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Electric and magnetic interaction of dyonic D-branes and odd spin structure

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Abstract

We present a general description of electromagnetic RR interactions between pairs of magnetically dual D-branes, focusing on the interaction of a magnetically charged brane with an electrically charged one. In the boundary state formalism, it turns out that while the electric-electric and/or magnetic-magnetic interaction corresponds to the usual RR even spin structure, the magnetic-electric interaction is described by the RR odd spin structure. As representative of the generic case of a dual pair of p and 6-p branes, we discuss in detail the case of the self-dual 3-brane wrapped on a T_6/Z_3, which looks like an extremal dyonic black hole in 4 dimensions.

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I. INTRODUCTION

Since Polchinski provided us with a powerfull σ-model technique for studing non perturbative phenomena in String theory [1], a number of interesting relations between String theory, Supergravity and Super Yang-mills theory have been understood. In particular, the already known family of solitonic p-brane solutions of Type IIA and IIB supergravities in 10 dimensions [2] are recognized to be described, in the String theory framework, by D-branes.

A number of fascinating issues like black holes entropy and non-perturbative properties of Super Yang-Mills theory in diverse dimensions have been adressed in this context and partially answered, promoting D-branes and their dynamics [3,4] to one of the most promising and interesting parts of String theory to be studied. In particular, using the boundary state formalism [5], many properties of D-branes have been efficiently studied both in the covariant [6–9] and in the light-cone [10] formalisms.

In this paper we are going to study some interesting aspects of the electromagnetic interactions between pairs of dual D-branes, which corresponds to the RR configuration of the exchanged closed superstring. In the boundary state formalism, one has to further consider the two possible GSO signs, referred to as even and odd spin structure respectively. It is well known that the even spin structure encodes the standard interaction between two both electrically or both magnetically charged objects. We call this the diagonal interaction. An electrically charged object can also interact with a magnetically charged one. This interaction is more difficult to describe because the gauge potential fields cannot be globally defined. We call it the off-diagonal interaction, which also occurs in the general case of two dyonic objects [11], carrying each both electric and magnetic charge (beside it, they will also have the diagonal interaction, of course). A general theoretical framework for describing the off-diagonal interaction has been developed long ago in ref. [12]. In Sect. II, we review shortly this general framework, which is in fact very well suited for discussing the brane's interactions, and we will show that some recently derived results for dyons in various dimensions [13] are naturally obtained within this scheme.

In Sect. III we show that the general results of Sect. II for the off-diagonal interaction are exactly reproduced in String theory within the boundary state formalism, by an expression of the amplitude in the RR configuration corresponding to the odd spin structure. According to the general framework, one has to consider the off-diagonal interaction of, say, one brane with a pair of a brane and an antibrane (it is like having a Dirac string between the two members of the pair). In Sect. IV we consider in particular the interesting case of the D3-brane of the type IIB theory, which is self-dual in $d = 10$ dimensions, that is both electrically and magnetically charged with respect to the self-dual RR 4-form present in the massless spectrum of the theory. We evaluate explicitly their diagonal and off-diagonal interactions. In Sect. V, we consider a wrapped 3-brane [7,8] in the interesting compactification over the T_6/Z_3 orbifold [14], leading in 4 dimensions to an $N = 2$ effective Supergravity theory

[15]. We show how the single electric and magnetic charge in ten dimensions is reinterpreted from the 4-dimensional non-compact spacetime point of view as a variety of possible dyonic charges, all satisfying Dirac's quantization condition, depending on the orientation of the brane in the compact space. It is rather amusing to see how the odd spin structure string computation automatically encodes this feature.

Let us end this introduction by remembering that, from the analysis of refs. [7,8], the 3-brane on the T_6/Z_3 orbifold does not couple to (4-dimensional) scalars; rather, it only couples to gravity and to the $U(1)$ gauge field with equal strength, the total static diagonal interaction being zero, as appropriate for BPS states. Thus, being a source of equal strengh for gravity and Maxwell fields, and nothing else, it looks like a Reissner-Nordström black hole in 4-dimensional spacetime.

II. INTERACTIONS OF CHARGES, MONOPOLES AND DYONS

As well kown, the electromagnetic potential generated by a magnetic monopole cannot be defined everywhere; in the case of a p-extended object in d spacetime dimensions, there exists a Dirac hyperstring on which the potential is singular. As a consequence, the phase shift of another electrically charged q-dimensional extended object along a closed trajectory in this monopole background, which would be a gauge-invariant quantity if the potential were well defined, suffers from an ambiguity. In fact, the requirement that the phase-shift should remain unchanged mod 2π leads to the famous Dirac quantization condition $eg = 2\pi n$.

It is possible to define a mod 2π gauge-invariant phase shift also for open trajectories by considering a pair of charge and anti-charge instead of a single charge. Since an anti-charge travelling forward in time is equivalent to a charge travelling backward, this system can in fact be considered as a single charge describing a closed trajectory *. The phase-shift for such a setting in the monopole background is then a gauge-invariant quantity (provided Dirac's quantization condition holds). Actually, this is the setting that can be most easily analyzed in the String theory framework, since it corresponds to D-branes moving with constant relative velocities. Indeed the available techniques for computing explicitely branes interactions allow us to deal only with rectilinear trajectories, more in general with hyperplanes as world surfaces.

The phase-shift for a system of a charge and an anti-charge moving along two parallel straight trajectories in a monopole background is a special case of the general analysis carried

* If one consider only the usual electric-electric part of the interaction, one can even consider a single infinite straight trajectory; the corresponding phase-shift is gauge-invariant provided we require any gauge transformation to vanish at infinity.

out in ref. [12] that we shall briefly review.

We will consider dual pairs of branes, namely p-branes and $(d-4-p)$-branes (with d being the dimension of the corresponding spacetime). It is convenient to describe the interactions formally in the Euclidean signature (which can be then continued to the Lorentz one). With such a metric one can consider closed world surfaces of the branes, as they would correspond, in Lorentz spacetime, to brane-antibrane pairs, as explained above.

The world surface $\Sigma_{(p+1)}$ of the p-brane is $(p+1)$-dimensional and it couples to the $(p+1)$-form gauge potential $A_{(p+1)}$. We introduce the notation:

$$\int_{\Sigma_{(p+1)}} A_{(p+1)} \equiv \Sigma_{(p+1)} \cdot A_{(p+1)} \ . \tag{1}$$

This can be rewritten as

$$\Sigma_{(p+1)} \cdot A_{(p+1)} = \Sigma_{(p+2)} \cdot F_{(p+2)} \ , \tag{2}$$

where F is the field strength $F_{(p+2)} = \nabla A_{(p+1)}$ and $\Sigma_{(p+2)}$ is an arbitrary $(p+2)$-dimensional surface whose boundary $\partial\Sigma_{(p+2)}$ is $\Sigma_{(p+1)}$. In formulae:

$$\Sigma_{(p+2)} \cdot \nabla A_{(p+1)} = \partial\Sigma_{(p+2)} \cdot A_{(p+1)} = \Sigma_{(p+1)} \cdot A_{(p+1)} \ . \tag{3}$$

The diagonal (electric-electric and/or magnetic-magnetic) interaction of two p-branes, whose world surfaces are $\Sigma'_{(p+1)}$ and $\Sigma_{(p+1)}$ respectively, can be written as

$$I_D = (e'e + g'g)\, \Sigma'_{(p+2)} \cdot P\Sigma_{(p+2)} = (e'e + g'g)\, \Sigma'_{(p+1)} \cdot D\Sigma_{(p+1)} \ , \tag{4}$$

where e, e' (g, g') are the electric (magnetic) charges carried by the two branes, D is the propagator, that is the inverse of the Laplace-Beltrami operator $\Delta = \partial\nabla + \nabla\partial$, i.e. $\Delta D = 1$, and $P = \nabla D \partial$. In the Euclidean path-integral, this interaction appears at the exponent, namely the integrand is e^{-I_D}.

