Metastable supergravity vacua with F and D supersymmetry breaking

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Abstract

We study the conditions under which a generic supergravity model involving chiral and vector multiplets can admit viable metastable vacua with spontaneously broken supersymmetry and realistic cosmological constant. To do so, we impose that on the vacuum the scalar potential and all its first derivatives vanish, and derive a necessary condition for the matrix of its second derivatives to be positive definite. We study then the constraints set by the combination of the flatness condition needed for the tuning of the cosmological constant and the stability condition that is necessary to avoid unstable modes. We find that the existence of such a viable vacuum implies a condition involving the curvature tensor for the scalar geometry and the charge and mass matrices for the vector fields. Moreover, for given curvature, charges and masses satisfying this constraint, the vector of F and D auxiliary fields defining the Goldstino direction is constrained to lie within a certain domain. The effect of vector multiplets relative to chiral multiplets is maximal when the masses of the vector fields are comparable to the gravitino mass. When the masses are instead much larger or much smaller than the gravitino mass, the effect becomes small and translates into a correction to the effective curvature. We finally apply our results to some simple classes of examples, to illustrate their relevance.

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1   Introduction

Recently substantial progress has been achieved in understanding how spontaneous super-symmetry breaking could be realized in a phenomenologically and cosmologically viable way in string inspired supergravity models. On one hand, from the microscopic string point of view, the structure of the Kähler potential is well understood at leading order in the weak coupling and low energy expansions [1, 2], and also the effects leading to non-trivial superpotentials are now believed to be reasonably well understood [3, 4, 5], although the identification of viable models still remains an open problem. Actually, it has been suggested that perhaps one should not look for a unique candidate that would be singled out for some reason, but rather for a statistical distribution in the landscape of possible vacua [6]. On the other hand, from the macroscopic supergravity point of view, much progress has been made in understanding the structure that the Kähler potential and the superpotential need to have in order for the theory to admit a phenomenologically acceptable vacuum. It has also been argued that the tuning of the cosmological constant could be realized in a very economical and transparent way in a theory including two sectors, one of them admitting an AdS supersymmetric vacuum and the other one breaking supersymmetry and adding a positive contribution to the potential [7].

Spontaneous supersymmetry breaking can be triggered both by the F auxiliary fields of chiral multiplets and the D auxiliary fields of vector multiplets. It is however believed that the qualitative seed for the breaking must come from chiral multiplets, and that vector multiplets may only affect the quantitative aspects of it. The reason for this is that in standard situations the values of the D's turn out to be proportional to the values of the F's at any stationary point of the superpotential. This is true both in rigid [8] and local supersymmetry [9], and relies essentially on the holomorphicity of the superpotential. The only situation where this relation can be possibly avoided is in the presence of a genuine field-independent Fayet-Iliopoulos term. However, although in global supersymmetry this is a natural possibility, in local supersymmetry it is highly constrained due to the fact that the usual Fayet-Iliopoulos term is not invariant in the presence of gravity [10]. In fact, it turns out that even in the presence of a constant Fayet-Iliopoulos term associated to a gauged R symmetry, the D's are still proportional to the F's, as long as the superpotential does not vanish [11], as is required in order to achieve a finite supersymmetry breaking scale with vanishing cosmological constant. Nevertheless, despite this relation between the F and D auxiliary fields, there are scenarios in which the D-terms may play a crucial rôle besides the F-terms in producing a viable vacuum.

The situation in models where supersymmetry breaking is dominated by chiral multiplets is by now well understood. It is known that the form of the Kähler potential imposes crucial restrictions on the possibility of getting a viable vacuum, that is, a metastable stationary point with a very tiny cosmological constant. It was for example argued in several ways that just the dilaton modulus could not lead to a viable situation [12], unless subleading corrections to its Kähler potential become large [13, 14], and therefore the perturbative control over the theory is compromised. Similarly, it was realized that the compactification volume modulus could instead dominate the supersymmetry breaking,

but only if subleading corrections to its Kähler potential are taken into account [15, 16]. In two recent papers [17, 18], we have addressed in a more systematic way the general question of whether it is possible to translate more directly onto the parameters of a given theory the condition of flatness, related to the cancellation of the cosmological constant, and the condition of stability, related to the stabilization of all the fluctuation modes. We found that, in the simplest case of supergravity models involving only chiral multiplets, it is possible to answer to this question in a remarkably sharp and simple way. It turns out that there exists a strong necessary condition on the value that the curvature of the Kähler geometry is allowed to take. Moreover, the Goldstino vector of F auxiliary fields gets constrained not only on its length, but also on its direction. More precisely, the form of the Riemann tensor must present valleys where the curvature takes a value that is below a certain threshold, and the Goldstino direction cannot be too far away from those directions, how far this can be being fixed by the value of the curvature scalars.

The results of [17, 18] provide a criterium that allows to discriminate to some extent between promising and non-promising models by knowing the approximate form of their Kähler potential only, independently of the form of the superpotential. Indeed, they were shown to have very useful and general implications on the possibility of realizing in a viable way supersymmetry breaking when it is dominated by the F auxiliary fields. These implications are particularly striking for string models, where the moduli sector is identified with the hidden sector. The main reason for this is that in these models the Kähler geometry has constant curvature at leading order in the coupling and derivative expansions, so that the constraints translate very directly into restrictions on the parameters of the Lagrangian. For instance, the lower bound found for the curvature was used to explain in a more robust way why the dilaton cannot dominate supersymmetry breaking, whereas the volume modulus can dominate it but in a way that is very sensitive to small subleading corrections to the Kähler potential. A similar strategy has also been used to explore the statistics of supersymmetry breaking vacua in certain classes of string models [19]. Finally, there have also been studies adopting a complementary viewpoint and developing more efficient tools to study algebraically the vacua allowed by a given theory with fixed Kähler potential and superpotential [20].

The situation in models where supersymmetry breaking is significantly affected not only by chiral multiplets, but also by vector multiplets, is more complex and less understood. The fact that the D-term contribution to the scalar potential is positive definite, (unlike the F-term contribution, which has an indefinite sign), suggests that they may play an important rôle in the stabilization of the scalar fields triggering spontaneous supersymmetry breaking. Note that this distinction is essentially implied by Lorentz invariance, which allows scalar fields to have a generic potential leading to non-vanishing VEVs, but requires vector fields to have a restricted potential leading to vanishing VEVs. As a result, the squared masses of all the charged fields tend to be increased by the gauging. Actually there are two types of effects. The first is that Goldstone fields corresponding to global symmetries of the ungauged theory get absorbed by gauge fields through the Higgs mechanism when the coupling is switched on, and thereby obtain a positive physical squared mass. The second is that the non-Goldstone fields also receive a positive correction to their

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squared masses due to the gauging, which can therefore help in stabilizing them. These possibilities have been recently explored for instance in [8, 21, 22, 23, 24, 25, 26, 27], but a more systematic and quantitative understanding of the features of this more general situation is still missing. The aim of this work is to try to fill this gap by performing a general study from which one could extract information that can be directly used in practice to build viable models.

The main goal of this paper is thus to generalize the study performed in [17, 18] to the more general class of supergravity theories involving not only chiral multiplets but also vector multiplets that gauge isometries of the Kähler geometry. This can actually be done by following the same strategy as in [17, 18]. Conceptually, the main novelties lie in the fact that besides the Kähler potential and the superpotential, one must also specify the Killing vectors defining the isometries that are gauged and the gauge kinetic function defining the couplings. Technically the task is substantially more complicated due to the fact that the Goldstino vector now involves not only F auxiliary fields but also D auxiliary fields, and also that the scalar potential has a more complicated structure. Nevertheless, it is possible to find the exact generalization of the flatness and stability conditions of [17], but their implications can be worked out in detail only in certain specific regimes, due to the increased algebraic complication of the problem. In general, the effect of vector multiplets alleviates the necessary conditions for flatness and stability, compared to the case involving only chiral multiplets. However, the associated gauge symmetries also restrict the variety of models, as the superpotential should be gauge invariant up to Kähler transformations. We will actually show that in a variety of situations, the net qualitative effect of vector multiplets is to reduce the effective curvature felt by the chiral multiplets. In certain situations, this can be more sharply interpreted as coming from a correction to the effective Kähler potential induced by the presence of vector multiplets. As already mentioned, corrections to the Kähler potential can be crucial for the existence of a viable vacuum in certain situations. In this respect, there is a very interesting distinction between corrections induced by extra chiral multiplets and those induced by vector multiplets: the former can either increase or decrease the curvature, since they have an indefinite sign, whereas the latter (as we will show in the body of the paper) always decrease it.

A final comment is in order regarding the issue of implementing the idea of getting a metastable Minkowski minimum through an uplifting sector that breaks supersymmetry in a soft way. It is clear that such a sector will have to contain some light degrees of freedom, providing also some non-vanishing F and/or D auxiliary field. Models realizing an F-term uplifting are easy to construct. A basic precursor of such models was first constructed in [28] and then further exploited, for instance, in [29]. More recently, a variety of other examples have been constructed, where the extra chiral multiplets have an O' Raifeartaigh like dynamics, which is either genuinely postulated from the beginning [30, 31] or effectively derived from the dual description of a strongly coupled theory [32] admitting a metastable supersymmetry breaking vacuum as in [33]. Actually, a very simple and general class of such models can be constructed by using as uplifting sector any kind of sector breaking supersymmetry at a scale much lower than the Planck scale [17]. Models realizing a D-term uplifting, on the other hand, are difficult to achieve. The

natural idea of relying on some Fayet-Iliopoulos term [34] does not work, due to the already mentioned fact that such terms must generically be field-dependent in supergravity, so that the induced D is actually proportional to the available charged F's. It is then clear that there is an obstruction in getting D much bigger than the F's. Most importantly, if the only charged chiral multiplet in the model is the one of the would-be supersymmetric sector (which is supposed to have vanishing F) then also D must vanish, implying that a vector multiplet cannot act alone as an uplifting sector [35, 36]. This difference between F-term and D-term uplifting is once again due to the basic fact that chiral multiplets can dominate supersymmetry breaking whereas vector multiplets cannot.

The paper is organized as follows: In section 2, we present a brief summary of the relevant features of gauge invariant supergravity models. In section 3 we discuss the general properties of the vacuum and the interplay between the spontaneous breaking of supersymmetry and gauge symmetries. In section 4 we study various types of relations that exist between D and F auxiliary fields. In section 5 we derive a necessary condition for stability, and combine it with the flatness condition to define a set of general constraints that are necessary for the existence of a viable vacuum. In section 6 we elaborate on the possible strategies that can be used to derive more concrete implications from these constraints, and in particular on the different ways in which the relation between D and F auxiliary fields can be taken into account. In sections 7, 8 and 9 we then pursue three different approaches to this problem, which are based respectively on a dynamical relation, a kinematical relation and a kinematical bound between the D and the F auxiliary fields. In section 10 we illustrate the relevance of our general results with some examples of string inspired models. In section 11, we conclude with a qualitative summary of our results.

2 Gauge invariant supergravity models

In this section we will briefly review the main features of general supergravity models with minimal supersymmetry in four dimensions [37, 38], emphasizing those particular aspects that will be relevant for our analysis. We will use the notation of [39] and set $M_{\mathrm{P}} = 1$.

Consider first a supergravity theory with n chiral multiplets Φ^i. The two-derivative Lagrangian is specified by a single real Kähler function $G(\Phi^k, \Phi^{k\dagger})$ [1]. The Kähler geometry of the manifold spanned by the complex scalar fields is determined by the metric $g_{i\bar{j}} = G_{i\bar{j}}$, which can be used to raise and lower chiral indices [2]. It can happen that this theory has a group of some number m of global symmetries, compatibly with supersymmetry. These are generated by holomophic Killing vectors $X_a^i(\Phi^k)$, in the sense that a generic symmetry transformation with infinitesimal real constant parameters λ^a is implemented by the operator $\delta = \lambda^a(X_a^i \partial_i + X_a^{\bar{i}} \partial_{\bar{i}})$. The chiral superfields transform as $\delta\Phi^i = \lambda^a X_a^i(\Phi^k)$. It is then clear that the condition for these transformations to represent an invariance of

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[1]The function G is related to the more commonly used Kähler potential K and superpotential W by the equation $G = K + \log|W|^2$. This decomposition is however ambiguous, due to the Kähler symmetry transforming $K \to K + F + \bar{F}$ and $W \to e^{-F}W$, and leaving G invariant. As already mentioned, W cannot vanish at a non-supersymmetric Minkowski minimum, and the function G is therefore well defined.

[2]Subscripts on scalar quantities denote ordinary derivatives with respect to the fields.

the theory is that the function G should be invariant: $\delta G = 0$. This implies:

$$X_a^i G_i + X_a^{\bar{i}} G_{\bar{i}} = 0 \,. \tag{2.1}$$

This equation is valid at any point of the scalar manifold and one can therefore derive additional conditions by taking derivatives of it. The corresponding information is most conveniently extracted by using covariant derivatives ∇_i, in terms of which holomorphicity of the vectors X_a^i implies $\nabla_i X_a^{\bar{j}} = 0$ or $\nabla_i X_{aj} = 0$. Taking one derivative of eq. (2.1), one finds that:

$$X_{ai} + X_a^k \nabla_i G_k + G_k \nabla_i X_a^k = 0 \,. \tag{2.2}$$

Taking two derivatives, and using the fact that the metric is covariantly constant, one finds instead:

$$\nabla_i X_{a\bar{j}} + \nabla_{\bar{j}} X_{ai} = 0 \,. \tag{2.3}$$

Using the definition of the Riemann tensor and the holomorphicity of the Killing vectors we know that $\nabla_i \nabla_{\bar{j}} X_{ap} = R_{i\bar{j}p\bar{q}} X_a^{\bar{q}}$. With the help of this relation, and taking a suitable combination of four derivatives of (2.1), one also finds:

$$\left(X_a^m \nabla_m + X_a^{\bar{n}} \nabla_{\bar{n}} \right) R_{i\bar{j}p\bar{q}} + R_{r\bar{j}p\bar{q}} \nabla_i X_a^r + R_{i\bar{s}p\bar{q}} \nabla_{\bar{j}} X_a^{\bar{s}} + R_{i\bar{j}t\bar{q}} \nabla_p X_a^t + R_{i\bar{j}p\bar{u}} \nabla_{\bar{q}} X_a^{\bar{u}} = 0 \,. \tag{2.4}$$

The conditions (2.3) and (2.4) can be rephrased in terms of Lie derivatives as $\mathcal{L}_{X_a} g_{i\bar{j}} = 0$ and $\mathcal{L}_{X_a} R_{i\bar{j}p\bar{q}} = 0$, and show that each symmetry is associated to an isometry of the scalar manifold [3].

