Primordial magnetic fields from inflation?

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Abstract

The hot plasma above the electroweak scale contains (hyper) charged scalar particles which are coupled to Abelian gauge fields. Scalars may interact with gravity in a non-conformally invariant way and thus their fluctuations can be amplified during inflation. These fluctuations lead to creation of electric currents and produce inhomogeneous distribution of charge density, resulting in the generation of cosmological magnetic fields. We address the question whether these fields can be coherent at large scales so that they may seed the galactic magnetic fields. Depending upon the mass of the charged scalar and upon various cosmological (critical fraction of energy density in matter, Hubble constant) and particle physics parameters we found that the magnetic fields generated in this way are much larger than vacuum fluctuations. However, their amplitude on cosmological distances is found to be too small for seeding the galactic magnetic fields.

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I. FORMULATION OF THE PROBLEM

The idea that our galaxy could possess a magnetic field dates back to the (independent but simultaneous) works of Fermi [1] (motivated by the origin of high energy cosmic rays) and Schwinger [2] (motivated by the origin of galactic synchrotron emission). Since then, a lot of work has been done both from the experimental and theoretical side. Listening to observations [3,4] large scale cosmological magnetic fields can be estimated from Faraday rotation effects and Zeeman splitting of (hyperfine) spectral lines. Listening to theory the main puzzle is connected with the dynamical origin of large scale magnetic fields.

It is usually assumed that the observed magnetic fields were (exponentially) amplified by galactic dynamo mechanism from pre-existing seed magnetic fields, coherent over scales of the order of 100 pc at the time of galaxy formation. The amplitude of the necessary seed fields is quite uncertain and depends on many details of the dynamo mechanism [3,5] and on cosmological parameters [6,7]. Typical vaules of the seeds range from 10^{-15} to 10^{-25} Gauss at the decoupling epoch.

In general, two types of ideas are considered for the explanation of the seed magnetic fields. The first one is related to astrophysics. For instance, a Biermann battery mechanism can be postulated at the level of protogalaxy [8] and this will possibly lead to a mean current able to provide a source term for the evolution equations of the magnetic fields in the galactic plasma. The observation of large scale magnetic fields associated with clusters of galaxies [9] also suggested the appealing possibility of connecting (inter) galactic magnetic fields with active galactic nuclei [10] (in this picture the dynamo mechanism is not essential since the rotation of a cluster is much smaller than the rotation of a galaxy). The second type of ideas relies on the the interplay between particle physics and cosmology at different moments in the life of the Universe. In this framework various models were discussed such as cosmological defects [11], phase transitions [12] electroweak anomaly [13], inflation [6,14], string cosmology [15], temporary electric charge non-conservation [16] or breaking of the gauge invariance [6].

The main problem of most particle physics mechanisms of the origin of seed fields is how to produce them coherently on *cosmological* scales. All causal proposals (related, for example, to the bubble collisions at the phase transitions) may produce sufficiently large magnetic fields only on sub-horizon scales. Inflation, quite effective in producing density fluctuations at super-horizon scales, fails to amplify directly the vacuum fluctuations of the electromagnetic field because of its conformally invariant coupling to gravity.

An attractive and very economical idea on the possible primordial origin of the galactic magnetic fields was suggested in [6]. In short, it is based on the following observation. While the coupling of electromagnetic field to the metric and to the charged fields is conformally invariant (this is not necessarily true in the models with dynamical dilaton [15]), the coupling of the charged scalar field to gravity is not. Thus, vacuum fluctuations of the charged scalar field can be amplified during inflation at super-horizon scales, leading potentially to non-trivial correlations of the electric currents and charges over cosmological distances. The fluctuations of electric currents, in their turn, may induce magnetic fields through Maxwell equations at the corresponding scales. The role of the charged scalar field may be played by the Higgs boson which couples to the hypercharge field above the electroweak phase transition; the generated hypercharged field is converted into ordinary magnetic field at the temperatures of the order of electroweak scale.

No detailed computations were carried out in [6] in order to substantiate this idea. The suggestion of [6] was further developed quite recently in [17] for the standard electroweak theory with an optimistic conclusion that large scale magnetic fields can be indeed generated. In [18] a supersymmetric model was studied.

The aim of the present paper is to re-analyze this proposal and compute the amplitude and the spectrum of seed magnetic fields that may be generated because of amplification of zero-point fluctuations of the scalar field during inflation. Our set-up resembles the one of Ref. [17]. We suppose that an inflationary phase was followed by a radiation dominated phase and we compute the charged particle production associated with the change of the metric. This allows to define the spectrum and magnitude of the current fluctuations at the

beginning of the radiation era. Taking the current distribution as an initial condition, we study the plasma-physics problem of the relaxation of such an initial condition. We compute finally the magnetic fields, which survived possibly until the time of galaxy formation.

Depending upon the mass of the charged scalar and upon various cosmological (critical fraction of energy density in matter, Hubble constant) and particle physics parameters we found that the magnetic fields generated in this way are much larger than vacuum fluctuations, in agreement with qualitative conclusions of refs. [6,17,18]. However, in contrast with [17], their amplitude on cosmological distances is found to be too small, by many orders of magnitude, in order to seed the galactic magnetic fields. We trace back this difference in the conclusions to the discrepancy in the results obtained for the Bogoliubov coefficients (appearing in the problem of scalar particle production) and to the treatment of the relaxation of electric currents in conducting media during the radiation dominated epoch.

The plan of our paper is the following. In Section II we will discuss the amplification of the charged scalar field in an expanding cosmological background. In Section III we will study the connection between the amplification of the charged scalar and the production of charge and current density fluctuations. In Section IV we will develop a curved space description of the evolution of charge and current fluctuations within a kinetic (Landau-Vlasov) approach. We will apply our analysis to the physical case of an ultra-relativistic plasma prior to decoupling. Section V contains some phenomenological applications of our formalism. We will mainly be concerned with the generation of large scale magnetic fields in various models of cosmological evolution and with the possible occurrence of charge density and current fluctuations at large scales. Section VI contains our concluding remarks.

II. AMPLIFICATION OF A COMPLEX SCALAR FIELD DURING INFLATION

In this paper we will only consider scalar electrodynamics. Possible generalizations of our results to theories containing more scalar fields (for example, for electroweak theory or its extensions) are straightforward. Since fermions are conformally coupled to gravity, their

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gravitational production is too small to generate any substantial seed magnetic fields [17].

Consider the action of a (massive) charged scalar field minimally coupled to the background geometry and to the electromagnetic field (hypercharge field, if the standard model is assumed):

$$S = \int d^4x \sqrt{-g}[(\mathcal{D}_\mu \phi)^* \mathcal{D}^\mu \phi - m^2 \phi^* \phi - \frac{1}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}], \tag{2.1}$$

where $\mathcal{D}_\mu = \partial_\mu - ie\mathcal{A}_\mu$, $\mathcal{F}_{\mu\nu} = \partial_{[\mu}\mathcal{A}_{\nu]}$, $g_{\mu\nu}$ is the four-dimensional metric and g its determinant.

We will suppose that the background geometry is described, in conformal coordinates, by a Friedmann-Robertson-Walker (FRW) line element

$$g_{\mu\nu} = a^2 \eta_{\mu\nu}, \quad ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2], \tag{2.2}$$

where $\eta_{\mu\nu}$ is the usual Minkowski metric.

It is convenient to introduce rescaled fields $\Phi = a\phi$ and $A_\mu = a\mathcal{A}_\mu$. Correspondingly, we denote by $\vec{E}$ and $\vec{B}$ the electric and magnetic fluctuations in curved space. They are related to the usual flat space fields $\vec{\mathcal{E}}$ and $\vec{\mathcal{B}}$ by a time-dependent rescaling involving the scale factor:

$$\vec{E} = a^2 \vec{\mathcal{E}}, \quad \vec{B} = a^2 \vec{\mathcal{B}}. \tag{2.3}$$

In terms of the rescaled fields the action is

$$S = \int d^3x \, d\eta [\eta^{\mu\nu} D_\mu \Phi^* D_\nu \Phi + (\frac{a''}{a} - m^2 a^2)\Phi^* \Phi - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}], \tag{2.4}$$

where the prime denotes differentiation with respect to the conformal time coordinate η and $D_\mu = \partial_\mu - ieA_\mu$, $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$. This is simply the action of electrodynamics in flat spacetime with a time dependent mass term for the scalar field. From this form of the action it is obvious that there is no direct amplification of electromagnetic fields during inflation. Moreover, for conformal coupling of the scalar field to gravity the term proportional to a'' is absent and the scalar particle production is supressed by the charged boson mass.