Consider now what we call the off-diagonal interaction of two mutually dual branes, a p-brane and a $(d-4-p)$-brane, in $d = 2(q+1)$ dimensions (the case $p = q-1$ is self dual):

$$I_{off-D} = eg'\Sigma'_{(d-2-p)} \cdot {}^*P\Sigma_{(p+2)} + e'g\Sigma_{(p+2)} \cdot {}^*P\Sigma'_{(d-2-p)} \ . \tag{5}$$

Here ${}^*F = \epsilon/2^q F$ means the Hodge dual of a form F, obtained by contracting its components with the antisymmetric tensor. It is crucial to observe that the Hodge duality operation depends on the dimension $d = 2(q+1)$ of spacetime (that we shall suppose to be even in any case). In fact, the ϵ tensor satisfies $(\epsilon/2^q)^2 = (-1)^{q+1}\,1$ and $\epsilon^T = (-1)^{q+1}\epsilon$. Using these properties, one can see that $P + (-1)^{q+1}{}^*P^* = 1$ in the space of antisymmetric tensors, as it is equivalent to the Hodge decomposition. Therefore ${}^*P + P^* = {}^*1$. Now, the insertion of the ^{*1} between $\Sigma'_{(d-2-p)}$ and $\Sigma_{(p+2)}$ yields a contact term given by their intersection number;

assuming by a "Dirac veto" that this number is zero, we get ${}^*P \doteq -P^*$. Finally, transposing the second term in eq. (5) and using the above properties, we get finally

$$\begin{aligned} I_{off-D} &= (eg' + (-1)^q e'g)\, \Sigma'_{(d-2-p)} \cdot {}^*P\Sigma_{(p+2)} \\ &= \frac{1}{2}\, (eg' + (-1)^q e'g) \left(\Sigma'_{(d-2-p)} \cdot {}^*P\Sigma_{(p+2)} + (-1)^q \Sigma_{(p+2)} \cdot {}^*P\Sigma'_{(d-2-p)} \right) \,. \end{aligned} \tag{6}$$

In order for the path integral over $e^{iI_{off-D}}$ to be well defined, it is necessary to impose the Dirac quantization condition [13]

$$(eg' + (-1)^q e'g) = 2\pi n \,. \tag{7}$$

The point is that I_{off-D} depends on the (supposed irrelevant) choice of the unphysical $\Sigma'_{(d-2-p)}$, which is only constrained to have the physical brane world surface $\Sigma'_{(d-3-p)}$ as its boundary: $\partial\Sigma'_{(d-2-p)} = \Sigma'_{(d-3-p)}$. However, the path-integral integrand is in this case $e^{iI_{off-D}}$ and this has no ambiguity. Indeed,

$$I_{off-D} = (2\pi n)\Sigma'_{(d-2-p)} \cdot {}^*\nabla D\Sigma_{(p+1)} \,. \tag{8}$$

Now, if we change $\Sigma'_{(d-2-p)}$ keeping its boundary fixed, the ensuing change of I_{off-D} can be written as $\delta I_{off-D} = (2\pi n)\partial\mathcal{V}_{(d-1-p)} \cdot {}^*\nabla D\Sigma_{(p+1)}$, where the boundary of $\mathcal{V}_{(d-1-p)}$ is the union of the old $\Sigma'_{(d-2-p)}$ and the new one. By integrating by parts, using $\nabla^* = {}^*\partial$ and $\partial\Sigma_{(p+1)} = 0$ since we consider closed world surfaces, we get

$$\delta I_{off-D} = (2\pi n)\mathcal{V}_{(d-1-p)} \cdot {}^*\Sigma_{(p+1)} = 2\pi(\text{integer}) \,, \tag{9}$$

since $\mathcal{V}_{(d-1-p)} \cdot {}^*\Sigma_{(p+1)}$ is the intersection number of the closed hypersurface $\Sigma_{(p+1)}$ and the hypervolume $\mathcal{V}_{(d-1-p)}$ and is therefore an integer. Notice that relaxing the Dirac veto, eq. (6) is a consistent expression provided $eg' + (-1)^q e'g = 4\pi n$.

The above properties remain valid also when we compactify some of the dimensions, in particular compactifying 6 (the directions $a, a+1$, $a = 4, 6, 8$) of the $d = 10$ dimensions in String theory. Objects whose extended dimensions are wrapped in the compactified directions will appear point-like in the 4-dimensional spacetime. In particular, as anticipated, we will be interested in the sequel in the case of the $D3$-brane, occuring in Type IIB String theory, compactified on the orbifold T_6/Z_3. The 3-brane of Type IIB is a special case since it is both electrically and magnetically charged with respect to the self-dual RR 4-form; this peculiarity will be relevant in our study giving rise, both before and after the compactification, to a dyonically charged state. From the 4-dimensional spacetime point of view, this will look like the interaction of two dyons, whose values of electric and magnetic charges turn out to be dictated by the brane's different orientations in the compact directions. For instance, if the two (off-diagonally) interacting branes are parallel in the compact directions, then it is easy to see (we will be explicit in the following) that $I_{off-D} = (2\pi n)\Sigma'_{(d-2-p)} \cdot {}^*\nabla D\Sigma_{(p+1)} = 0$

and this will be interpreted in 4 dimensions by saying that there is no off-diagonal interaction between to "parallel" dyons, that is having the same ratio (magnetic charge)/(electric charge). In fact, two such dyons behave with respect to each other as purely electrically charged particles.

It is amusing to notice that although the Dirac quantization condition is automatically implemented, as we said, once the off-diagonal interaction is correctly normalized in 10 dimensions, it might look somewhat non-obvious at first sight in 4 dimensions, due to the non-intuitive features of compact spaces. We will explore the ensuing pattern of charge quantization in the following sections.

In the following, we are going to consider the off-diagonal interaction of two pairs of 3-branes-antibranes, wrapped on the compact space and moving linearly in spacetime (the brane's parameters will be labeled by B, the antibrane's ones by A and the index $i = 1, 2$ labels the two pairs). We will take the trajectories in spacetime to describe a line in the (t, x) plane. In each of the two pairs, the brane and the antibrane are parallel to each other. This means that each pair is described by two parallel 4-dimensional hyperplanes, three directions being compact and specified by the angles $\theta_a^{(i)}$ $(a = 4, 6, 8)$, which are common to the brane and the antibrane, in each of the three tori which compose T_6 and one direction $w^{(i)}$ in the plane (t, x). In the Lorentz spacetime, the (t, x) direction $w^{(i)}$ is specified by an hyperbolic angle, the rapidity $v^{(i)}$ $(w_t^{(i)} = \sinh v^{(i)}\ ,\ \ w_x^{(i)} = \cosh v^{(i)})$. The (t, x) trajectory of the brane of the pair i is taken in the positive t-direction and is located at position $y_B^{(i)}, z_B^{(i)}$ in the transverse (y, z) plane, while the trajectory of the antibrane is taken in the negative t-direction and is located at position $y_A^{(i)}, z_A^{(i)}$. It is convenient to introduce a complex variable $\xi = y + iz$. The positions of the brane and the antibrane of the two pairs in the transverse (y, z) plane is depicted in Fig. 1.

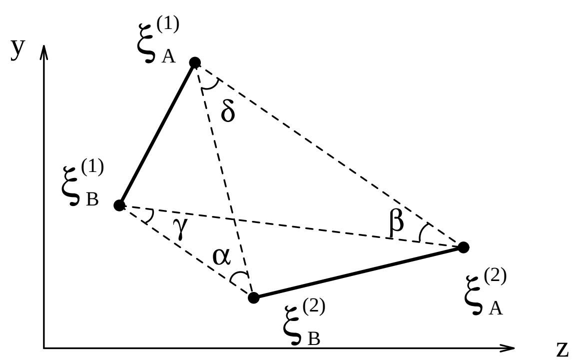

Fig. 1

According to the general construction, the diagonal and off-diagonal interactions I_D and I_{off-D} are given by eqs. (4) and (6) respectively. In order to integrate along the hypersurfaces, let us suppose first that the angles $\theta_a^{(2)}$ are different from the angles $\theta_a^{(1)}$.