Consider now the possibility of gauging such isometries with the introduction of vector multiplets. The corresponding supergravity theory will then include n chiral multiplets Φ^i and m vector multiplets V^a. Its two-derivative Lagrangian is specified by a real Kähler function $G(\Phi^k, \Phi^{k\dagger}, V^a)$, determining in particular the scalar geometry, m holomorphic Killing vectors $X_a^i(\Phi^k)$, generating the isometries that are gauged, and an m by m matrix of holomorphic gauge kinetic functions $H_{ab}(\Phi^k)$, defining the gauge couplings. There exists a general and systematic way of promoting the globally invariant action of the chiral multiplet theory to a locally invariant Lagrangian involving also the vector multiplets. This can be done in superfields by generalizing the real constant transformation parameters λ^a to chiral superfield parameters Λ^a, and asking the transformation operator δ to act also on the vector fields, as $\delta = \Lambda^a X_a^i \partial_i + \bar{\Lambda}^a X_a^{\bar{i}} \partial_{\bar{i}} - i(\Lambda^a - \bar{\Lambda}^a)\partial_a$. The transformations of the chiral and vector multiplets read then $\delta \Phi^i = \Lambda^a X_a^i(\Phi^k)$ and $\delta V^a = -i(\Lambda^a - \bar{\Lambda}^a)$. The minimal coupling between chiral and vector multiplets turn ordinary derivatives into covariant derivatives, and induces a new contribution to the scalar potential coming from the vector auxiliary fields D^a, in addition to the standard one coming from the chiral auxiliary fields F^i. The condition for the action to be invariant under the just mentioned

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[3]Notice that for constant curvature Kähler manifolds, for which $\nabla_k R_{i\bar{j}p\bar{q}} = 0$, the conditions (2.4) represent linear constraints on the derivatives of the Killing vectors.

local transformations is clearly that G must be invariant: $\delta G = 0$ [4]. This implies now the conditions:

$$G_a = -i \, X_a^i \, G_i = i \, X_a^{\bar{i}} \, G_{\bar{i}} \,. \tag{2.5}$$

In addition, the gauge kinetic function H_{ab} must have an appropriate behavior under gauge transformations, in such a way as to cancel possible gauge anomalies Q_{abc} and to lead to a consistent quantum effective action. More precisely, $\mathrm{Re}\, H_{ab}$ must be invariant, whereas $\mathrm{Im}\, H_{ab}$ must have a variation that exactly matches the coefficient of Q_{abc} [5]. In general, such anomalies have a quite involved structure and can depend on the scalar fields. Their general form has been studied in [41] in the limit of rigid supersymmetry, and more recently in [42] in the context of local supersymmetry. The functional form that H_{ab} is allowed to take is then strongly constrained and linked to the form of the anomaly Q_{abc}. More precisely, the anomaly cancellation condition $\delta H_{bc} = i \, \lambda^a Q_{abc}$ implies the functional relations

$$X_a^i \nabla_i H_{bc} = i \, Q_{abc} \,. \tag{2.6}$$

Now, since the quantities X_a^i and H_{bc} are both holomorphic, these equations are subject to a strong integrability condition. Indeed, by taking first derivatives of the relations (2.6), one finds that the real function Q_{abc} must actually be constant. The functional form of the gauge kinetic function is then essentially fixed by the transformation rules of the scalar fields, in such a way that its variation is constant. This means that the possibility of canceling anomalies through local Wess-Zumino terms, which always exists in general, is quite strongly constrained by supersymmetry. In the low-energy effective theories underlying string models, for instance, it is realized in the special form of the Green-Schwarz mechanism [43]. In that case, some fields that were neutral at tree-level and present in H_{ab} acquire a non-trivial transformation law due to one-loop corrections to the Kähler potential. In addition, there can also be one-loop threshold corrections to the gauge kinetic function that depend on charged fields. These two effects account for anomaly cancellation in all the known situations.

The expectation values of the real and imaginary parts of H_{ab} define respectively the inverse couplings and the θ-angles for the vector fields. The former defines also a metric for the gauge fields, which can be used to raise and lower vector indices:

$$h_{ab} = \mathrm{Re} H_{ab} \,. \tag{2.7}$$

The expectation values of the first and second derivatives of H_{ab} define additional relevant parameters of the theory. As a consequence of the holomophicity of H_{ab}, it is possible to

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[4]This means that the Kähler potential K and the superpotential W must be invariant only up to a local Kähler transformation, associated with a gauging of the R symmetry.

[5]In general, there can also be generalized Chern-Simons terms, which can be added to the Lagrangian compatibly with supersymmetry. These can contribute to the anomalous variation of the action and allow therefore to generalize a bit the way in which anomaly cancellation is realized. See [40] for discussions of this point. Here we shall simply interpret Q_{abc} as the residual quantum anomaly that is left after having considered possible generalized Chern Simons terms.

parametrize these through the following two quantities:

$$h_{abi} = \nabla_i \mathrm{Re} H_{ab}, \qquad h_{abij} = \nabla_i \nabla_j \mathrm{Re} H_{ab}. \tag{2.8}$$

The condition (2.6) for anomaly cancellation implies then the relation:

$$X_a^i h_{bci} = \frac{i}{2} Q_{abc}. \tag{2.9}$$

The theory is most conveniently formulated by using the superconformal formalism [44], with a chiral compensator multiplet Φ. The Lagrangian reads then simply [6]:

$$\mathcal{L} = \int d^4\theta \left[-3 \exp\left\{ -\frac{1}{3} G(\Phi^k, \Phi^{k\dagger}, V^a) \right\} \right] \Phi^\dagger \Phi + \left(\int d^2\theta\, \Phi^3 + \mathrm{h.c.} \right)$$
$$+ \left(\int d^2\theta\, \frac{1}{4} H_{ab}(\Phi^k) W^{a\alpha} W_\alpha^b + \mathrm{h.c.} \right). \tag{2.10}$$

The kinetic Lagrangians for the scalar and the gauge fields involve respectively the covariant derivative $\mathcal{D}_\mu \Phi^i = \partial_\mu \Phi^i - X_a^i(\Phi^k) A_\mu^a$ and the field strengths $F_{\mu\nu}^a$, and read:

$$\mathcal{L}_{\mathrm{kin}} = -\, g_{i\bar{j}} \mathcal{D}^\mu \phi^i \mathcal{D}_\mu^* \phi^{\bar{j}} - \frac{1}{4}\, h_{ab}\, F^{\mu\nu a} F_{\mu\nu}^b. \tag{2.11}$$

The scalar potential can be computed by integrating out the auxiliary fields. To do so, it is convenient to gauge fix the redundant superconformal symmetries by setting the scalar component of the compensator to $e^{G/6}$ and the fermion component to 0, and to parametrize its non-trivial auxiliary component as $e^{G/6}F$. The auxiliary field Lagrangian is then easily computed, and the corresponding equations fix $F = e^{G/2}(1 - 1/3\, G^k G_k)$ and

$$F^i = -\, e^{G/2}\, g^{i\bar{j}}\, G_{\bar{j}}^-, \qquad D^a = -\, h^{ab}\, G_b. \tag{2.12}$$

Substituting these expressions back into the auxiliary field Lagrangian, the scalar potential is found to be:

$$V = e^G \left(G^k G_k - 3 \right) + \frac{1}{2} G^a G_a. \tag{2.13}$$

Using the relation (2.5) and the expression $m_{3/2} = e^{G/2}$ for the gravitino mass, one can finally rewrite this in the standard way as

$$V = -3\, m_{3/2}^2 + g^{i\bar{j}}\, F_i F_{\bar{j}} + \frac{1}{2} h^{ab} D_a D_b. \tag{2.14}$$

where

$$F_i = -m_{3/2}\, G_i, \tag{2.15}$$

$$D_a = i\, X_a^i\, G_i = -i\, X_a^{\bar{i}}\, G_{\bar{i}}. \tag{2.16}$$

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[6]The D and F densities contain now, besides the terms appearing in rigid supersymmetry, also terms depending on the graviton and the gravitino, but these will not be relevant for us.

The relation (2.16) shows that the D_a can be geometrically identified with the Killing potentials. Indeed, taking one derivative one finds that the Killing vectors can be written in the following way, which automatically solves the Killing equation (2.3):

$$X_{ai} = i\,\nabla_i D_a\,, \qquad X_{a\bar{\imath}} = -i\,\nabla_{\bar{\imath}} D_a\,. \tag{2.17}$$

Taking two derivatives of the relation (2.16), one obtains instead a two-index tensor characterizing the local "charges" of the chiral multiplets

$$q_{ai\bar{\jmath}} = \frac{i}{2}\big(\nabla_i X_{a\bar{\jmath}} - \nabla_{\bar{\jmath}} X_{ai}\big) = \nabla_i \nabla_{\bar{\jmath}} D_a\,. \tag{2.18}$$

Notice finally that by taking derivatives of (2.5), one also deduces that $X_{ai} = -i\,G_{ai}$, $X_{a\bar{\imath}} = i\,G_{a\bar{\imath}}$ and $q_{ai\bar{\jmath}} = -G_{ai\bar{\jmath}}$.

3   Vacuum and spontaneous symmetry breaking

The vacuum of the theory is associated to a stationary point of the scalar potential (2.13). The scalar fields take in general non-vanishing vacuum expectation values, and local supersymmetry and the gauge symmetries can thus be spontaneously broken. The cosmological constant scale is identified with the vacuum energy, and we will assume that it is adjusted, through a tuning of parameters in the effective Lagrangian, to its very small measured value, which is negligible with respect to the gravitino mass scale $e^{G/2}$. We shall therefore impose from the beginning that on the vacuum $V = 0$, implying the flatness condition:

$$-3 + G^i G_i + \frac{1}{2}\,e^{-G} G^a G_a = 0\,. \tag{3.1}$$

The stationarity conditions correspond to requiring that $\nabla_i V = 0$, and they are given by:

$$G_i + G^k \nabla_i G_k + e^{-G}\Big[G^a\Big(\nabla_i - \frac{1}{2}G_i\Big)G_a + \frac{1}{2}h_{abi}G^a G^b\Big] = 0\,. \tag{3.2}$$

The $2n$-dimensional mass matrix for small fluctuations of the scalar fields around the vacuum has two different n-dimensional blocks, which can be computed as $m_{i\bar{\jmath}}^2 = \nabla_i \nabla_{\bar{\jmath}} V$ and $m_{ij}^2 = \nabla_i \nabla_j V$. Using the flatness and stationarity conditions, one finds, after a straightforward computation [45, 46]:

$$
\begin{aligned}
m_{i\bar{\jmath}}^2 = {}& e^G\Big[g_{i\bar{\jmath}} - R_{i\bar{\jmath}p\bar{q}}G^p G^{\bar{q}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k\Big] \\
&+ \Big[-\frac{1}{2}\Big(g_{i\bar{\jmath}} - G_i G_{\bar{\jmath}}\Big)G^a G_a + \Big(G_{(i}h_{ab\bar{\jmath})} + h^{cd}h_{aci}h_{bd\bar{\jmath}}\Big)G^a G^b \\
&\quad - 2\,G^a G_{(i}\nabla_{\bar{\jmath})}G_a - 2\,G^a h^{bc}h_{ab(i}\nabla_{\bar{\jmath})}G_c + h^{ab}\nabla_i G_a \nabla_{\bar{\jmath}} G_b + G^a \nabla_i \nabla_{\bar{\jmath}} G_a\Big]\,,
\end{aligned} \tag{3.3}
$$

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\begin{aligned}
m_{ij}^2 = {}& e^G\Big[2\,\nabla_{(i}G_{j)} + G^k \nabla_{(i}\nabla_{j)}G_k\Big] \\
&+ \Big[-\frac{1}{2}\Big(\nabla_{(i}G_{j)} - G_i G_j\Big)G^a G_a + \Big(G_{(i}h_{abj)} + h^{cd}h_{aci}h_{bdj} - \frac{1}{2}h_{abij}\Big)G^a G^b \\
&\quad - 2\,G^a G_{(i}\nabla_{j)}G_a - 2\,G^a h^{bc}h_{ab(i}\nabla_{j)}G_c + h^{ab}\,\nabla_i G_a \nabla_j G_b\Big]\,.
\end{aligned} \tag{3.4}
$$

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The m-dimensional mass matrix for the vector fields can instead be read off from the kinetic term of the scalar fields, and has the form:

$$M^2_{ab} = 2\,g_{i\bar{j}}X^i_a X^{\bar{j}}_b = 2\,g_{i\bar{j}}\nabla^i G_a \nabla^{\bar{j}}G_b\,. \tag{3.5}$$

In general the theory displays a spontaneous breakdown of both supersymmetry and gauge symmetries. These two breakings can happen at independent scales and along different directions in field space, and this is the main reason for the increased complexity of this analysis. Supersymmetry breaking is realized through a fermionic version of the Higgs mechanism, in which the gravitino field absorbs a linear combination of the fermion fields ψ^i and ψ^a of the chiral and vector multiplets, the would-be Goldstino fermion. The relevant combination can be read off from the mixing between the gravitino and the chiral and vector multiplet fermions, and is given by:

$$\eta = \frac{i}{\sqrt{3}}e^{G/2}G_i\psi^i + \frac{1}{\sqrt{6}}G_a\psi^a\,. \tag{3.6}$$

The Goldstino is thus associated with the vector $\eta_\xi = (ie^{G/2}G_i/\sqrt{3}, G_a/\sqrt{6})$ in the space of chiral and vector multiplet spinors $\psi^\xi = (\psi^i, \psi^a)$: $\eta = \eta_\xi\psi^\xi$. The norm squared of this vector is given by $\eta_\xi\eta^\xi = e^G G^i G_i/3 + G^a G_a/6 = m^2_{3/2}$, due to the flatness condition (3.1), and the scale of supersymmetry breaking can therefore be associated with the gravitino mass $m_{3/2}$. On the other hand, gauge symmetry breaking is realized through an ordinary bosonic Higgs mechanism, in which the gauge fields A^a_μ absorb linear combinations of the scalar fields ϕ^i of the chiral multiplets, the would-be Goldstone bosons. The relevant combinations can be read off from the mixing between the gauge fields and the chiral multiplet scalar fields, and are given by:

$$\sigma_a = X_{ai}\,\phi^i + X_{a\bar{i}}\,\phi^{\bar{i}}\,. \tag{3.7}$$

The Goldstone bosons are thus associated to the vectors $\sigma_{a\alpha} = (X_{ai}, X_{a\bar{i}})$ in the space of chiral multiplet scalars $\phi^\alpha = (\phi^i, \phi^{\bar{i}})$: $\sigma_a = \sigma_{a\alpha}\phi^\alpha$. The scalar product between two of these vectors is given by $\sigma_{a\alpha}\sigma^\alpha_b = 2X^i_a X_{bi} = M^2_{ab}$, and the scales of gauge symmetry breaking are therefore controlled by the gauge field mass matrix M_{ab}.

In general, supersymmetry and gauge symmetry breaking occur in an entangled way because of two reasons. The first is that the directions of the breakings are in general not orthogonal, and the second is that the breaking scales are not necessarily well separated. This situation simplifies (and the two types of breakings disentangle) only whenever the Goldstino and the Goldstone directions are nearly orthogonal or the gravitino and the gauge field masses are hierarchically different. In the first case, the two breaking disentangle just because they involve different sets of fields, whereas in the second they decouple because of the very large mass difference of the relevant fields.