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To compute the magnetic field fluctuations we will use a perturbative approach. Namely, we will firstly compute the scalar particle production omitting completely the coupling of the scalar field to the gauge field (i.e. neglecting the back reaction, as it can be checked to be, a posteriori, self-consistent).

The classical evolution equations for Φ in the case $A_\mu = 0$ are simply given by:

$$\Phi'' - \nabla^2 \Phi - \frac{a''}{a}\Phi + m^2 a^2 \Phi = 0. \tag{2.5}$$

We will often use also decomposition of the complex scalar field in terms of two real fields, $\Phi = (\Phi_1 + i\Phi_2)/\sqrt{2}$.

Once the background geometry is specified, the amplified inhomogeneities in the field Φ can be computed. Suppose, as in [17], that the history of the Universe consists of two different epochs. A primordial phase, whose background evolution is not exactly known, and a radiation dominated phase where the scale factor $a(\eta)$ evolves linearly in conformal time. A continuous (and differentiable) choice of scale factors is then represented by

$$a_i(\eta) = \left(-\frac{\eta}{\eta_1}\right)^{-\alpha}, \quad \eta < -\eta_1,$$

$$a_r(\eta) = \frac{\alpha\eta + (\alpha+1)\eta_1}{\eta_1}, \quad \eta \geq -\eta_1, \tag{2.6}$$

where α is some effective exponent parametrizing the dynamics of the primordial phase of the Universe. Notice that if $\alpha = 1$ we have that the primordial phase coincides with a de Sitter inflationary epoch. In practice we will consider $\alpha = 1$ or slightly deviating from it.

As a result of the change in the behaviour of the scale factor occurring at $-\eta_1$ the modes of Φ will be parametrically amplified. Defining $x_1 = k\eta_1$ and $\mu = m\eta_1$ we can write the Bogoliubov coefficients for $\alpha = 1$ (standard inflation) and for cosmologically interesting case $x_1 \ll 1$ (long ranged fluctuations) and $\mu \ll 1$ (small scalar mass, potentially giving rise to large scalar fluctuations):

$$\alpha_k = e^{i\frac{\pi}{8}}\sqrt{\pi}\left\{-\frac{x_1^{-\frac{3}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{i\,x_1^{-\frac{1}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \left[\frac{1}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{(i-1)\mu^{1/4}}{\sqrt{2}\Gamma(\frac{1}{4})}\right]\sqrt{x_1}\right\} + \mathcal{O}(\mu^{\frac{5}{4}}),$$

$$\beta_k = e^{-i\frac{\pi}{8}}\sqrt{\pi}\left\{\frac{x_1^{-\frac{3}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} - \frac{i\,x_1^{-\frac{1}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \left[-\frac{1}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{(i+1)\mu^{1/4}}{\sqrt{2}\Gamma(\frac{1}{4})}\right]\sqrt{x_1}\right\} + \mathcal{O}(\mu^{\frac{5}{4}}). \tag{2.7}$$

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More general expressions can be found in Appendix A where we also give details about the calculation. The terms kept in the expansion for small x_1 and μ is such that the unitarity condition for the Bogoluibov coefficients is satisfied: $|\alpha_k|^2 - |\beta_k|^2 = 1$. This property of the truncation is quite important: when discussing charge density fluctuations we will see that interesting cancellations between the leading terms arise.

In the opposite limit, for $k > 1/\eta_1$ the mixing coefficients are exponentially suppressed as $\exp[-\lambda k \eta_1]$ [19]. The coefficient λ depends upon the details of the transition between the inflationary and radiation dominated phases. The existence of an exponential suppression in the mixing coefficients is quite important from a general point of view: it ensures gentle ultra-violet properties for the physical quantities we ought to compute.

We should mention that the leading contribution to the amplification coefficients has been computed in different contexts [20]. However, we are not only interested in the leading behaviour of the Bogoliubov coefficients but also in the corrections whose contribution is relevant when the leading contribution cancels, as in the case of charge density fluctuations (see the following Section).

III. CHARGE AND CURRENT DENSITY FLUCTUATIONS

The Bogoliubov coefficients obtained in the previous Section specify the probability of charged particle creation. The fluctuations in the scalar field induce also fluctuations in the electric current associated with the $U(1)$ symmetry of our action. The fluctuations in the charge and current density act as a source term for the evolution of the fluctuations in the gauge fields.

We will ignore, for the moment, the effect of the plasma conductivity which is essential for the calculation of induced magnetic fields (it will be discussed in the following section) and we will simply consider the structure of the current-current correlators. In the curved space, the conservation equation for the current is given by:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}j^\mu) = 0, \tag{3.1}$$

where

$$j^\mu = ie(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*). \tag{3.2}$$

It is convenient to introduce the rescaled current

$$J^\mu = \sqrt{-g} g^{\mu\nu} j_\nu, \tag{3.3}$$

which can be expressed as

$$J^\mu = e\left[\Phi_2 \partial^\mu \Phi_1 - \Phi_1 \partial^\mu \Phi_2\right] \tag{3.4}$$

in terms of conformal fields. Notice that in this last expressions the index appearing in the derivatives is raised (or lowered) using the Minkowski metric and not the curved metric.

If we average J^μ on the vacuum we have clearly that $\langle J^\mu \rangle = 0$. However, the fluctuations of the same quantity for two space-time separated points are not zero. By defining the two-point functions of the field operators Φ_1 and Φ_2

$$\mathcal{G}(x,y) = \langle \Phi_1(x)\Phi_1(y) \rangle = \langle \Phi_2(x)\Phi_2(y) \rangle, \tag{3.5}$$

the two-point function of the charge and current density fluctuations can be written as

$$\langle J_\mu(x) J_\nu(y) \rangle = 2e^2 \left\{ \mathcal{G}(x,y) \frac{\partial^2}{\partial x^\mu \partial y^\nu} \mathcal{G}(x,y) - \frac{\partial}{\partial y^\nu} \mathcal{G}(x,y) \frac{\partial}{\partial x^\mu} \mathcal{G}(x,y) \right\}, \tag{3.6}$$

where $x \equiv (\vec{x}, \eta)$ and $y \equiv (\vec{y}, \tau)$ (η and τ are two different conformal times). In the case of the vacuum (no amplification took place) the two-point functions are simply

$$\mathcal{G}(x,y) = \frac{1}{2(2\pi)^3} \int \frac{d^3 k}{k_0} e^{ik(x-y)}. \tag{3.7}$$

In the case of flat space-time the current density fluctuations can be expressed as

$$\langle J_\mu(x) J_\nu(y) \rangle = \frac{e^2}{4(2\pi)^6} \int \frac{d^3 p}{p_0} \int \frac{d^3 k}{k_0} (p_\mu - k_\nu)^2 e^{i(k+p)(x-y)}. \tag{3.8}$$

When the background passes from an inflationary phase to a radiation dominated phase we have instead that the two-point functions of the field operators can be written as

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$$\mathcal{G}(x,y) = \int \frac{d^3 k}{(2\pi)^3} \Big[|\alpha_k|^2 g_k(\eta) g_k^*(\tau) + |\beta_k|^2 g_k^*(\eta) g_k(\tau)$$
$$+ \alpha_k \beta_k^* g_k(\eta) g_k(\tau) + \alpha_k^* \beta_k g_k^*(\eta) g_k^*(\tau) \Big] e^{-i\vec{k}\cdot\vec{r}}, \tag{3.9}$$

where $g_k(\eta)$ are given by Eqs. (A.12) and where $\vec{r} = \vec{x} - \vec{y}$.

