On initial conditions for the Hot Big Bang

F. Bezrukov

Max-Planck-Institut für Kernphysik,
PO Box 103980, 69029 Heidelberg, Germany
Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary prospect 7a, Moscow 117312, Russia
E-mail: `Fedor.Bezrukov@mpi-hd.mpg.de`

D. Gorbunov

Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary prospect 7a, Moscow 117312, Russia
E-mail: `gorby@ms2.inr.ac.ru`

M. Shaposhnikov

Institut de Théorie des Phénomènes Physiques,
École Polytechnique Fédérale de Lausanne,
CH-1015 Lausanne, Switzerland
E-mail: `Mikhail.Shaposhnikov@epfl.ch`

ABSTRACT: We analyse the process of reheating the Universe in the electroweak theory where the Higgs field plays a role of the inflaton. We estimate the maximal temperature of the Universe and fix the initial conditions for radiation-dominated phase of the Universe expansion in the framework of the Standard Model (SM) and of the νMSM — the minimal extension of the SM by three right-handed singlet fermions. We show that the inflationary epoch is followed by a matter dominated stage related to the Higgs field oscillations. We investigate the energy transfer from Higgs-inflaton to the SM particles and show that the radiation dominated phase of the Universe expansion starts at temperature $T_r \simeq (3-15) \times 10^{13}\,\text{GeV}$, where the upper bound depends on the Higgs boson mass. We estimate the production rate of singlet fermions at preheating and find that their concentrations at T_r are negligibly small. This suggests that the sterile neutrino Dark Matter (DM) production and baryogenesis in the νMSM with Higgs-driven inflation are low energy phenomena, having nothing to do with inflation. We study then a modification of the νMSM, adding to its Lagrangian higher dimensional operators suppressed by the Planck scale. The role of these operators in Higgs-driven inflation is clarified. We find that these operators do not contribute to the production of *Warm Dark Matter* (WDM) and to baryogenesis. We also demonstrate that the sterile neutrino with mass exceeding 100 keV (a Cold Dark Matter (CDM) candidate) can be created during the reheating stage of the Universe in necessary amounts. We argue that the mass of DM sterile neutrino should not exceed few MeV in order not to overclose the Universe.

KEYWORDS: inflation, physics of the early universe, dark matter, cosmological neutrinos.

arXiv:0812.3622v2 [hep-ph] 19 Jun 2009

Contents

1. Introduction

The statement that the Universe was dense and hot in the past is an established experimental fact. It follows from existence of the Cosmic Microwave Background radiation (CMB), which has a perfect Planck spectrum, and from accordance of predictions of the Big Bang Nucleosynthesis (BBN) with observations. The latter tells that the Universe had the temperature of at least few MeV. Whether the Universe was even hotter is an open question, which hardly can be answered by experimental means and thus is biased by theoretical prejudice.

According to the current theory of inflation (for a recent review see [1]) the early evolution of the Universe can be roughly divided into three parts. During the first, inflationary stage, the Universe expands exponentially and becomes nearly flat. At this stage relic gravity waves and matter perturbations, leading to structure formation, are generated. During the second, reheating stage, the energy stored in the inflaton field is transferred to the fields of the Standard Model and other (hypothetical) particles (if they exist). The third stage is the radiation dominated Universe in nearly thermal equilibrium for most of the SM particles. The starting moment of this stage t_r corresponds to a maximal temperature of the Universe $T_{\max}$, and this is the onset of the standard Hot Big Bang.

The system in thermal equilibrium is completely characterised by temperature T and chemical potentials μ_i for exactly conserved quantum numbers $\mathcal{Q}_i$; the corresponding operators $\hat{\mathcal{Q}}_i$ obey $[\hat{\mathcal{Q}}_i, \hat{\mathcal{H}}] = 0$, where $\hat{\mathcal{H}}$ is the Hamiltonian of the system. For the expanding Universe the precise thermal equilibrium never exists. To describe the state of the Universe at $T \sim T_{\max}$ the set of operators $\hat{\mathcal{Q}}_i$ should be supplemented by approximately conserved operators $\hat{\mathcal{Q}}_A$, whose rate of change is much smaller than the rate of the Universe expansion. Thus, to follow the Universe evolution at later times, $t > t_r$, one can use the ordinary kinetic approach based on Boltzmann equations (or equations for density matrix, if coherent quantum effects are essential) with initial density matrix

$$\rho_0 \propto \exp\left(-\frac{\hat{\mathcal{H}}}{T_{\max}} - \sum_i \frac{\mu_i}{T_{\max}} \hat{\mathcal{Q}}_i - \sum_A \frac{\mu_A}{T_{\max}} \hat{\mathcal{Q}}_A \right) . \tag{1.1}$$

The magnitude of the maximal temperature $T_{\max}$ together with the set of values of the chemical potentials μ_i, μ_A can be called *the initial conditions for the Hot Big Bang.* If they are known, the further evolution can be completely specified by the standard methods of kinetic theory.

Clearly, to find the initial conditions for the Big Bang one has to know what are the relevant particle degrees of freedom at $T < T_{\max}$ (in particular, if any new particles beyond those already present in the SM exist), or, in other words, what is the Hamiltonian $\hat{\mathcal{H}}$. The knowledge of the Hamiltonian would allow to determine the set of conserved $\hat{\mathcal{Q}}_i$ or nearly conserved $\hat{\mathcal{Q}}_A$ operators and identify the relevant chemical potentials. Now, to determine $T_{\max}$ and $\mu_{i,A}$ the interaction of the inflaton with the fields in $\hat{\mathcal{H}}$ must be known, and the physics of reheating must be elucidated.

Basically, to find the initial conditions for the Big Bang one should have at hand the theory which is valid up to the scale of inflation. There are quite a number of proposals for these types of theories, based on different ideas about physics beyond the SM. These ideas include low energy supersymmetry and Grand Unification, small, large or infinite extra dimensions (see e.g. [2] and [3] for reviews) and many others. Clearly, any model of

physics beyond the SM, must be able to explain the observed phenomena that cannot be addressed by the SM physics. They include neutrino masses and oscillations, the existence of dark matter in the Universe, baryon asymmetry, inflation, and accelerated expansion of the Universe at present.[1]

The most economical particle physics model which is capable of solving in a unified way all these problems of the SM is the νMSM (Neutrino Minimal Standard Model) of [4, 5]. This theory is nothing but the SM augmented by three relatively light (lighter than Z boson) right-handed singlet fermions. If the dilaton field is added to the νMSM, the theory can be made scale-invariant at the quantum level by a specific renormalization procedure [6, 7]. The spontaneous breaking of the scale invariance leads then to generation of all mass parameters, including the Newton's gravity constant. Higgs mass is stable against quantum corrections, cosmological constant is equal to zero, while dark energy, leading to the late acceleration of the Universe, appears if general relativity is replaced by the unimodular gravity [8]. Different phenomenological and cosmological aspects of this theory, together with the study of how to search for new particles, can be found in refs. [4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In [17] it was argued that this model may be valid all the way up to the Planck scale (for a similar argument in a related theory, see ref. [20]). Many of the parameters of this model are already fixed or constrained by existing cosmological observations and particle physics experiments.

