum_{n=0}^{\infty}(\mp)^n \left\{\sinh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\
&\qquad\qquad \left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} ,
\end{aligned}$$

For the fermions in the R$\pm$ sectors, the results are similar,

$$\begin{aligned}
&< \psi^0(z)\bar{\psi}^0(\bar{z}) >_{R\pm} = < \psi^1(z)\bar{\psi}^1(\bar{z}) >_{R\pm} = \\
&= F_o^R(\pm) - i\sum_{n=0}^{\infty}(\mp)^n \left\{\cosh 2[(v_1-v_2)n - v_2]\frac{q^{2n} e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\
&\qquad\qquad \left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^{2n} e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} , \\
&< \psi^0(z)\bar{\psi}^1(\bar{z}) >_{R\pm} = < \psi^1(z)\bar{\psi}^0(\bar{z}) >_{R\pm} = \\
&= G_o^R(\pm) + i\sum_{n=0}^{\infty}(\mp)^n \left\{\sinh 2[(v_1-v_2)n - v_2]\frac{q^{2n} e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\
&\qquad\qquad \left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^{2n} e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} ,
\end{aligned}$$

with additional zero mode contributions,

$$\begin{aligned}
F_o^R(+) &= -\frac{i}{2}\frac{\cosh(v_1+v_2)}{\cosh(v_1-v_2)} , \quad F_o^R(-) = -\frac{i}{2}\frac{\sinh(v_1+v_2)}{\sinh(v_1-v_2)} , \\
G_o^R(+) &= -\frac{i}{2}\frac{\sinh(v_1+v_2)}{\cosh(v_1-v_2)} , \quad G_o^R(-) = -\frac{i}{2}\frac{\cosh(v_1+v_2)}{\sinh(v_1-v_2)} .
\end{aligned}$$

As usual, world-sheet supersymmetry means (here for osc.) a relation between the odd fermions and the derivative of the bosons

$$< \partial X^\mu(z)\bar{X}^\nu(\bar{z}) > = \frac{1}{2} < \psi^\mu(z)\bar{\psi}^\nu(\bar{z}) >_{R-} \quad . \tag{24}$$

There are also non vanishing equal-point correlators, which can be computed in the same way. They can also be deduced from the previous ones using the b.c. to reflect left and right movers at the boundaries.

The correlators can be actually expressed in terms of twisted ϑ-functions. To understand this, consider the rescaled combinations of fermions $\psi^\pm = e^{\mp v_2}(\psi^0 \pm \psi^1)$, satisfying usual b.c. on one brane and twisted b.c. on the other brane

$$\begin{aligned}
\psi^\pm(z) &= -i\bar{\psi}^\mp(\bar{z}) \ , \quad \tau = 0 \ \Leftrightarrow \ z = \bar{z} \ , \\
\psi^\pm(z) &= -ie^{\pm 2v}\bar{\psi}^\mp(\bar{z}) \ , \quad \tau = l \ \Leftrightarrow \ z = \bar{z} + 2il \ .
\end{aligned}$$

The propagators $P^s_{(\pm)}(z-\bar{z}) =< \psi^\pm(z)\bar{\psi}^\pm(\bar{z}) >_s$ should accordingly have appropriate periodicity conditions on the covering torus with modulus 2il from which the cylinder can be obtained by the involution $z \doteq \bar{z} + 2il$.

In fact, under the shift $w \to w + m + 2iln$ on the covering torus, the propagators transform as

$$
\begin{aligned}
P^{R+}_{(\pm)}(w + m + 2iln) &= e^{i\pi n} e^{\pm 2nv} P^{R+}_{(\pm)}(w) \ , \\
P^{R-}_{(\pm)}(w + m + 2iln) &= e^{\pm 2nv} P^{R-}_{(\pm)}(w) \ , \\
P^{NS+}_{(\pm)}(w + m + 2iln) &= e^{i\pi m} e^{i\pi n} e^{\pm 2nv} P^{NS+}_{(\pm)}(w) \ , \\
P^{NS-}_{(\pm)}(w + m + 2iln) &= e^{i\pi m} e^{\pm 2nv} P^{NS-}_{(\pm)}(w) \ ,
\end{aligned}
$$

These properties, together with the universal local behavior $P^s_{(\pm)}(w) \to 1/(4\pi w)$, imply that for the even spin structures

$$
P^s_{(\pm)}(w) = \frac{1}{4\pi} \frac{\vartheta_s(w \pm i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)}{\vartheta_s(\pm i\frac{v}{\pi}|2il)\vartheta_1(w|2il)} \ . \tag{25}
$$

3.2 Axion

For the axion, the polarization tensor is transverse and antisymmetric, and can be taken to be $G_{ij} = 1/2\epsilon_{ijk}p^k/p$. Only the odd spin structure can contribute in this case because of the antisymmetry of G_{ij}. Notice that in the twisted sector of the Z_3 orbifold, there are only two fermionic zero modes in the 2-3 pair, and the amplitude could be non vanishing since there is the possibility of soaking up these two zero modes with the vertex operator.