4   Chiral versus vector auxiliary fields

The chiral and vector auxiliary fields are given by $F_i = -e^{G/2}G_i$ and $D_a = -G_a$. These relations are enforced by the equations of motion of the auxiliary fields, and are therefore

true at any point of the scalar field space, and not only at the stationary points of the potential. These two kinds of auxiliary fields are however not completely independent from each other. There is a first relation between the F_i and the D_a, which is of "kinematical" nature, in the sense that it is satisfied as a functional relation valid at any point of the scalar field space. This first relation comes from (2.5) and is a consequence of the gauge invariance of G. It reads:

$$D_a = -i\,\frac{X_a^i}{m_{3/2}}F_i = i\,\frac{X_a^{\bar{i}}}{m_{3/2}}F_{\bar{i}}\,. \tag{4.1}$$

This relation shows that the D_a are actually linear combinations of the F_i, with coefficients of order $\mathcal{O}(M_{aa}/m_{3/2})$. One can then derive a simple bound on the sizes that the D_a can have relative to the F_i. Indeed, using the inequality $|a^i b_i| \leq \sqrt{a^i a_i}\sqrt{b^j b_j}$, one deduces from the relation (4.1) that:

$$|D_a| \leq \frac{1}{\sqrt{2}}\frac{M_{aa}}{m_{3/2}}\sqrt{F^i F_i}\,. \tag{4.2}$$

There is also a second relation between the F_i and the D_a, which is instead of "dynamical" nature, in the sense that it is valid only at the stationary points of the potential. It comes from considering a suitable linear combination of the stationarity conditions (3.2) along the direction X_a^i, that is, imposing stationarity with respect to those particular field variations that correspond to complexified gauge transformations: $X_a^i \nabla_i V = 0$. This corresponds essentially to extract the information concerning stationarity of those scalar fields that are absorbed by the gauge field in the Higgs mechanism. To derive this relation, one starts by contracting (3.2) with X_a^i. The result can be simplified by using eq. (2.2). Using also the relations (2.9), (2.16) and (3.5) one gets:

$$-\nabla_i X_{a\bar{j}}\,F^i F^{\bar{j}} - i\left(F^i F_i - m_{3/2}^2\right)D_a - \frac{i}{2}\,M_{ab}^2\,D^b + \frac{i}{2}Q_{abc}D^b D^c = 0\,. \tag{4.3}$$

The real part of this equation is identically satisfied at any point. Indeed, the real part of the first term vanishes due to the Killing condition (2.3), whereas that of the last three terms vanishes trivially due to the fact that the quantities D_a, M_{ab}^2 and Q_{abc} are real. This just reflects the fact that the potential V is invariant under gauge transformations at any point: $\mathrm{Re}(X_a^i \nabla_i V) = \delta V = 0$. The imaginary part of this equation is instead non-trivial, as V is not invariant under imaginary gauge transformations. Using the definition (2.18), we get the following quadratic relation between the D_a and the F_i [9, 47] (see also [48]):

$$q_{ai\bar{j}}\,F^i F^{\bar{j}} - \frac{1}{2}\left[M_{ab}^2 + 2\left(F^i F_i - m_{3/2}^2\right)h_{ab}\right]D^b + \frac{1}{2}\,Q_{abc}D^b D^c = 0\,. \tag{4.4}$$

It is worth discussing what happens in the limit of rigid supersymmetry. In order to do so, it is important to note that all the formulae that have been derived up to now in this section are actually valid for any value of the cosmological constant, and not only for the case where it vanishes. This means that the sizes of the auxiliary fields F_i and D_a are not necessarily limited by the flatness condition. We can then consider the limit of rigid supersymmetry, $m_{3/2} \to 0$ and $M_P \to \infty$, while keeping all the other quantities

fixed. In that limit, the kinematical constraint (4.2) becomes trivial. This reflects the fact that in the rigid limit, the functional forms of the F_i and the D_a simplify and become independent of each other. On the other hand, the dynamical relation (4.4) persists in the rigid limit, although it gets substantially simplified. Indeed, the second and third terms in the relation (4.4) can be neglected, since by dimensional analysis they are suppressed by powers of M_P. Therefore we are left with: $q_{ai\bar{j}} F^i F^{\bar{j}} - 1/2\, M_{ab}^2 D^b + 1/2\, Q_{abc} D^b D^c = 0$, and we can conclude that the kinematical bound (4.2) is of gravitational origin, whereas the dynamical relation (4.4) is of non-gravitational origin. In the light of this observation, it should be emphasized that the obstruction in getting a non-zero D in situations with vanishing F should be interpreted as a constraint coming from the dynamical relation (4.4) rather than from the kinematical relation (4.1), since it is due to the holomorphicity of the superpotential and not to any particular feature associated to gravity [7]. It is clear that this obstruction also translates into a natural tendency for D auxiliary fields to be smaller than F auxiliary fields [8].

We are going to discuss now two situations in which the relation between the F_i and the D_a simplifies. They correspond to the two different limits where the breaking of supersymmetry and gauge symmetries happen at well separated scales and therefore disentangle.

4.1 Heavy vector limit

A first limit in which the situation substantially simplifies is when the gauge bosons masses are much bigger than the gravitino mass: $M_{ab} \gg m_{3/2}$. In this limit (and assuming that all the other quantities in the problem remain finite) one finds that the kinematical bound (4.2) becomes irrelevant, whereas the dynamical relation (4.4) implies that the D_a are small and given by

$$D^a \simeq 2\, M^{-2ab} q_{bi\bar{j}} F^i F^{\bar{j}}\,. \tag{4.5}$$

More precisely, the flatness condition implies that $F_i \sim \mathcal{O}(m_{3/2})$, and the expression (4.5) has thus the form of a sum of terms that are $\mathcal{O}(m_{3/2}/M_{ab})$, with $\mathcal{O}(q_{ai\bar{j}})$ coefficients. The simplification of the relation (4.4) in this limit is due to the fact that gauge symmetries are broken at a much higher scale than supersymmetry. It is then a good approximation to integrate out the vector multiplets in a supersymmetric way at the scale defined by M_{ab}, and consider supersymmetry breaking within a low-energy effective supergravity theory involving only chiral multiplets. The F_i represent then the dominant supersymmetry breaking effects that are directly induced by chiral multiplets, whereas the D_a can now be understood as encoding the subleading corrections to supersymmetry breaking effects that are indirectly induced by virtual heavy vector fields [8].

Note also that due to the kinematical relation (4.1), the linear combination of chiral auxiliary fields $X_a^i F_i$ (which corresponds to the vector auxiliary fields D_a) must be small.

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[7]Nevertheless, it is curious that in the presence of an anomaly cancelled by a Wess-Zumino term, eq. (4.4) does not seem to display automatically this obstruction and one apparently needs to invoke (4.1).

[8]See for instance [49] for a discussion on the possibility of achieving D larger than F in the simplest class of theories with global supersymmetry and linearly realized gauge symmetries.

One gets then the m restrictions $X_a^i F_i \simeq 0$ as a consequence of the fact that, in the limit considered here, m of the n chiral multiplets are absorbed through a supersymmetric Higgs mechanism, and only $n - m$ gauge-invariant combinations of chiral multiplets remain as light fields in the low energy effective action that is used to describe supersymmetry breaking effects. Note that this indirectly implies that the vector fields can be much heavier than the gravitino (without any other parameter blowing up) only if the number of chiral multiplets is higher than the number of vector multiplets: $n > m$.

It is possible to verify that one correctly reproduces (4.5) by integrating out explicitly the vector fields. This can be done by solving the vector multiplet equations of motion directly in superfields. In the limit we are considering terms involving supercovariant derivatives can be neglected, since on dimensional grounds they would give subleading contributions. This means that one can neglect the gauge kinetic term in (2.10) and the equations of motion of the vector superfields are simply:

$$G_a(\Phi_i, \Phi_i^\dagger, V_a) \exp\left\{-\frac{1}{3} G(\Phi_i, \Phi_i^\dagger, V_a)\right\} \Phi^\dagger \Phi \simeq 0. \tag{4.6}$$

These represent algebraic equations involving the vector superfields V_a, the chiral superfields Φ_i, and the compensator superfield Φ. To solve them and integrate out the vector superfields, one needs to choose a gauge for the superfield gauge transformations. A convenient choice is the super-unitary gauge, where the would-be Goldstone superfields are set to zero. At the linearized level, this means setting $G_a \simeq 0$ (the $\simeq$ sign being related to the fact that this is possible only in the exactly supersymmetric limit). One can then solve eq. (4.6) and express the V^a in terms of the physical Φ^i orthogonal to X_a^i and Φ. Substituting the results back into the Lagrangian, one would then find the whole effective Lagrangian for the light chiral multiplets. Except for very special models, this is however difficult to carry out explicitly in practice, due to the fact that arbitrary powers of V^a appear in the equation. As a consequence, it is in general not possible to solve (4.6) exactly at the superfield level to determine the complete effective Lagrangian in closed form. One can however solve it approximately by assuming that the superfields V^a are small, and expanding it in powers of V^a. At first order in the expansion, one finds that this is consistent if $G_a \simeq 0$, as is the case in the super-unitary gauge we are using. At second order, the equation for V^a is linear and can be solved to determine the leading order effect of the vector multiplet. In doing so one finds the following expression:

$$\left[G_{ab} - \frac{1}{3} G_a G_b\right] V^b + G_a \simeq 0. \tag{4.7}$$

Taking into account that the quantity $G_{ab} - G_a G_b/3$ has a vacuum expectation value given by $M_{ab}^2/2$, which is by assumption large, one finally finds:

$$V^a \simeq -2M^{-2ab} G_b. \tag{4.8}$$

In this equation, V^a and G_b are superfields, whereas M^{-2ab} are numbers. Taking finally the D component of this equation, and recalling that $G_{ai\bar{j}} = -q_{ai\bar{j}}$, one recovers eq. (4.5). One can also use the result (4.8) to compute the leading order correction that is induced by the heavy vector multiplets in the effective action for the light chiral multiplets. It is

given by $\Delta G \simeq -M^{-2ab}G_a G_b$ [9]. Notice finally that the F component of the gauge fixing condition $G_a \simeq 0$ reproduces instead the constraint $X_a^i F_i \simeq 0$, since $G_{ai} = iX_{ai}$.

4.2 Light vector limit

Another limit in which the situation also simplifies is when the gauge bosons masses are much smaller than the gravitino mass: $M_{ab} \ll m_{3/2}$. In this limit (and assuming that all the other quantities in the problem remain finite) the kinematical bound (4.2) becomes very stringent and implies that the D_a are small. By the flatness condition we get then $F^i F_i \simeq 3\,m_{3/2}^2$, and the dynamical relation (4.4) simplifies to

$$D_a \simeq \frac{1}{2}\,m_{3/2}^{-2}\,q_{ai\bar{j}}F^i F^{\bar{j}}\,. \tag{4.9}$$

Note that since $F_i \sim \mathcal{O}(m_{3/2})$, this expression has the form of a sum of terms that are $\mathcal{O}(1)$, with $\mathcal{O}(q_{ai\bar{j}})$ coefficients. But from the kinematical bound we know that the D_a must actually be small, more precisely $\mathcal{O}(M_{ab})$. This implies that there must be a cancellation of the leading order contribution of the various terms: $q_{ai\bar{j}}F^i F^{\bar{j}} \simeq 0$. This shows that the vector multiplets can be very light only if the charged chiral multiplets contribute to supersymmetry breaking in a suitably aligned way. It also indirectly implies that, unless the charges $q_{ai\bar{j}}$ become somehow degenerate, the vector fields can be much lighter than the gravitino only if the number of chiral multiplets is higher than the number of vector multiplets: $n > m$.

The fact that the relation between the D_a and the F_i simplifies in this case can be partly understood from the fact that in this limit supersymmetry is broken at a scale much higher than the scale of gauge symmetry breaking. The D_a must then be small, as a consequence of the fact that supersymmetry breaking is dominated by the F_i of the chiral multiplets. On the other hand, in order to approximately preserve gauge invariance, the vacuum expectations values of the latter cannot be arbitrary, but must rather satisfy m approximate relations, which are identified with the conditions $q_{ai\bar{j}}F^i F^{\bar{j}} \simeq 0$.

5 Constraints from flatness and stability

We want now to study the constraints that can be put on the theory by imposing the flatness condition, guaranteeing the vanishing of the cosmological constant, and the stability condition, enforcing the positivity of the scalar mass matrix. In order to do so, it is useful to introduce the $2n$-dimensional vector of scalar fields $\phi^\alpha = (\phi^i, \phi^{\bar{i}})$. Using this notation, the complete $2n$-dimensional mass matrix for the scalar fields can be written in terms of the blocks (3.3) and (3.4) as

$$m_{\alpha\bar{\beta}}^2 = \begin{pmatrix} m_{i\bar{j}}^2 & m_{ij}^2 \\ m_{\bar{i}\bar{j}}^2 & m_{\bar{i}j}^2 \end{pmatrix}\,. \tag{5.1}$$

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[9]The correction $D^a D_a/2$ to the scalar potential is not included in this result, and corresponds to a subleading effect coming from the vector field kinetic terms.

In order to study the restrictions imposed by the requirement that the physical squared mass of the scalar fields are all positive, it is necessary to take appropriately into account the spontaneous breaking of gauge symmetries. In this process m of the scalars, the would-be Goldstone bosons, are absorbed by the gauge fields and turned into their longitudinal components. In the unitary gauge, these modes are disentangled from the remaining physical scalar fields, and their m-dimensional mass matrix coincides with the mass matrix M_{ab}^2 of the gauge fields. Since this matrix is by construction positive definite, we do not need to worry about these directions for the stability of the physical scalar fields. What we have instead to impose is that the mass matrix $\tilde{m}_{\alpha\bar{\beta}}^2$ of the remaining $2n - m$ physical scalar fields is positive definite. This complicates the analysis with respect to the case where vector multiplets are absent, because the matrix $\tilde{m}_{\alpha\bar{\beta}}^2$ is defined by a complicated real projection of the Hermitian matrix $m_{\alpha\bar{\beta}}^2$ onto the subspace orthogonal to the would-be Goldstone directions $X_a^\alpha = (X_a^i, X_a^{\bar{i}})$.

Fortunately, there exist a simple way to avoid the extra complications introduced by the gauge fixing procedure. It relies on the observation that the would-be Goldstone modes correspond, before the gauge fixing, to flat directions of the unphysical mass matrix $m_{\alpha\bar{\beta}}^2$, and get their physical mass through their kinetic mixing with the gauge bosons [10]. This means that positivity of the physical mass matrix $\tilde{m}_{\alpha\bar{\beta}}^2$ implies semi-positivity of the unphysical mass matrix $m_{\alpha\bar{\beta}}^2$. This condition is much simpler to study, and we will therefore take it as our starting point.

To extract some useful information from the condition that the mass matrix (5.1) should be semi-positive definite, we can now use the same strategy as in [17]. Instead of requiring that $m_{\alpha\bar{\beta}}^2 Z^\alpha Z^{\bar{\beta}} \geq 0$ for any direction Z^α, we can look only along some special directions, and see if we can obtain simple necessary conditions. Note first that combining the conditions coming from the two independent directions $Z_1^\alpha = (z^i, z^{\bar{i}})$ and $Z_2^\alpha = (iz^i, -iz^{\bar{i}})$ defined by any complex vector z^i, one deduces that $m_{i\bar{j}}^2 z^i z^{\bar{j}} \geq 0$. This means that if $m_{\alpha\bar{\beta}}^2$ is semi-positive definite, then the principal block $m_{i\bar{j}}^2$ must necessarily be semi-positive definite as well. Our strategy will then be, as in [17], to look for a suitable vector z^i leading to a simple constraint on the parameters of the theory.