Consider then the Fourier transform of $D_d(r) = \int d^dk/(2\pi)^d \tilde{D}(k) e^{ikr}$ and write $\tilde{D}(k) = 1/k^2 = \int_0^\infty dl e^{-lk^2}$. The integration along the planes in the compact space and along the (t,x) plane will result in putting to zero all the compact and the (t,x) components of the momentum k. Hence, after those integrations, the propagator D will be reduced to the Fourier transform of $\tilde{D}$ where only k_y, k_z are different from zero, that is the two dimensional propagator D_2 in the plane (y,z). Thus, the only possible derivatives occurring in the previous equation will be in the (y,z) plane. Actually, by doing the integration over l as the last one, the other integrations factorize into the product of integrations along the planes (t,x), (y,z) and the three compact planes $(a, a+1)$ respectively.

In the diagonal case, the integration in the (t,x) plane gives

$$(w^{(1)} \cdot w^{(2)}) \int dt^{(1)} \int dt^{(2)} \int \frac{dk_t dk_x}{(2\pi)^2} e^{i(t^{(1)}w^{(1)} - t^{(2)}w^{(2)})\cdot k} e^{-l(k_t^2 + k_x^2)} = \frac{w^{(1)} \cdot w^{(2)}}{|w^{(1)} \wedge w^{(2)}|} = \coth(v_1 - v_2)$$

where $w^{(i)}$ represents the direction of the i branes trajectories in the (t,x) plane. The integrations in the $(a, a+1)$ planes give instead, as we will see in Sect. V,

$$\frac{\prod_a L_a^{(1)} L_a^{(2)}}{\mathrm{Vol}(T_6/Z_3)} \prod_a \cos\left(\theta_a^{(1)} - \theta_a^{(2)}\right) .$$

This factor (times the 10-dimensional charges $e'e + g'g$) is interpreted in 4-dimensions as the dyon charge combination $e^{(1)}e^{(2)} + g^{(1)}g^{(2)}$. It is convenient to introduce the two-dimensional *complex* propagator, whose real part is $D_2(\xi, \xi') = \mathrm{Re}\mathcal{D}_2(\xi, \xi')$ (λ is an infrared cut-off)

$$\mathcal{D}_2(\xi, \xi') = \frac{1}{2\pi} \ln \frac{\xi - \xi'}{\lambda} . \tag{10}$$

The remaining integrations in the (y,z) plane are over the straight lines joining the brane in $\xi_B^{(i)}$ and the antibrane in $\xi_A^{(i)}$ for each of the two pairs $i = 1, 2$, and give

$$\begin{aligned} &\left(e^{(1)}e^{(2)} + g^{(1)}g^{(2)}\right) \int_{\xi_B^{(1)}}^{\xi_A^{(1)}} d\xi^{(1)} \cdot \partial_{\xi^{(1)}} \int_{\xi_B^{(2)}}^{\xi_A^{(2)}} d\xi^{(2)} \cdot \partial_{\xi^{(2)}} \mathrm{Re}\mathcal{D}_2(\xi^{(1)}, \xi^{(2)}) = \\ &= \frac{\left(e^{(1)}e^{(2)} + g^{(1)}g^{(2)}\right)}{2\pi} \mathrm{Re} \ln \left(\frac{\xi_A^{(1)} - \xi_A^{(2)}}{\xi_B^{(1)} - \xi_A^{(2)}} \cdot \frac{\xi_B^{(1)} - \xi_B^{(2)}}{\xi_A^{(1)} - \xi_B^{(2)}} \right) . \end{aligned} \tag{11}$$

In the off-diagonal case, the integration in the (t,x) plane gives

$$(w^{(1)} \wedge w^{(2)}) \int dt^{(1)} \int dt^{(2)} \int \frac{dk_t dk_x}{(2\pi)^2} e^{i(t^{(1)}w^{(1)} - t^{(2)}w^{(2)})\cdot k} e^{-l(k_t^2 + k_x^2)} = \frac{w^{(1)} \wedge w^{(2)}}{|w^{(1)} \wedge w^{(2)}|} = \pm 1 .$$

The result is therefore ± 1 (the degenerate case where the trajectories (1) and (2) are parallel should be taken to be zero). The integrations in the $(a, a+1)$ planes give instead

$$\frac{\prod_a L_a^{(1)} L_a^{(2)}}{\mathrm{Vol}(T_6/Z_3)} \prod_a \sin\left(\theta_a^{(1)} - \theta_a^{(2)}\right) .$$

This factor (times the 10-dimensional charges $eg' + e'g$) is interpreted in 4-dimensions as the dyon charge combination $e^{(1)}g^{(2)} - g^{(1)}e^{(2)} = 2\pi n$. The remaining integrations in the (y, z) plane give in this case (for $n = 1$)

$$\begin{aligned}
&\left(e^{(1)}g^{(2)} - g^{(1)}e^{(2)}\right) \int_{\xi_B^{(1)}}^{\xi_A^{(1)}} d\xi^{(1)} \wedge \partial_{\xi^{(1)}} \int_{\xi_B^{(2)}}^{\xi_A^{(2)}} d\xi^{(2)} \cdot \partial_{\xi^{(2)}} \mathrm{Re}\mathcal{D}_2(\xi^{(1)}, \xi^{(2)}) = \\
&\quad = \mathrm{Im} \ln \left(\frac{\xi_A^{(1)} - \xi_A^{(2)}}{\xi_B^{(1)} - \xi_A^{(2)}} \cdot \frac{\xi_B^{(1)} - \xi_B^{(2)}}{\xi_A^{(1)} - \xi_B^{(2)}} \right) \\
&\quad = \beta - \alpha = \delta - \gamma
\end{aligned} \tag{12}$$

(keeping the same sign convention for the angles, see Fig. 1).

There are here two important observation that we can make. First, considering pairs of branes-antibranes automatically eliminates any infrared divergence. Second, the off-diagonal interaction is given by the difference of the angles by which any curve joining $\xi_B^{(1)}$ and $\xi_A^{(1)}$ is seen from $\xi_B^{(1)}$ and $\xi_A^{(1)}$, or viceversa. We thus see explicitely that I_{off-D} is defined modulo 2π. Concluding, the total diagonal and off-diagonal interactions are given by

$$I_D = \frac{\left(e^{(1)}e^{(2)} + g^{(1)}g^{(2)}\right)}{\tanh(v^{(1)} - v^{(2)})} \mathrm{Re}\mathcal{D}_2 \ , \tag{13}$$

$$I_{off-D} = \pm \left(e^{(1)}g^{(2)} - g^{(1)}e^{(2)}\right) \mathrm{Im}\mathcal{D}_2 \ , \tag{14}$$

with

$$\mathcal{D}_2 = \ln \left(\frac{\xi_A^{(1)} - \xi_A^{(2)}}{\xi_B^{(1)} - \xi_A^{(2)}} \cdot \frac{\xi_B^{(1)} - \xi_B^{(2)}}{\xi_A^{(1)} - \xi_B^{(2)}} \right) \ .$$

Notice the interesting fact that in $d = 2(q+1) = 10$, where the gauge field is a $q = 4$ even form, the 3-brane is a dyon in the sense that it has $e = g = \mu_3 = \sqrt{2\pi}$ and that it has both a diagonal and an off-diagonal interaction with itself. In fact, the off-diagonal interaction is in this case proportional to $e^{(1)}g^{(2)} + e^{(2)}g^{(1)}$ (whereas for q odd it is proportional to $e^{(1)}g^{(2)} - e^{(2)}g^{(1)}$) and different from zero also for $e^{(1)} = e^{(2)}$, $g^{(1)} = g^{(2)}$. On the contrary, for $d = 2(q + 1) = 4$, where the gauge field is a $q = 1$ odd form, two "parallel" dyons having $e^{(1)} = e^{(2)}$ and $g^{(1)} = g^{(2)}$ do not have any off-diagonal interaction, the latter beeing proportional to $e^{(1)}g^{(2)} - e^{(2)}g^{(1)}$.