After integrating by parts the two-derivative bosonic term appearing in the contraction (21), and using world-sheet supersymmetry (24), the result simplifies to

$$
\mathcal{M}^{R-}_{ax} = \frac{i}{8}\cos\theta \left[-\partial_\tau < p\cdot X(z) p\cdot \bar{X}(\bar{z}) > + \frac{1}{2}(k^2 - q^2) \right] \ . \tag{26}
$$

However, since $\partial_\tau|_l = \partial_\tau|_{l'} - \partial_{l'}|_\tau$ the final amplitude is a total derivative ($Z_B Z_F^{R-} = 2\sinh v$ for the twisted sector of Z_3) and vanishes even in the twisted sector of the Z_3 orbifold

$$
\begin{aligned}
\mathcal{A}_{ax} &= \frac{i}{4}\cos\theta \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} (\partial_\tau - \partial_{l'}) \left\{ e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} > \right\} \\
&= 0 \ .
\end{aligned} \tag{27}
$$

3.3 Dilaton

For the dilaton, the polarization tensor is $G_{ij} = \delta_{ij} - p^i p^j/p^2$. Only the even spin structures will now contribute, because of the symmetry of G_{ij}. Again, the two-derivative bosonic term in the contraction is conveniently integrated by parts.

In this case, we shall analyze the large distance limit, in which it is enough to keep only leading terms for $l \to \infty$ in the propagators. In this limit, the bosonic exponential reduces to

$$
< e^{ip\cdot X} >= \left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} \ , \tag{28}
$$

with the boosted energies $p^{(1,2)} = p\gamma_{1,2}(1 - V_{1,2}\cos\theta) = p(\cosh v_{1,2} - \sinh v_{1,2}\cos\theta)$.

After some complicated algebra, one finds for the contractions (keeping a subleading term in the NS± sectors because of a possible enhancement coming from the partition function)

$$\mathcal{M}_{dil}^{R+} = \frac{1}{4p^2}\left[(k^2-q^2) - 2p^2\cos\theta\tanh v\right]\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right) , \tag{29}$$

$$\mathcal{M}_{dil}^{NS\pm} = \frac{1}{4p^2}\left[(k^2-q^2) \mp 8e^{-2\pi l}p^2\cos\theta\sinh v\right]\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right) . \tag{30}$$

Using eq. (28) for $< e^{ip\cdot X} >$ and integrating by parts in the final amplitude, one finds the following rules for the τ and l' poles in the contraction:

$$\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \doteq -\frac{1}{4}\frac{q^2}{p^{(2)2}} , \quad \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \doteq -\frac{1}{4}\frac{k^2}{p^{(1)2}} . \tag{31}$$

These imply $\mathcal{M}_{dil}^{R+} = \mathcal{M}_{dil}^{NS\pm} = 0$, and thus at large distances

$$\mathcal{A}_{dil} = 0 . \tag{32}$$

3.4 Graviton

For the graviton, the polarization tensor is taken to be symmetric transverse and traceless, $G_{ij} = h_{ij} = h_{ji}$, $p^i h_{ij} = h^i_i = 0$, and has two independent components. In this case, things are more complicated, but one can proceed essentially as for the dilaton. One obtains for $l \to \infty$

$$\mathcal{M}_{grav}^{R+} = -\frac{1}{4}\left[h_{ij}k^ik^j - p\tanh v h_{i1}k^i\right]$$
$$-V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}$$
$$+V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} , \tag{33}$$

$$\mathcal{M}_{grav}^{NS\pm} = -\frac{1}{4}\left[h_{ij}k^ik^j \mp 4e^{-2\pi l}\left(p\sinh 2v h_{i1}k^i - p^2\sinh^2 v h_{11}\right)\right]$$
$$-V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}$$
$$+V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} . \tag{34}$$