In the case of theories with both supersymmetry and gauge symmetries, there exist two types of special complex directions z^i one could look at. The first is the direction G^i, which is associated with the Goldstino direction in the subspace of chiral multiplet fermions. The second is identified with the directions X_a^i, which are instead associated with the Goldstone directions in the space of chiral multiplet scalars. Consider first the direction G^i. Starting from (3.3) and using the fact that $G^i \nabla_i G_a = G_a$, the flatness condition (3.1), and also the stationarity condition (3.2) to simplify terms involving $\nabla_i G_j$,

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[10]It is straightforward to verify that the Goldstone directions X_a^α correspond indeed to flat directions of the mass matrix $m_{\alpha\bar{\beta}}^2 = \nabla_\alpha \nabla_{\bar{\beta}} V$. To see this note that, in the notation we are using in this section, the operator generating real gauge transformations is just $\delta = \lambda^a X_a^\alpha \nabla_\alpha$. The condition of gauge invariance of the potential V, namely $\delta V = 0$, implies thus that $X_a^\alpha \nabla_\alpha V = 0$. Since this relation is valid at any point, one can take another derivative and deduce the new condition $\nabla_{\bar{\beta}} X_a^\alpha \nabla_\alpha V + X_a^\alpha \nabla_{\bar{\beta}} \nabla_\alpha V = 0$. Now, evaluating this expression at a stationary point, where $\nabla_\alpha V = 0$, one finds $m_{\alpha\bar{\beta}}^2 X_a^{\bar{\beta}} = 0$, implying in particular that $m_{\alpha\bar{\beta}}^2 X_a^\alpha X_a^{\bar{\beta}} = 0$.

one finds, after a long but straightforward computation:

$$m_{i\bar{j}}^2 G^i G^{\bar{j}} = e^G \Big[6 - R_{i\bar{j}p\bar{q}} G^i G^{\bar{j}} G^p G^{\bar{q}} \Big] + \Big[-2\, G^a G_a + h^{cd} h_{aci} h_{bd\bar{j}} G^i G^{\bar{j}} G^a G^b \Big] \quad (5.2)$$
$$+ e^{-G} \Big[M_{ab}^2 G^a G^b - \frac{3}{4} Q_{abc} G^a G^b G^c - \frac{1}{2} \big(G^a G_a \big)^2 + \frac{1}{4} h_{ab}{}^i h_{cdi} G^a G^b G^c G^d \Big].$$

The condition $m_{i\bar{j}}^2 G^i G^{\bar{j}} \geq 0$ is then the generalization of the condition derived in [17] for theories involving only chiral multiplets. We believe that also in this case this condition captures the most significant restrictions associated to stability, insofar it comes from the scalar and pseudoscalar partners of the Goldstino, which are generically expected to be the lightest and most dangerous modes.

Consider next the directions X_a^i. Using the fact that $X_a^i \nabla_i G_b = i/2\, M_{ab}^2$, as well as the condition (2.2) to simplify the terms involving $\nabla_i G_j$, one finds:

$$m_{i\bar{j}}^2 X_a^i X_a^{\bar{j}} = e^G \Big[M_{aa}^2 - i\, q_{ai\bar{j}} \Big(X_a^i G^{\bar{j}} - X_a^{\bar{j}} G^i \Big) + q_{ai}{}^k q_{a\bar{j}k} G^i G^{\bar{j}} - R_{i\bar{j}p\bar{q}} X_a^i X_a^{\bar{j}} G^p G^{\bar{q}} \Big]$$
$$+ \Big[\frac{1}{4} M_{aa}^4 - \Big(q_{bi\bar{j}} X_a^i X_a^{\bar{j}} + \frac{1}{2} Q_{abc} M_a^{2c} \Big) G^b - \frac{1}{4} \Big(M_{aa}^2 - 2\, G_a G_a \Big) G^b G_b$$
$$- M_{ab}^2 G_a G^b + \frac{1}{4} Q_{abd} Q_{ac}{}^d G^b G^c + \frac{1}{2} Q_{abc} G_a G^b G^c \Big]. \quad (5.3)$$

The conditions $m_{i\bar{j}}^2 X_a^i X_a^{\bar{j}} \geq 0$ represent in principle new additional constraints, which are absent in theories involving only chiral multiplets. We believe however that these conditions do not capture any extremely relevant new information, as they involve essentially the complex partners of the Goldstone scalars, which are not expected to be very dangerous [11].

We are now ready to use the conditions of flatness and stability at the stationary point to find some algebraic constraints applying to any supergravity theory, and which are simple enough to represent an interesting restriction on the functions G, h_{ab} and X_a that specify the theory. Recall that the flatness condition is given by eq. (3.1) and takes the form:

$$-3 + G^i G_i + \frac{1}{2} e^{-G} G^a G_a = 0. \quad (5.4)$$

The necessary conditions for stability are more involved and correspond to requiring that the expressions (5.2) and (5.3) should be semi-positive. This leads to two kinds of algebraic conditions. Note however that (5.2) depends only on M_{ab}^2 and not on $q_{ai\bar{j}}$, and has a simple tensor structure, whereas (5.3) depends both on M_{ab}^2 and $q_{ai\bar{j}}$, and has a much more complicated tensor structure. As we already mentioned, we believe that only the former leads to a truly significant condition so we will therefore take as our necessary

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[11]This fact can be explicitly seen in the simplest case of models involving just one chiral and one vector field. In this case there are only two mass eigenvalues, which must therefore be in one-to-one correspondence with the two complex directions X and G'. One of these eigenvalues does not vanish and is clearly associated to G', whereas the other does vanish and is associated to X. It is then straightforward to verify that the non-zero eigenvalue is exactly captured by (5.2), and that (5.3) is just proportional to it.

condition for meta-stability the condition $m_{i\bar{j}}^2 G^i G^{\bar{j}} \geq 0$, which implies:

$$R_{i\bar{j}p\bar{q}} G^i G^{\bar{j}} G^p G^{\bar{q}} \leq 6 + e^{-G} \left[-2\, G^a G_a + h^{cd} h_{aci} h_{bd\bar{j}} G^i G^{\bar{j}} G^a G^b \right]$$
$$+ e^{-2G} \left[M_{ab}^2\, G^a G^b - \frac{3}{4}\, Q_{abc}\, G^a G^b G^c \right. \tag{5.5}$$
$$\left. - \frac{1}{2} \left(G^a G_a \right)^2 + \frac{1}{4}\, h_{ab}{}^k h_{cdk}\, G^a G^b G^c G^d \right].$$

The flatness condition (5.4) and the stability constraint (5.5) represent a strong restriction on the theory. They generalize the results derived in [17] and [18] to the more general case where both chiral and vector multiplets participate significantly to supersymmetry breaking. In the following, to work out more explicitly the implications of these constraints, we will mostly focus on the more tractable case where the gauge kinetic functions are constant and diagonal, so that $h_{ab} = g_a^{-2}\delta_{ab}$, $h_{ai} = 0$ and $h_{aij} = 0$. In this situation, one has also necessarily $Q_{abc} = 0$. The flatness and stability conditions take in this case the following simple form:

$$\begin{cases} g_{i\bar{j}} F^i F^{\bar{j}} = 3\, m_{3/2}^2 - \dfrac{1}{2}\, h_{ab}\, D^a D^b \,, \\[2mm] R_{i\bar{j}p\bar{q}}\, F^i F^{\bar{j}} F^p F^{\bar{q}} \leq 6\, m_{3/2}^4 + \left(M_{ab}^2 - 2\, m_{3/2}^2 h_{ab} \right) D^a D^b - \dfrac{1}{2}\, h_{ab} h_{cd}\, D^a D^b D^c D^d \,. \end{cases} \tag{5.6}$$

Besides this, it is important to remember that the values of the F_i and the D_a are not completely independent, but rather related by eq. (4.4). In the simpler situation considered here, this relation implies, after using the flatness condition, that [12]:

$$q_{ai\bar{j}}\, F^i F^{\bar{j}} = \frac{1}{2} \left[M_{ab}^2 + 4\, m_{3/2}^2\, h_{ab} \right] D^b - \frac{1}{2}\, h_{ab} h_{cd}\, D^b D^c D^d \,. \tag{5.7}$$

Recall finally that the auxiliary fields F_i and D_a are related to the complex directions G^i and X_a^i by the relations $F_i = -m_{3/2} G_i$ and $D_a = i X_a^i G_i$. This means that the F_i are associated to the direction of supersymmetry breaking, whereas the D_a are associated to the scalar products between this direction and the gauge symmetry breaking direction. Whenever the D_a are small, the gauge symmetry breaking effects related to the vector multiplets get weaker. More precisely, in the limit $g_a D_a \to 0$ the eqs. (5.6) reduce to those emerging in the absence of vector multiplets. This is not at all obvious, since in this limit the D-terms still give a non-vanishing contribution to $m_{i\bar{j}}^2$. As can be seen from (3.3) this contribution is however given simply by $g_a^2 X_{ai} X_{a\bar{j}}$, and therefore vanishes along the direction G^i, as $g_a G^i X_{ai} = i g_a D_a$ and this vanishes in the limit $g_a D_a \to 0$. On the other

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[12]Note that using the relations (5.7) it is possible to rewrite the quadratic term $(M_{ab}^2 - 2\, m_{3/2}^2 h_{ab}) D^a D^b$ in (5.6) as $2\, q_{ai\bar{j}} D^a F^i F^{\bar{j}} - 2\, g_{i\bar{j}} h_{ab} F^i F^{\bar{j}} D^a D^b$. One might then be tempted to further rewrite this as $2(Z_{i\bar{j}ab} - g_{i\bar{j}} h_{ab}) F^i F^{\bar{j}} D^a D^b$, in terms of the new quantity $Z_{i\bar{j}ab} = D_{(a}^{-1} \nabla_i \nabla_j D_{b)}$, which does however not have any clear interpretation. This would allow us to rewrite the conditions (5.6) as $\mathcal{G}_{\xi\bar{\theta}} \eta^\xi \eta^{\bar{\theta}} = m_{3/2}^2$ and $\mathcal{R}_{\xi\bar{\theta}\chi\bar{\psi}} \eta^\xi \eta^{\bar{\theta}} \eta^\chi \eta^{\bar{\psi}} \leq 2/3\, m_{3/2}^4$, in terms of the Goldstino vector $\eta_\xi = (iF^i/\sqrt{3}, D^a/\sqrt{6})$ and some generalized "metric" and "curvature" tensors. Unfortunately, it does not seem to be possible to write the conditions (5.7) in any illuminating way in terms of η_ξ. One could then ignore (5.7) and treat all the components of η_ξ as independent variables, to define a weaker set of equations, in the same spirit as in [18], although we will not follow this direction here.

hand, this limit implies also some constraints among the chiral multiplets. If the masses of the vectors are small ($g_a M_{ab} \to 0$), then the kinematical relation between D_a and F_i implies $g_a X_a^i F_i \to 0$, whereas if they are large ($g_a M_{ab} \to \infty$), the dynamical relation (5.7) between D_a and F_i implies instead $g_a q_{ai\bar{j}} F^i F^{\bar{j}} \to 0$.

Finally we would like to briefly comment on the special case of models involving just one chiral multiplet and one vector multiplet. In this case the physical mass matrix has just 1 entry and the condition for it to be positive is given exactly by (5.5). In this case, the symbol $\leq$ in (5.5) and (5.6) can therefore be replaced by $<$, and the condition is not only necessary but also sufficient.

6 Analysis of the constraints

The flatness and stability conditions (5.6) represent our main result regarding the constraints put on gauge invariant supergravity theories by the requirement that they should admit a phenomenologically and cosmologically viable vacuum. Along this section we will address the problem of working out more concretely the implications of these constraints.

We assume for simplicity that the gauge group consists of only Abelian $U(1)$ factors. In such a case, it is always possible to choose the basis of vector fields so that the mass matrix of the gauge bosons is diagonal, $M_{ab}^2 = M_a^2 \delta_{ab}$. The gauge kinetic function is assumed to be diagonal as well, and therefore the gauge couplings can be written as $h_{ab} = g_a^{-2} \delta_{ab}$. We can then choose the special parametrization of scalar fields that corresponds to normal coordinates for the Kähler geometry, in which the Kähler metric becomes $\delta_{I\bar{J}}$. Similarly, we can rescale the vector fields to define flat indices also for vector indices, in such a way that the vector metric becomes just δ_{AB}. This rescaling simply amounts to including a factor g_a for each vector index a, and one has for instance $D_A = g_a D_a$ and $M_A^2 = g_a^2 M_a^2$. In this way, no explicit dependence on g_a is left in the formulae.

It is convenient to introduce new variables, obtained by suitably rescaling the auxiliary fields with the gravitino mass as follows:

$$f^I = \frac{F^I}{\sqrt{3}\, m_{3/2}}, \qquad d^A = \frac{D^A}{\sqrt{6}\, m_{3/2}}. \tag{6.1}$$

Note that the flat capital indices of these new fields are raised and lowered with the diagonal metrics $\delta_{I\bar{J}}$ and δ_{AB}. In addition, we also find convenient to introduce the following new quantities, defined by rescaling the Killing vectors and its derivatives by the vector masses:

$$v_A^I = \frac{\sqrt{2} X_A^I}{M_A}, \qquad T_{AI\bar{J}} = \frac{q_{AI\bar{J}}}{M_A}. \tag{6.2}$$

Finally, we shall introduce the following dimensionless parameters, measuring the hierarchies between the scales of gauge symmetry and supersymmetry breaking:

$$\rho_A = \frac{M_A}{2\, m_{3/2}}. \tag{6.3}$$

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In terms of the quantities we have just introduced, the conditions (5.6) can be rewritten in the following form:

$$\begin{cases} \delta_{I\bar{J}} f^I f^{\bar{J}} = 1 - \sum_A d_A^2 \,, \\ R_{I\bar{J}P\bar{Q}} f^I f^{\bar{J}} f^P f^{\bar{Q}} \leq \dfrac{2}{3} + \dfrac{4}{3} \sum_A \left(2\,\rho_A^2 - 1 \right) d_A^2 - 2 \sum_{A,B} d_A^2 d_B^2 \,, \end{cases} \tag{6.4}$$

whereas the dynamical relation (4.4), kinematical relation (4.1) and kinematical bound (4.2) read now:

$$d_A = \sqrt{\frac{3}{2}} \, \frac{\rho_A \, T_{AI\bar{J}} f^I f^{\bar{J}}}{\rho_A^2 - 1/2 + 3/2 \, \delta_{I\bar{J}} f^I f^{\bar{J}}} \,, \tag{6.5}$$

$$d_A = i\rho_A v_A^I f_I \,, \tag{6.6}$$

$$|d_A| \leq \rho_A \sqrt{\delta_{I\bar{J}} f^I f^{\bar{J}}} \,. \tag{6.7}$$

Notice that the bound (6.7) and the flatness condition (6.4) imply, together, the following limits on the sizes of the auxiliary fields:

$$\sqrt{\delta_{I\bar{J}} f^I f^{\bar{J}}} \in \left[\frac{1}{\sqrt{1 + \sum_B \rho_B^2}}, 1 \right], \qquad \sqrt{\sum_A d_A^2} \in \left[0, \sqrt{\frac{\sum_B \rho_B^2}{1 + \sum_B \rho_B^2}} \right]. \tag{6.8}$$

The size of the d_A auxiliary field is therefore limited by the kinematical bound for small ρ_A and by the flatness condition for large ρ_A.