It turns out from our analysis that the d=10 off-diagonal interaction, proportional to $e_{10}g_{10}$, becomes automatically proportional to $e_4^{(1)}g_4^{(2)} - e_4^{(2)}g_4^{(1)}$ upon compactification down to d=4. This happens because the off-diagonal interaction is proportional to the factor $\prod_a \sin(\theta_a^{(1)} - \theta_a^{(2)})$, which is zero when the branes (1) and (2) are seen by a non-compact observer to be parallel in the sense that $e^{(1)} = e^{(2)}$ and $g^{(1)} = g^{(2)}$. All of this will be explicitly shown in Sect. V. More in general, notice that the off-diagonal interaction between two dyons (1) and (2) is symmetric both for q even and for q odd, under the exchange of every quantum number, $(1) \leftrightarrow (2)$. In fact, the transverse (y, z) contribution to the amplitude, that is $\mathcal{D}_2$, is symmetric, $\mathcal{D}_2(1,2) = \mathcal{D}_2(2,1)$, whereas each pair of the remaining non-transverse directions (t,x) and $(a,a+1)$ gives an antisymmetric contribution; therefore, since $e^{(1)}g^{(2)} + (-1)^q e^{(2)}g^{(1)}$ is symmetric for q even and antisymmetric for q odd, the total amplitude turns out to be symmetric in both cases (see eq. (6)).

III. THE INTERACTIONS IN STRING THEORY

As already noticed, the diagonal electric-electric and/or magnetic-magnetic interaction between two p-branes is a well defined quantity also for open trajectories. In this case, in fact, there is no strict necessity of considering interactions among pairs of brane-antibrane (although this is advisable to avoid infrared problems). In string theory, the diagonal even interaction of just one brane at $\xi^{(1)}$ and one brane at $\xi^{(2)}$ is computed within the boundary state formalism to be [1,3–9]

$$\mathcal{A}_D = \frac{\mu_p^2}{16} \sum_{\alpha\ even} < v^{(1)}, \theta_a^{(1)}, \xi^{(1)} | \int_0^\infty dl e^{-lH} | v^{(2)}, \theta_a^{(2)}, \xi^{(2)} >_\alpha , \tag{15}$$

where $|... >$ is the boundary state representing the p-brane, H is the closed superstring hamiltonian, μ_p is the RR charge of the p-brane and the factor 1/16 comes from our conventional normalization. The even spin structure corresponding to the case $\alpha = RR+$ (meaning the RR closed superstring sector and the GSO projection sign = 1) represents the diagonal electromagnetic interaction, whereas the two NSNS spin structures $\alpha = NS\pm$ represent the gravitational one. The main features of this diagonal amplitude are reviewed in Sect. IV.

Let us stress here that only the even spin structure contributes. In fact, in the odd spin structure case, even if the rapidity tilt $v^{(1)} - v^{(2)}$ and the angle tilt $\theta_a^{(1)} - \theta_a^{(2)}$ prevent the occurrence of fermionic zero modes in the directions (t,x) and $(a,a+1)$ with $a = 4,6,8$, there still remain fermionic zero modes in the (y,z) transverse directions. The amplitude therefore vanishes since there is no insertion of operators to soak up those zero modes.

Now we show that also the off-diagonal interaction can be expressed in String theory within the boundary state formalism. In this case, as we have seen, it is necessary to consider at least the interaction of a brane-antibrane pair, say located at $\xi^{(1)}_{B,A}$, with one brane (or antibrane) located at $\xi^{(2)}$. According to the previous general description, this interaction is expressed by an integral over a Dirac string joining $\xi^{(1)}_B$ and $\xi^{(1)}_A$, which we represent parametrically by $\xi^{(1)}(s)$, $s = (0,1)$.

The expression of the off-diagonal odd amplitude is the following:

$$\mathcal{A}_{off-D} = \frac{\mu_p^2}{16} \int_0^1 ds < v^{(1)}, \theta_a^{(1)}, \xi^{(1)}(s) | J(s)\bar{J}(s) \int_0^\infty dl e^{-lH} | v^{(2)}, \theta_a^{(2)}, \xi^{(2)} >_{RR-} , \tag{16}$$

where the subscript $RR-$ means that the braket is evaluated in the RR odd spin structure. Here $J, \bar{J}$ represent the left and right moving "supercurrents": $J = \partial X^\mu \psi_\mu$ and $\bar{J} = \bar{\partial} X^\mu \bar{\psi}_\mu$.

Along the Dirac string, $\partial, \bar{\partial} = \partial_s \mp i\partial_\tau$, where ∂_τ is the normal derivative, that is along the direction τ orthogonal to the Dirac string; τ is therefore the (Euclidean) world sheet evolution time of the closed superstring.

The odd spin structure case is now different from zero due to the supercurrent insertion. In fact since the odd amplitude vanishes unless there is the proper fermionic zero modes insertion, only the part of the insertion containing $\psi_y\bar{\psi}_z$ (or z, y interchanged) will contribute (for this reason the result would be the same also inserting the complete supercurrent including also the ghost part). Since the boundary conditions essentially identify ψ and $\bar{\psi}$, we see that, due to the anticommuting properties of the fermionic coordinates,

$$< (1)|J(s)\bar{J}(s)\int_0^\infty dle^{-lH}|(2) >_{RR-} = 2i < (1)|(\partial_s y\partial_\tau z - \partial_s z\partial_\tau y)\int_0^\infty dl\psi_y^0\bar{\psi}_z^0 e^{-lH}|(2) >_{RR-} \ .$$

Now, in the odd spin structure case the contribution of the fermionic and bosonic oscillator modes is equal to 1, since the bosonic modes' contribution is exactly the inverse of the fermionic modes' one. Moreover, only the non-oscillator part of the inserted supercurrents contributes: the fermions are necessarily zero modes as already explained and give an antisymmetric result; consequently, we are left with an antisymmetric bosonic correlation which is zero except for the non-oscillator part. Thus it remains only the non-oscillator modes, both bosonic and fermionic, contribution. The rest of the discussion now follows closely the general pattern described in Sect. II. The part of that contribution from the coordinates directions (t, x) and $(a, a+1)$ with $a = 4, 6, 8$ gives a position independent factor, which after compactification can be reinterpreted as the dyon charge combination $e^{(1)}g^{(2)} - e^{(2)}g^{(1)}$. This will be explicitely discussed in Sect. V. It is interesting to notice that the contribution of the fermionic non-oscillator modes is essential in providing the correct "numerators" in the resulting expressions.

The position dependence of the amplitude comes from the (y, z) non-oscillator modes contribution. The fermionic zero modes' (y, z) contribution, with our normalization, is equal to $1/2$, due to the insertion of $\psi_y^0\bar{\psi}_z^0$. We further notice that for the bosonic modes $ds(\partial_s y, \partial_s z) = (dy, dz)$ along the integration line, and that as an operator $(\partial_\tau y, \partial_\tau z) = -(\partial_y, \partial_z)$; therefore $ds(\partial_s y\partial_\tau z - \partial_s z\partial_\tau y) = dy\partial_z - dz\partial_y \equiv d\xi \wedge \partial_\xi$. Moreover, for the transverse bosonic modes $< \xi^{(1)}(s)| \int_0^\infty dle^{-lH}|\xi^{(2)} >= D_2(\xi^{(1)}(s), \xi^{(2)})$, whereas the remaining non-transverse part of the amplitude gives $\pm i$. Finally we obtain

$$\int_0^1 ds < (1)|J(s)\bar{J}(s)\int_0^\infty dle^{-lH}|(2) >_{RR-}^{(y,z)} = \int_{\xi_B^{(1)}}^{\xi_A^{(1)}} d\xi^{(1)} \wedge \partial_{\xi^{(1)}} D_2(\xi^{(1)}, \xi^{(2)}) \ , \tag{17}$$

which reproduces precisely the expected result for the off-diagonal interaction, see Sect. II.

As a final comment, one could also suspect that the odd spin structure contribution, eq. (16), might somehow automatically come from general world sheet supersymmetry considerations. In fact, it is known [16] that the occurrence of the supercurrent insertion is dictated by the occurrence of the socalled supermoduli, which indeed are expected in the odd spin structure case. Actually, in the cylinder case there is only one modulus, the previously introduced l, and thus one would expect only one supermodulus and one supercurrent insertion. However in our case we are obliged to consider simoultaneously the interaction of a brane and antibrane pair with a given brane (or antibrane). Thus it is not surprising to see the occurrence of the pair of supercurrents J and $\bar{J}$ as if the brane-antibrane pair would entail the torus, rather than cylinder, topology.

Let us stress that in any case it is a fact that the boundary state amplitude eq. (16) reproduces exactly the correct result for the off-diagonal electric-magnetic interaction.

