SENSOR ARRAY CALIBRATION VIA TRACKING WITH THE EXTENDED KALMAN FILTER

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ABSTRACT

Starting with a randomly distributed sensor array with unknown sensor orientations, array calibration is needed before target localization and tracking can be performed using classical triangulation methods. In this paper, we assume that the sensors are only capable of accurate direction of arrival (DOA) estimation. The calibration problem cannot be completely solved given the DOA estimates alone, since the problem is not only rotationally symmetric but also includes a range ambiguity. Our approach to calibration is based on tracking a single target moving at a constant velocity. In this case, the sensor array can be calibrated from target tracks generated by an extended Kalman filter (EKF) at each sensor. A simple algorithm based on geometrical matching of similar triangles will align the seperate tracks and determine the sensor positions and orientations relative to a reference sensor. Computer simulations show that the algorithm performs well even with noisy DOA estimates at the sensors.

1. INTRODUCTION

The problem of localization and tracking by passive sensor arrays arises in numerous practical applications [1]-[4], [6]-[9]. Likewise, two-dimensional bearings-only target motion analysis (TMA) has been studied extensively [1]-[6]. One of the most familiar situations is tracking by a single moving observer, which monitors the bearing angle of an acoustic source (target) that is assumed to be moving with constant velocity. Figure 1 depicts the geometrical configuration of the problem in 2D, where the sensor motion is unconstrained in the x-y plane. Even though the configuration appears to be intuitively simple, the tracking problem is not easy to solve, because the problem is intrinsically nonlinear.

Sensor array calibration is a generalization of this problem in which the sensor positions as well as the target track have to be determined. In this broader problem, the target is not constrained to move on a straight line and can assume a complex motion. This estimation problem is again nonlinear and unfortunately not amenable to linear analysis techniques. Moreover, if the sensors have random orientation references, calibration requires not only localizing the sensors but also identifying their orientation angles. Estimation is performed using only the noisy DOA (direction of arrival) estimates and hence the tracking part of the problem inherently includes range and rotational ambiguities. The range ambiguity problem is illustrated in Fig. 2, where it is shown that two different targets may have the same DOAs at all times. Rotational

Prepared through collaborative participation in the Advanced Sensors Consortium sponsored by the U.S. Army Research Laboratory under Cooperative Agreement DAAL01-96-2-0001.

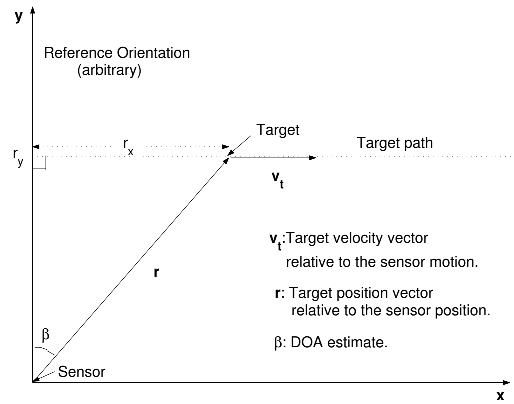

Figure 1: Geometrical configuration of the two dimensional tracking problem, where sensor and target are assumed to lie in the same plane. The target track is assumed to be parallel to the x-direction, and the sensor reference orientation is arbitrarily chosen to be the y-direction. Note that the sensor is situated at the origin.

ambiguity is more evident since adding or subtracting the same constant to all of the reference angles would not have any effect. These ambiguities necessitate a criterion for the observability of the target, which was studied by Nardone and Aidala [5].

In this paper, an extended Kalman filter is used to calibrate the sensor array for a target moving with a constant velocity. The pertinent filter equations of state and measurement are shown. The extended Kalman filter equations are formulated in the so-called modified polar coordinates (MPC for 2D, modified spherical coordinates-MSC for 3D) [1]- [4],[12]- [13], which decouples the observable and unobservable variables in the state vector [2]. This decoupling prevents the possibility of an ill-conditioned covariance matrix and hence provides stability to the filter. The MPC (or MSC) extended Kalman filter is favored over a Cartesian coordinate formulation and some other pseudo-linear solutions since its estimates are asymptotically unbiased [10, 11]. Moreover, as will be shown later, the extended Kalman filter can determine the sensor positions and orientations accurately for a target moving with constant velocity, by exploiting the inherent range and rotational ambiguities. The constant velocity assumption simplifies the problem considerably, and might be feasible in the calibration stage if it were possible to drive a known calibration target through the sensor field to generate data for the EKF calibration algorithm.