In [21] it was found that the Higgs boson of the SM can play a role of the inflaton, if its non-minimal coupling to the gravity Ricci scalar is large enough. Exactly the same mechanism works in νMSM and its scale-invariant version with dilaton [8]. The evolution of the dilaton in the latter model with phenomenologically interesting choice of parameters happens to be irrelevant for inflation.

Reference [21] provides a rough upper limit on the maximal temperature of the Universe. The aim of this work is to demonstrate that the problem of the initial conditions for the Big Bang can be solved unambiguously in the SM and in the νMSM, and to find these initial conditions. To this end we consider in detail how the energy stored in Higgs-inflaton gets transferred to the SM and νMSM degrees of freedom. This allows to make a refined estimate of the reheat temperature and to fix the concentrations of the singlet fermions before the hot stage. We show that the abundances of new particles are too small to influence the low temperature baryogenesis in the νMSM studied in [5, 12, 18],[2] and low temperature dark matter production worked out in [13, 14, 19].[3]

The Lagrangian of the SM or of the νMSM, which can be considered as the effective theories, can contain all sorts of higher dimensional operators, suppressed by the Planck mass. Therefore, we consider the influence of these operators on inflation and on production of singlet fermions of the νMSM. We find that these operators are definitely not essential for baryogenesis and for dark matter production, if mass of the lightest sterile neutrino is below $100\,\mathrm{keV}$. In other words, the conclusion that the production of *WDM* sterile neutrinos with mass in the keV region must be due to their mixing with active neutrinos is a robust consequence of the νMSM. Since the presence of higher-dimensional operators

---

[1]Perhaps, the observed accelerated expansion of the Universe should not necessarily be included in this list as it may be irrelevant for the early stages of the Universe evolution we are interested in this work.

[2]See [22] for a suggestion to use singlet fermion oscillations for leptogenesis.

[3]For an original proposal of sterile neutrino as a dark matter candidate see [23, 24, 25], earlier computations of sterile dark matter abundance can be found in [23, 25, 26, 27].

looks to be a generic phenomenon, we argue that the DM sterile neutrinos have to be lighter than few MeV in order not to overclose the Universe.

The paper is organized as follows. In section 2 we review the mechanism of inflation based on the Higgs boson of the Standard Model, and determine the relevant interactions of the Higgs-inflaton with the other fields of the SM. In section 3 we analyse different processes reheating the Universe after inflation and estimate the maximal temperature $T_{\max}$. In section 4 we discuss the approximate conservation laws in the SM and the νMSM and define the operators $\hat{\mathcal{Q}}_A$. Then we estimate the values of chemical potentials μ_A generated by renormalizable interactions existing in the νMSM and the SM. In section 5 we add to the theory higher dimensional operators and analyse their influence on Higgs-driven inflation and on the generated values of the chemical potentials. In section 6 we study the effects of CP-violation at the reheating stage. Section 7 contains conclusions.

2. Higgs-driven inflation

The inflationary model with the Higgs boson as the inflaton [21, 28] adds the non-minimal coupling with gravity to the action of the SM (or νMSM)

$$S_J = S_{\text{SM}} + \int d^4x \sqrt{-g} \left(-\frac{M^2}{2} R - \xi \Phi^\dagger \Phi R \right) . \tag{2.1}$$

Here S_{SM} is the SM action, M is some mass parameter, which is nearly equal to the Planck mass in our case, R is the scalar curvature, Φ is the Higgs doublet, and ξ is a constant fixed by the requirement of correct scale of the CMB fluctuations. Index "J" stands for the "Jordan frame" action. Action (2.1) contains all possible terms of dimension 4 without higher derivatives.[4] In this section we review shortly the inflation analysis of [21, 28] and introduce some formulas important for the study of the reheating period.

The only part of the action relevant for inflation is the scalar sector. In the unitary gauge with $\Phi(x) = \frac{1}{\sqrt{2}}\binom{0}{v+h(x)}$ it has the form

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M^2 + \xi h^2}{2} R + \frac{\partial_\mu h \partial^\mu h}{2} - \frac{\lambda}{4} \left(h^2 - v^2 \right)^2 \right\} , \tag{2.2}$$

and Higgs vacuum expectation value is $v = 246\,\text{GeV}$. Another part we analyse later on, while we always stick to the unitary gauge for simplicity.

The conformal transformation. The simplest way to work with this action is to get rid of the non-minimal coupling to gravity by making the conformal transformation from the Jordan frame to the Einstein frame (see, e.g. [29, 30]):

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} , \quad \Omega^2 = \frac{M^2 + \xi h^2}{M_P^2} , \tag{2.3}$$

---

[4]One could also add other dimension 4 terms like R^2, $R_{\mu\nu}R^{\mu\nu}$, etc., but they lead to terms with higher derivatives in the equations of motion and, therefore, lead to additional degrees of freedom, which should be dealt with in some special way. We do not consider such extensions here.

where $M_P \equiv 1/\sqrt{8\pi G_N} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass. This transformation leads to a non-minimal kinetic term for the Higgs field. So, it is also convenient to replace h with new canonically normalised scalar field χ by making use of

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}} \; . \tag{2.4}$$

Finally, the action in the Einstein frame is

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\} \; , \tag{2.5}$$

where $\hat{R}$ is calculated using the metric $\hat{g}_{\mu\nu}$ and the potential is rescaled with the conformal factor

$$U(\chi) = \frac{1}{\Omega^4 [h(\chi)]} \frac{\lambda}{4} \left[h^2(\chi) - v^2 \right]^2 \; . \tag{2.6}$$

We will a bit ambiguously write potential U and scale factor Ω as functions of either h or χ, which should not lead to misreadings, as far as h and χ can be expressed one through another in a unique way. Figure 1 illustrates the connection between the Higgs field in the Jordan frame, h, and the Higgs field in the Einstein frame, χ. For $\xi \gg 1$, the solution of eq. (2.4) can be approximated in two major regions,[5] separated by

$$X_{\mathrm{cr}} \equiv \sqrt{\frac{2}{3}} \frac{M_P}{\xi} \; .$$

Namely,

$$\chi \simeq \begin{cases} h & \text{for} \quad h < X_{\mathrm{cr}} \; , \\ \sqrt{\frac{3}{2}} M_P \log \Omega^2(h) & \text{for} \quad X_{\mathrm{cr}} < h \; . \end{cases} \tag{2.7}$$

Note that for analysis of reheating we will need only the field values smaller than $M_P/\sqrt{\xi}$, where the logarithm in (2.7) can be expanded in the following way

$$\chi \simeq \sqrt{\frac{3}{2}} \frac{\xi h^2}{M_P} \qquad \text{for} \quad X_{\mathrm{cr}} < h \ll \frac{M_P}{\sqrt{\xi}} \; . \tag{2.8}$$

Using relations (2.7) we can explicitly write the potential as (here we assume $v \ll M_P/\xi$)

$$U(\chi) \simeq \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } \chi < X_{\mathrm{cr}} \; , \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } X_{\mathrm{cr}} < \chi \; . \end{cases} \tag{2.9}$$

Again, in the region interesting for reheating, the potential can be approximated by the quadratic potential

$$U(\chi) \simeq \frac{\omega^2}{2}\chi^2 \qquad \text{for} \quad X_{\mathrm{cr}} < \chi \ll \sqrt{\frac{3}{2}} M_P \; , \tag{2.10}$$

where the "inflaton mass" ω is

$$\omega \equiv \sqrt{\frac{\lambda}{3}} \frac{M_P}{\xi} \; . \tag{2.11}$$

Figure 2 shows schematically the potential (2.9).