IV. D3-BRANES IN 10 DIMENSIONS

In this section, we make more explicit the content of the formulae of Sect. III by briefly reviewing a series of results obtained in ref. [7,8] about the dynamics of D3-branes in 10 dimensions, using the boundary state formalism. In particular we will consider the precise structure of the amplitude for the scattering of two moving of such D3-branes with an arbitrary orientation putting in evidence the various contributions coming from the four different spin structures arising in a closed string channel computation.

Let us start from a 3-brane configuration with Neumann boundary conditions in the directions $t = X^0$ and X^a, and Dirichlet in $x = X^1, y = X^2, z = X^3$ and X^{a+1}, with $a = 4, 6, 8$. The coordinates X^a, X^{a+1} will eventually become compact. Consider then two of these 3-branes moving with velocities $V^{(1)} = \tanh v^{(1)}$, $V^{(2)} = \tanh v^{(2)}$ along the 1 direction, at transverse positions $\vec{Y}^{(1)}$, $\vec{Y}^{(2)}$, and tilted in $a, a+1$ planes with generic angles $\theta_a^{(1)}$ and $\theta_a^{(2)}$.

The cylinder amplitude in the closed string channel is just a tree level propagation between the two boundary states, which are defined to implement the boundary conditions defining the branes:

$$\mathcal{A} = \frac{\mu_3^2}{16} \int_0^\infty dl \sum_\alpha < v^{(1)}, \theta_a^{(1)}, \vec{Y}^{(1)} | e^{-lH} | v^{(2)}, \theta_a^{(2)}, \vec{Y}^{(2)} >_\alpha \ . \tag{18}$$

As stated before, there are two sectors, RR and NSNS, corresponding to periodicity and antiperiodicity of the fermionic fields around the cylinder, and after the GSO projection there are four spin structures, $R\pm$ and $NS\pm$, corresponding to all the possible periodicities of the fermions on the covering torus.

The configuration space boundary state can be written as the a product of delta functions enforcing the boundary conditions for the center of mass position operator X_o^μ, that is a Fourier superposition of momentum states:

$$|v, \theta_a, \vec{Y} >_B = \delta\left(\cosh v (X_o^1 - Y^1) - \sinh v X_o^0\right) \delta\left(X_o^2 - Y^2\right) \delta\left(X_o^3 - Y^3\right)$$
$$\prod_a \delta\left(\cos\theta_a (X_o^a - Y^a) + \sin\theta_a X_o^{a+1}\right) |v, \theta_a >$$

$$= \int \frac{d^6\vec{k}}{(2\pi)^6} e^{i\vec{k}_B\cdot\vec{Y}} |v,\theta_a> \otimes |k_B> \ , \tag{19}$$

with the boosted and rotated momentum

$$k_B^\mu = (\sinh v k^1, \cosh v k^1, k^2, k^3, \cos\theta_a k^a, \sin\theta_a k^a) \ .$$

Integrating over the momenta and taking into account momentum conservation which for non-vanishing $v \equiv v^{(1)} - v^{(2)}$ and $\theta_a \equiv \theta_a^{(1)} - \theta_a^{(2)}$ forces all the Dirichlet momenta but k^2, k^3 to be zero, the amplitude factorizes into a bosonic (B) and a fermionic (F) partition functions:

$$\begin{aligned}\mathcal{A} &= \frac{\mu_3^2}{2\sinh|v|\prod_a 2\sin|\theta_a|} \int_0^\infty dl \int \frac{dk^2 dk^3}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-k_B^2 l} \sum_\alpha Z_B Z_F^\alpha \\ &= \frac{\mu_3^2}{2\sinh|v|\prod_a 2\sin|\theta_a|} \int_0^\infty \frac{dl}{4\pi l} e^{-\frac{b^2}{4l}} \sum_\alpha Z_B Z_F^\alpha \ ,\end{aligned} \tag{20}$$

where $\mu_3 = \sqrt{2\pi}$ is the 3-brane tension, $\vec{b} = \vec{Y}_T^{(1)} - \vec{Y}_T^{(2)}$ ($b = |\xi^{(1)} - \xi^{(2)}|$) is the transverse impact parameter (in the $2,3$ directions) and

$$Z_{B,F}^\alpha = < v^{(1)}, \theta_a^{(1)} | e^{-lH} | v^{(2)}, \theta_a^{(2)} >_{B,F}^\alpha \ .$$

In the above expression, only the oscillator modes of the string coordinates X^μ appear, since we have already integrated over the center of mass coordinate. Notice also that world-sheets with $l \ll b^2$ give a negligible contribution to the amplitude, and in the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute.

Notice finally that the amplitude $\mathcal{A}$ can be written, in agreement with the fact that it corresponds to a phase-shift, as a world sheet integral

$$\mathcal{A} = \mu_3^2 \int d\tau \prod_a \int d\xi_a \int_0^\infty dl (4\pi l)^{-3} e^{-\frac{r^2}{4l}} \frac{1}{16} \sum_\alpha Z_B Z_F^\alpha \tag{21}$$

in terms of the true distance

$$r = \sqrt{\vec{b}^2 + \sinh^2 v\tau^2 + \sum_a \sin^2\theta_a \xi_a^2} \ .$$

In the limit $v, \theta_a \to 0$, translational invariance along the directions $1, a$ is restored and the integral over the world-sheet produces simply the volume V_{3+1} of the 3-branes.

The remaining part of the boundary state has been explicitly constructed in ref. [7] (see also [8]); after the GSO projection, the even part of total partition function was found to be ($\eta(2il)$ being the Dedekin function)

$$Z_B = \eta(2il)^4 \frac{2i\sinh v}{\vartheta_1(i\frac{v}{\pi}|2il)} \prod_a \frac{2\sin\theta_a}{\vartheta_1(\frac{\theta_a}{\pi}|2il)} \ , \tag{22}$$

$$\begin{aligned}Z_F^{even} = \eta(2il)^{-4} \Big\{ &\vartheta_2(i\frac{v}{\pi}|2il) \prod_a \vartheta_2(\frac{\theta_a}{\pi}|2il) \\ &-\vartheta_3(i\frac{v}{\pi}|2il) \prod_a \vartheta_3(\frac{\theta_a}{\pi}|2il) + \vartheta_4(i\frac{v}{\pi}|2il) \prod_a \vartheta_4(\frac{\theta_a}{\pi}|2il) \Big\} \ .\end{aligned} \tag{23}$$

The even part of the amplitude represents the usual interplay of the RR attraction and NSNS repulsion, leading to the well known BPS cancellation of the interaction between two parallel D-branes (vanishing like v^4 for small velocities). In the large distance limit $(b, l \to \infty)$, the behavior of the partition functions is

$$\begin{aligned} & Z_B \to 1 \ , \\ & Z_F^{even} \to 2\cosh v \prod_a 2\cos\theta_a - 2\left(2\cosh 2v + \sum_a 2\cos 2\theta_a\right) \ . \end{aligned}$$

As we have seen in Sect. III, the odd part encodes instead the electric-magnetic off-diagonal RR interaction; due to the supercurrent insertion carrying the fermion fields ψ^2, ψ^3, it does not vanish, since the (2,3) fermionic zero modes are soaked up:

$$Z_F^{odd} = \eta(2il)^{-4}\vartheta_1(i\frac{v}{\pi}|2il) \prod_a \vartheta_1(\frac{\theta_a}{\pi}|2il) \ .$$

Notice that in the odd spin structure, the oscillator's contribution cancel between fermions and bosons by world sheet supersymmetry, and

$$Z_B Z_F^{odd} = 2i\sinh v \prod_a 2\sin\theta_a \ .$$

Remember also that the bosonic coordinates present in these supercurrents alter the non-oscillator part of the bosonic partition function precisely in the right way to allow the interpretation of Sect. II.

Summarizing, the diagonal interaction between two 3-branes at the positions $\xi^{(1)}$ and $\xi^{(2)}$ in the transverse (2,3) plane is, at large distances,

$$I_D = \mu_3^2 \coth v \prod_a \cot\theta_a D_2|\xi^{(1)} - \xi^{(2)}| \ , \tag{24}$$

where

$$D_d(r) = \int \frac{d^dk}{(2\pi)^d}\frac{e^{i\vec{k}\cdot\vec{r}}}{k^2} = \int_0^\infty dl\,(4\pi l)^{-\frac{d}{2}}\,e^{-\frac{r^2}{4l}}$$

is the Green function in d dimensions.