The extended Kalman MPC state vector consists of four variables: bearing rate, range rate divided by the range, bearing, and

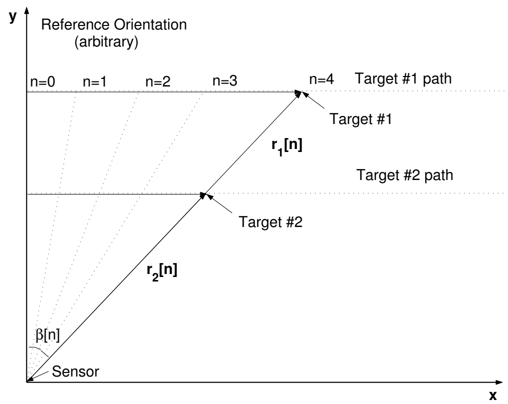

Figure 2: Two different targets may have the same DOA at all times. It is impossible to distinguish target #1 from target #2 given the DOA measurements alone.

reciprocal of the range as compared to its Cartesian counterpart, whose state vector has velocity and range as state variables [2]. With the MPC state variables, it will be demonstrated with computer simulations that the extended Kalman filter scales the target track when the initial range information is not available. Basic rotations to align the target tracks from multiple sensors will result in sensor orientations whereas simple geometrical triangle similarity arguments will culminate in the scaled relative sensor positions[1].

In Section 2, the filter properties are given and our approach to the solution is revealed. Section 3 accounts for the details of the sensor calibration scheme and offers an alternative method for calibration. Section 3 also provides simulations demonstrating the efficacy of our approach. Section 4 extends the solution to 3D using the modified spherical coordinates. Conclusions are given in the last section.

2. THE EKF FILTER PROPERTIES

The MP state vector is given by

$$\mathbf{y}(t) \triangleq \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} = \begin{bmatrix} \dot{\beta}(t) \\ \dot{r}(t)/r(t) \\ \beta(t) \\ 1/r(t) \end{bmatrix} \tag{1}$$

where

$$\begin{aligned} r(t) &= \|\mathbf{r}(t)\| = \sqrt{r_x^2(t) + r_y^2(t)} \\ \beta(t) &= \tan^{-1}\left[\frac{r_x(t)}{r_y(t)}\right] + \eta(t) \end{aligned} \tag{2}$$

depict the relative target range and bearing angle, respectively. $\eta(t)$ denotes the additive Gaussian noise to the bearing measurements. Development of the EKF filter equations can be found at [1]-[2],[4] and hence are not reproduced here.

The extended Kalman filter requires four inputs in order to produce its desired output, which is the tracked position of the target. These inputs are the noisy bearings angle measurement vector, the initial state vector, the initial error covariance matrix, and a relative acceleration vector. The relative acceleration vector is set to zero since the target is assumed to be moving with constant velocity. The angle measurement vector is generated by Eq. (2) and when the sensor is stationary it can be written as (refer to the configuration in Fig. 1)

$$\tan(\beta(t)) = \frac{v_t}{r_y} t = \alpha t \tag{3}$$

The initial state vector and covariance matrix, on the other hand, are initialized as follows

$$\mathbf{y}(0|0) = [(\beta(2) - \beta(1))/T \quad 0 \quad \beta(2) \quad 1/R_o]^T \tag{4}$$

$$\mathbf{P}(0|0) = \begin{bmatrix} 2\sigma_e^2(1)/T^2 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & \sigma_e^2(1) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

where T is the sampling period, R_o is our initial distance estimate to the target, $\sigma_e^2(n)$ is the time varying variance of the additive Gaussian angle measurement noise, and γ is a free parameter of our choice.

An important feature of the extended Kalman filter is that if our initial target range estimate, R_o, is not specified correctly, the filter generates a scaled target track (Fig. 3). It is easy to prove this property by looking at Eq. (3). If r_y is not specified correctly, since α only depends on the DOA estimates, v_k will be scaled accordingly. As mentioned above, it is not possible to overcome the range ambiguity without having an absolute distance measure. However, this property of the extended Kalman filter enables us to use any value of R_o without losing the track information.