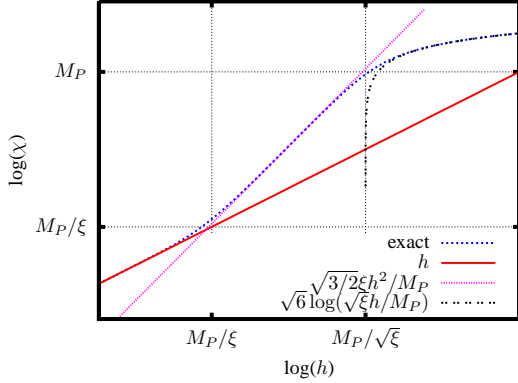[5]Exact analytic solution exists, but is not really enlightening.

Figure 1: Dependence of χ (the Einstein frame Higgs field) on h (the Jordan frame Higgs field), logarithmic scale.
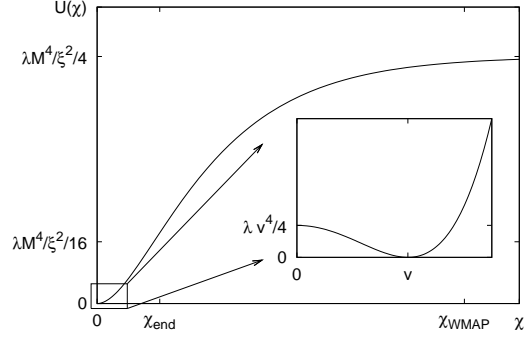
Figure 2: Effective potential in the Einstein frame. The insert magnification is not to scale.

Inflationary phase. The potential (2.9) is exponentially flat for large field values, and provides the slow roll inflation. Analysis of the inflation in the Einstein frame[6] can be performed in the standard way using the slow-roll approximation. The slow roll parameters (in notations of [1]) are easier to express analytically as functions of the field h using (2.4) and (2.6), instead of the field χ,

$$\epsilon = \frac{M_P^2}{2} \left(\frac{dU/d\chi}{U} \right)^2 = \frac{M_P^2}{2} \left(\frac{U'}{U} \frac{1}{\chi'} \right)^2 , \tag{2.12}$$

$$\eta = M_P^2 \frac{d^2 U/d\chi^2}{U} = M_P^2 \frac{U'' \chi' - U' \chi''}{U \chi'^3} , \tag{2.13}$$

where $'$ denotes derivative with respect to h. Slow roll ends at $\epsilon \simeq 1$, which corresponds to the value h_{end}. The perturbation modes of WMAP [34] scale $k/a_0 = 0.002/\text{Mpc}$ left horizon when the field value equals h_{WMAP}. The latter is determined by the number of inflation e-foldings,

$$N = \int_{h_{\text{end}}}^{h_{\text{WMAP}}} \frac{1}{M_P^2} \frac{U}{U'} \left(\chi' \right)^2 dh . \tag{2.14}$$

To generate proper amplitude of the density perturbations the potential should satisfy at h_{WMAP} the normalization condition

$$U/\epsilon = 24\pi^2 \Delta_{\mathcal{R}}^2 M_P^4 \simeq (0.0276 M_P)^4 . \tag{2.15}$$

For usual quartic potential inflation this condition fixes the coupling constant λ, while in our case this allows to find the value for ξ for *any* given value of λ. The inflationary predictions (see, e.g., [1]) for the CMB spectrum parameters are then given by the expressions for spectral index n_s and tensor-to-scalar perturbation ratio r,

$$n_s = 1 - 6\epsilon + 2\eta , \quad r = 16\epsilon , \tag{2.16}$$

also calculated at h_{WMAP}.

---

[6]The same results can be obtained in the Jordan frame [31, 32, 33].

In the case of the standard Higgs potential $V(h) = \frac{\lambda}{4}(h^2 - v^2)^2$ we get for the slow roll parameters [28] (in the limit $h^2 \gtrsim M_P^2/\xi \gg v^2$, $\xi \gg 1$, exact expressions can be found in [29]),

$$\epsilon \simeq \frac{4M_P^4}{3\xi^2 h^4} \;, \quad \eta \simeq \frac{4M_P^4}{3\xi^2 h^4}\left(1 - \frac{\xi h^2}{M_P^2}\right) \;. \tag{2.17}$$

Inflation ends in at $h_{\text{end}} \simeq (4/3)^{1/4} M_P/\sqrt{\xi} \simeq 1.07 M_P/\sqrt{\xi}$ (and $\chi_{\text{end}} \simeq 0.94 M_P$). The number of e-foldings is (2.14)

$$N = \frac{3}{4}\left[\frac{h_{\text{WMAP}}^2 - h_{\text{end}}^2}{M_P^2/\xi} + \log\frac{1 + \xi h_{\text{end}}^2/M_P^2}{1 + \xi h_{\text{WMAP}}^2/M_P^2}\right] \;, \tag{2.18}$$

leading to $h_{\text{WMAP}} \simeq 9.14 M_P/\sqrt{\xi}$. Thus, the WMAP normalization (2.15) requires (for $N = 59$)

$$\xi \simeq 47000\sqrt{\lambda} \;, \tag{2.19}$$

where λ is the Higgs boson self coupling constant, taken at *inflationary scale*. Note, that we retained here the logarithmic term in (2.18), which was left out in [21]. This, together with WMAP5 value for normalization (2.15), changed the numerical value in the relation (2.19). This does not significantly change the spectral index and tensor to scalar ratio.

The spectral index is $n_s \simeq 1 - 8(4N + 9)/(4N + 3)^2$, and the tensor-to-scalar perturbation ratio is $r \simeq 192/(4N + 3)^2$.