In order to define properly the charge and current density fluctuations at different scales it is helpful to introduce an averaging procedure both over space and over time. In the case of the charge density fluctuations we will define the charge fluctuations inside a patch of volume L^3 and within a typical time $\mathcal{T}$ as

$$Q_{L,\mathcal{T}}^2 = \frac{1}{\mathcal{T}^2} \int d^3 x \int d^3 y \int d\eta \int d\tau \langle J_0(\vec{x},\eta) J_0(\vec{y},\tau) \rangle W_{L,\mathcal{T}}(\vec{x}, \eta - \xi_0) W_{L,\mathcal{T}}(\vec{y}, \tau - \xi_0), \tag{3.10}$$

where $W_{L,\mathcal{T}}$ are the smearing functions selecting the contribution of the correlator inside a given space-time region. We can choose the smearing functions to be, for instance, the so-called top hat function which has a sharp edge. It is defined as $W_{L,\mathcal{T}}(\vec{x},\eta) = 1$ for $|\vec{x}| \leq L$, $|\eta| < \mathcal{T}$ and it is equal to zero otherwise. This choice is, however, not so useful for our case. In fact, one can show that the Fourier transform of the top hat profile goes to zero, for large k, as $(kr_0)^{-3}$. Unfortunately this behaviour (ultimately related to the shrap edge of the profile) could create spurious effects in the charge and current density fluctuations. A better choice is, for our purposes, the gaussian smearing function

$$W_{L,\mathcal{T}}(\vec{x},\eta) = e^{-\frac{|\vec{x}|^2}{2L^2} - \frac{\eta^2}{2\mathcal{T}^2}}. \tag{3.11}$$

This expression smears the high frequency modes more efficiently than the top hat function. By inserting Eqs. (3.9) into Eq. (3.10) we obtain, after regulatization over time, in the limit $L \ll m^{-1}$ and for $\xi_0 \geq \eta_1$

$$Q_{L,\mathcal{T}}^2 \simeq \frac{e^2}{(2\pi)^5} \int d^3 x \int d^3 y \int d^3 k \int d^3 p |\alpha_p \beta_k - \alpha_k \beta_p|^2 e^{-\frac{|\vec{x}|^2}{2L^2} - \frac{|\vec{y}|^2}{2L^2}} e^{i(\vec{k}+\vec{p})\cdot(\vec{x}-\vec{y})}. \tag{3.12}$$

A similar procedure can be carried on in the case of the current density fluctuation.

As we will see in the next Section, the relevant quantity to be computed in order to estimate the size of the gauge field fluctuations is given by the trace of

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$$\langle (\vec{\nabla} \times \vec{J})_k (\vec{\nabla} \times \vec{J})_l \rangle = \epsilon_{ika}\epsilon_{jlb}\frac{\partial^2}{\partial x^a \partial y^b}\langle J_i(\vec{x}, \eta) J_j(\vec{y}, \tau)\rangle. \tag{3.13}$$

Therefore, the regularized quantity we are interested in is

$$(\vec{\nabla} \times \vec{J})^2_{L,\mathcal{T}} = \frac{e^2}{\mathcal{T}^2}\int d^3x \int d^3y \int d\eta \int d\tau \langle (\vec{\nabla} \times \vec{J})_k (\vec{\nabla} \times \vec{J})_k \rangle W_{L,\mathcal{T}}(\vec{x}, \eta - \xi_0) W_{L,\mathcal{T}}(\vec{y}, \tau - \xi_0). \tag{3.14}$$

After regularization over time we find, using Eq. (3.9) into Eq. (3.14) in the limit $m\xi_0^2/\eta_1 \ll 1$, that for $L \gg m^{-1}$ and for $\xi_0 \geq \eta_1$ our correlator becomes

$$(\vec{\nabla} \times \vec{J})^2_{L,\mathcal{T}} \simeq \frac{e^2}{(2\pi)^5}\frac{1}{m^2}\left(\frac{\eta_1}{\xi_0}\right)^2 \int d^3x \int d^3y \int d^3k \int d^3p \mathcal{D}(k,p)e^{-\frac{|\vec{x}|^2}{2L^2} - \frac{|\vec{y}|^2}{2L^2}}e^{i(\vec{k}+\vec{p})\cdot(\vec{x}-\vec{y})}, \tag{3.15}$$

where

$$\mathcal{D}(k,p) = [k^2 p^2 - (\vec{k} \cdot \vec{p})^2]|\alpha_k + \beta_k|^2|\alpha_p + \beta_p|^2. \tag{3.16}$$

IV. MAGNETIC FIELDS FROM CHARGE AND CURRENT DENSITY FLUACTUATIONS

Once we know the charge and current fluctuations at the beginning of the radiation dominated epoch we can compute the induced fluctuations in the electromagnetic fields from the Maxwell equations. The complete solution of this problem is hardly possible in a general case because of the complicated (and model dependent) dynamics of the inflaton decay and of the reheating of the Universe after inflation. We shall use an extremely simplified picture of this process. Namely, we will suppose that the plasma right after inflation is in thermal equilibrium with some temperature T, up to small (at super-horizon scales) perturbations of the distribution functions leading to charge and current fluctuations. Though this picture may be, in general, far from reality we expect it to reproduce the physics of the magnetic field generation quite accurately, because the rate of equilibration of the electromagnetic reaction is large compared with the rate of the Universe expansion after inflation and may

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be bigger than the rate of the inflaton decay. In fact, our result depends essentially on the density of the charged particles and their typical momentum only. Thus they may be qualitatively applicable even to situations with large deviations from thermal equilibrium.

Within this set-up the problem can be related to the relaxation of an initial perturbation in the distribution function. Consider an equilibrium homogeneous and isotropic conducting plasma, characterized by a distribution function $f_0(p)$ common for both positively and negatively charged ultrarelativistic particles (for example, electrons and positrons) . Suppose now that this plasma is slightly perturbed, so that the distribution functions are

$$f_+(\vec{x}, \vec{p}, \eta) = f_0(p) + \delta f_+(\vec{x}, \vec{p}, \eta), \quad f_-(\vec{x}, \vec{p}, \eta) = f_0(p) + \delta f_-(\vec{x}, \vec{p}, \eta), \tag{4.1}$$

where $+$ refers to positrons and $-$ to electrons, and $\vec{p}$ is the conformal momentum. The Vlasov equation defining the curved-space evolution of the perturbed distributions can be written as

$$\frac{\partial f_+}{\partial \eta} + \vec{v} \cdot \frac{\partial f_+}{\partial \vec{x}} + e(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_+}{\partial \vec{p}} = \left(\frac{\partial f_+}{\partial \eta}\right)_{\text{coll}}, \tag{4.2}$$

$$\frac{\partial f_-}{\partial \eta} + \vec{v} \cdot \frac{\partial f_-}{\partial \vec{x}} - e(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_-}{\partial \vec{p}} = \left(\frac{\partial f_-}{\partial \eta}\right)_{\text{coll}}, \tag{4.3}$$

where the two terms appearing at the right hand side of each equation are the collision terms. This system of equation represents the curved space extension of the Vlasov-Landau approach to plasma fluctuations [21,22]. All particle number densities here are related to the comoving volume.

Notice that, in general $\vec{v} = \vec{p}/\sqrt{\vec{p}^2 + m_e^2 a^2}$. In the ultra-relativistic limit $\vec{v} = \vec{p}/|\vec{p}|$ and the Vlasov equations are conformally invariant. This implies that, provided we use conformal coordinates and rescaled gauge fields, the system of equations which we would have in flat space [23] looks exactly the same as the one we are discussing in a curved FRW (spatially flat) background [24].

The evolution equations of the gauge fields coupled to the two Vlasov equations can be written as

$$\vec{\nabla} \cdot \vec{E} = e \int d^3p [f_+(\vec{x}, \vec{p}, \eta) - f_-(\vec{x}, \vec{p}, \eta)],$$

$$\vec{\nabla} \times \vec{E} + \vec{B}' = 0,$$

$$\vec{\nabla} \cdot \vec{B} = 0,$$

$$\vec{\nabla} \times \vec{B} - \vec{E}' = \int d^3p \vec{v} [f_+(\vec{x}, \vec{p}, \eta) - f_-(\vec{x}, \vec{p}, \eta)]. \tag{4.4}$$

Now, if $\delta f_\pm(\vec{x}, \vec{p}, \eta)$ at the beginning of the radiation dominated epoch η_0 are known (their magnitude follows from our computation of the Bogoluibov coefficients), and $E(\vec{x}, \eta_0) = B(\vec{x}, \eta_0) = 0$ initially, the magnetic field at later times can be found from Eqs. (4.2)–(4.4).

This problem is solved in the Appendix B in a relaxation time approximation for the collision integral and under the assumption of small fluctuations in the distribution functions. Electric fields, as well as the charge density perturbations quickly relax to zero during the time given by the inverse conductivity of the plasma, $\sigma \sim n_q/(\langle p \rangle \nu)$, where $n_q \sim T^3$ is the density of the charged particles and $\langle p \rangle \sim T$ is their average momentum, ν is the frequency of collisions [1]. As for the magnetic fields, for the most interesting case of large scales ($k^2 \ll \sigma/\eta$ the result does not depend on the frequency of collisions ν and reads

$$\vec{B} \sim \frac{\langle p \rangle}{\alpha n_q} \vec{\nabla} \times \vec{J}, \tag{4.5}$$

where α is the fine structure constant.