One can use the same equivalence relations (31) as before to write $\mathcal{M}_{grav}^s$ in a τ, l'-independent form; but in any case, $\mathcal{A}_{grav} \neq 0$. The general structure of the amplitude is

$$\mathcal{A}_{grav} = \frac{1}{\sinh v}\int \frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}I_1 I_2 \sum_s Z_B Z_F^s \mathcal{M}_{grav}^s , \tag{35}$$

and involves three independent functions of the momenta

$$\mathcal{M}_{grav}^s = B^s(p,k,q) + q^2 C_1^s(p,k,q) + k^2 C_2^s(p,k,q) .$$

The kinematical integrals over the two proper times τ, l' can be easily evaluated, finding the usual dual structure with a double serie of poles

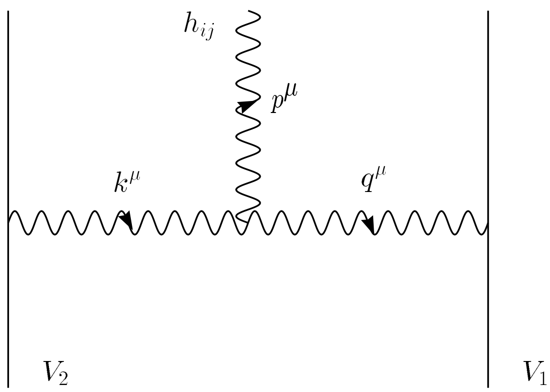
$$I_1 = \int_0^\infty d\tau e^{-\frac{q^2}{2}\tau} \left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{q^2}{8\pi}]\Gamma[-\frac{p^{(2)2}}{2\pi}+1]}{\Gamma[\frac{q^2}{8\pi} - \frac{p^{(2)2}}{2\pi}+1]} \underset{p\to 0}{\longrightarrow} -\frac{2}{q^2} \ ,$$
$$I_2 = \int_0^\infty dl' e^{-\frac{k^2}{2}l'} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{k^2}{8\pi}]\Gamma[-\frac{p^{(1)2}}{2\pi}+1]}{\Gamma[\frac{k^2}{8\pi} - \frac{p^{(1)2}}{2\pi}+1]} \underset{p\to 0}{\longrightarrow} -\frac{2}{k^2} \ .$$

The last limit is required by the eikonal approximation ($p \ll M = 1$) and selects the massless part of the states emitted by the branes.

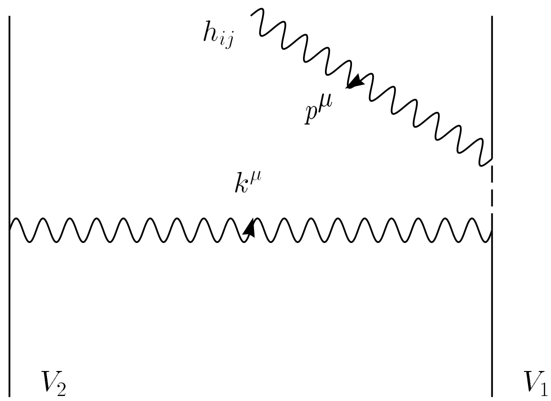
Finally, the amplitude assumes a simple field theory form

$$\mathcal{A}_{grav} = \frac{4}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \left\{ B^s \frac{1}{q^2k^2} + C_1^s \frac{1}{k^2} + C_2^s \frac{1}{q^2} \right\} \ . \tag{36}$$

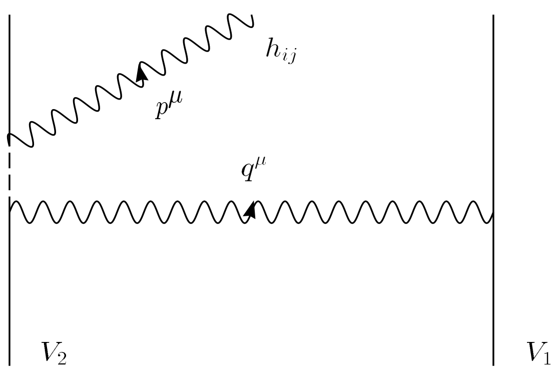
The graphical interpretation of the three contributions B^s, C_1^s and C_2^s is the following



$$\Rightarrow \quad B^s \frac{1}{q^2k^2} \quad (\tau, l' \neq 0)$$



$$\Rightarrow \quad C_1^s \frac{1}{k^2} \quad (\tau = l, l' = 0)$$



$$\Rightarrow \quad C_2^s \frac{1}{q^2} \quad (\tau = 0, l' = l)$$

The B^s factor corresponds to an annihilation process occurring far away from both branes, whereas the C_1^s and C_2^s factors correspond to absorption-emission bremsstrahlung-like processes occurring on the first and the second brane respectively.