To derive the implications of the constraints (6.4), one should take into account the fact that f^I and d^A are not independent variables. There are then several possible strategies that one can follow. A first possibility is to use the the dynamical relation (6.5) to write the d_A in terms of the f^I. One can then consider the parameters $R_{I\bar{J}P\bar{Q}}$, M_A and $T_{AI\bar{J}}$ as fixed and take as free variables the f^I. In this way the equations have a reasonably simple tensor structure, but they become of higher order in the variables. As a consequence, they can in principle be solved only in special limits on the values of the parameters, like for instance the limits of heavy and light vectors, where the relation (6.5) simplifies and the effects induced by the d_A represent only small corrections. A second possibility is to use the kinematical relation (6.6) to write the d^A in term of the f^I. One can then consider the parameters $R_{I\bar{J}P\bar{Q}}$ and X_A^I as fixed, and the quantities f^I as free variables. The equations have then the same structure as for a theory with only chiral multiplets, with at most quartic terms, but their tensor structure is substantially more involved. Therefore in general the equations cannot be solved, and this type of analysis is efficient only in special cases, where $R_{I\bar{J}P\bar{Q}}$ and/or X_A^I have special properties. A third possibility is to impose only the kinematical bound (6.7) to restrict the values of the d^A in terms of the f^I. One can then consider the parameters $R_{I\bar{J}P\bar{Q}}$ and M_A as fixed, and the quantities f^I and d^A as free variables subject only to a bound. By doing so, one clearly looses a substantial amount of information, and the resulting conditions will therefore be weaker than the original ones, although still necessary. This is similar in spirit to what was done in [18] to study cases with only chiral multiplets but with a Kähler geometry leading to a complicated structure of the Riemann tensor $R_{I\bar{J}P\bar{Q}}$.

It is clear that switching from the dynamical relation (6.5) to the kinematical relation (6.6) and finally to the kinematical bound (6.7) represents a gradual simplification of the formulae, which is also accompanied by a loss of information. As a consequence, the three different types of strategies we just described, which are based on the use of these three different relations, will be tractable over an increasingly larger domain of parameters, but this will be accompanied by a gradual weakening of the implied constraints. In the following sections we will perform more explicitly these different analyses and see what kind of information can be obtained with each of them.

7   Exploiting the dynamical relation between d_A and f_I

As already discussed in section 4, whenever the masses M_A of the vector multiplets are all much larger or much smaller than $m_{3/2}$, that is $\rho_A \gg 1$ or $\rho_A \ll 1$, the situation simplifies. In those limits (and assuming that the quantities $T_{AI\bar{J}}$ remain finite) the auxiliary fields d_A become small, and the dynamical relation (6.5) simplifies. It becomes then convenient to use it and to apply the first strategy outlined in section 6.

For the purpose of this section, it is useful to rewrite the conditions (6.4) in a slightly different form, which exhibits in a better way the effect of the vector multiplets with respect to the situation involving only chiral multiplets. To do so, we define new rescaled variables in such a way to reabsorb the non-trivial right-hand side of the flatness condition, and to transfer all the non-trivial effect of the vector multiplets into the right-hand side of the stability condition. This can be done by introducing the following new quantities:

$$z^I = \frac{f^I}{\sqrt{1 - \sum_A d_A^2}} \, . \tag{7.1}$$

Using these new variables z^I instead of the f^I, the flatness and stability conditions (6.4) can be rewritten as:

$$\begin{cases} \delta_{I\bar{J}} \, z^I z^{\bar{J}} = 1 \, , \\ R_{I\bar{J}P\bar{Q}} \, z^I z^{\bar{J}} z^P z^{\bar{Q}} \leq \dfrac{2}{3} \, K(d_A^2, \rho_A^2) \, , \end{cases} \tag{7.2}$$

where

$$K(d_A^2, \rho_A^2) = 1 + 4 \, \frac{\sum_A \rho_A^2 d_A^2 - \left(\sum_A d_A^2 \right)^2}{\left(1 - \sum_B d_B^2 \right)^2} \, . \tag{7.3}$$

Due to the non-linearity of the change of variable (7.1), the dynamical relation (6.5) becomes more complicated and does not allow us to explicitly express the variables d_A in terms of the new z_I. Similarly, the kinematical relation (6.6) and the kinematical bound (6.7) also get modified. More explicitly, these three relations between auxiliary fields take

now the following forms:

$$d_A \frac{1 + \rho_A^2 - 3/2 \sum_B d_B^2}{1 - \sum_B d_B^2} = \sqrt{\frac{3}{2}} \, \rho_A \, T_{AI\bar{J}} \, z^I z^{\bar{J}} \,, \tag{7.4}$$

$$\frac{d_A}{\sqrt{1 - \sum_B d_B^2}} = i \rho_A v_A^I z_I \,, \tag{7.5}$$

$$\frac{|d_A|}{\sqrt{1 - \sum_B d_B^2}} \leq \rho_A \,. \tag{7.6}$$

This formulation of the constraints becomes identical to the previous one for small d_A. It presents however some advantages. Note in particular that in the limit $d_A \ll 1$ the new variables z^I defined by (7.1) approximately coincide with the f^I, and can be therefore efficiently used to study the leading order effects due to vector multiplets.

We will now show that for the cases we are going to study in this section, the conditions (7.2) can be rewritten in the form:

$$\begin{cases} \delta_{I\bar{J}} \, z^I z^{\bar{J}} = 1 \,, \\ \tilde{R}_{I\bar{J}P\bar{Q}} \, z^I z^{\bar{J}} z^P z^{\bar{Q}} \leq \dfrac{2}{3} \,. \end{cases} \tag{7.7}$$

These equations have the same functional form as the ones found in the case with only chiral multiplets. However the relevant variables z^I are rescaled with respect to the original variables f_I, and the coefficients $\tilde{R}_{I\bar{J}P\bar{Q}}$ are shifted with respect to the components of the Riemann tensor $R_{I\bar{J}P\bar{Q}}$.

7.1 Heavy vector limit

In the limit $\rho_A \gg 1$, the relation (7.4) implies that the d_A are small and given by:

$$d_A \simeq \sqrt{\frac{3}{2}} \rho_A^{-1} \, T_{AI\bar{J}} \, z^I z^{\bar{J}} \,. \tag{7.8}$$

In the small d_A limit the function K in (7.3) can then be simplified by keeping only the leading term and evaluating it by using eq. (7.8). This results in a quartic dependence on the variables z_i. One can then rewrite the conditions in the form (7.7), with an effective curvature tensor given by:

$$\tilde{R}_{I\bar{J}P\bar{Q}} \simeq R_{I\bar{J}P\bar{Q}} - 2 \sum_A \left(T_{AI\bar{J}} T_{AP\bar{Q}} + T_{AI\bar{Q}} T_{AP\bar{J}} \right). \tag{7.9}$$

In addition, note that since $d_A \simeq 0$, the kinematical relation (7.5) implies the m constraints $v_A^I z_I \simeq 0$.

These formulae show that in the limit in which the vector multiplets are heavy, they give two kinds of effects. On one hand, they induce a correction to the Kähler curvature for the chiral multiplets. On the other hand, they reduce the number of relevant variables z^I coming from the chiral multiplets, as some of the directions are associated to the would-be Goldstone chiral multiplets that are absorbed by the vector multiplets. It is clear from

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the form of the new tensor (7.9) that the correction on the curvature induced by the vector multiplets is negative, so the net effect is (compared with the situation with just chiral fields) to help in fulfilling the bound (7.7). Note that the correction in (7.9) is not necessarily small and can be of $\mathcal{O}(1)$. Actually, even for $T_{AI\bar{J}} \sim \mathcal{O}(1)$, one has $z_I \sim \mathcal{O}(1)$ and $d_A \sim \mathcal{O}(\rho_A^{-1})$, so that the ratio $d_A/z_I \sim \mathcal{O}(\rho_A^{-1})$ is still small, as assumed.

We can understand this result also from a slightly different point of view. We see from (7.9) that the correction induced by a heavy vector field to the Kähler curvature is negative and of order M_A^{-2}. This is by assumption small compared to $m_{3/2}^{-2}$, but it can still be large compared to 1 (in Planck units). The original curvature R of the chiral multiplet theory is instead of order Λ^{-2}, where Λ is the scale of the most relevant higher-dimensional operator. One must certainly have $\Lambda \gg m_{3/2}$, and gravitational effects correspond to $\Lambda \sim 1$. The effects due to vector multiplets can thus really compete with the curvature effects due to chiral multiplets, or even dominate over them. In particular, when the original curvature effects are of gravitational origin, the vector multiplet effects become significant when $M_A \sim 1$. This is typically what happens for spontaneously broken anomalous $U(1)$ symmetries in string models.

As we explained in section 4, in the limit $\rho_A \gg 1$ it should be possible to integrate out the vector multiplets and reinterpret their net effect through a correction to the Kähler potential of the chiral multiplets. It is interesting to verify that such a procedure indeed reproduces (7.9). To do so, we can use superfields and follow the same steps as in subsection 4.1. Recall in particular that the correction to the potential G was shown to be equal to $\Delta G \simeq -\sum_A M_A^{-2} G_A^2$. Since in flat indices the Riemann tensor is given by the fourth derivative of the potential G, the corresponding correction to the curvature is:

$$\Delta R_{I\bar{J}P\bar{Q}} \simeq -\sum_A M_A^{-2} \nabla_I \nabla_{\bar{J}} \nabla_P \nabla_{\bar{Q}} (G_A^2) \,. \tag{7.10}$$

Taking into account that $\nabla_I \nabla_J G_A = 0$ and $\nabla_I \nabla_{\bar{J}} G_A = -q_{AI\bar{J}}$, one recovers then the second term in the expression (7.9). Notice however that this simplified derivation of (7.9) is valid only under the assumption that the correction is small, whereas as already emphasized, (7.9) is actually valid also in more general situations where the correction can be sizable.

7.2   Light vector limit

In the limit $\rho_A \ll 1$, the bound (7.6) and the relation (7.4) imply that the d_A are small and given by

$$d_A \simeq \sqrt{\frac{3}{2}} \, \rho_A \, T_{AI\bar{J}} \, z^I z^{\bar{J}} \,. \tag{7.11}$$

Again, as in the previous subsection, we can simplify the function K by keeping only the quartic term in z_i. This allows us to finally rewrite the conditions (7.2) in the form (7.7), with an effective curvature tensor now given by:

$$\tilde{R}_{I\bar{J}P\bar{Q}} \simeq R_{I\bar{J}P\bar{Q}} - 2\sum_A \rho_A^4 \left(T_{AI\bar{J}} T_{AP\bar{Q}} + T_{AI\bar{Q}} T_{AP\bar{J}} \right) \,. \tag{7.12}$$

Therefore, the net effect of the vector multiplets is as before to reduce the Kähler curvature (with respect to the case with only chiral multiplets). In addition, the number of relevant directions in field space is again reduced, since the kinematical bound (7.6) together with the dynamical relation (7.11) imply the m constraints $T_{AI\bar{J}} z^I z^{\bar{J}} \simeq 0$.

7.3   Arbitrary vector masses

One may wonder whether it is possible to generalize the analyses done in subsections 7.1 and 7.2 to the case of arbitrary vector masses. In particular, the crucial question is whether the effect of vector multiplets could again be essentially encoded into a modification of the Riemann tensor. It is clear from the structure of the equations (7.2) that in general this is not going to be the case. However, we will see that the original constraints imply some other weaker constraints, which have indeed this form but contain less information except in the cases of large or small masses.

In the case of only one vector multiplet, the flatness condition guarantees that the numerator in (7.4) never vanishes and stays always positive. Moreover, the kinematical bound (7.6) implies that it reaches its minimal value for $d = \rho/\sqrt{1 + \rho^2}$. Using this result, we can then get an upper bound for the quantity $|d|/(1 - d^2)$ by evaluating the numerator at its minimum. In this way we find that:

$$\frac{|d|}{1 - d^2} \leq \sqrt{\frac{3}{2}} \frac{\rho \left(1 + \rho^2\right)}{1 + \rho^2/2 + \rho^4} \left|T_{I\bar{J}} z^I z^{\bar{J}}\right|. \tag{7.13}$$

We can now find an upper bound to the function K by neglecting the negative term in (7.3) and using the bound (7.13). We can finally substitute this upper bound for K in the conditions (7.2) and deduce a simpler but weaker set of the conditions. These have once again the form (7.7), where now:

$$\tilde{R}_{I\bar{J}P\bar{Q}} = R_{I\bar{J}P\bar{Q}} - 2\,\epsilon(\rho)\left(T_{I\bar{J}} T_{P\bar{Q}} + T_{I\bar{Q}} T_{P\bar{J}}\right), \tag{7.14}$$

with a function $\epsilon(\rho)$ given by:

$$\epsilon(\rho) = \frac{\rho^4 \left(1 + \rho^2\right)^2}{\left(1 + \rho^2/2 + \rho^4\right)^2}. \tag{7.15}$$

This expression is valid for arbitrary ρ and in the limits $\rho \gg 1$ and $\rho \ll 1$ it correctly reproduces (7.9) and (7.12).

In the more general case of several vector multiplets, the situation is more complicated, due to the fact that the numerator in the left-hand side of the dynamical relation (7.4) can vanish [13]. For this reason it is no longer possible to find a simple bound of the type (7.13) valid for all values of the parameters ρ_A. One can however still derive a useful bound in a large domain of the space of values of the parameters ρ_A. To see which domain should be considered, note that the kinematical bound implies that:

$$1 + \rho_A^2 - \frac{3}{2}\sum_B d_B^2 \geq \frac{1 + \rho_A^2 - \sum_B \rho_B^2/2 + \rho_A^2 \sum_B \rho_B^2}{1 + \sum_B \rho_B^2}. \tag{7.16}$$

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[13]It is however clear that whenever this numerator becomes small, also the right-hand side of the equation must do so.