V. ESTIMATES OF MAGNETIC FIELD

We are now ready to estimate the produced current and charge density fluctuations. We will firstly discuss the case of magnetic fields assuming that the primordial background

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[1] All quantities here are given for conformal time. For instance the curved space conductivity σ is related to the flat space one, σ_c , through a rescaling which involves the scale factor : $\sigma = a\sigma_c$. Once we use our rescaled system n, T and σ do not change for adiabatic Universe expansion, provided that the effective number of massless degrees of freedom is constant. This is different from Ref. [17] where ordinary (flat space) conductivity scales as temperature squared.

is of de Sitter or quasi-de Sitter type. We will then move to estimate the charge density fluctuations.

A. De Sitter case

From Eq. (3.15) we see that the correlation function of current density fluctuations is determined by the sum of the Bogoliubov coefficients. From Eq. (2.7) we have

$$|\alpha_k + \beta_k|^2 |\alpha_p + \beta_p|^2 \sim \left(\frac{\pi^2}{\Gamma(3/4)^4} \sin^4 \frac{\pi}{8}\right) \mu^{-1} |k\eta_1|^{-3} |p\eta_1|^{-3}, \tag{5.1}$$

which implies

$$(\vec{\nabla} \times \vec{J})_{L,\mathcal{T}}^2 = \left(\frac{\eta_1}{\xi_0}\right)^2 \left(\frac{e^2}{(2\pi)^5} \frac{\pi^2}{\Gamma(3/4)^2} \sin^4 \frac{\pi}{8}\right) \frac{k_1^4}{\mu^3} \mathcal{I}(L), \tag{5.2}$$

where

$$\mathcal{I}(L) = \int d^3x \int d^3y \int d^3k \int d^3p \frac{[k^2 p^2 - (\vec{k} \cdot \vec{p})^2]}{k^3 p^3} e^{-\frac{|\vec{x}|^2}{2L^2}} e^{-\frac{|\vec{y}|^2}{2L^2}} e^{i(\vec{k}+\vec{p})\cdot(\vec{x}-\vec{y})} \simeq 0.9 \times (2\pi)^5 L^2. \tag{5.3}$$

The final result for our correlator is

$$\frac{(\vec{\nabla} \times \vec{J})_{L,\mathcal{T}}^2}{V^2} \simeq \left(0.9 \frac{e^2 \pi^2}{\Gamma(3/4)^4} \sin^4 \frac{\pi}{8}\right) \frac{k_1^4}{L^4} \mu^{-3}, \tag{5.4}$$

where $V \sim L^3$ is the typical volume of a region of size L; we take $\xi_0 \sim \eta_1$ (larger values of ξ_0 give even smaller values of the magnetic fields).

In order to get an estimate of the magnetic field, one should specify the density of the charged particles and their average momentum, see (4.5). A most realistic estimate would be to take $n \sim T^3$ and $\langle p \rangle \sim T$, where T is the reheating temperature. Then we obtain for the gauge field fluctuations

$$\frac{B^2}{T^4} = 6.26 \left(\frac{m}{M_P}\right)^{-3} \left(\frac{H_1}{M_P}\right)^5 \frac{1}{(LT)^4}. \tag{5.5}$$

In Eq. (5.5) L is the coherence scale of the magnetic field. The ratio of the magnetic field to the T^2 is roughly constant during the Universe expansion (if dynamo effects and the galaxy

collapse are discarded as well as annihilation of heavy particles) and may be taken at any time, provided the coherence length L is taken at the same epoch. We will take $L \sim 1$ Mpc [6,17] at the present microwave background temperature [2], which gives $LT \sim 3 \times 10^{25}$. For a galactic mass perturbation (of the order of 10^{12} solar masses, including dark matter) the typical length scale is of the order of $1.9 \times (\Omega_0 h^2)^{-1/2}$ Mpc [25]. The gravitational collapse of the protogalaxies enhances the magnetic flux frozen in the primeval galactic patch by roughly a factor of the order of 10^3.

The obtained value for the magnetic field should be compared with the values of the magnetic field necessary to seed the galactic dynamo mechanism. The differential rotation of the galaxy introduces a parity violating term in the MHD equations (the dynamo term). The effect of this term exponentially amplify the seed magnetic field by a factor $e^{\Gamma t}$ [3,4]. In this amplification factor enter two numbers: the galactic age (of the order of 10 Gyrs and the dynamo amplification rate (Γ) whose estimate is rather uncertain: values of the order 0.3–0.5 Gyr for Γ^{-1} are present in the literature [3,5]. Following a recent analysis [7] we have that the required value for the seed field can be expressed as

$$\frac{B_{\mathrm{dec}}}{T_{\mathrm{dec}}^2} \geq 5 \times 10^{-17}, \ \ \text{for} \ \ \Gamma^{-1} = 0.5 \, \mathrm{Gyr},$$
$$\frac{B_{\mathrm{dec}}}{T_{\mathrm{dec}}^2} \geq 2 \times 10^{-25}, \ \ \text{for} \ \ \Gamma^{-1} = 0.3 \, \mathrm{Gyr}. \tag{5.6}$$

where T_{dec} is the decoupling temperature. These values have been obtained in the case of $\Omega_0 \sim 0.3$, $\Omega_\Lambda \sim 0.7$ and $h = 0.65$ as a fiducial set of cosmological parameters [32,33]. Notice that in the case of a flat Universe with $\Omega_0 = 1$ we would get values of $B_{\mathrm{dec}}/T_{\mathrm{dec}}^2$ close to 10^{-15}.

We want now to compare these values with the parameter space described by our estimate. If we take $m = 100$ GeV (the lower mass limit for the Higgs boson) and if we assume

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[2]Equally, in agreement with the analysis in Ref. [7], one can take $L \sim 100$ pc at the galaxy formation time, which gives essentially the same value for LT.

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$H_1/M_P \simeq 10^{-6}$ (i.e. the maximal H_1 compatible with microwave background anisotropies) we have that

$$\frac{B_{\text{dec}}}{T_{\text{dec}}^2} \sim 5.77 \times 10^{-40}, \tag{5.7}$$

twenty orders of magnitude smaller than the required value in order to seed the magnetic field of our galaxy. One can argue that by lowering the mass this estimate will improve. This is not the case. By taking $m \sim 1$ MeV (which is not realistic at all) we get $B_{\text{dec}}/T_{\text{dec}}^2 \sim 10^{-32.5}$, still too small to be relevant. Notice, finally, that to tune H_1 does not help: since it can only get smaller that 10^{-6} it can only make the value of the seed even smaller than the values we just mentioned.

The most conservative estimate of the magnetic field can be obtained with the asumption that the number density of the charged particles in the plasma is the number of gravitationally created scalars,

$$n_{gr} = \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \sim \frac{1}{8\pi\Gamma^2(3/4)} \left(\frac{H_1}{m}\right)^{\frac{1}{2}} H_1^3 \log\left(\frac{H_1}{m}\right) \tag{5.8}$$

and their average momentum is simply $\langle p \rangle \sim H_1 \sim 100 \text{GeV}$, specific for the gravitational production. If this is indeed the case, the value of the magnetic field B is larger by a factor

$$8\pi\Gamma^2(3/4)\left(\frac{m}{M_P}\right)^{\frac{1}{2}}\left(\frac{M_P}{H_1}\right)^{\frac{3}{2}}, \tag{5.9}$$

if compared with the estimate (5.7). Following these considerations the obtained magnetic field becomes

$$\frac{B}{T^2} \sim 10^2 \left(\frac{H_1}{m}\right)\frac{1}{(LT)^2}. \tag{5.10}$$

Even with the most optimistic numbers (i.e. $m \sim 100$ GeV and $H_1 \sim 10^{-6}M_P$) we get that B/T^2 can be, at most, 10^{-38} over scales relevant for the dynamo action, with a minute gain, with respect to the estimate of Eq. (5.7). In priciple, one can think that even if the produced fields are too weak to turn-on the galactic dynamo they could be of some relevance for other processes occurring at different epochs in the life of the Universe. For

instance, the electroweak horizon at the time of the electroweak phase transition [29,30] gives $L_{ew}T_{ew} \sim 10^{16}$. This would imply from Eq. (5.10) that $B_{ew}/T_{ew}^2 \sim 10^{-19}$. At the electroweak epoch the smallest coherence lenght of magnetic fields is set by the diffusivity. Around the diffusivity scale $L_{diff}T_{ew} \sim 10^8$ [29]. Thus the obtained magnetic field can be, at most, $B_{ew} \sim 10^{-3}T_{ew}^2$. In order to have sizable effects on the phase diagram of the electroweak transition we should have, at least, that $B_{ew}/T_{ew}^2 \geq 0.2$ [31] so the produced fields are also too small in this context.