The number N of e-foldings is fixed from the post-inflation history of the Universe described in section 3. We show there that the inflationary stage is followed by the matter dominated epoch, corresponding to oscillations of Higgs-inflaton with frequency ω, defined in (2.11). The radiation dominated era starts at effective temperature T_r, given by (3.13). Then, the number of e-foldings is (see [35])

$$N = 62 - \log\frac{k}{a_0 H_0} - \log\frac{10^{16}\,\text{GeV}}{U^{1/4}(\chi_{\text{WMAP}})} + \log\frac{U^{1/4}(\chi_{\text{WMAP}})}{U^{1/4}(\chi_{\text{end}})} - \frac{1}{3}\log\frac{U^{1/4}(\chi_{\text{end}})}{\rho^{1/4}(T_{max})}$$
$$\simeq 60.4 - \log\frac{k}{a_0 H_0} - \frac{1}{6}\log\frac{X_{\text{cr}}}{X_r} \;. \tag{2.20}$$

Here the present Hubble parameter is $H_0 = 0.7/(3000\,\text{Mpc})$, $U(\chi_{\text{WMAP}}) \simeq \frac{\lambda M_P^4}{4\xi^2}$, $\rho(T_{max})$ is the energy density at the beginning of the hot stage, X_r is in the range (3.10). Then, we get

$$N \simeq 59 \;, \quad n_s \simeq 0.97 \;, \quad r \simeq 0.0034 \;. \tag{2.21}$$

The predicted values are well within one-sigma border of allowed region of parameter space, see figure 3.

Effective couplings in the inflationary domain. The inflation and reheating of the Universe occur at energy scales much larger, than the electroweak scale. This calls for the study of radiative corrections to the inflationary potential. A qualitative discussion of the influence of loop effects on inflation can be found in [21]. A number of explicit computations (giving in some cases conflicting results) has been reported recently [36, 37, 38, 39, 40]. The conclusion of [37, 38, 39, 40] is that Higgs-driven inflation is a viable phenomenon in a

certain interval of Higgs masses $m_{\min} < m_H < m_{\max}$. The values of $m_{\min}$ and $m_{\max}$ found in [39] are:

$$m_{\min} = [126.1 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5] \text{ GeV} , \qquad (2.22)$$

$$m_{\max} = [193.9 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.1176}{0.002} \times 0.1] \text{ GeV} . \qquad (2.23)$$

with a theoretical uncertainty $\delta_{\text{theor}} = \pm 2 \text{ GeV}$. The similar result for $m_{\min}$ was found in an earlier paper [38] and also in [40].

What is essential for the present study of reheating, is the magnitude of the coupling constants in the relevant energy domain $\sim M_P/\xi$. So, an appropriate renormalization group running of the coupling constants should be taken into account. Specifically, one should use the M_P/ξ scale value for the electroweak coupling constant

$$\alpha_W^{-1} \simeq 43 . \qquad (2.24)$$

and the corresponding numbers for the strong and U(1) gauge couplings.

The dependence of the value of the scalar self-coupling λ at the scale M_P/ξ on the Higgs boson mass is illustrated in figure 4 (see ref. [39] for detailed description). It can be seen, that out of

Figure 3: The allowed WMAP+BAO+SN region for inflationary parameters (r, n_s), adopted from [34]. The green box is our predictions supposing 59 e-foldings of inflation. Black and white dots are predictions of usual chaotic inflation with $\lambda\phi^4$ and $m^2\phi^2$ potentials, HZ is the Harrison-Zeldovich spectrum.

the window (2.22,2.23) for Higgs masses the inflationary scale Higgs self-interaction starts to behave badly at the energy scale of inflation. For large Higgs masses it becomes large and thus leads to strong coupling. For small Higgs masses it gets negative and leads to instability of the electroweak vacuum. The analysis of the present paper is not applicable very close to the boundaries of the allowed region. If the Higgs mass is approaching (2.22) or (2.23) one has to redo the analysis including higher order radiative corrections to the Higgs potential, what is beyond the scope of this paper.

Yet another remark concerns applicability of perturbation theory at high momenta. As was found in [41] and in [42, 43], the perturbation theory, which is used throughout the calculations of reheating, is inapplicable for momenta above M_P/ξ in a potential of the form (2.6). However, at reheating the energy is mostly contained in particles with momenta $\sim \lambda M_P/\xi$ (see below), which interact weakly. Thus, the details of the formulation of the theory for high momenta are irrelevant for present calculations.

3. Reheating in Higgs-driven inflation
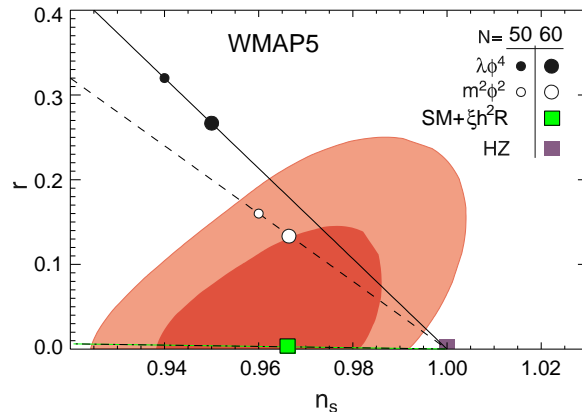
3.1 Qualitative picture

The scalar potential for the Higgs field[7] in the Einstein frame exhibits three qualitatively different behaviours, leading to three stages of the Universe expansion. The first, inflationary stage, corresponding to the flat potential at $\chi > M_P$, has been already discussed in section 2. The second specific region of the scalar field values is

Figure 4: λ at the scale M_P/ξ depending on the Higgs boson mass m_H for $m_t = 169.1, 171.2, 173.3\,\mathrm{GeV}$ (from upper to lower graph). Varying e-foldings number and an error in the WMAP normalization measurement one introduces changes invisible on the graph.

$$M_P > \chi > X_{\mathrm{cr}} = \sqrt{\frac{2}{3}}\frac{M_P}{\xi} \, , \qquad (3.1)$$

where the scalar potential is essentially *quadratic,* see (2.10). The slow roll inflation terminates at $\chi \sim M_P$ with the onset of the oscillations of the scalar field. Since the effective inflaton mass ω is non-zero for these field values, the exponential expansion of the Universe is changed to the power low, corresponding to matter domination. The amplitude of the Higgs field during this stage is decreased due to expansion of the Universe and due to particle creation. At last, for $\chi < X_{\mathrm{cr}}$ we are certainly in the radiation-dominated epoch: the potential for the Higgs field (2.9) does not contain any essential mass parameters and thus is scale-invariant; the scalar self-coupling and couplings of the Higgs field to the fields of the SM are relatively large, and lead to a rapid energy transfer from the coherent oscillations to relativistic particles. Assuming the instant conversion of the energy of coherent oscillations to relativistic degrees of freedom of the SM, we get a lower bound on reheating temperature $T_{\mathrm{reh}} \gtrsim 1.5 \times 10^{13}\,\mathrm{GeV}$ (see [21, 28]).

However, as we show in this section, creation of particles happens to be important even for $\chi > X_{\mathrm{cr}}$, and the reheat temperature is higher. In what follows we will be mostly interested in the very moment, when matter dominated expansion is replaced by the radiation dominated one. That is when energy in coherent oscillations of the scalar field is equal to energy collected by SM particles. We will characterise this moment by an effective temperature T_r (the "r" stands for "radiation dominance"). It is determined by equating of the would-be thermally equilibrium energy of SM plasma described by T_r to its actual energy. The real thermal equilibrium is achieved at somewhat lower temperature $T_{\mathrm{reh}} < T_r$.