3.5 Large distances

It is interesting to compare the string theory results to a field theory computation in the limit of large impact parameters $\vec{b}$.

For the axion and the dilaton, there is no coupling in supergravity allowing the emission process, and therefore the vanishing of the string amplitude is understood. For the annihilation term of the graviton, there are three possible diagrams in supergravity, involving the exchange of vectors, dilatons and gravitons. Their contributions in the eikonal approximation are

$$\begin{aligned} &B_\phi^{NS} \sim -a^2 h_{ij} k^i k^j \ , \\ &B_{V_\mu}^{R} \sim e^2 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right] \ , \\ &B_{g_{\mu\nu}}^{NS} \sim -M^2 \left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right] \ . \end{aligned} \tag{37}$$

The annihilation part of the string amplitude in the various compactification schemes is instead the following:
0-brane: untwisted sector & 3-brane on $T_2 \otimes T_4/Z_2, T_6$
One finds an exponential enhancement from the partition functions in the NSNS sector,

$$Z^{R+} - Z^{NS+} + Z^{NS-} \to 16 \cosh v - 4 \cosh 2v - 12 \ , \quad Z^{NS+} + Z^{NS-} \to 2e^{2\pi l} \ ,$$

and in the final result we recognize a cancellation of the leading order between the RR vector and the NSNS dilaton and graviton exchange:

$$\begin{aligned} &B_{grav}^{R} = 4 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right] \ , \\ &B_{grav}^{NS} = -\left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right] \\ &\qquad\quad -3 h_{ij} k^i k^j \ , \\ &\Rightarrow B_{grav} \sim V^4 h_{ij} k^i k^j + V^3 p h_{i1} k^i + V^2 p^2 h_{11} \ . \end{aligned} \tag{38}$$

0-brane: twisted sector
In this case, there is no enhancement in the NSNS sector,

$$Z^{R+} - Z^{NS+} - Z^{NS-} \to 4 \cosh v - 4 \ , \quad Z^{NS+} - Z^{NS-} \to 0 \ ,$$

and the cancellation of the leading order occurs between the RR vector and NSNS dilaton exchange:

$$\begin{aligned} &B_{grav}^{R} = \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right] \ , \\ &B_{grav}^{NS} = -h_{ij} k^i k^j \ , \\ &\Rightarrow B_{grav} \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11} \ . \end{aligned} \tag{39}$$

3-brane on T_6/Z_3
In this case there is again an enhancement in the NSNS sector,

$$Z^{R+} - Z^{NS+} + Z^{NS-} \to 4 \cosh v - 4 \cosh 2v \ , \quad Z^{NS+} + Z^{NS-} \to 2e^{2\pi l} \ ,$$

and the cancellation is between the RR vector and the NSNS graviton exchange:

$$B^{R}_{grav} = \left[\cosh v h_{ij}k^i k^j - p \sinh v h_{i1}k^i\right] \ ,$$
$$B^{NS}_{grav} = -\left[\cosh 2v h_{ij}k^i k^j - 2p \sinh 2v h_{i1}k^i + 2p^2 \sinh^2 v h_{11}\right] \ ,$$
$$\Rightarrow B_{grav} \sim V^2 h_{ij}k^i k^j + V p h_{i1}k^i + V^2 p^2 h_{11} \ . \tag{40}$$

The patterns of cancellation in the various cases confirm the interpretation in terms of low energy supermultiplets coming from the computation of the potential.

Collinear emission

In the case of collinear emission, that is for $\theta = 0$, the results simplify a lot and one finds

$$B_{grav} \sim V^n h_{ij}k^i k^j \ , \ \ C_{1grav} = C_{2grav} = 0 \ , \tag{41}$$

with $n = 2, 4$ depending on the amount of supersymmetry left over. Also, in this case the contributions C_1 and C_2 from bremsstrahlung-like processes vanish identically.

3.6 Radiated energy

To conclude, let us compute the average energy radiated when two D-branes pass each other at impact parameter $\vec{b}$. This is given by

$$< p > \sim \int \frac{d^3\vec{p}}{p} p \left|\mathcal{A}\right|^2 \ .$$

For $\theta = 0$ and $V \ll 1$ one obtains

$$\mathcal{A} \sim V^{n-1} g_s l_s f(\frac{p \cdot b}{V}) e^{-\frac{p \cdot b}{V}} \ ,$$

where f is a slowly varying function and $n = 2, 4$. Notice that the emission is exponentially suppressed for $p \sim p_{max} = V/b$. By dimensional analysis one finds finally

$$< p > \sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3} \ . \tag{42}$$

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