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The right-hand side of (7.16) is positive if

$$\rho_A^2 \geq \frac{1}{4}\left(-1 - 2\sum_{B\neq A}\rho_B^2 + \sqrt{4\left(\sum_{B\neq A}\rho_B^2\right)^2 + 12\sum_{B\neq A}\rho_B^2 - 15}\right).$$ (7.17)

The function in the right hand side of the inequality (7.17) vanishes when $\sum_{B\neq A}\rho_B^2 = 2$. For lower values, $\sum_{B\neq A}\rho_B^2 < 2$, the function is negative and the condition is thus always satisfied. On the other hand, for larger values, $\sum_{B\neq A}\rho_B^2 > 2$, the function is positive and there is thus a non-trivial restriction on the parameters. It actually turns out that in this case this function grows monotonically up to the maximal value of $1/2$, so if $\rho_A^2 > 1/2$ the condition (7.17) is automatically satisfied. This shows that the numerator in (7.16) can vanish only in very special valleys of parameters space, where some of the ρ_A are small and some other are large. On the contrary, if all of them are either not too small or no too large, then this problem does not appear. We can then restrict our analysis to the following two regions of parameter space:

$$I_> = \left\{\vec{\rho}\,\middle|\,\rho_A^2 \geq \frac{1}{2}, \forall A\right\}, \qquad I_< = \left\{\vec{\rho}\,\middle|\,\rho_A^2 \leq 2, \forall A\right\}.$$ (7.18)

In the domain $I_< \cup I_>$ one can then deduce the following simple upper bound for (7.4):

$$\frac{|d_A|}{1 - \sum_B d_B^2} \leq \sqrt{\frac{3}{2}}\frac{\rho_A\left(1 + \sum_B\rho_B^2\right)}{1 + \rho_A^2 - \sum_B\rho_B^2/2 + \rho_A^2\sum_B\rho_B^2}\left|T_{AI\bar{J}}z^I z^{\bar{J}}\right|.$$ (7.19)

This is the obvious generalization of the bound (7.13) for the case of more than one vector multiplet. One can then proceed as before, and use this to derive an upper bound for the function K, which can then be substituted in the original conditions to deduce a simpler but weaker set of constraints. In this way one finds once again (7.7), with an effective curvature of the form

$$\tilde{R}_{I\bar{J}P\bar{Q}} = R_{I\bar{J}P\bar{Q}} - 2\sum_A\epsilon(\rho_A)\left(T_{AI\bar{J}}T_{AP\bar{Q}} + T_{AI\bar{Q}}T_{AP\bar{J}}\right),$$ (7.20)

where $\epsilon(\rho_A)$ is now given by

$$\epsilon(\rho_A) = \frac{\rho_A^2\left(1 + \sum_B\rho_B^2\right)^2}{\left(1 + \rho_A^2 - \sum_B\rho_B^2/2 + \rho_A^2\sum_B\rho_B^2\right)^2}.$$ (7.21)

This expression correctly reproduces (7.9) and (7.12) in the limits $\rho_A \gg 1$ and $\rho_A \ll 1$. It also reproduces (7.15) in the case of a single vector multiplet, since in that case $I_< \cup I_>$ coincides with the whole parameter space.

7.4 Simple scalar geometries

It was shown in [17, 18] that in the case of theories with only chiral multiplets the conditions of flatness and stability could be solved exactly for certain particular classes of scalar manifolds leading to a simple structure for the Riemann tensor. This is for instance the case when the scalar manifold factorizes in a product of one-dimensional scalar manifolds (factorizable scalar manifolds), or when it is a symmetric coset group manifold of the

form G/H (symmetric scalar manifolds). It is then illustrative and physically interesting to apply the results derived in this subsection to these simple examples.

For factorizable manifolds, the metric is diagonal and therefore the structure of the Riemann tensor simplifies to $R_{I\bar{J}P\bar{Q}} = R_I\,\delta_{I\bar{J}P\bar{Q}}$. Assuming then also that the Killing potentials are separable, so that $T_{AI\bar{J}} = q_{AI}/M_A\,\delta_{I\bar{J}}$, the corrected effective curvature appearing in (7.20) takes the following simple form:

$$\tilde{R}_{I\bar{J}P\bar{Q}} = R_I\,\delta_{I\bar{J}P\bar{Q}} - 2\sum_A \epsilon(\rho_A)\frac{q_{AI}q_{AP}}{M_A^2}\left(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\right). \tag{7.22}$$

This is no longer separable, but the problem can however still be solved exactly. Indeed, the flatness and stability conditions (7.7) reduce to

$$\begin{cases} \sum_I |z^I|^2 \simeq 1\,, \\ \sum_{I,J}\tilde{R}_{IJ}\,|z^I|^2\,|z^J|^2 \leq \dfrac{2}{3}\,, \end{cases} \tag{7.23}$$

where the n-dimensional matrix $\tilde{R}_{IJ}$ is given by

$$\tilde{R}_{IJ} = R_I\,\delta_{IJ} - 4\sum_A \epsilon(\rho_A)\frac{q_{AI}\,q_{AJ}}{M_A^2}\,. \tag{7.24}$$

The values of the variables z^I minimizing the stability condition subject to the flatness constraint are easily found to be $|z^I|^2 = \sum_J(\tilde{R}^{-1})_{IJ}/\sum_{R,S}(\tilde{R}^{-1})_{RS}$. Substituting these values into the stability bound, one finds then the following necessary condition:

$$\sum_{I,J}(\tilde{R}^{-1})_{IJ} \geq \frac{3}{2}\,. \tag{7.25}$$

Whenever the correction to the curvature induced by the vector multiplets is small, the inverse of the matrix (7.24) is easily found and the above curvature constraint simplifies to:

$$\sum_A \epsilon(\rho_A)\frac{(q_{AI}R_I^{-1})^2}{M_A^2} \gtrsim \frac{1}{4}\left(\frac{3}{2} - \sum_I R_I^{-1}\right). \tag{7.26}$$

It is then clear that in a case where $\sum_I R_I^{-1}$ is only slightly lower than $3/2$, the effect of vector multiplets can help in satisfying the bound.

For maximally symmetric scalar manifolds, the Riemann tensor is related to the metric and takes the form $R_{I\bar{J}P\bar{Q}} = R_{\text{all}}/2\,(\delta_{I\bar{J}}\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\delta_{P\bar{J}})$, where R_{all} is an overall curvature scale. Assuming then as before that the Killing potentials are diagonal, $T_{AI\bar{J}} = q_{AI}/M_A\,\delta_{I\bar{J}}$, the corrected effective curvature takes the form:

$$\tilde{R}_{I\bar{J}P\bar{Q}} = \left(\frac{R_{\text{all}}}{2} - 2\sum_A \epsilon(\rho_A)\frac{q_{AI}\,q_{AP}}{M_A^2}\right)\left(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\right). \tag{7.27}$$

This is still of the maximally symmetric form, and the flatness and stability conditions (7.7) can then be written in the form:

$$\begin{cases} \sum_I |z^I|^2 \simeq 1\,, \\ \sum_{I,J}\Delta\tilde{R}_{I\bar{J}}\,|z^I|^2\,|z^J|^2 \geq R_{\text{all}} - \dfrac{2}{3}\,, \end{cases} \tag{7.28}$$

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where the n-dimensional matrix $\Delta \tilde{R}_{I\bar{J}}$ is given by

$$\Delta \tilde{R}_{I\bar{J}} = 4 \sum_A \epsilon(\rho_A) \frac{q_{AI}\, q_{AJ}}{M_A^2}\,. \tag{7.29}$$

The values of the z^I that optimize the stability bound taking into account the flatness condition are again easily found, and read in this case $|z^I|^2 = \sum_J (\Delta \tilde{R}^{-1})_{IJ}/\sum_{R,S}(\Delta \tilde{R}^{-1})_{RS}$. Substituting these values into the stability bound, one finds then the following necessary condition:

$$\sum_{R,S}(\Delta \tilde{R}^{-1})_{RS} \leq \left(R_{\text{all}} - \frac{2}{3}\right)^{-1}\,. \tag{7.30}$$

This means that whenever $R_{\text{all}} \geq 2/3$, there need to be vector fields with sufficiently low mass and large charges to avoid unstable modes. For instance, in the particular case with a single vector multiplet, the necessary condition is just $q_{\text{max}}/M \geq 1/2\sqrt{R_{\text{all}} - 2/3}$.

8 Exploiting the kinematical relation between d_a and f_I

Whenever the Riemann tensor and the charge matrix have simple tensor structures, the kinematical relation (6.6) can become useful in the task of solving the constraints (6.4). This is the case, for instance, when the scalar manifold is factorizable and the isometries that are gauged are each aligned along a single one-dimensional submanifold. In such a situation the Riemann tensor is diagonal and has the form $R_{I\bar{J}P\bar{Q}} = R_I\, \delta_{I\bar{J}P\bar{Q}}$ and each Killing vector has a single non-vanishing component, $X_A^I = X^I\, \delta_A^I$. In this situation it is convenient to use the second strategy outlined in section 6. In this case the number of chiral and vector multiplets are equal: $m = n$ and we can then identify the chiral and vector indices $I, J, \ldots$ and $A, B, \ldots$, keeping in mind that simpler cases with less vector multiplets can be described by simply setting some of the X^I to 0 [14].

The crucial simplification in this case is that, due to the kinematical relation (4.1), each vector auxiliary field d_I is proportional to the corresponding chiral auxiliary field f_I. For each one-dimensional subspace of the scalar manifold, the ratio of the two auxiliary fields is actually fixed by the mass of the corresponding vector field: $|d_I|/|f_I| = \rho_I$. The orientation of the Goldstino direction along the n one-dimensional subspaces of the scalar manifold is instead arbitrary. It can be parametrized by n variables x_i related to the absolute sizes of $|f_i|$ and $|d_i|$ in each of these subspaces. A convenient choice is to define:

$$x_I = \sqrt{|f_I|^2 + |d_I|^2}\,. \tag{8.1}$$

---

[14]It should be emphasized that this simple example where the space factorizes in a number of sectors with one pair of chiral and vector multiplets each is rather restrictive. Indeed, the requirement of gauge invariance of G severely restricts the possible form of the superpotential W. In the case where both K and W are gauge invariant, W can only be a constant, implying that non-zero f_I and d_I can be induced only through a non-trivial K. A more general possibility is that K and W have gauge variations that compensate each other. This fixes however almost uniquely the form of W in terms of the form of K, and one finds again a very similar situation.

The flatness and stability conditions can then be rewritten in the following very simple form:

$$
\begin{cases}
\sum_I x_I^2 = 1 \,, \\
\sum_{I,J} \tilde{R}_{IJ}\, x_I^2\, x_J^2 + \dfrac{2}{3} \sum_I E_I\, x_I^2 \leq \dfrac{2}{3}\,,
\end{cases}
\tag{8.2}
$$

where

$$
\tilde{R}_{IJ} = \frac{R_I\, \delta_{IJ} + 2\, \rho_I^2 \rho_J^2}{\left(1 + \rho_I^2\right)\left(1 + \rho_J^2\right)}\,, \qquad
E_I = \frac{2\, \rho_I^2 \left(1 - 2\, \rho_I^2\right)}{\left(1 + \rho_I^2\right)}\,.
\tag{8.3}
$$

The problem can now be solved along the same lines as in [17, 18]. The constraint (8.2) can be interpreted as an upper bound on the function $f(x_I) = \sum_{I,J} R_{IJ}\, x_I^2\, x_J^2 + 2/3 \sum_I E_I\, x_I^2$, where the real variables x_I range from 0 to 1 and are subject to the constraint $\sum_K x_K^2 = 1$. In particular, the inequality (8.2) implies that $f_{\min} < 2/3$, where $f_{\min}$ is the minimum value of $f(x_I)$ within the allowed range for the x_I. Finding $f_{\min}$ is a constrained minimization problem which can be solved in the standard way using Lagrangian multipliers. It is straightforward to show that the values of the variables at the minimum are given by

$$
x_I^2 = \frac{1}{\sum_{P,Q} \tilde{R}_{PQ}^{-1}} \left[\sum_J \tilde{R}_{IJ}^{-1} - \frac{1}{3} \sum_{J,K,L} \tilde{R}_{IJ}^{-1} \tilde{R}_{KL}^{-1} \left(E_J - E_L\right) \right].
\tag{8.4}
$$

The condition $f_{\min} < 2/3$ then implies that the constraint on the curvatures takes the form:

$$
\sum_{I,J} \tilde{R}_{IJ}^{-1}\left(1 - E_I\right) - \frac{1}{6} \sum_{I,J,P,Q} \tilde{R}_{IJ}^{-1} \tilde{R}_{PQ}^{-1}\left(E_J - E_P\right) E_Q \geq \frac{3}{2}\,.
\tag{8.5}
$$

The expression (8.5) represents the main result of this section. It correctly reduces to the condition $\sum_I R_I^{-1} > 3/2$ in the limit where all the gauge couplings are switched off. The matrices $\tilde{R}_{IJ}$ are positive definite and reduce to $R_I \delta_{IJ}$ at zero coupling, whereas the parameters E_I can be either positive or negative but vanish at zero coupling.

Using the expression of $\tilde{R}_{IJ}$, the inequality (8.5) can actually be rewritten more explicitly in terms of the inverse curvatures R_I^{-1}. The result takes the form:

$$
\sum_I \alpha_I\, R_I^{-1} + \sum_{I,J} \beta_{IJ}\, R_I^{-1} R_J^{-1} + \sum_{I,J,K} \gamma_{IJK}\, R_I^{-1} R_J^{-1} R_K^{-1} \geq \frac{3}{2}\,,
\tag{8.6}
$$

where the coefficients α_I, β_{IJ} and γ_{IJK} are positive and depend on the mass parameters ρ_I as follows:

$$
\begin{aligned}
\alpha_I &= 1 + 4\, \rho_I^6\,, \\
\beta_{IJ} &= \frac{8}{3}\left(\rho_I^2 - \rho_J^2\right)^2 \left(1 - \rho_I^2 - \rho_J^2 + \rho_I^4 + \rho_J^4 - 2\rho_I^2 \rho_J^2 + 2\, \rho_I^4 \rho_J^2 + 2\, \rho_J^4 \rho_I^2 + \rho_I^4 \rho_J^4\right), \\
\gamma_{IJK} &= \frac{16}{3}\left(\rho_I^2 - \rho_J^2\right)^2 \left(\rho_I^2 - \rho_K^2\right)^2 \left(\rho_J^2 - \rho_K^2\right)^2.
\end{aligned}
\tag{8.7}
$$

Note that in the limit $\rho_I \to 0$ the constraint (8.6) reduces to the correct condition $\sum_I R_I^{-1} \geq 3/2$ that was found in [17, 18] for the case of theories with only chiral multiplets. Actually, as all the coefficients α_I, β_{IJ} and γ_{IJK} are positive and, in particular, $\alpha_I > 1$ then the condition (8.6) is less stringent than the condition found in that case. Note also that for large ρ_I the bound is trivially satisfied.

8.1 One pair of chiral and vector multiplets

In the simplest case of models involving 1 chiral multiplet and 1 vector multiplet, the situation is particularly simple. The flatness condition fixes $x = 1$. The auxiliary fields can then be parametrized as $|f| = \cos\delta$ and $|d| = \sin\delta$, where the angle δ is completely fixed by the parameter ρ as $\rho = \tan\delta$. The condition (8.6) involves in this case only the first term so is linear in the inverse curvatures, and it implies the constraint:

$$R \leq \frac{2}{3}\left(1 + 4\tan^6\delta\right). \tag{8.8}$$

This shows that the stability condition can always be satisfied for sufficiently large values of δ; more precisely, one needs $\delta \in [\delta_{\min}, \pi/2]$, where

$$\delta_{\min} = \begin{cases} 0, & R < \dfrac{2}{3}, \\ \arctan\left[\dfrac{3}{8}\left(R - \dfrac{2}{3}\right)\right]^{1/6}, & R > \dfrac{2}{3}. \end{cases} \tag{8.9}$$

As expected, the bound $R < 2/3$ found in the limit of vanishing δ (in which the effect of the vector multiplet is negligible, and we recover the result found for just one chiral field), get corrected for non-vanishing δ. Actually when the effect of the vector multiplet starts becoming important the bound gets milder and eventually trivializes for large δ. Note however that the first correction appears only at sixth order in δ.