To determine T_r, let us neglect first the effects of particle creation and consider the evolution of the Universe with the Higgs field in the interval (3.1). The Friedman equation reads:

$$H^2(t) = \frac{1}{3M_P^2}\left[\frac{\omega^2}{2}\chi^2(t) + \frac{1}{2}\dot{\chi}^2(t)\right] \, , \qquad (3.2)$$
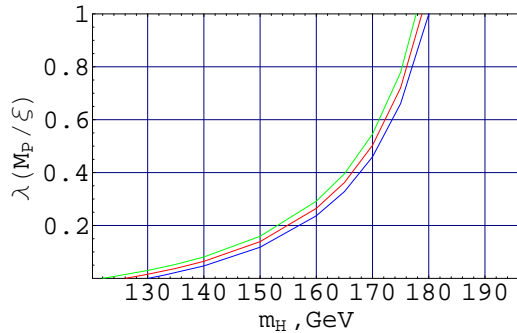
---

[7]We will use the names "Higgs boson" and "inflaton" interchangeably, depending on the context.

and leads to a matter dominated expansion regime

$$a \propto t^{2/3} \; , \tag{3.3}$$

$$\chi(t) = X(t) \cos \left[\omega(t - t_o) \right] \; , \tag{3.4}$$

$$H(t) = \frac{\sqrt{\lambda}}{3\sqrt{2}\,\xi} X(t) = \frac{2}{3t} \; , \qquad\qquad X(t) = 2\sqrt{2}\frac{\xi}{\sqrt{\lambda}}\frac{1}{t} \; . \tag{3.5}$$

Here t is the physical time, a is the scale factor, H is the Hubble parameter, $X(t)$ is the amplitude of the background inflaton field oscillations, t_o here gives the arbitrary phase of the oscillations, and ω is defined in (2.11). This solution is approximate. It is only reliable for $H \ll \omega$, when the change of the scale factor is small during one oscillation. The amplitude reaches the critical value X_{cr} at the critical time

$$t \approx t_{\mathrm{cr}} \equiv \frac{2\xi}{\omega} \; . \tag{3.6}$$

To determine the particle production, we will consider the solution (3.5) as an external background. This approximation breaks down when the energy of created relativistic particles is comparable with the energy of the scalar field (inflaton zero mode)

$$\rho_{\mathrm{inf}} = \frac{\omega^2}{2} X^2 = \frac{\lambda}{4} X_{\mathrm{cr}}^2 X^2 \; . \tag{3.7}$$

This moment will give us the temperature T_r we are interested in.

To start with, we describe on the qualitative level various processes which occur during the reheating stage at $t < t_{\mathrm{cr}}$ and single out the most important ones. Further, we analyse these processes in detail.

The main mechanism draining energy from the inflaton zero mode is creation of the particles directly from coupling to the Higgs-inflaton. In the background approximation the inflaton field (3.4) can be considered as an external source of all other fields. This source has the form of the varying-with-time masses of all the particles (this includes the propagating modes of the Higgs field itself). Only particles with large couplings to the Higgs field can be created effectively by this mechanism. These are the gauge bosons and top quark. Their masses in the region (2.8) are

$$m_W^2(\chi) = \frac{g^2}{2\sqrt{6}} \frac{M_P |\chi|}{\xi} \; , \tag{3.8}$$

$$m_t(\chi) = y_t \sqrt{\frac{M_P |\chi(t)|}{\sqrt{6}\xi}} \; \mathrm{sign}\, \chi \; . \tag{3.9}$$

Here $g^2/4\pi = \alpha_W$ is the weak coupling constant, and $y_t = \sqrt{2}m_t/v$ is top quark Yukawa. However, exactly due to large couplings they are heavy and are still non-relativistic. The production of such particles does not change the equation of state from the non-relativistic matter to radiation. That change happens eventually only due to creation of the relativistic secondary particles (such as light leptons or quarks) via decays or scatterings of the heavy particles. A competing (but slightly slower) process is the direct creation of relativistic Higgs excitations, which happens because the potential (2.9) is not exactly quadratic near the origin.[8]

---

[8]The non-linearity of the potential (2.9) at large field values $\chi \sim M_P$ is relevant for particle creation only during a short period at the very early time, that may be neglected.

As a result, the generic picture of the reheating process is the following. As far as the inflaton "mass" ω is smaller than the gauge boson (3.8) or top quark mass (3.9) for $\chi \gtrsim X_{\mathrm{cr}}$, creation of the gauge bosons or top quarks is possible only at the moments, when the inflaton field crosses zero (when $\chi(t) \lesssim X_{\mathrm{cr}}$). During each zero crossing some gauge bosons and top quarks are created. At first, when the concentration of the created particles is small (occupation numbers $n_{\mathbf{k}} \ll 1$), the creation rate is constant (see Appendix A.2). At this stage the created W bosons are non-relativistic and decay into light SM fermions (which are relativistic). The decay rate, however, changes with time with the decreasing amplitude of the inflaton oscillations. The decay process sustains some quasi constant density of the created bosons (Appendix B.1). It stops when the decay rate becomes smaller than the production rate, which happens at the inflaton oscillation amplitude (3.21). Up to this moment no significant energy transfer from the inflaton to radiation happens. Then the generation process accelerates, being enhanced by the stochastic parametric resonance (occupation numbers $n_{\mathbf{k}} > 1$, and the concentration of W bosons rises. The energy transfer into the light SM fermions proceeds now mainly via $WW \rightarrow f\bar{f}$ annihilation, (see Appendix B.2). This process rapidly transfers all the energy to radiation, resulting in transition from the matter domination expansion $a \propto t^{2/3}$ to the radiation domination $a \propto t^{1/2}$ at field amplitude only slightly smaller than (3.21). This should be considered as a refined upper bound[9] on critical X. The lower bound is given by the slower energy transfer mechanism — the generation of the Higgs bosons on close-to-vicinity nonlinearities of the potential (2.9) (see paragraph C), which yields transition at the oscillation amplitude (3.27). We do not analyse the production of the top quarks in the present work, because their contribution is smaller than that of W bosons. This is because the parametric resonance enhancement is absent for fermions due to Pauli exclusion principle.

To summarise, the matter-radiation transition happens when the amplitude of the inflaton oscillations is somewhere in the region

$$3.7 \left(\frac{\lambda}{0.25}\right)^{1/2} X_{\mathrm{cr}} < X_r < 40 \left(\frac{\lambda}{0.25}\right) X_{\mathrm{cr}} \,. \tag{3.10}$$

The temperature T_r is estimated as follows,

$$g_* \frac{\pi^2}{30} T_r^4 \simeq \frac{\omega^2 X_r^2}{2} = \frac{\lambda}{4} X_{\mathrm{cr}}^2 X_r^2 \,, \tag{3.11}$$

where $g_* \sim 100$ is the effective number of degrees of freedom of the SM. This gives for (3.10)

$$1.4 \times 10^{-5} M_P < T_r < 4.5 \times 10^{-5} \left(\frac{\lambda}{0.25}\right)^{1/4} M_P \,, \tag{3.12}$$

or

$$3.4 \times 10^{13} \,\mathrm{GeV} < T_r < \left(\frac{\lambda}{0.25}\right)^{1/4} 1.1 \times 10^{14} \,\mathrm{GeV} \,. \tag{3.13}$$

The coupling constant λ here is taken at the inflationary scale. Its dependence on the physical Higgs mass is presented in figure 4. Note, that at $T = T_r$ the particle distributions are not yet fully thermal.