B. Quasi de Sitter case

Up to now we assumed that the primordial phase in the evolution of the Universe was of pure de Sitter type. In this subsection we are going to study what happens is this requirement is relaxed. In principle we know that during the inflationary phase the scalar field slightly decreases its value and, consequently, the inflationary curvature scale is not exactly constant but mildly decreasing as a function of time. In order to account for the decrease in the curvature and in the scalar field we can define the two slow-rolling parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \lambda = \frac{\ddot{\phi}}{H\dot{\phi}}, \tag{5.11}$$

where ϕ denotes, in this Section, the inflaton and $H = \dot{a}/a$ is the Hubble parameter (the over-dot denotes differentiation with respect to the cosmic time). The important point for our purposes is that the slight decrease in the curvature corrects the evolution equation for the charged scalar. More specifically, we know that the dependence on the curvature appears in the mode function as a''/a. It is a simple exercise to show that

$$\frac{a''}{a} = 2a^2H^2(1 - \frac{\epsilon}{2}), \tag{5.12}$$

which implies that the slow-rolling corrections will make the index appearing in the Hankel functions slightly larger

$$\rho = \frac{3}{2} + \epsilon. \tag{5.13}$$

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the case $\epsilon = 0$ corresponds to the pure de Sitter case. Clearly, from our general expressions of the Bogoliubov coefficients (see Eq.(A.16)) an increase in ρ implies an increase of the Bogoliubov coefficients in the infra-red part of the spectrum.

Therefore we want to estimate the magnetic fields in the case where the index ν is kept generic. For this purpose, from Eq. (A.16) we have that

$$|\alpha_k + \beta_k|^2 |\alpha_p + \beta_p|^2 \simeq \left(2^{4\rho-6} (\rho - \frac{1}{2})^4 \frac{\Gamma(\rho)^4}{\Gamma(3/4)^4} \sin^4 \frac{\pi}{8} \right) \mu^{-1} |k\eta_1|^{-2\nu} |p\eta_1|^{-2\rho}. \tag{5.14}$$

Following the same steps as in the pure de Sitter case we can estimate that

$$\frac{B^2}{T^4} \simeq \left(\frac{2^{4\rho-6}(\rho-\frac{1}{2})^4 \Gamma^4(\rho)}{2\Gamma^4(3/4)e^2} \sin^4 \frac{\pi}{8} \right) \left(\frac{m}{M_P} \right)^{-3} \left(\frac{H_1}{M_P} \right)^{2+2\rho} (L\,T)^{4\rho-10}. \tag{5.15}$$

If ρ could be larger than 1.5 the magnetic fields would also be larger at the relevant scales.

There are two relevant bounds on ρ. The first and obvious one comes from the energy density stored in the charged scalar modes. The energy density stored in the scalar field modes goes as $mk^3 |\beta_k|^2$. This means, in the case of generic ρ that the energy density (in critical units) is

$$\left(\frac{m}{H_1} \right)^{\frac{1}{2}} \left(\frac{H_1}{M_P} \right)^{(\rho+\frac{1}{2})} \left(\frac{k}{T} \right)^{3-2\rho}. \tag{5.16}$$

If we take $m \sim 100$ GeV and $\rho \sim 2$ we see that this expression still gives a value 10^{-6} at the decoupling scale. Larger values of ρ could induce further anisotropies in the microwave background. So we will assume $1.5 < \rho < 2$ which would be already enough to increase magnetic fields according to Eq. (5.15).

The second class of bounds stems from the fact that ρ is connected with the slow rolling parameters which are constrained. In fact, the contribution to the scalar spectral index deduced from the COBE data could be written, in terms of the slow-rolling parameters, as [34,35]

$$n = 1 - 4\epsilon + 2\lambda. \tag{5.17}$$

The values of ϵ and λ are different depending upon the different models of background evolution, namely upon the different analytical forms of the inflationary potential driving

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inflation. This can be appreciated by writing the equations of motion of the inflaton in the slow-rolling approximation

$$3H\dot{\phi} + \frac{\partial V}{\partial \phi} \simeq 0,$$
$$M_P^2 H^2 \simeq V. \tag{5.18}$$

By using these two equations we can re-express ϵ and λ as

$$\epsilon = \frac{M_P^2}{6}\left(\frac{\partial \ln V}{\partial \phi}\right)^2,$$
$$\lambda = -\frac{M_P^2}{6}\left(\frac{\partial \ln V}{\partial \phi}\right)^2 + \frac{M_P^2}{3V^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right). \tag{5.19}$$

Clearly the values of ϵ and λ depend upon the value of ϕ. For instance, we could estimate the value of ϵ coinciding with the value of the field 50 e-folds before the end of inflation (corresponding to the moment where the large scales went out of the horizon) [35]. In this case, for a power-law potential $V \sim \phi^q$ we have that

$$\epsilon = \frac{q}{q+200}, \quad \lambda = \frac{q-2}{q+200}. \tag{5.20}$$

In the case of an exponential potential of the form $V = \exp\left[6\phi^2/(pM_P^2)\right]$ we have that $\epsilon = \lambda = 1/p$. Consequently, from Eq. (5.17), we have that $n = 1 - (2+q)/100$ for power-law potentials, $n = 1 - 2/p$ for exponential potentials. The scale-invariant spectrum as it has been observed by the DMR experiment aboard the COBE satellite [36,37] the spectral index can lie in the range $0.9 \leq n \leq 1.5$. In order to have more magnetic fields we should increase ρ, namely we should go for large ϵ. The variation of the spectral constrains the maximal value of ϵ. So if we take 0.9 as the minimal value for n we would have from Eq. (5.20) that the q (for a power-law potential) is $q = 8$. But this would imply that the maximal ϵ is 0.03 and our effective ρ will be 1.53. Too small to give relevant consequences in Eq. (5.15) for the magnetic fields generation. Similar conclusions could be reached in the case of power-law potentials. By playing with the value of ρ it is not possible to enhance the value obtained for large scale magnetic fields.

C. Charge density fluctuations

On the basis of the kinetic discussion the modes which survive in the plasma are the transverse ones. The charge density fluctuations, being associated with the longitudinal modes will be dissipated quite quickly in a typical time of the order of the inverse temperature. Still it is interesting to check if the charge density fluctuations are small. From our expression of the Bogoliubov coefficients given in Eq. (2.7) we have that

$$|\alpha_p \beta_k - \alpha_k \beta_p|^2 = \frac{\pi^2}{2\Gamma(1/4)^2 \Gamma(3/4)^2} \frac{|p\eta_1|}{|k\eta_1|^3}. \tag{5.21}$$

We can insert this last expression into Eq. (3.12) and perform the integrations. The integrations over x and y are trivial. The integration over the moduli over the momenta leads to a non trivial integral which can be exactly computed:

$$\int_0^\infty \frac{dz}{z} e^{-z^2} \int_0^\infty w^2 dw e^{-w^2} \sinh 2wz = -1/2. \tag{5.22}$$

Defining then the electric charge density as

$$n_e = \frac{Q_{L,T}}{L^6} \tag{5.23}$$

we obtain that

$$\frac{n_e}{n_\gamma} \sim 10^{-2} \left(\frac{H_1}{M_P}\right)^{1/2} (LT)^{-2}, \tag{5.24}$$

where $n_\gamma \sim T^3$ is the photon density. This value, for a length scale corresponding to the horizon at decoupling, would give $n_e/n_\gamma \sim 10^{-58}$. We want to stress that the bounds [38,39] on the electric charge fluctuations were derived by assuming that charge fluctuations would induce electric fields coherent over the whole horizon. These fields would cause some observable effect in the microwave background so that a constraint on the charge density could be derived. Again, on the basis of kinetic treatment of plasma fluctuations, we can say that electric fields dissipate as soon as the the plasma becomes conducting. Therefore the effects on the microwave background are not present.

VI. CONCLUSIONS

In this paper we discussed the amplification and the fate of the fluctuations of a charged scalar field in the inflationary Universe scenario. These fluctuations may lead eventually to the generation of some magnetic fields in the Universe. We found that the produced magnetic field are always much smaller than the most optimistic lower bounds required in order to seed the galactic dynamo mechanism. Thus, the inflationary production of charged scalars is unlikely to be responsible for the observed galactic magnetic fields.