8.2 Two pairs of chiral and vector multiplets

The next-to-simplest case is the case of models involving 2 chiral multiplets and 2 vector multiplets. The solution of the flatness condition can in this case be parametrized in terms of an arbitrary angle θ so that $x_1 = \cos\theta$ and $x_2 = \sin\theta$. The auxiliary fields can then be written as $f_1 = \cos\theta\cos\delta_1$, $d_1 = \cos\theta\sin\delta_1$, $f_2 = \sin\theta\cos\delta_2$, $d_2 = \sin\theta\sin\delta_2$, where the angles $\delta_{1,2}$ are completely fixed by $\rho_{1,2} = \tan\delta_{1,2}$. The condition (8.6) involves in this case the first two terms, which are linear and quadratic in the curvatures, and implies a rather complicated constraint involving the curvatures $R_{1,2}$ and the angles $\delta_{1,2}$. From its structure, and the fact that the relevant coefficients α_I and β_{IJ} are positive, it is however clear that this constraint is milder than the constraints that would arise for each pair of chiral and vector multiplets on its own. For example, in the particular case where the two sectors are identical one would find the same constraint as for a single basic sector with a pair of chiral and vector multiplet, but with an effective curvature reduced by a factor of 2, as is also the case when only chiral multiplets are present.

8.3 Several pairs of chiral and vector multiplets

In the more general case of models involving an arbitrary number of pairs of chiral and vector multiplets, the situation is even more complicated. Nevertheless it is interesting to point out that for situations where the parameters ρ_I satisfy certain properties, it is possible to derive a condition that is linear, rather than cubic, in the inverse curvatures

R_I^{-1}. More precisely, this is the case in the limit where all the ρ_I are such that $\rho_I^2 \le 1/2$. In that case, the quantity E_I is positive definite, so then it is possible to get a new (but weaker) condition by disregarding the term involving E_I from the conditions (8.2). Following the same procedure that was used to derive eq. (8.6), we get the necessary condition:

$$\sum_I \left(1 + \rho_I^2\right)^2 R_I^{-1} \ge \frac{3}{2}. \tag{8.10}$$

This means that in this domain of parameters, namely $\rho_I^2 \le 1/2$, and the approximation considered here, the net effect of the vector multiplets is to effectively reduce the curvatures R_I by a factor $(1 + \rho_I^2)^{-2}$, that is, $\tilde{R}_I = \left(1 + \rho_I^2\right)^{-2} R_I$.

9 Exploiting the kinematical bound between d_a and f_I

In more general cases, for which the analyses of sections 7 and 8 cannot be applied, one may try to work out the implications of the constraints (6.4) by applying the third strategy outlined in section 6. This consists in considering both the f^I and the d^A as independent variables, constrained only by the kinematical bound (6.7). This approach is simple and can be worked out in full generality; however it clearly ignores a substantial amount of information concerning the actual dynamical and kinematical relations between the f^I and the d^A. The resulting implications will therefore be weaker than the ones derived in the previous sections. Note in this respect that the kinematical bound represents a significant constraint only when $\rho_A \lesssim 1$, whereas it becomes trivial when $\rho_A \gg 1$. We therefore expect that the condition resulting from this analysis will be stronger for small ρ_A and will become weaker and weaker for increasing ρ_A.

For this analysis, it is convenient to use the new variables z^I defined in (7.1) instead of the f_I, and also similarly rescaled variables ϵ_A instead of the d_A. More precisely, we consider the following change of variables:

$$z^I = \frac{f^I}{\sqrt{1 - \sum_A d_A^2}}, \qquad \epsilon_A = \frac{d_A}{\sqrt{1 - \sum_B d_B^2}}. \tag{9.1}$$

Using these new variables, the flatness and stability conditions (7.2) can be rewritten in the simple form:

$$\begin{cases} \delta_{I\bar{J}} z^I z^{\bar{J}} = 1, \\ R_{I\bar{J}P\bar{Q}} z^I z^{\bar{J}} z^P z^{\bar{Q}} \le \frac{2}{3} K(\epsilon_A^2, \rho_A^2), \end{cases} \tag{9.2}$$

where now

$$K(\epsilon_A^2, \rho_A^2) = 1 + 4 \sum_A \rho_A^2 \, \epsilon_A^2 - 4 \sum_{A,B} \epsilon_A^2 \, \epsilon_B^2 + 4 \sum_{A,B} \rho_A^2 \, \epsilon_A^2 \, \epsilon_B^2, \tag{9.3}$$

and the kinematical bound (6.7) becomes simply:

$$|\epsilon_A| \le \rho_A. \tag{9.4}$$

It is now clear from (9.2) that the most favorable situation is when the variables z^I minimize the function $R_{I\bar{J}P\bar{Q}}\, z^I z^{\bar{J}} z^P z^{\bar{Q}}$ and the variables ϵ_A maximize the function $K(\epsilon_A^2, \rho_A^2)$. The maximization of $K(\epsilon_A^2, \rho_A^2)$ with respect to ϵ_A should be done taking into account the bound (9.4):

$$K_{\text{best}}(\rho_A^2) = \max\left\{ K(\epsilon_A^2, \rho_A^2) \,\Big|\, \epsilon_A^2 \le \rho_A^2 \right\}. \tag{9.5}$$

The computation of this quantity is complicated by the fact that the function $K(\epsilon_A^2, \rho_A^2)$ depends on all the variables ϵ_A^2, and that its maximum may lie inside region $\epsilon_A^2 < \rho_A^2$ or at the boundary $\epsilon_A^2 = \rho_A^2$, depending on the values of the parameters ρ_A^2. It is then a bit laborious, although straightforward, to characterize the constrained maximum of the function $K(\epsilon_A^2, \rho_A^2)$ over the full range of the parameters ρ_A^2. Once this maximal value $K_{\text{best}}(\rho_A^2)$ has been found, one can substitute it in the stability condition to get the following necessary conditions:

$$\begin{cases} \delta_{I\bar{J}}\, z^I z^{\bar{J}} = 1\,, \\ \tilde{R}_{I\bar{J}P\bar{Q}}\, z^I z^{\bar{J}} z^P z^{\bar{Q}} \le \dfrac{2}{3}\,, \end{cases} \tag{9.6}$$

where

$$\tilde{R}_{I\bar{J}P\bar{Q}} = K_{\text{best}}(\rho_A^2)^{-1} R_{I\bar{J}P\bar{Q}}\,. \tag{9.7}$$

Again, from the structure of this constrain we see that the net effect of the vector multiplets is to reduce the effective curvature that is perceived by the chiral multiplets, in this case by the multiplicative factor K_{best}^{-1}, (which is clearly smaller than 1). Unfortunately, it does not seem to be possible to find a closed expression for K_{best} in general. We will thus first examine in detail the simplest models with 1 and 2 vector multiplets, and then discuss what can be said about the general case with n vector multiplets.

9.1   One vector field

In the presence of one vector field, the function (9.3) takes the form

$$K(\epsilon^2, \rho^2) = 1 + 4\,\rho^2\,\epsilon^2 + 4\left(\rho^2 - 1\right)\epsilon^4\,. \tag{9.8}$$

The maximum of this function of ϵ^2 within the region $[0, \rho^2]$ sits either at the stationary point $\rho^2/(2 - 2\rho^2)$ or at the boundary point ρ^2, depending on the value of ρ^2. More precisely, if we define the two domains

$$I_1 = \left[0, \frac{1}{2}\right], \qquad I_2 = \left[\frac{1}{2}, +\infty\right[, \tag{9.9}$$

one finds:

$$\epsilon^2{}_{\text{best}} = \begin{cases} \dfrac{1}{2}\dfrac{\rho^2}{1 - \rho^2}\,, & \rho^2 \in I_1\,, \\[2mm] \rho^2\,, & \rho^2 \in I_2\,. \end{cases} \tag{9.10}$$

The corresponding maximal value of the function K is then:

$$K_{\text{best}} = \begin{cases} \dfrac{1 - \rho^2 + \rho^4}{1 - \rho^2}, & \rho^2 \in I_1, \\[2mm] 1 + 4\,\rho^6, & \rho^2 \in I_2. \end{cases} \tag{9.11}$$

Note that when a single chiral multiplet is present, the kinematical relation fixes $\epsilon^2 = \rho^2$, and one has to take the second branch of the expression (9.11) for any value of ρ. On the other hand, when two or more chiral multiplets are present, one can in principle have $\epsilon^2 < \rho^2$, and one needs to consider the first branch of (9.11) for small values of ρ.

9.2   Two vector fields

In the presence of two vector fields, the function (9.3) takes the form

$$K(\epsilon_{1,2}^2) = 1 + 4\left(\rho_1^2\,\epsilon_1^2 + \rho_2^2\,\epsilon_2^2\right) + 4\left(\rho_1^2 - 1\right)\epsilon_1^4 + 4\left(\rho_2^2 - 1\right)\epsilon_2^4$$
$$+ 4\left(\rho_1^2 + \rho_2^2 - 2\right)\epsilon_1^2\,\epsilon_2^2. \tag{9.12}$$

The problem is symmetric, and we can thus assume without loss of generality that $\rho_1^2 \leq \rho_2^2$. The maximum of the function (9.12) with respect to $(\epsilon_1^2,\epsilon_2^2)$ within the region $[0, \rho_1^2] \times [0, \rho_2^2]$ sits at the points $(\rho_1^2/(2-2\rho_1^2), 0)$ and $(\rho_1^2, \rho_2^2 + \rho_1^2(\rho_1^2 + \rho_2^2 - 2)/(2 - 2\rho_2^2))$, or at the boundary points $(\rho_1^2, 0)$ and (ρ_1^2, ρ_2^2). More precisely, the relevant domains turn out to be:

$$I_A = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[0, \frac{1}{2}\right],\, \rho_2^2 \in \left[0, \rho_1^2\right] \right\}, \tag{9.13}$$

$$I_B = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[\frac{1}{2}, 2\right],\, \rho_2^2 \in \left[0, \frac{2 - \rho_1^2}{1 + \rho_1^2}\rho_1^2\right] \right\}, \tag{9.14}$$

$$I_C = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[\frac{1}{2}, \frac{2}{\sqrt{7}}\right],\, \rho_2^2 \in \left[\frac{2 - \rho_1^2}{1 + \rho_1^2}\rho_1^2, \rho_1^2\right] \right\}, \tag{9.15}$$

$$I_D = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[\frac{2}{\sqrt{7}}, 2\right],\, \rho_2^2 \in \left[\frac{2 - \rho_1^2}{1 + \rho_1^2}\rho_1^2, \frac{1 - \rho_1^2 + \sqrt{1 + 14\rho_1^2 - 7\rho_1^4}}{4}\right] \right\}, \tag{9.16}$$

$$I_E = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[\frac{2}{\sqrt{7}}, 2\right],\, \rho_2^2 \in \left[\frac{1 - \rho_1^2 + \sqrt{1 + 14\rho_1^2 - 7\rho_1^4}}{4}, +\infty\right[\right\}, \tag{9.17}$$

$$I_F = \left\{ (\rho_1^2, \rho_2^2) \,\Big|\, \rho_1^2 \in \left[2, +\infty\right[,\, \rho_2^2 \in \left[0, +\infty\right[\right\}, \tag{9.18}$$

and the constrained maximum is located at the following points:

$$(\epsilon_1^2, \epsilon_2^2)_{\text{best}} = \begin{cases} \left(\dfrac{1}{2}\dfrac{\rho_1^2}{1 - \rho_1^2}, 0\right), & (\rho_1^2, \rho_2^2) \in I_A, \\[3mm] (\rho_1^2, 0), & (\rho_1^2, \rho_2^2) \in I_B, \\[3mm] \left(\rho_1^2, \dfrac{1}{2}\dfrac{\rho_2^2 + \rho_1^2(\rho_1^2 + \rho_2^2 - 2)}{1 - \rho_2^2}\right), & (\rho_1^2, \rho_2^2) \in I_C \cup I_D, \\[3mm] (\rho_1^2, \rho_2^2), & (\rho_1^2, \rho_2^2) \in I_E \cup I_F. \end{cases} \tag{9.19}$$

The corresponding maximal value of the function $K(\epsilon_{1,2}^2)$ is:

$$K_{\text{best}} = \begin{cases} \dfrac{1 - \rho_1^2 + \rho_1^4}{1 - \rho_1^2}, & (\rho_1^2, \rho_2^2) \in I_A, \\[2ex] 1 + 4\,\rho_1^6, & (\rho_1^2, \rho_2^2) \in I_B, \\[2ex] \dfrac{1 - \rho_2^2 + \rho_2^4 + \rho_1^2(\rho_1^2 + 2)\rho_2^2(\rho_2^2 - 2) - 2\rho_1^6\rho_2^2 + \rho_1^8}{1 - \rho_2^2}, & (\rho_1^2, \rho_2^2) \in I_C \cup I_D, \\[2ex] 1 + 4(\rho_1^6 + \rho_2^6) + 4\rho_1^2\rho_2^2(\rho_1^2 + \rho_2^2 - 2), & (\rho_1^2, \rho_2^2) \in I_E \cup I_F. \end{cases} \tag{9.20}$$

9.3 Arbitrary number of vector fields

For a larger number of vector multiplets, the maximization problem becomes more and more complex, and it does not seem to be possible to find the general solution. Nevertheless, it turns out to be possible to make some quantitative analysis also in this more general case, by either making some assumptions on the parameters ρ_A or by weakening the conditions.

In the particular case where all the parameters ρ_A are such that $\rho_A > 1$ it is clear from the form of the expression (9.3) that the constrained maximum of the function $K(\epsilon_A^2, \rho_A^2)$ is located at the corner of the parameter space where all the ϵ_a^2 are maximal, and therefore $\epsilon_{A\,\text{best}}^2 = \rho_A^2$. In terms of the variables d_A, this means that the preferred situation is the one in which $d_A^2 = \rho_A^2/(1 + \sum_B \rho_B^2)$. Note that this value is in general less than the maximal value that each d_A^2 is individually allowed to take by the kinematical bound, but their sum is instead equal to the maximal value $\sum_A d_A^2 = \sum_A \rho_A^2/(1 + \sum_B \rho_B^2)$ it is allowed to take. In other words, the optimal situation is realized in one particular direction in the space of the d_A such that $\sum_A d_A^2$ is maximal. The corresponding value of the function K is in this case:

$$K_{\text{best}} = 1 + 4\sum_A \rho_A^4 - 4\sum_{A,B}\rho_A^2\rho_B^2 + 4\sum_{A,B}\rho_A^4\rho_B^2. \tag{9.21}$$

Note that this expression correctly reproduces that last branches of the complete results (9.11) and (9.20) for 1 and 2 vector multiplets.