---

[9]More accurate analytical estimate of outcome of the scattering processes is hardly possible, since particle masses vary quite rapidly.

3.2 W boson production

Let us start with description of boson production by the external oscillating source (3.4). We write the equation of motion with the mass given by (3.8). Actually, with formula (3.8) we ignore the exact behaviour of the mass in time intervals when $\chi < X_{\rm cr}$: then the potential is quartic, so the zero mode evolution also deviates from (3.4). Hence, the jump in the derivative in m_W^2 is "smoothed". This introduces an additional cutoff in the spectrum of produced W bosons, which can be safely neglected. To simplify the computation, we also replace the vector boson by a scalar particle. We will take into account three polarizations of the vector boson in the final formula only.

Evolution of the mode $\phi_{\mathbf{k}}$ with conformal momentum $\mathbf{k}$ is governed by the equation

$$\ddot{\phi}_{\mathbf{k}} + 3H\dot{\phi}_{\mathbf{k}} + \left(\frac{\mathbf{k}^2}{a^2} + m_W^2(t) \right) \phi_{\mathbf{k}} = 0 \ . \tag{3.14}$$

This equation can be solved in the adiabatic approximation except for the moments when $m_W(t)$ is close to zero. The reason is that $m_W(t)$ is larger than the frequency of the background field ω except for small background $\chi \lesssim X_{\rm cr}$. The solution of equation (3.14) in the adiabatic approximation is (see Appendix A.1)

$$\frac{\phi_{\mathbf{k}}}{a^{3/2}} = \frac{\alpha_{\mathbf{k}}^j}{\sqrt{2k_0}} e^{-i \int_0^t k_0 dt} + \frac{\beta_{\mathbf{k}}^j}{\sqrt{2k_0}} e^{+i \int_0^t k_0 dt} \ , \tag{3.15}$$

where parameters $\alpha_{\mathbf{k}}^j$, $\beta_{\mathbf{k}}^j$ remain constant between the moments t_j, corresponding to zero background field $\chi(t_j) = 0$.

In the vicinity of the moments $\chi(t_j) = 0$ the mass $m_W(t)$ becomes small compared to the background (source) frequency ω and particle creation can take place. Exact solution in this region (see Appendix A.1) allows for matching $\alpha_{\mathbf{k}}^{j+1}, \beta_{\mathbf{k}}^{j+1}$ to $\alpha_{\mathbf{k}}^j, \beta_{\mathbf{k}}^j$. For the change in the occupation numbers $n_{\mathbf{k}}^j \equiv |\beta_{\mathbf{k}}^j|^2$ we have

$$\delta n_{\mathbf{k}}^j \equiv n_{\mathbf{k}}^{j+1} - n_{\mathbf{k}}^j = \frac{|R_{\mathbf{k}}|^2}{1 - |R_{\mathbf{k}}|^2} + \frac{2|R_{\mathbf{k}}|^2}{1 - |R_{\mathbf{k}}|^2} n_{\mathbf{k}}^j + \frac{2|R_{\mathbf{k}}|}{1 - |R_{\mathbf{k}}|^2} \cos \theta_{\rm tot}^j \sqrt{n_{\mathbf{k}}^j (n_{\mathbf{k}}^j + 1)} \ , \tag{3.16}$$

where $R_{\mathbf{k}}$ is a decreasing with $\mathbf{k}$ function defined in (A.15) (see also figure 5), and $\theta_{\rm tot}^j$ is the $\mathbf{k}$-dependent phase (A.17).

If the occupation numbers $n_{\mathbf{k}}^j$ on the right hand side of eq. (3.16) are small, $n_{\mathbf{k}}^j \ll 1$, it describes a simple particle creation with constant rate. This is certainly the case if the particles, generated at each zero crossing, decay before the next zero crossing, or scatter and change momentum $\mathbf{k}$ to some value where the generation coefficients in (3.16) are small. If not, eq. (3.16) describes a resonance like production of bosons. There exist several regimes, depending on the model. The usual ("narrow") parametric resonance emerges when $n_j \gg 1$ and the phase $\theta_{\rm tot}^j / \pi$ changes by an integer number between t_j and t_{j+1}. In this case one has a pronounced exponential behaviour for $n_{\mathbf{k}}$, located in narrow regions of momentum, which are defined by $\Delta\theta_{\rm tot}^j \equiv \theta_{\rm tot}^{j+1} - \theta_{\rm tot}^j \simeq n\pi$. Another resonance situation is realized when

$$\Delta\theta_{\rm tot}^j \gg \pi \ . \tag{3.17}$$

This regime is called "stochastic resonance" [44]. In this case the jumps at moments t_j may be in either direction, but, on average, they also lead to exponential growth, though

a slower one. The regime (3.17) is realised for $X \gtrsim (\lambda/g^2)X_{\mathrm{cr}}$ (see (A.17), (A.8)), which is always true in our case. In this regime the particle generation proceeds for any momenta with not too small $|R_{\mathbf{k}}|$ (otherwise the last term easily spoils growth of the occupation number), which implies low momenta.

So, while the number of created particles is small, $n_{\mathbf{k}} \ll 1$, the creation of the W^+ bosons from (3.16) proceeds in the linear regime and can be approximated as (see Appendix A.2)

$$\frac{d(a^3 n_{W^+})}{dt} \approx 3 \cdot a^3 A \frac{\alpha_W}{2\pi^2} \omega^2 X_{\mathrm{cr}}^2 X \; , \tag{3.18}$$

with numeric coefficient $A \simeq 0.0455$. The created particles are essentially non-relativistic. For concentrations of other gauge bosons we have the obvious relations $n_{W^+} = n_{W^-}$, $n_Z = n_{W^+}/\cos^2\theta_W$, where θ_W is the weak mixing angle.

When the occupation number becomes larger, the production is enhanced due to the Bose statistics, and is becoming approximately exponential (Appendix A.3)

$$\frac{d(a^3 n_W)}{dt} \sim a^3 2\omega B n_W \; , \tag{3.19}$$

where the numerical coefficient $B \simeq 0.045$. These particles are also created with nonrelativistic momenta. Note again, that here the backreaction of the particles on the condensate is neglected.

3.3 Transfer into relativistic particles

Now let us analyse how decay and scattering of the W bosons influence their generation, described in the section 3.2.