ACKNOWLEDGMENTS

APPENDIX A: BOGOLIUBOV COEFFICIENTS

Defining Φ_1 and Φ_2 as the real and imaginary part of Φ as

$$\Phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2), \tag{A.1}$$

we assume that the background geometry evolves from $\eta \to -\infty$ to $\eta \to +\infty$ for instance, according to Eq. (2.6). In both limits we can define a Fourier expansion of Φ_1 and Φ_2 in terms of two distinct orthonormal sets of modes. By then promoting the classical fields to quantum mechanical operators in the Heisenberg representation we can write, for $\eta \to -\infty$

$$\Phi_1^{\text{in}}(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}}[a_k f_k(\eta)e^{i\vec{k}\cdot\vec{x}} + a_{-k}^\dagger f_k^*(\eta)e^{-i\vec{k}\cdot\vec{x}}],$$

$$\Phi_2^{\text{in}}(\vec{x}, \eta) = \int \frac{d^3p}{(2\pi)^{3/2}}[b_p f_p(\eta)e^{i\vec{p}\cdot\vec{x}} + b_{-p}^\dagger f_p^*(\eta)e^{-i\vec{p}\cdot\vec{x}}]. \tag{A.2}$$

where the two sets of creation and annihilation operators (i.e. $[a_k, a_{-k}^\dagger]$ and $[b_p, b_{-p}^\dagger]$) are mutually commuting. As $\eta \to +\infty$ Φ_1 and Φ_2 can be expanded in a second orthonormal set of modes

$$\Phi_1^{\text{out}}(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}}[\tilde{a}_k g_k(\eta)e^{i\vec{k}\cdot\vec{x}} + \tilde{a}_{-k}^\dagger g_k^*(\eta)e^{-i\vec{k}\cdot\vec{x}}],$$

$$\Phi_2^{\text{out}}(\vec{x}, \eta) = \int \frac{d^3p}{(2\pi)^{3/2}}[\tilde{b}_p g_p(\eta)e^{i\vec{p}\cdot\vec{x}} + \tilde{b}_{-p}^\dagger g_p^*(\eta)e^{-i\vec{p}\cdot\vec{x}}]. \tag{A.3}$$

Since both sets of modes are complete the old modes can be expressed in terms of the new ones

$$f_k(\eta) = \alpha_k g_k(\eta) + \beta_k g_k^*(\eta). \tag{A.4}$$

This transformation, once inserted back into Eq. (A.2), implies that

$$\tilde{a}_k = \alpha_k a_k + \beta_k^* a_{-k}^\dagger. \tag{A.5}$$

Notice that in order to preserve the scalar products in the old and new sets of orthonormal modes we have that the two complex numbers α_k and β_k are subjected to the constraints $|\alpha_k|^2 - |\beta_k|^2 = 1$. Exactly the same discussion applies for the field operator Φ_2. Eq.

(A.5) is nothing but the well known Bogoliubov-Valatin transformation; α_k and β_k are the Bogoliubov coefficients parametrizing the mixing between positive and negative frequency modes.

In order to ensure the continuity of the operators Φ_1 and Φ_2 we have to match continuously the old mode functions with the new ones. During the primordial phase of the Universe the evolution equations satisfied by the mode functions is

$$\frac{d^2 f_k}{d\eta^2} + \left[k^2 - \frac{\alpha(\alpha+1)}{\eta^2} + \frac{\mu}{\eta_1^2}(\frac{\eta_1}{\eta})^{2\alpha} \right] f_k = 0, \tag{A.6}$$

where $\alpha = 1$ for a pure de Sitter background $\mu = m/(H_1 a_1) = m\eta_1$. Notice that with H we will denote the Hubble factor in cosmic time (as usual the relation between the Hubble factor in conformal time, e.g. $\mathcal{H}$ and the Hubble factor in cosmic time is $H = \mathcal{H}/a$; during the de Sitter epoch $\mathcal{H} \sim \eta^{-1}$). Notice that if the mass of the charged scalar is not of Planckian magnitude, $\mu \ll 1$. Moreover, in the limit where $\mu \geq 1$ one can argue that the amplified fluctuations will be exponentially suppressed. Since we want to explore the situation where the mass of the scalar field is of electroweak order we will always be (quite safely) in the limit $\mu \ll 1$.

The exact solution of Eq. (A.6) which reduces, in the limit $\eta \to -\infty$, to the usual positive frequency Minkowski space solution is given by [26]

$$f_k(\eta) = \frac{1}{\sqrt{2k}} \ p \ \sqrt{-x} \ H_\rho^{(1)}(-x), \tag{A.7}$$

where $x = k\eta$ and $H_\rho^{(1)}$ is the first order Hankel function. Again, in the pure de Sitter case, $\rho = 3/2\sqrt{1 - (4/9)\mu^2}$. Since μ is typically small, $\rho \simeq 3/2$ in the pure de Sitter case. We denoted with p a phase factor which we choose such that

$$p = \sqrt{\frac{\pi}{2}} \ e^{i\frac{\pi}{4}(1+2\rho)}. \tag{A.8}$$

With this choice of p we have that $f_k(\eta) \sim e^{-ik\eta}/\sqrt{2k}$ for $\eta \to -\infty$ [26].

After the radiation dominated phase sets in (for $\eta > -\eta_1$) the evolution equation obeyed by the mode functions $g_k(\eta)$ is given by

$$\frac{d^2 g_k}{d\eta^2} + [k^2 + \frac{\mu^2(\eta + 2\eta_1)^2}{\eta_1^4}]g_k = 0. \tag{A.9}$$

This last equation can be easily recast in the form of a parabolic cylinder equation [26,27].

Defining $\gamma = \mu/\eta_1^2$ we can introduce two new quantities, namely

$$z = \sqrt{2\gamma}(\eta + 2\eta_1), \quad q = \frac{k^2}{2\gamma}. \tag{A.10}$$

Consequently, Eq. (A.10) becomes

$$\frac{d^2 g_k}{dz^2} + [q + \frac{z^2}{4}]g_k = 0, \tag{A.11}$$

which is one of the canonical forms of the parabolic cylinder equation [26,27]. The exact solutions which reduce to positive and negative frequency modes for $\eta \to +\infty$ are

$$g_k(\eta) = \frac{1}{(2\gamma)^{1/4}} e^{i\frac{\pi}{8}} D_{-iq-\frac{1}{2}}(ie^{-i\frac{\pi}{4}}z),$$

$$g_k^*(\eta) = \frac{1}{(2\gamma)^{1/4}} e^{-i\frac{\pi}{8}} D_{iq-\frac{1}{2}}(e^{-i\frac{\pi}{4}}z), \tag{A.12}$$

where D_σ are the parabolic cylinder functions in the Whittaker's notation [27]. Notice that with our choice of normalizations we have, in the limit $z \gg |q|$ and for $k^2\eta_1 \ll m$, that [26,27]

$$g_k(\eta) \sim \sqrt{\frac{\eta_1}{2m\eta}} e^{-\frac{i}{2}\frac{m\eta^2}{\eta_1}}. \tag{A.13}$$

In view of the actual calculation it is worth recalling the exact expressions of the parabolic cylinder functions (in the Whittaker form) in terms of confluent hypergeometric (Kummer) functions $_1F_1(a,b,x)$:

$$D_{iq-\frac{1}{2}}(e^{-i\frac{\pi}{4}}z) = \sqrt{\pi}2^{\frac{iq}{2}-\frac{1}{4}} e^{\frac{iz^2}{4}} \left[\frac{1}{\Gamma(\frac{3}{4} - \frac{iq}{2})} {}_1F_1(\frac{1}{4} - \frac{iq}{2}, \frac{1}{2}, -\frac{iz^2}{2}) \right.$$

$$\left. - \frac{z(1-i)}{\Gamma(\frac{1}{4} - \frac{iq}{2})} {}_1F_1(\frac{3}{4} - \frac{iq}{2}, \frac{3}{2}, -\frac{iz^2}{2}) \right],$$

$$D_{-iq-\frac{1}{2}}(ize^{-i\frac{\pi}{4}}) = \sqrt{\pi}2^{-\frac{iq}{2}-\frac{1}{4}} e^{-i\frac{z^2}{4}} \left[\frac{1}{\Gamma(\frac{3}{4} + \frac{iq}{2})} {}_1F_1(\frac{1}{4} + \frac{iq}{2}, \frac{1}{2}, \frac{iz^2}{2}) \right.$$

$$\left. - \frac{z(1+i)}{\Gamma(\frac{1}{4} + \frac{iq}{2})} {}_1F_1(\frac{3}{4} + \frac{iq}{2}, \frac{3}{2}, \frac{iz^2}{2}) \right]. \tag{A.14}$$

The Bogoliubov coefficients are obtained from

$$f_k(-\eta_1) = \alpha_k g_k(-\eta_1) + \beta_k g_k^*(-\eta_1),$$

$$f_k'(-\eta_1) = \alpha_k g_k'(-\eta_1) + \beta_k g_k^{*'}(-\eta_1), \tag{A.15}$$

which is a system of two equations in the two unknowns α_k and β_k.