The off-diagonal interaction between a 3-brane at transverse position $\xi^{(2)}$ and a pair of 3-brane and antibrane at $\xi_B^{(1)}$ and $\xi_A^{(1)}$ is instead the same all distances and given by

$$I_{off-D} = \pm\mu_3^2 \int_{\xi_B^{(1)}}^{\xi_A^{(1)}} d\xi^{(1)} \wedge \partial_{\xi^{(1)}} D_2|\xi^{(1)} - \xi^{(2)}| \ . \tag{25}$$

V. D3-BRANE ON T_6 AND T_6/Z_3

In this section we shall apply the general construction that we have introduced to the case of the Type IIB 3-brane wrapped on the orbifold T_6/Z_3. Compactifying the directions $a, a+1, a=4,6,8$ on T_6, one gets $N=8$ supersymmetry, which is further broken down to $N=2$ by the Z_3 moding, and this configuration was shown in ref. [7] to correspond to a solution of the low energy effective $N=2$ supergravity with no coupling to any scalar.

T_6/Z_3 is the orbifold limit of a CY manifold with Hogde numbers $h_{1,1}=9$ and $h_{1,2}=0$. The standard counting of hyper and vector multplets for Type IIB compactifications tells us that $n_V = h_{1,2}$ and $n_H = h_{1,1}+1$ [17] and the 4-dimensional low energy effective theory we are left with is therefore $N=2$ supergravity coupled to 10 hypermultiplets and 0 vector multiplets (see [15] and references therein). In particular, the only vector field arising in the compactification, namely the graviphoton, comes from the self-dual RR 4-form $C_{\mu\nu\rho\sigma}$ under which the D3-brane is already charged in 10 dimensions.

As explicitly shown in ref. [7,8], the 3-brane wrapped on T_6/Z_3 does not couple to the hypers (as it must be) and has both an electric and a magnetic charge with respect to the graviphoton, consistently with the fact that the 3-brane is selfdual in 10 dimensions. This can be seen by analyzing the velocity dependence of the large distance behavior of the scattering amplitude for two of these 3-branes moving with constant velocities in the 4-dimensional non-compact spacetime, in which they look point-like. The boundary state decribing this 3-brane wrapped on T_6/Z_3 can be obtained from the one constructed for the non-compact 3-brane essentially through the usual quantization of the momentum along a compact direction.

More precisely, recall that the T_6/Z_3 orbifold is constructed identifying points in the covering $T_6 = T_2 \times T_2 \times T_2$ which are connected by Z_3 rotations in the 3 $a, a+1$ planes corresponding to each of the T_2's [14]. Notice that $h_{1,2}=0$ means that the number of complex deformations is 0 in this case, consistently with the fact that the Z-orbifold procedure "freezes out" any possible freedom in the choice of the 3 T_2's [17]. This reflects into the fact that the 3-brane configuration we consider *must* have one Neumann and one Dirichlet direction in each of the 3 T_2's and is therefore wrapped on a 3-cycle which is "democratically" embedded in $(T_2)^3$.

Let us start concentrating on a single T_2 factor, then. The only lattice compatible with the eventual Z_3 moding is the triangular one, with modulus $\tau = Re^{i\frac{\pi}{3}}$, as in Fig. 2. The lattice of windings $\bar{L} = L_x + iL_y$ is given by $\bar{L} = m\tau + nR = \frac{R}{2}(2n+m) + i\frac{\sqrt{3}}{2}Rm$, with m, n integers, that is

$$L_x = \frac{R}{2}N_x \ , \quad L_y = \frac{\sqrt{3}}{2}RN_y \ ,$$

where N_x, N_y are integers of the same parity. The lattice of momenta is as usual determined by the requirement that the plane wave $e^{ip\cdot X}$ is well defined when X is shifted by a vector belonging to the winding lattice, and one finds

$$p_x = \frac{2\pi}{R} n_x \ , \quad p_y = \frac{2\pi}{\sqrt{3}R} n_y \ ,$$

where n_x, n_y are again integers of the same parity.

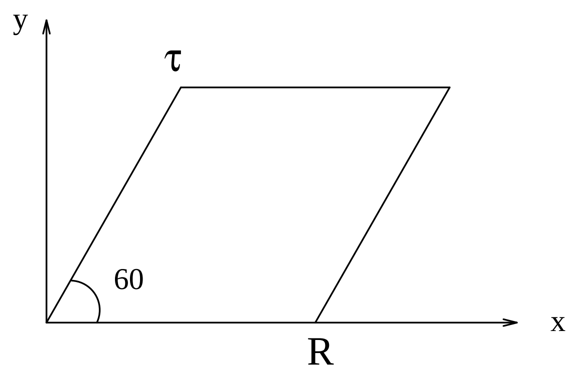

Fig. 2

We choose in each of the T_2 an arbitrary Dirichlet direction x' at angle θ with the x direction and an orthogonal Neumann direction y' at angle $\Omega = \theta + \frac{\pi}{2}$ with the x direction, and fix its lenght. This amounts to choose an arbitrary vector $\bar{L}$ in the winding lattice, which is identified by the pair (N_x, N_y) or, more conveniently for the following, by the orthogonal pair $(\bar{n}_y, -\bar{n}_x)$, which corresponds to the orthogonal direction of allowed momenta (see Fig. 3). In this way

$$\begin{aligned} &L_x = -L \sin\theta \ , \quad L_y = L \cos\theta \ , \\ &\cos\theta = -\frac{\sqrt{3}R}{2L}\, \bar{n}_x \ , \quad \sin\theta = -\frac{R}{2L}\, \bar{n}_y \ . \end{aligned}$$

where

$$L \equiv |\bar{L}| = \frac{R}{2}\sqrt{\bar{n}_y^2 + 3\bar{n}_x^2} \ .$$

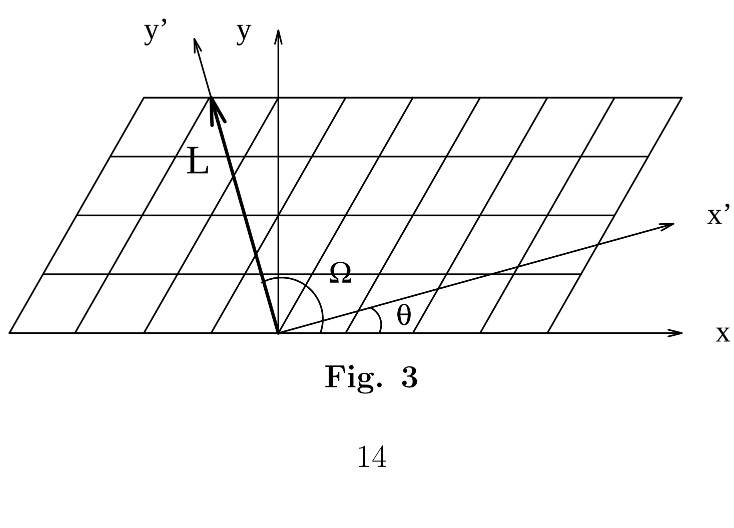

Fig. 3

We are now interested in the bosonic non oscillator modes contribution to the whole picture and let us start, for semplicity, remembering the result for the non-compact case. The boundary state for the bosonic non oscillator modes in a given $a, a+1$ pair is

$$
\begin{aligned}
|\vec{Y}>_B &= \delta\left(X_o' - Y'\right)|0> \\
&= \int \frac{dp_x dp_y}{(2\pi)} e^{-i(p_x\cdot Y_x + p_y\cdot Y_y)} \delta\left(\cos\theta p_y - \sin\theta p_x\right)|p_x, p_y> \ .
\end{aligned} \tag{26}
$$