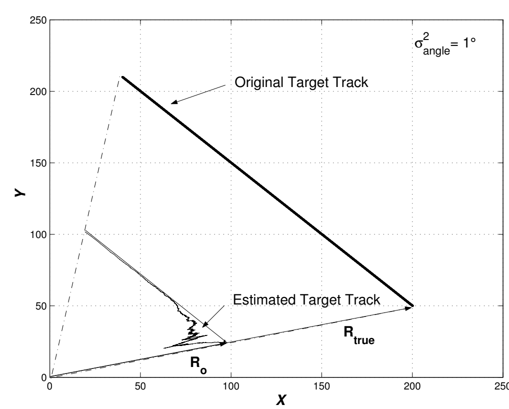

Figure 3: If R_o was larger than R_{true}, the estimated target track would form a bigger triangle that is again similar to the triangle formed by the true track. Note that the sensor is situated at the origin and $\gamma = 1$.

In the simulation of Fig. 3, a sampling period of $T = 0.002$s was used for illustrative purposes. In a more realistic situation that imposes low power constraints on the sensors, high sampling rates are not desirable. The extended Kalman filter has also been tested for low sampling rates. Figure 4 shows that it still performs reasonably well in tracking the target even though that sampling period was $T = 1$s and the DOA measurements were disturbed by additive Gaussian noise with standard deviation $\sigma_{angle}^2 = 1°$.

[1]It should be noted that the range ambiguity can not be circumvented given DOA measurements alone. It will be required to have one true range to reach the absolute localization of the sensor array.

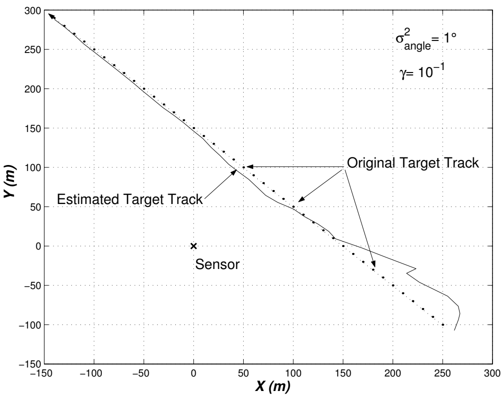

Figure 4: Target is moving with v_x =10m/s and v_y =10m/s. $N = 40$ samples are taken during 40s, corresponding to a sampling period T=1s. $\gamma = 10^{-1}$ gave the best track for this case.

3. SENSOR ARRAY CALIBRATION USING EXTENDED KALMAN FILTER

In this section, the sensor array will be calibrated using the tracks generated by the extended Kalman filter along with a simple geometrical procedure. Figure 5 illustrates the approach used in determining the sensor positions where A_0B_0 and A_1B_1 are the track estimates of the reference sensor and sensor 1, respectively. Since the track R_1 and $\Delta\beta_1$ are known, the angles ϕ_1 and α_1 can be determined. Then, on the reference track R_0, we can find triangulate the sensor position (x_{s1}, y_{s1}) by intersecting two lines from A_0 (A_0A_1) and B_0 (B_0B_1) using the angles ϕ_1 and α_1, respectively. In order to find the reference angle for sensor 1, $\Delta O_0A_1B_1$ is shifted without any rotation on (x_{s1}, y_{s1}) so that O_0 coincides with the point at (x_{s1}, y_{s1}). Then, the rotation angle that will align the triangle $\Delta O_0A_1B_1$ and the triangle $\Delta O_1A_1B_1$ is sought.

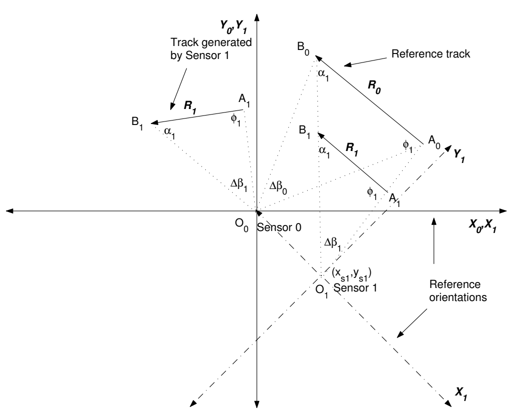

Figure 5: Triangle $\Delta O_0A_1B_1$ is similar to $\Delta O_1A_0B_0$ by the scaling property of the extended Kalman filter. Since the reference track A_0B_0 and the track A_1B_1 estimated by sensor 1 are known, (x_{s1}, y_{s1}) can be determined geometrically. Once the sensor position is determined, sensor orientation is found by simply finding the rotation angle that aligns the estimated track and the reference track in the configuration shown.