By solving this system we obtain an exact expression for the Bogoliubov coefficients which is, in general a function of two variables : $\mu = m\eta_1$ and $x_1 = k\eta_1$. Since $\mu \ll 1$ we can expand the exact result in this limit and we obtain, in the case of a generic Bessel index ρ,

$$\alpha_k = \pi e^{i\frac{\pi}{8}}\left\{\frac{i}{\sqrt{2}\Gamma(\frac{3}{4})}S_2(x_1,\rho)\mu^{-\frac{1}{4}} + \frac{(1+i)}{2\Gamma(\frac{1}{4})}[S_1(x_1,\rho) + S_2(x_1,\rho)]\mu^{\frac{1}{4}}\right\} + \mathcal{O}(\mu^{\frac{5}{4}}),$$

$$\beta_k = \pi e^{-i\frac{\pi}{8}}\left\{-\frac{i}{\sqrt{2}\Gamma(\frac{3}{4})}S_2(x_1,\rho)\mu^{-\frac{1}{4}} + \frac{(i-1)}{2\Gamma(\frac{1}{4})}[S_1(x_1,\rho) + S_2(x_1,\rho)]\mu^{\frac{1}{4}}\right\} + \mathcal{O}(\mu^{\frac{5}{4}}), \tag{A.16}$$

where $S_1(x_1,\rho)$ and $S_2(x_1,\rho)$ contain the explicit dependence upon the Hankel's functions:

$$S_1(x_1,\rho) = e^{i\frac{\pi}{4}(1+2\rho)}H_\rho^{(1)}(x_1),$$

$$S_2(x_1,\rho) = \sqrt{x_1}e^{i\frac{\pi}{4}(1+2\rho)}\left[(\rho + \frac{1}{2})\frac{H_\rho^{(1)}(x_1)}{\sqrt{x_1}} - \sqrt{x_1}H_{\rho+1}^{(1)}(x_1)\right]. \tag{A.17}$$

If we are now specifically interested in the pure de Sitter phase we can insert the value $\rho = 3/2$ into Eq. (A.17). Then, we can insert the obtained expressions into Eq. (A.16) and we obtain the wanted Bogoliubov coefficients reported in Eqs. (2.7).

Notice that it would not be correct to use the asymptotic solutions (like the one reported in Eq. (A.13)) in order to compute the Bogoliubov coefficients[3]. In fact Eq. (A.13) can be viewed as the a WKB-type solution of Eq. (A.11) which can be also written as

$$\frac{d^2 g_k}{d\eta^2} + \omega_k^2(\eta)g_k = 0, \quad \omega_k^2(\eta) = k^2 + m^2 a^2(\eta). \tag{A.18}$$

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[3]We disagree with the calculation of Ref. [17] where the matching has been performed using approximate mode functions. In our case, for small μ the leading behaviour of the Bogoliubov coefficient goes as $\mu^{-1/4}$. In the case of [17] it goes as $\mu^{-1/2}$, an artifact of the WKB approximation.

By now postulating a WKB-type solution we have that

$$g_k(\eta) = \frac{1}{2W(\eta)} e^{-i \int^\eta W(\eta') d\eta'}. \tag{A.19}$$

By now inserting the trial solution back to Eq. (A.18) we get $W(\eta)$ is specified by the following non-linear relation

$$W^2(\eta) = \omega_k^2(\eta) - \frac{1}{2}\left[\frac{W''}{W} - \frac{3}{2}\left(\frac{W'}{W}\right)^2\right]. \tag{A.20}$$

This equation can be solved by iteration. If we keep the lowest order we get that

$$W_0(\eta) \simeq \omega_k(\eta) \tag{A.21}$$

and by using the explicit expression of the scale factor during the radiation dominated epoch we exactly get Eq. (A.13). This solution is valid provided the corrections to the exact expression of $W(\eta)$ are small, namely, provided, from eq, (A.20)

$$\omega_k^2(\eta) \gg \frac{1}{2}\left[\frac{W_0''}{W_0} - \frac{3}{2}\left(\frac{W_0'}{W_0}\right)^2\right]. \tag{A.22}$$

This last inequality, using the explicit expression of $\omega_k(\eta)$, implies that

$$k^2\eta^2 + m^2\eta^2\left(\frac{\eta + 2\eta_1}{\eta_1}\right) \gg 1. \tag{A.23}$$

Now we can see that this inequality is clearly satisfied for $\eta \to +\infty$. However, for $\eta \sim \eta_1$, this inequality would imply $m\eta_1 > 1$ (since $k\eta_1 < 1$). Therefore, in order to be consistent with the requirement that $m\eta_1 < 1$ we have to use the WKB-type solution only for large (positive) η.

APPENDIX B: VLASOV-LANDAU APPROACH TO ELECTROMAGNETIC FIELD FLUCTUATIONS.

The purpose of this appendix is to give details concerning the derivation of the relation between the electromagnetic field fluctuations and the initial fluctuations in the current (or charge) density profile.

By subtracting Eqs. (4.2) and (4.3) we obtain the equations relating the fluctuations of the distributions functions of the charged particles present in the plasma to the induced gauge field fluctuations:

$$\frac{\partial}{\partial \eta} f(\vec{x}, \vec{p}, t) + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} f(\vec{x}, \vec{p}, t) + 2e\vec{E} \cdot \frac{\partial f_0}{\partial \vec{p}} = -\nu(p) f,$$

$$\vec{\nabla} \cdot \vec{E} = e \int d^3 p f(\vec{x}, \vec{p}, \eta),$$

$$\vec{\nabla} \times \vec{E} + \vec{B}' = 0,$$

$$\vec{\nabla} \cdot \vec{B} = 0,$$

$$\vec{\nabla} \times \vec{B} - \vec{E}' = \int d^3 p \vec{v} f(\vec{x}, \vec{p}, \eta), \tag{B.1}$$

where $f(\vec{x}, \vec{p}, \eta) = \delta f_+(\vec{x}, \vec{p}, \eta) - \delta f_-(\vec{x}, \vec{p}, \eta)$ and $\nu(p)$ is a typical frequency of collisions.

We can solve this system [23] by taking the Fourier transform of the space-dependent quantities and the Laplace transform of the time-dependent quantities:

$$\vec{E}_{\vec{k}\omega} = \int_0^\infty d\eta e^{i\omega\eta} \int d^3 x e^{-i\vec{k}\cdot\vec{x}} \vec{E}(\vec{x}, \eta),$$

$$\vec{B}_{\vec{k}\omega} = \int_0^\infty d\eta e^{i\omega\eta} \int d^3 x e^{-i\vec{k}\cdot\vec{x}} \vec{B}(\vec{x}, \eta),$$

$$f_{\vec{k}\omega} = \int_0^\infty d\eta e^{i\omega\eta} \int d^3 x e^{-i\vec{k}\cdot\vec{x}} f(\vec{x}, \vec{p}, \eta). \tag{B.2}$$

We have now to specify, at the initial time, the form of the perturbed distribution function which can be derived from the amplification studied in the previous Section. We will call $g_{\vec{k}}(\vec{p})$ the initial profile of the distribution function. Eq. (B.1) can then be re-written as

$$-g_{\vec{k}}(\vec{p}) + i(\vec{k} \cdot \vec{v} - \omega) f_{\vec{k}\omega}(\vec{p}) + 2e\vec{E}_{\vec{k}\omega} \cdot \frac{\partial f_0}{\partial \vec{p}} = -\nu f, \tag{B.3}$$

$$i\vec{k} \cdot \vec{E}_{\vec{k}\omega} = e \int f_{\vec{k}\omega}(\vec{p}) d^3 p, \tag{B.4}$$

$$i\vec{k} \cdot \vec{B}_{\vec{k}\omega} = 0, \tag{B.5}$$

$$\vec{B}_{\vec{k}\omega} = \frac{1}{\omega} \vec{k} \times \vec{E}_{\vec{k}\omega}, \tag{B.6}$$

$$i\omega(1 - \frac{k^2}{\omega^2})\vec{E}_{\vec{k}\omega} + \frac{i}{\omega}\vec{k}(\vec{k} \cdot \vec{E}_{\vec{k}\omega}) = \int d^3 p \, \vec{v} \, f_{\vec{k}\omega}(\vec{p}), \tag{B.7}$$

where eq. (B.7) has been obtained by using Eq. (B.6) in the (Fourier and Laplace) transformed of the last of Eqs. (B.1). The Gauss constraint at $\eta = 0$ implies that

$$i\vec{k} \cdot \vec{E}_0(\vec{k}) = e \int d^3p \; g_{\vec{k}}(\vec{p}). \tag{B.8}$$

If we start, at the initial time, with a given profile of fluctuations fluctuation the Gauss constraint determines the initial value of the electric field. The magnetic field fluctuations are consistently equal to zero.