At small concentrations of the bosons the main process is their decay. The (average) decay width of the SM gauge boson is

$$\Gamma \approx 0.8 \alpha_W \langle m_W \rangle \; , \tag{3.20}$$

The mass here is approximated as averaged over inflaton background field oscillations. Comparing (3.20) with the production rate (3.19), we find that for

$$X > \frac{2}{0.64\,\pi} \frac{B^2 \lambda}{\alpha_W^3} X_{\mathrm{cr}} \approx 40 \left(\frac{\lambda}{0.25} \right) X_{\mathrm{cr}} \; . \tag{3.21}$$

the decay is a more rapid process and prevents the exponential regime to start.

The energy transfer rate is then balanced by the linear production (3.18) and the decay rate (3.20). In Appendix B.1 we show that the energy transfer to the relativistic modes is negligible for this period.

When the decay process becomes inefficient, the concentration grows, leading to enhancement of the production approaching exponential behaviour (3.19). At the same time the main process responsible for the energy transfer to light particles becomes the annihilation of the W bosons, which is proportional to density squared. The relevant process is the annihilation into two fermions $WW \to f\bar{f}$ via t-channel fermion exchange. The estimate of the cross section for the process is

$$\sigma \approx \left(\frac{g}{\sqrt{2}} \right)^4 \frac{2N_l + 2N_q N_c}{8\pi} \frac{1}{\langle m_W^2 \rangle} \approx 10\pi \frac{\alpha_W^2}{\langle m_W^2 \rangle} \; , \tag{3.22}$$

where $N_l = 3$ is the number of lepton generations, $N_q = 2 + 1/4$ is the effective number of quark generations (virtual t-quark contribution is suppressed by $(m_W/m_t)^2 \sim 1/4$), and $N_c = 3$ is the number of colours.

Then, the equality between generation and annihilation of the W bosons is reached at

$$n_{\text{scatter}} = \frac{2B\omega}{\sigma} \ . \tag{3.23}$$

The energy drain into the relativistic modes is

$$\frac{d}{dt} \left(a^4 \rho \right) = 2a^4 \cdot \sqrt{\langle m_W^2 \rangle} \cdot \sigma n_{\text{scatter}}^2 \ . \tag{3.24}$$

The integral is saturated at late times and gives nearly immediate transfer of all the energy into the relativistic modes after the regime (3.21) finishes.

One may note, that this approximation may overestimate W boson production, and it may actually proceed at a smaller rate, because of several reasons. First, the occupation number $n_{\mathbf{k}}$ is not too large (B.9), and resonance regime is not fully reached. Second, the exponent in (3.19) is actually the upper limit. Careful analysis may reveal that the process is slower, so the transfer to the relativistic degrees of freedom happens later (at lower X) than (3.21), the latter should be considered as the upper bound on X_r. The lower bound is then given by a slower process of generation of the Higgs field excitations.

3.4 Higgs production

Another particles produced during the inflaton oscillations are inflaton excitations (Higgs particles). Really, mass of the excitations $\delta\chi$ in the background χ is given by $m_\chi^2 = U''(\chi)$, which is, approximately

$$m_\chi^2(t) = \begin{cases} \omega^2 & \text{for} \quad \chi(t) > \frac{\omega}{\sqrt{3\lambda}} \ , \\ 3\lambda\chi^2(t) & \text{for} \quad \chi(t) < \frac{\omega}{\sqrt{3\lambda}} \ , \end{cases} \tag{3.25}$$

where $\chi(t)$ is given[10] by (3.4). Thus, we need to solve the same equation (3.14), but now with the mass (3.25). The details of the solution are given in Appendix C. The produced particles are relativistic, with energy $E \sim \frac{1}{2}\sqrt{3\lambda}X$, and with energy balance equation density

$$\frac{d(a^3\rho)}{dt} \simeq a^3 \frac{\omega^5}{2\pi^3} \ . \tag{3.26}$$

This competes with the inflaton (zero mode) energy density ρ_{inf} at

$$X = \frac{M_P}{\xi} \left(\xi \frac{2\sqrt{6}\lambda}{33\pi^3} \right)^{1/3} \approx 3.7 \left(\frac{\lambda}{0.25} \right)^{1/2} \left(\frac{\xi}{47000\sqrt{\lambda}} \right)^{1/3} X_{\text{cr}} \ . \tag{3.27}$$

This provides the lower bound on the moment of transition to the radiation dominated epoch X_r.

---

[10]Of course, this is a rough approximation for $X \gg X_{\text{cr}}$, because we should in principle solve the equation of motion for χ in the exact potential. But at large $X \gg X_{\text{cr}}$ the time, system spent in the region $\chi < X_{\text{cr}}$, is small, and the background solution can be simply approximated by (3.4).

3.5 Fermion production from Higgs decay

For completeness, let us also calculate the number of light fermions generated at the re-heating stage by the inflaton-Higgs field. The result here does not apply to heavy fermions (top quark). The latter abundance is of small interest: before thermalization it is in any way not larger than that of the gauge bosons, and after thermalization top quarks are generated rather fast.

We analyse here the production of the light fermions by the Higgs condensate decay due to the Yukawa interactions. The latter are of the form (3.9) with some small Yukawa y instead of y_t. The exact treatment would require the solution of Dirac equation with the time-dependent mass (3.9). To make an order-of-magnitude estimate, we will replace the source $\propto \sqrt{\sin(\omega t)}$ by the simpler one, $\propto \sin(\omega t)$. Though the spectra of the produced fermions are different (the spectrum is monochromatic for the sinusoidal source), their total numbers are similar. After this substitution the fermion time-dependent mass term becomes:

$$
y\sqrt{\frac{M_P}{\sqrt{6}\xi X}}X\sin(\omega t)\bar{\psi}\psi\ . \tag{3.28}
$$

In the lowest order of perturbation theory the rate of fermion production here is equivalent to the decay rate of the system of scalar particles with mass ω, concentration $n = \omega X^2/2$ and effective Yukawa constant $y\sqrt{\frac{M_P}{\sqrt{6}\xi X}}$:

$$
\frac{d}{dt}(a^3 n_\psi) = a^3 \frac{\omega X^2}{2\sqrt{6}}\omega\frac{y^2}{8\pi}\left(\frac{M_P}{\xi X}\right)\ , \tag{3.29}
$$

(and the same formula for the antiparticles $n_{\bar{\psi}}$). An elementary computation leads to the constant physical particle density during the matter dominated expansion,

$$
n_\psi = y^2\frac{\omega^2 M_P}{16\pi}\frac{1}{\sqrt{3\lambda}} = y^2\frac{\sqrt{\lambda}\xi}{32\sqrt{2}\pi}X_{\rm cr}^3 \approx 80\left(\frac{\lambda}{0.25}\right)y^2 X_{\rm cr}^3\ . \tag{3.30}
$$

It is convenient to compare n_ψ with the entropy after the end of the matter domination stage

$$
s_r = g_*\frac{4\pi^2}{90}T_r^3 = g_*\frac{4\pi^2}{90}\left(\frac{30\lambda}{4\pi^2 g_*}X_{\rm cr}^2 X_r^2\right)^{3/4}
$$
$$
\approx \left[2.9\left(\frac{\lambda}{0.25}\right)^{3/2}\div 100\left(\frac{\lambda}{0.25}\right)^{9/4}\right]X_{\rm cr}^3\ , \tag{3.31}
$$

where we adopted (3.12) for the range of T_r. The resulting abundance is in the range

$$
\Delta_\psi \equiv \frac{n_\psi}{s_r} \approx \left[0.8\left(\frac{\lambda}{0.25}\right)^{1/2}\div 28\left(\frac{\lambda}{0.25}\right)^{5/4}\right]y^2\ . \tag{3.32}
$$

These results are used below for an estimate of primordial abundance of the sterile neutrinos in the νMSM.