The δ-function selects momenta parallel to the Dirichelet direction we have chosen. Indeed if ω is the direction of the generic $\vec{p}$ momentum, the argument of the δ-function becomes proportional to $\sin(\theta-\omega)$. Using of the normalization

$$
< p_x^{(1)}, p_y^{(1)}|p_x^{(2)}, p_y^{(2)} >= (2\pi)^2 \delta\left(p_x^{(1)} - p_x^{(2)}\right)\delta\left(p_y^{(1)} - p_y^{(2)}\right) \ ,
$$

one recovers the following vacuum amplitude

$$
\begin{aligned}
< \theta^{(1)}, \vec{Y}^{(1)}|e^{-lH}|\theta^{(2)}, \vec{Y}^{(2)} >_B &= \int dp_x dp_y e^{-i(p_x\cdot\Delta Y_x + p_y\cdot\Delta Y_y)} \times \\
&\quad \times\delta\left(\cos\theta^{(1)}p_y - \sin\theta^{(1)}p_x\right)\delta\left(\cos\theta^{(2)}p_y - \sin\theta^{(2)}p_x\right) \\
&= \frac{1}{\sin|\theta^{(1)} - \theta^{(2)}|} \ .
\end{aligned} \tag{27}
$$

In discretizing this result we adopt the following strategy. Let us begin by supposing $\theta^{(1)} \neq \theta^{(2)}$. First we substitute in eq. (27) the previously derived expressions for the discretized quantities $\vec{p}$ and θ and extract some jacobians from the Dirac δ-functions, obtaining

$$
< \theta^{(1)}, \vec{Y}^{(1)}|e^{-lH}|\theta^{(2)}, \vec{Y}^{(2)} >_B \ = \frac{L(\theta^{(1)})L(\theta^{(2)})}{(\sqrt{3}/4)R^2} \sum_{\substack{n_x,n_y \\ \text{same par}}} \delta\left(\bar{n}_x^{(1)}n_y - \bar{n}_y^{(1)}n_x\right)\delta\left(\bar{n}_x^{(2)}n_y - \bar{n}_y^{(2)}n_x\right) \ .
$$

Since in this case the solution of the condition enforced by the δ-functions is $n_x = n_y = 0$, all the momenta are zero and the exponential drops as in the continuum case.

The Dirac δ-function containing only integers can now be turned to a Kroneker one; however, since the latter is insensitive to an integer rescaling whereas the former transforms with an integer jacobian, we shall keep an arbitrary integer constant in this step:

$$
\delta\left(\bar{n}_x^{(1)}n_y - \bar{n}_y^{(1)}n_x\right)\delta\left(\bar{n}_x^{(2)}n_y - \bar{n}_y^{(2)}n_x\right) = N\delta_{\bar{n}_x^{(1)}n_y, \bar{n}_y^{(1)}n_x}\delta_{\bar{n}_x^{(2)}n_y, \bar{n}_y^{(2)}n_x} = N\delta_{n_x,0}\delta_{n_y,0} \ .
$$

Therefore †, with $\mathrm{Vol}(T_2) = (\sqrt{3}/2)R^2$

$$
< \theta^{(1)}, \vec{Y}^{(1)}|e^{-lH}|\theta^{(2)}, \vec{Y}^{(2)} >_B = N\frac{L(\theta^{(1)})L(\theta^{(2)})}{\mathrm{Vol}(T_2)}.
$$

† Notice that we consistently take $\sum_{\substack{n_x,n_y \\ \text{same par}}} \delta_{n_x,0}\delta_{n_y,0} = \frac{1}{2}$.

The integer N is fixed to 1 by the requirement that for $\theta^{(1)} = \theta^{(2)}$ the amplitude reduces to the "winding" $L^2/\mathrm{Vol}(T_2)$. Actually, in order to achieve the above limit, an infinite $L(\theta)$ is in general required because of the discreteness of the allowed angles, even if for strictly parallel branes finite $L(\theta)$'s are possible. Indeed, $L(\theta^{(1)})L(\theta^{(2)})\sin|\theta^{(1)} - \theta^{(2)}| = |\bar{n}_x^{(1)}\bar{n}_y^{(2)} - \bar{n}_y^{(1)}\bar{n}_x^{(2)}|\mathrm{Vol}(T_2)$. In this way the continuum and discrete results differ by the integer jacobian $|\bar{n}_x^{(1)}\bar{n}_y^{(2)} - \bar{n}_y^{(1)}\bar{n}_x^{(2)}|$ (which vanishes for $\theta^{(1)} = \theta^{(2)}$). The final result is then

$$< \theta^{(1)}, \vec{Y}_1|e^{-lH}|\theta^{(2)}, \vec{Y}_2 >_B = \frac{L(\theta^{(1)})L(\theta^{(2)})}{\mathrm{Vol}(T_2)} = \frac{|\bar{n}_x^{(1)}\bar{n}_y^{(2)} - \bar{n}_y^{(1)}\bar{n}_x^{(2)}|}{\sin|\theta^{(1)} - \theta^{(2)}|} \; . \tag{28}$$

The above result could have been obtained starting directly from the compact boundary state, that is, by *first* discretizing the continuum boundary state (26) and *then* computing the amplitude. The correct discrete boundary state turns out to be

$$|\vec{Y} >_B = L(\theta) \sum_{\substack{n_x, n_y \\ \text{same par}}} \frac{1}{(\sqrt{3}/2)R^2} e^{-\frac{2\pi}{R}i(n_x Y_x + n_y/\sqrt{3}Y_y)} \delta\left(\bar{n}_x n_y - \bar{n}_y n_x\right)|n_x, n_y > \; , \tag{29}$$

and reproduces correctly eq. (28) with the definition

$$< n_x, n_y|m_x, m_y >= \sqrt{3}R^2 \delta_{n_x,m_x}\delta_{n_y,m_y} \; .$$

Postponing for the moment the Z_3 identification, let us now consider as an instructive intermediate result the case of T_6. The result eq. (28) can be generalized in a straightforward way giving for the total contribution from the compact part of the bosonic non oscillator modes

$$< \theta_a^{(1)}, \vec{Y}^{(1)}|e^{-lH}|\theta_a^{(2)}, \vec{Y}^{(2)} >_B = \frac{V(B_1)V(B_2)}{\mathrm{Vol}(T_6)} \; , \tag{30}$$

where $V(B_1)$, $V(B_2)$ are the volumes of the two 3-branes. This factor can be reabsorbed in the definition of a 4-dimensional $\hat{\mu}_3$ (from now on $\theta_a^{(1)} - \theta_a^{(2)} \equiv \theta_a$)

$$\hat{\mu}_3^2 \equiv \mu_3^2 \frac{V(B_1)V(B_2)}{\mathrm{Vol}(T_6)} = 2\pi \prod_a \frac{|\bar{n}_a^{(1)}\bar{n}_{a+1}^{(2)} - \bar{n}_{a+1}^{(1)}\bar{n}_a^{(2)}|}{\sin|\theta_a|} \; . \tag{31}$$

The contribution of the fermions doesn't change during the compactification and the amplitude (20) becomes in this case

$$\mathcal{A} = \frac{\hat{\mu}_3^2}{\sinh|v|} \int_0^\infty \frac{dl}{4\pi l} e^{-\frac{b^2}{4l}} \frac{1}{16} \sum_s Z_B Z_F^s \; , \tag{32}$$

and can be rewritten this time as a *one* dimensional world-sheet integral

$$\mathcal{A} = \hat{\mu}_3^2 \int d\tau \int dl (4\pi l)^{-\frac{3}{2}} e^{-\frac{r^2}{4l}} \frac{1}{16} \sum_s Z_B Z_F^s \; , \tag{33}$$

in terms of the 4-dimensional distance

$$r = \sqrt{\vec{b}^2 + \sinh^2 v\tau^2}\ .$$

Eqs. (24) for the large distance diagonal interaction between two branes at the positions $\xi^{(1)}$ and $\xi^{(2)}$, and (25) for the scale-independent off-diagonal interaction between a brane at transverse position $\xi^{(2)}$ and a pair of brane and antibrane at $\xi_B^{(1)}$ and $\xi_A^{(1)}$, modify to

$$I_D = \alpha_{even} \coth v D_2 |\xi^{(1)} - \xi^{(2)}|\ , \tag{34}$$

$$I_{off-D} = \pm\alpha_{odd} \int_{\xi_B^{(1)}}^{\xi_A^{(1)}} d\xi^{(1)} \wedge \partial_{\xi^{(1)}} D_2 |\xi^{(1)} - \xi^{(2)}|\ , \tag{35}$$

with

$$\alpha_{even} = \hat{\mu}_3^2 \prod_a \cos\theta_a\ ,$$
$$\alpha_{odd} = \hat{\mu}_3^2 \prod_a \sin\theta_a\ .$$