We can now separate the electric field in its polarizations parallel and transverse to the direction of propagation of the fluctuation. The transverse current provides a source for the evolution of transverse electric field fluctuations

$$i\omega(1 - \frac{k^2}{\omega^2})\vec{E}^{\perp}_{\vec{k}\omega} = e \int d^3p f_{\vec{k}\omega}(\vec{p})\vec{v}_{\perp}, \tag{B.9}$$

whereas the charge fluctuations provide a source for the evolution of longitudinal electric field fluctuations

$$i\vec{k} \cdot \vec{E}^{\parallel}_{\vec{k}\omega} = e \int d^3p f_{\vec{k}\omega}(\vec{p}). \tag{B.10}$$

In Eqs. (B.9) and (B.10) we defined the longitudinal part of the electric field fluctuations and the transverse electric field as

$$\vec{E}^{\perp}_{\vec{k}\omega} = \vec{E}^{\perp}_{\vec{k}\omega} - \frac{\vec{k}}{|\vec{k}|^2}(\vec{k} \cdot \vec{E}_{\vec{k}\omega}), \quad \vec{E}^{\parallel}_{\vec{k}\omega} = \frac{\vec{k}}{|\vec{k}|^2}(\vec{E}_{\vec{k}\omega} \cdot \vec{k}). \tag{B.11}$$

The solution of Eq. (B.3) is given by

$$f_{\vec{k}\omega}(\vec{p}) = \frac{1}{i(\vec{k} \cdot \vec{v} - \omega - i\nu)}[g_{\vec{k}}(\vec{p}) - 2e\vec{v} \cdot \vec{E}_{\vec{k}\omega}\frac{\partial f_0}{\partial p}], \tag{B.12}$$

where we used that $\partial f_0/\partial \vec{p} \equiv \vec{v}\partial f_0/\partial p$. The longitudinal and transverse components of the electric fluctuations can be obtained by inserting Eq. (B.12) into Eqs. (B.9)-(B.10)

$$|\vec{E}^{\parallel}_{\vec{k}\omega}| = \frac{e}{ik \; \epsilon_{\parallel}} \int d^3p \frac{g_{\vec{k}}(\vec{p})}{i(\vec{k} \cdot \vec{v} - \omega - i\nu)}, \tag{B.13}$$

$$\vec{E}^{\perp}_{\vec{k}\omega} = \frac{e \; \omega}{\omega^2\epsilon_{\perp} - k^2} \int d^3p \vec{v}^{\perp}\frac{g_{\vec{k}}(\vec{p})}{(\vec{k} \cdot \vec{v} - \omega - i\nu)}, \tag{B.14}$$

where $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are, respectively, the longitudinal and transverse part of the polarization tensor

$$\epsilon_{\parallel}(k,\omega) = 1 - \frac{2e^2}{k^2} \int d^3p \frac{\vec{k}\cdot\vec{v}}{(\vec{k}\cdot\vec{v} - \omega - i\nu)} \frac{\partial f_0}{\partial p}, \tag{B.15}$$

$$\epsilon_{\perp}(k,\omega) = 1 - \frac{e^2}{\omega} \int d^3p \frac{\vec{v}_{\perp}^2}{(\vec{k}\cdot\vec{v} - \omega - i\nu)} \frac{\partial f_0}{\partial p}. \tag{B.16}$$

Now, the general expression for the generated magnetic field is

$$\vec{B}_{\vec{k}\omega} = \frac{e}{\omega^2 \epsilon_{\perp}(k,\omega) - k^2} \int d^3p [\vec{v} \times \vec{k}] \frac{g_{\vec{k}}(\vec{p})}{(\vec{k}\cdot\vec{v} - \omega - i\nu)}. \tag{B.17}$$

The space-time evolution of the magnetic fluctuations can be determined by performing the inverse Laplace and Fourier transforms:

$$\vec{B}(\vec{x},\eta) = \int e^{-i\omega\eta} \frac{e \, d\omega}{\omega^2 \epsilon_{\perp}(k,\omega) - k^2} \int d^3k e^{i\vec{k}\cdot\vec{x}} [\vec{v} \times \vec{k}] \int d^3p \frac{g_{\vec{k}}(\vec{p})}{(\vec{v}\cdot\vec{k} - \omega - i\nu)}. \tag{B.18}$$

In order to perform this integral, the explicit relations for the polarization tensors should be given. They depend on the equilibrium distribution function $f_0(p)$, which we take to be[4]

$$f_0(p) = \frac{n_q}{8\pi T^3} e^{-\frac{p}{T}}, \tag{B.19}$$

where T is the equilibrium temperature, p is the modulus of the momentum and n is the equilibrium (thermal) density of charged particles in the plasma. The normalization is chosen in such a way that $\int d^3p f_0(p) = n_q$.

Then we have for transverse polarization

$$\epsilon_{\perp}(k,\omega) = 1 + \frac{e^2 \, n_q}{2\omega k T} \left\{ [1 - \frac{(\omega + i\nu)^2}{k^2}] \ln \frac{k - \omega - i\nu}{k + \omega + i\nu} - 2\frac{\omega + i\nu}{k} \right\}, \tag{B.20}$$

and for the longitudinal polarization:

$$\epsilon_{\parallel}(k,\omega) = 1 + \frac{e^2 \, n_q}{k^2 T} \left\{ 2 + \frac{\omega + i\nu}{k} \ln \frac{k - \omega - i\nu}{k + \omega + i\nu} \right\}. \tag{B.21}$$

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[4] Notice that most of our considerations can be easily extended to the case of a Bose-Einstein or Fermi-Dirac distribution. What is important, in our context, is the analytical structure of the polarization tensors and this is the same for different distributions [28].

Consider now the case of very small momenta $k \ll \omega$ and $\omega \ll \nu$, relevant for long-ranged magnetic fields. Then the computation of the integral (B.18) in the large time limit and with the use of explicit form of the transverse polarization tensor in (B.20) gives [5]:

$$B(\vec{x}, \eta) \simeq \frac{T}{4\pi\alpha n_q} \exp\left(-k^2\eta/\sigma\right)\vec{\nabla} \times \vec{J}, \tag{B.22}$$

where σ is the plasma conductivity in the relaxation time approximation,

$$\sigma = \frac{2e^2 n_q}{\nu T} \tag{B.23}$$

and initial electric current is given by

$$\vec{J}(\vec{x}) = \int d^3 p\, \vec{v}\, g_{\vec{k}}(\vec{p}) \ . \tag{B.24}$$

In closing our discussion of the Vlasov equation we want to briefly comment about the validity of our approach. The obtained results assumed that the linearization of the Vlasov equation is consistent with the physical assumptions of our problem. This is indeed the case. In order to safely linearize the Vlasov equation we have to make sure that the perturbed distribution function of the charge fluctuations is always smaller than the first order of the perturbative expansion (given by the distribution of Eq. B.19). In other words we have to make sure that

$$|\delta f_+(\vec{x}, \vec{p}, \eta)| \ll f_0(\vec{p}), \quad |\delta f_-(\vec{x}, \vec{p}, \eta)| \ll f_0(\vec{p}). \tag{B.25}$$

These conditions imply that

$$\frac{e\vec{E}_{\vec{k}\omega}}{|\vec{k} \cdot \vec{v} - \omega - i\nu|} \cdot \frac{\partial f_0(\vec{p})}{\partial \vec{p}} \ll f_0(\vec{p}). \tag{B.26}$$

If we now define the relativistic plasma frequency as

$$\omega_p^2 = \frac{2\,e^2\,n_q}{3\,T}, \tag{B.27}$$

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[5]For small k, the equation $\omega^2\epsilon_\perp(k,\omega) - k^2 = 0$ defining the poles of the inverse Laplace transform implies $\omega \sim ik^2/\sigma$.

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we can see that the condition expressed by Eq. (A.20) can be restated, for modes $k \leq \omega_p$, as $|\vec{E}_{\vec{k},\omega}|^2 < n_q T$ (where we essentially took the square modulus of Eq. (B.26)). This last inequality expresses the fact that the energy density associated with the gauge field fluctuations should always be smaller than the critical energy density stored in radiation. The linear treatment of the Vlasov equation is certainly accurate provided the typical modes of the the field are smaller than the plasma frequency and provided the energy density in electric and magnetic fields is smaller than T^4, i.e. the energy density stored in the radiation background.

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