In finding the sensor position, it is not assumed that the reference track is the true track of the target. Hence, the range ambiguity is still present in the problem. However, once given a true range like the distance between two sensors or target track length, one can determine the absolute positions of the sensors with respect to some chosen reference sensor whose absolute position could be determined by some other means (e.g., GPS.) Another interesting feature is that the reference sensor angles can be determined by the DOA estimates alone without any range information.

In Fig. 6, the result of our approach is illustrated. A sampling period of $T = 0.5$s is used and 200 DOA samples are taken from a target moving on a straight line corresponding to a total observation time of 100s. The target track is disturbed to prevent perfect linearity and hence better accounts for realistic situations. The calibration is satisfactory considering the fact that only 200 samples are used. It should be noted that increasing the number of samples and the sampling frequency at the same time results in better estimates of the sensor positions if more accuracy is required. If more calibrating targets are available, averaging the resulting position estimates will increase the accuracy.

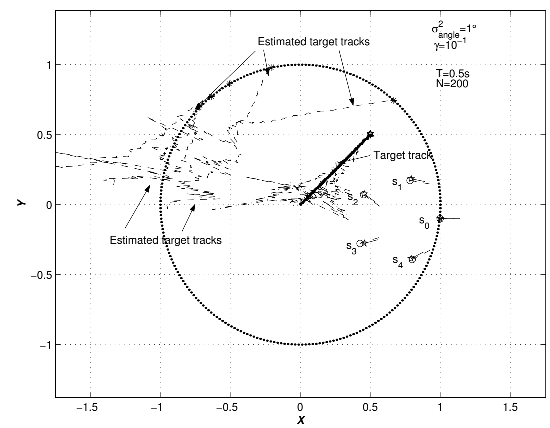

Figure 6: An array consisting of 5 sensors is calibrated using the extended Kalman filter. Orientations of the sensors are also shown along with the actual positions. Then, our estimates of the sensor position and orientation estimates are superposed.

4. EXTENSION TO THE 3D CASE

Filter equations for the 3D case are more complex than the equations for the 2D case but they follow the same structure. It can be shown that introducing the elevation angle also introduces a bias in the last state variable, which is the inverse range [1]. This bias may be removed, but the array calibration can be still achieved even in the presence of this bias[2]. It should be noted that the filter still preserves its scaling property and hence the rotation of similar triangles used in 2D can again be exploited for 3D. Figure 7 illustrates the track generated by the extended Kalman filter using the modified spherical coordinates with bias removal. The target is moving with $v_x = -25$m/s , $v_y = 25$m/s , $v_z = 25$m/s and $N = 300$ samples are used in obtaining the graph, corresponding to 90s of observation time.

[2] Biased track and the original target track also form a similar triangle.

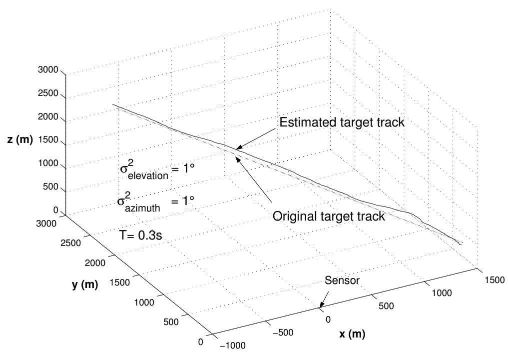

Figure 7: Tracking example in 3D.

5. CONCLUSIONS

The extended Kalman filter is used to train a sensor array with unknown position and orientations assuming one target with constant velocity. When the constant velocity assumption is imposed on the target, properties of the extended Kalman filter can be exploited to find the scaled relative sensor positions and relative orientations given the DOA measurements alone. In order to get rid of the range and rotational ambiguities of the tracking problem, one absolute range measure and a reference orientation must be given. This approach is simple and efficient as well as accurate as demonstrated by the simulations.

6. REFERENCES

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