Recalling (31) and noticing that

$$\cot\theta_a = \sqrt{3} \frac{3\bar{n}_a^{(1)}\bar{n}_a^{(2)} + \bar{n}_{a+1}^{(1)}\bar{n}_{a+1}^{(2)}}{\bar{n}_a^{(1)}\bar{n}_{a+1}^{(2)} - \bar{n}_{a+1}^{(1)}\bar{n}_a^{(2)}}\ ,$$

the two coupling can also be written as

$$\alpha_{even} = 2\pi \prod_a \sqrt{3}\left(3\bar{n}_a^{(1)}\bar{n}_a^{(2)} + \bar{n}_{a+1}^{(1)}\bar{n}_{a+1}^{(2)}\right)\ ,$$
$$\alpha_{odd} = 2\pi \prod_a \left(\bar{n}_a^{(1)}\bar{n}_{a+1}^{(2)} - \bar{n}_{a+1}^{(1)}\bar{n}_a^{(2)}\right)\ . \tag{36}$$

As expected, the orientation of the 3-branes in 10 dimensions affects the effective electric and magnetic couplings of the correspondig 0-branes in 4 dimensions. Notice that the Dirac quantization condition for the off-diagonal coupling α_{odd}, which is satisfied in 10 dimensions with the minimal allowed charges [1], remains satisfied in 4 with an integer which depends on the branes' orientation. This result can also be understood in terms of the relevant $N = 8$ supergravity. Notice in fact that

$$\prod_a \cos\theta_a = \frac{1}{4}\sum_{i=1}^{4} \cos\phi_i\ ,$$
$$\prod_a \sin\theta_a = -\frac{1}{4}\sum_{i=1}^{4} \sin\phi_i\ ,$$

with $\phi_i \equiv \phi_i^{(1)} - \phi_i^{(2)}$ and

$$\phi_1^{(1,2)} = \theta_4^{(1,2)} + \theta_6^{(1,2)} + \theta_8^{(1,2)}\ ,\quad \phi_2^{(1,2)} = -\theta_4^{(1,2)} - \theta_6^{(1,2)} + \theta_8^{(1,2)}\ ,$$
$$\phi_3^{(1,2)} = \theta_4^{(1,2)} - \theta_6^{(1,2)} - \theta_8^{(1,2)}\ ,\quad \phi_4^{(1,2)} = -\theta_4^{(1,2)} + \theta_6^{(1,2)} - \theta_8^{(1,2)}\ .$$

The effective couplings can thus be rewritten as

$$\alpha_{even} = \sum_{i=1}^{4} \left(e_i^{(1)} e_i^{(2)} + g_i^{(1)} g_i^{(2)} \right) \ ,$$
$$\alpha_{odd} = \sum_{i=1}^{4} \left(e_i^{(1)} g_i^{(2)} - g_i^{(1)} e_i^{(2)} \right) \ , \qquad (37)$$

with

$$e_i^{(1)} = \frac{\hat{\mu}_3}{2} \cos \phi_i^{(1)} \ , \quad e_i^{(2)} = \frac{\hat{\mu}_3}{2} \cos \phi_i^{(2)} \ ,$$
$$g_i^{(1)} = \frac{\hat{\mu}_3}{2} \sin \phi_i^{(1)} \ , \quad g_i^{(2)} = \frac{\hat{\mu}_3}{2} \sin \phi_i^{(2)} \ . \qquad (38)$$

This second consideration allows to keep track of the coupling to the various vector fields. In fact it happens that the ten vectors fields arising from dimensional reduction of the RR 4-form, couple to the brane only through four independent combinations of fields, with electric and magnetic charges parametrized by the four angles $\phi_i^{(1,2)}$. Since the electric and magnetic charges correponding to a given $\phi_i^{(1,2)}$ cannot vanish simultaneously, the 3-brane cannot decouple from any of the four effective gauge fields, in agreement with a pure Supergravity argument achieved in ref. [18]. From this point of view, the Dirac quantization condition, emerging clearly in (36), is to be understood on the sum of the couplings corresponding to the four independent $\phi_i^{(1,2)}$, and not on the charges with respect to the single fields.

The whole picture determines therefore a 4-parameter family of dyons which are inequivalent from the 4-dimensional point of view since they carry a different set of charges. Notice finally that when two of these branes have equal $\phi_i^{(1,2)}$'s (yielding vanishing ϕ_i's) their diagonal coupling no longer depends on the angles and the off-diagonal one vanish, as appropriate for identical dyons in $d = 4$ dimensions.

Let us discuss finally the orbifold case. As explained in ref. [7], the only effect of the Z_3 moding is to project the boundary state for T_6 onto its Z_3-invariant part. This projection can be easily performed by first computing the amplitude on T_6 with a relative twist z_a in the orientations, $\theta_a \rightarrow \theta_a + 2\pi z_a$, and then averaging finally on all the possible z_a's ‡.

Since the bosonic zero modes' contribution (30) does not depend explictly on the angles, the only modification introduced by the Z_3 moding is in the volume: $\mathrm{Vol}(T_6/Z_3) = 1/3\mathrm{Vol}(T_6)$. For the fermions, instead, one simply sets $\theta_a \rightarrow \theta_a + 2\pi z_a$; under this relative rotation one has correspondingly:

‡The twists z_a in the 3 $a, a+1$ planes satisfy $\sum_a z_a = 0$ in order to preserve at least one supersymmetry [14]. The allowed sets $\{z_a\}$ of relative twits can be taken to be $\{(0,0,0), (1/3, 1/3, -2/3), (2/3, 2/3, -4/3)\}$.

$$\begin{aligned}
\phi_1 &\to \phi_1 + 2\pi(z_4 + z_6 + z_8) = \phi_1 \ , \\
\phi_2 &\to \phi_2 + 2\pi(-z_4 - z_6 + z_8) = \phi_2 + 4\pi z_8 \ , \\
\phi_3 &\to \phi_3 + 2\pi(z_4 - z_6 - z_8) = \phi_3 - 4\pi z_4 \ , \\
\phi_4 &\to \phi_4 + 2\pi(-z_4 + z_6 - z_8) = \phi_4 + 4\pi z_6 \ .
\end{aligned}$$

The averaging procedure has the important consequence of projecting out the contribution depending on the non invariant ϕ_2, ϕ_3, ϕ_4, with respect to the T_6 case. Indeed,

$$\begin{aligned}
\alpha_{even} &= \hat{\mu}_3^2 \sum_{\{z_a\}} \prod_a \cos(\theta_a + 2\pi z_a) = \frac{\hat{\mu}_3^2}{4} \cos\phi_1 \ , \\
\alpha_{odd} &= \hat{\mu}_3^2 \sum_{\{z_a\}} \prod_a \sin(\theta_a + 2\pi z_a) = -\frac{\hat{\mu}_3^2}{4} \sin\phi_1 \ .
\end{aligned} \tag{39}$$

where the 1/3 of the averiging has canceled with the 3 coming from the volume of T_6/Z_3. Therefore, after the Z_3 moding, only one pair of electric and magnetic charges survives, consistently with the fact that, as already pointed out at the beginning of this section, only one vector field survives to the projection in the low energy effective theory, namely the graviphoton. The fact that the Dirac quantization condition still holds, like in the T_6 case, is due to the fact that (39) can be seen as the superposition of 3 pairs of 3-branes on T_6, with relative angles $\theta_a + 2\pi z_a$ instead of θ_a. For each pair (36) holds and so for their sum, that is (39).

Summarizing, the net effect of the wrapping of the 3-brane on T_6/Z_3 is therefore to obtain a 1-parameter family of dyons (rather than 4 as for T_6) whose effective couplings depend only on one combination of the relative angles between the whole 3-branes.

It is interesting to remark that the Z_3 projection, which reduces the 4 independent gauge fields to 1, is also responsible for the decoupling of the scalars fields from the 3-brane, as seen in ref. [7]. Thus, the 3-brane wrapped on T_6/Z_3 looks like an extremal R-N configuration, being a source of Gravity and Maxwell field only, the mass and the dyonic charge being equal in suitable units, i.e. $M^2 = e^2 + g^2$.

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