For completely generic values of the parameters ρ_A, it is possible to extract a weaker information from the constraints (9.2) by noticing that

$$K(\epsilon_A^2, \rho_A^2) \leq 1 + 4\,\rho^2\,\epsilon^2 + 4\,(\rho^2 - 1)\,\epsilon^4, \tag{9.22}$$

with:

$$\epsilon^2 = \sum_A \epsilon_A^2, \qquad \rho^2 = \sum_A \rho_A^2. \tag{9.23}$$

One can then use the upper bound (9.22) instead of the original expression (9.3) in the constraints. Doing so, one clearly looses any distinction among the various different multiplets, and one obtains instead a condition where the effect of the multiplets is somehow averaged. In fact, the expression (9.22) has the same form as the expression (9.8) that is

valid in the case of a single vector multiplet, but with ϵ^2 and ρ^2 given by eqs. (9.23). The constrained maximum of (9.22) within the range $\epsilon^2 \leq \rho^2$ is located at

$$
\epsilon^2{}_{\text{best}} = \begin{cases} \dfrac{1}{2}\dfrac{\rho^2}{1-\rho^2}, & \rho^2 < \dfrac{1}{2}, \\[4mm] \rho^2, & \rho^2 > \dfrac{1}{2}. \end{cases} \tag{9.24}
$$

In terms of the original variables, this means the optimal situation is the one in which all the d_A are such that the quantity $\sum_A d_A^2$ takes the maximal value $\sum_A \rho_A^2/(1 + \sum_B \rho_B^2)$ that it is allowed by the kinematical bound. As we already mentioned, there is a whole m-sphere of such directions, and no specific direction is singled out as optimal. This is due to the fact that in the present analysis any information distinguishing the different vector multiplets has been disregarded from the beginning. For the function K one finds

$$
K_{\text{best}} \leq \begin{cases} \dfrac{1 - \rho^2 + \rho^4}{1 - \rho^2}, & \rho^2 < \dfrac{1}{2}, \\[4mm] 1 + 4\,\rho^6, & \rho^2 > \dfrac{1}{2}. \end{cases} \tag{9.25}
$$

A last way to derive a weaker constraint that is valid for arbitrary values of the parameters ρ_A is to neglect the negative term in K. One is then left with a monotonically growing function of the variables ϵ_A, which obviously takes its maximal value at the boundary $\epsilon_A = \rho_A$ of the allowed domain. This leads to:

$$
K_{\text{best}} = 1 + 4\sum_A \rho_A^4 + 4\sum_{A,B} \rho_A^4\,\rho_B^2\,. \tag{9.26}
$$

This condition is completely general, and still keeps a distinction between the different vector multiplets. However, it is generically much weaker that the conditions we got in the previous analysis.

9.4 Simple scalar geometries

It is worth discussing more precisely the minimization of the term involving the effective Riemann tensor in the stability condition in the particular cases of separable or symmetric manifolds where, as already mentioned in section 7, the situation simplifies [17, 18].

For factorizable manifolds, the Riemann tensor can be written in terms of the n curvature scalars R_I of the one-dimensional factors and is just given by $R_{I\bar{J}P\bar{Q}} = R_I\,\delta_{I\bar{J}P\bar{Q}}$. The effective curvature that is relevant for the constraints (9.6) has then the same factorized form, and is given by $R_{I\bar{J}P\bar{Q}} = \tilde{R}_I\,\delta_{I\bar{J}P\bar{Q}}$, where

$$
\tilde{R}_I = K_{\text{best}}(\rho_A^2)^{-1} R_I\,. \tag{9.27}
$$

The values of the variables z^I optimizing (9.6) are given by $|z^I|^2 = \tilde{R}_I^{-1}/\sum_J \tilde{R}_J^{-1}$. In terms of the original variables, this means that the f^I align along the direction with highest total effective inverse curvature. One finds then the following necessary condition on the effective inverse curvature scalars:

$$
\sum_I \tilde{R}_I^{-1} \geq \frac{3}{2}\,. \tag{9.28}
$$

For maximally symmetric manifolds, the Riemann tensor can be written in terms of a single curvature scale R_{all} as $R_{I\bar{J}P\bar{Q}} = R_{\text{all}}/2\,(\delta_{I\bar{J}}\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\delta_{P\bar{J}})$. The effective curvature appearing in the flatness and stability conditions (9.6) remains again of the same form, and reads $\tilde{R}_{I\bar{J}P\bar{Q}} = \tilde{R}_{\text{all}}/2\,(\delta_{I\bar{J}}\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\delta_{P\bar{J}})$, where:

$$\tilde{R}_{\text{all}} = K_{\text{best}}(\rho_A^2)^{-1} R_{\text{all}}\,. \tag{9.29}$$

In this case, no particular direction in the variables z^I, or equivalently in the f^I, is singled out as optimal, due to the fact that the space is maximally symmetric and all the directions are thus equivalent. One finds however the necessary condition:

$$\tilde{R}_{\text{all}}^{-1} \geq \frac{3}{2}\,. \tag{9.30}$$

10 String inspired examples

In this section we will apply our results to the typical situations arising for the moduli sector of string models. In order to get a better idea of the usefulness of the conditions (6.4) for finding phenomenologically viable vacua in the presence of D-terms, we will study in more detail the form that these conditions take in the simplest case involving 1 chiral and 1 vector multiplet, and in the next-to-simplest case of 2 chiral multiplets with a factorized geometry and 1 vector multiplet. We will also briefly comment on how the idea of uplifting introduced in [7] fits into our study.

10.1 One chiral superfield and one isometry

The simplest possible situation arises in a low energy effective theory with one chiral superfield and one isometry, which is gauged with one vector multiplet. For moduli fields of string models, the prototype of Kähler potential describing such a situation is of the form:

$$K = -n \log\big(\Phi + \Phi^\dagger\big)\,. \tag{10.1}$$

This is a constant curvature manifold with $R = 2/n$. It has a global symmetry associated to the Killing vector $X = i\,\xi$, which can be gauged as long as the superpotential is also gauge invariant.

In this case, the flatness condition can be solved by introducing an angle δ and parametrizing the auxiliary fields defined in (6.1) as $f = \cos\delta$ and $d = \sin\delta$. The angle δ is, in this simple case, fixed by the ratio between the vector and the gravitino masses, the parameter ρ defined in (6.3):

$$\tan\delta = \rho\,. \tag{10.2}$$

In this simple situation, the stability condition is actually necessary and sufficient for the only non-trivial mass eigenvalue to be positive. We can thus change the $\leq$ sign to a $<$ sign, and write this condition as

$$R < \frac{2}{3}\big(1 + 4\,\rho^6\big)\,. \tag{10.3}$$

From this expression, it is clear that it is always possible to satisfy the stability condition with a large enough value of ρ. More precisely, recalling that $R = 2/n$, we get that

$$n > \frac{3}{1 + 4\,\rho^6}\,. \tag{10.4}$$

Note in particular that eq. (10.4) implies that whenever n is substantially less than 3, which is the critical value for stability in the absence of gauging, the contribution to supersymmetry breaking coming from the D auxiliary field must be comparable to the one coming from the F auxiliary field. This would be for instance the case for an effective theory based on the dilaton modulus, for which $n = 1$. On the other hand, if n is close or equal to 3, like in the case of an effective theory based on the volume modulus, the effect of D auxiliary fields can be still relevant even if supersymmetry breaking is dominated by the F auxiliary field.

It is important to note that in this case gauge invariance of G severely restricts the form of the superpotential, which can only be of the form $W = \alpha\,e^{-\beta\,\Phi}$. With these forms of K and W, it is easy to see that a satisfactory extremum can actually exist only if $n < 3$, as in the absence of the D-term we get a negative definite ($n < 3$) or positive definite ($n > 3$) scalar potential and we need the negative one to compete with the positive definite contribution due to the D-terms in order to stabilize the field. This model was studied in detail in [21] [15]. Of course, as was already noted in [21], the concept of uplifting a supersymmetric minimum using D-terms does not apply in this simple case as if we consider only one chiral field and one vector field the D-term is proportional to the only available F-term.

10.2   Two chiral superfields and one isometry

Another interesting and reasonably simple case is that of a low energy effective theory with two chiral superfields with a factorizable geometry and one isometry, which is gauged with one vector multiplet. For moduli fields in string models, one typically has a Kähler potential of the form

$$K = -n_1 \log\left(\Phi_1 + \Phi_1^\dagger\right) - n_2 \log\left(\Phi_2 + \Phi_2^\dagger\right). \tag{10.6}$$

The scalar manifold defined by this Kähler potential consists of two one-dimensional subspaces with constant scalar curvatures $R_1 = 2/n_1$ and $R_2 = 2/n_2$, and has actually two independent global symmetries, under which the chiral multiplets independently shift by an imaginary constant. We will consider the case in which we gauge only one linear combination of the isometries, defined by $X = (i\,\xi_1, i\,\xi_2)$. The most general superpotential allowed by gauge invariance of G is then of the form $W = e^{-\beta(\xi_1\Phi_1 + \xi_2\Phi_2)}W(\xi_2\Phi_1 - \xi_1\Phi_2)$.

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[15]In the more general case of a non-constant gauge kinetic function, which was considered in [21], the condition (10.3) gets modified. In terms of the parameter $\kappa = g'/(g\sqrt{K''})$, one finds:

$$R < \frac{2}{3}\left(1 + 4\,\rho^6\right) + 4\,\sqrt{3}\,\kappa\,\rho^4\sqrt{1 + \rho^2} + 4\,\kappa^2\,\rho^2\left(2 + \rho^2\right). \tag{10.5}$$

Again, this condition can always be satisfied for appropriate values of δ.

In this case the solution of the flatness condition can be parametrized by two angles δ and θ, with $f_1 = \cos\theta\cos\delta$, $f_2 = \sin\theta\cos\delta$ and $d = \sin\delta$. It is also useful to introduce the following two angles, defining the orientation of the Killing vector $X = (X^1, X^2)$ and the inverse curvature vector $R^{-1} = (R_1^{-1}, R_2^{-1})$:

$$\tan x = \frac{\xi_2}{\xi_1}, \qquad \tan r = \frac{n_2}{n_1}. \tag{10.7}$$

From the definitions of the vector mass, the gravitino mass, and the parameter ρ, one finds in this case the following relation between the angles δ and θ:

$$\frac{\tan\delta}{\cos(\theta - x)} = \rho. \tag{10.8}$$

The stability condition can then be written in the following form:

$$\left(R_1\cos^4\theta + R_2\sin^4\theta\right) + \frac{2}{3}(\rho^4 - \rho^6)\left(2\cos^2(\theta - x) - \frac{1}{1 - \rho^2}\right)^2 \le \frac{2}{3}\frac{1 - \rho^2 + \rho^4}{1 - \rho^2}. \tag{10.9}$$

In general, this condition cannot be solved exactly, since it is quartic. It is however easy to see that the first term in the left-hand side of (10.9) takes its minimal value when $\cos^2\theta = R_2/(R_1 + R_2)$, implying $\theta = r$, whereas the second term gets minimized when $\cos^2(\theta - x) = 1/[2(1 - \rho^2)]$ if $\rho \le 1/2$ and when $\theta = x$ if instead $\rho > 1/2$. One can then derive a lower bound for the left-hand side by minimizing the two terms separately, and derive in this way a simple necessary condition. Recalling that $R_1 = 2/n_1$ and $R_2 = 2/n_2$, one deduces in this way that:

$$n_1 + n_2 \ge 3 \begin{cases} \dfrac{1 - \rho^2}{1 - \rho^2 + \rho^4}, & \text{if } \rho \le 1/2, \\[2mm] \dfrac{1}{1 + 4\rho^6}, & \text{if } \rho > 1/2. \end{cases} \tag{10.10}$$

It is clear from this condition that for large enough ρ one can always satisfy the bound, the minimal value needed for ρ being determined by n_1 and n_2. Note also that the expression (10.10) reproduces the same bounds derived in section 9, as is clear from eq. (9.11).

An interesting particular case occurs when one of the shifts vanishes, say $\xi_1 = 0$. In such a situation, one finds $x = \pi/2$, and the condition (10.9) can be solved exactly. Actually it turns out that for values of ρ such that $3(n_1 + n_2) + 4n_1n_2\rho^4(1 - \rho^2) \le 0$, the bound can always be satisfied for some range of values of the angle θ. On the other hand, for values of ρ such that $3(n_1 + n_2) + 4n_1n_2\rho^4(1 - \rho^2) > 0$ the following condition must be satisfied:

$$n_1 + n_2 \ge 3 - 4n_2\rho^6 - \frac{4}{3}n_1n_2\rho^4(1 - \rho^2 + \rho^4). \tag{10.11}$$

Note also that in such a model, it is possible to stabilize the field Φ_1 in a supersymmetric way. This would correspond to $\theta = \pi/2$, and the situation would then become identical to the one described in the previous section. In this kind of model, if the field Φ_2 satisfy the bound (10.3), one could then use the sector containing Φ_2 and V to break supersymmetry and "uplift" to a Minkowski vacuum a supersymmetric AdS minimum for the sector involving Φ_1.

11   Conclusions

In this paper we have studied the constraints that can be put on gauge invariant super-gravity models from the requirement of the existence of a flat and metastable vacuum. Following the same strategy as in the previous analysis presented in [17, 18] for the simplest supergravity theories involving only chiral multiplets, we have considered the constraints implied by the flatness condition implying the vanishing of the cosmological constant and by a necessary but not sufficient condition for metastability, derived by looking at the particular complex direction in the scalar field space that corresponds to the Goldstino direction. These conditions define two algebraic constraints on the chiral and vector auxiliary fields F and D, and depend on the curvature of the Kähler potential, the mass of the vector fields and the derivatives of the gauge kinetic function. The major difficulty in solving more explicitly these constraints comes from the kinematical and the dynamical relations existing between the chiral and vector auxiliary fields. We have presented and followed three different methods to derive more explicitly the restrictions imposed by the flatness and stability constraints on the parameters defining the theory. These methods are based on the use of respectively the dynamical relation, the kinematical relation and a kinematical bound between the F and D auxiliary fields, and preserve a decreasing amount of the information contained in the original conditions. In this way, we were able to obtain several kinds of necessary conditions, which are relevant in different situations.

Our results can be summarized as follows: as expected, the presence of vector multi-plets, in addition to chiral multiplets, tends to alleviate the constraints with respect to a situations with only chiral multiplets. This is mainly due to the fact that the D-type aux-iliary fields give a positive definite contribution to the scalar potential, on the contrary of the F-type auxiliary fields, which give an indefinite sign contribution. The effect of vector multiplets is maximal when the gauge boson masses are comparable to the gravitino mass. When these two mass scales are instead hierarchically different, the effect of vector mul-tiplets is small and is encoded in a shift of the components of the Riemann tensor. More in general, we found through various types of analyses that the main effect of the vector multiplets is essentially to reduce the effective curvature felt by the chiral multiplets, and thereby to make the condition of metastability less constraining. On the other hand, the local symmetries associated to the vector multiplets and the corresponding Higgs mecha-nism controlling their spontaneous breaking imply that the presence of vector fields does not merely represent an extra generalizing ingredient, but also leads to restrictions.

We believe that the general results derived in this paper can be useful in discriminating more efficiently potentially viable models among those emerging, for instance, as low-energy effective descriptions of string models. We leave the exploration of this application for future work.

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