4. Initial conditions for the hot Big Bang

In section 3 we found that when the amplitude of the Higgs-inflaton drops below $3.8X_{\text{cr}}$ the matter-dominated expansion of the Universe is changed to the radiation dominated behaviour, which can be characterised at this moment by the effective temperature $T_r \simeq 3 \times 10^{13}$ GeV. It is this moment which can be considered as a starting point for the standard hot Big Bang: the later evolution of the system can be followed with the use of the SM or the νMSM Lagrangian and standard finite temperature equilibrium and non-equilibrium methods. As we discussed in the Introduction, to specify the system completely, one has also to determine the values of the chemical potentials corresponding to either exactly or approximately conserved quantum numbers. In this section we identify the most important operators and fix the chemical potentials for them.

Let us start with the SM. It has got three anomaly-free exactly conserved quantum numbers $\mathcal{Q}_\alpha = L_\alpha - \frac{1}{3}B$ (L_α is the lepton number of the generation α and B is the baryonic number). In addition to them, there are quite a number of approximately conserved different fermionic numbers, such as baryon number (broken by electroweak anomaly), asymmetry in the number of right-handed electrons and light quarks such as u and d (broken by small Yukawa couplings), etc. In the standard inflationary logic one concludes that all the quantum numbers — eigenvalues of corresponding operators — are exponentially small at the end of inflation (at $h \sim M_P$) and thus can be put to zero. What concerns the charges $\mathcal{Q}_\alpha$, they cannot be created in the process of reheating the Universe, analysed above, simply because they are exactly conserved. As for the other charges, such as asymmetry in the light quark flavours, their generation can only occur due to CP-violating effects. Therefore it is suppressed by the Jarlskog determinant [45], since the only source of CP-violation in the SM is related of the Kobayashi-Maskawa phase. Applying the argument of refs. [46, 47] to this case one concludes that asymmetries in all CP-odd operators at the beginning of the Big Bang are at most on the level of 10^{-22}. To summarize, in the SM all chemical potentials are negligibly small at the beginning of the Big Bang. At the same time, the CP-even operators (such as the abundance of fermions plus antifermions of a given type) equilibrate with the rate not smaller than $\alpha_W^2 T$, which exceeds the rate of the Universe expansion right after the beginning of the Big Bang. So, deviations from thermal equilibrium in the CP-even sector in the SM can be neglected as well.

Let us turn now to νMSM. The most general renormalizable Lagrangian containing the SM fields and three right-handed singlet fermions has the form:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - y_{\alpha I} \, \bar{L}_\alpha N_I \tilde{\Phi} - \frac{M_{IJ}}{2} \, \bar{N}_I^c N_J + \text{h.c.} \,, \qquad (4.1)$$

where $\mathcal{L}_{\text{SM}}$ is the Lagrangian of the SM, $y_{\alpha \text{I}}$ are new Yukawa couplings, Φ is the SM Higgs doublet, and $\tilde{\Phi}_i = \epsilon_{ij} \Phi_j^*$. In the contrary to the SM, there are no exactly conserved global quantum numbers in the model. To analyse the approximately conserved currents, at least the orders of magnitude of various parameters, entering eq. (4.1), have to be fixed. In the νMSM the Majorana masses of the singlet fermions are below the electroweak scale and, correspondingly, the new Yukawa coupling constants are smaller than those in the quark or charged lepton sector. Their values are constrained by cosmology, astrophysics and experiment.

The lightest out of the 3 neutral leptons, N_1, plays the role of the dark matter particle.[11] Its Yukawa couplings are bounded from above as

$$\sum |F_{\alpha 1}|^2 \lesssim 10^{-24} \tag{4.2}$$

from cosmological considerations [4, 14] related to DM production and from X-ray constraints on the radiative width of the DM sterile neutrino [10].

The heavier nearly degenerate singlet fermions $N_{2,3}$ (their common mass is denoted by M) fix the pattern of neutrino masses and mixings and produce baryon asymmetry of the Universe. Their Yukawa couplings satisfy:

$$F_2^2 = \frac{\kappa M m_{\mathrm{atm}}}{\epsilon v^2} \ , \tag{4.3}$$

where $m_{\mathrm{atm}} \approx 0.05\,\mathrm{eV}$ is the atmospheric neutrino mass difference, $\kappa = 1(2)$ for normal (inverted) neutrino mass hierarchy, $F_i^2 \equiv [y^\dagger y]_{ii}$, and $\epsilon = \frac{F_3}{F_2} < 1$. If $\epsilon \sim 1$, i.e. for the case when the couplings of singlet fermions to active leptons are similar, F_2^2 is at most $\sim 2 \times 10^{-13}$, corresponding to $M \sim M_W$. For the smallest possible value of parameter $\epsilon \simeq 7 \times 10^{-5}$ (a lower limit is coming from the requirement of successful baryogenesis, see figure 10 of [18]) one gets an absolute upper bound on F_2^2,

$$F_2^2 \lesssim 3 \times 10^{-11} \ , \tag{4.4}$$

roughly coinciding with the electron Yukawa coupling.

In the limit $F_i \to 0$ the sterile fermions completely decouple from the fields of the SM, and the νMSM contains an infinite number of exactly conserved operators, corresponding to a number of singlet fermions with any given momentum. Right after inflation these numbers are exponentially small and can be put to zero. Then, these singlet fermions are created as described in section 3.5.[12] With the use of (4.2,4.4,3.32) we get that the abundance of DM sterile neutrinos N_1 produced at the reheating stage is at most $\Delta_1 = n_{N_1}/s \simeq 7 \times 10^{-23}$, and the abundance of $N_{2,3}$ is at most 2×10^{-9}. These numbers are too small to play any role in the subsequent evolution of the Universe. These were the constraints on CP-even operators, the CP-asymmetries in left-right helicities are suppressed much stronger as the CP violating amplitudes must contain at least two extra powers of Yukawas.

To summarize, the initial condition for the Big Bang in the νMSM can be described by the density matrix

$$\hat{\rho}(0) = \hat{\rho}_{\mathrm{SM}} \otimes |0\rangle\langle 0| \ , \tag{4.5}$$

where $\hat{\rho}_{\mathrm{SM}} = Z_{\mathrm{SM}}^{-1} \exp(-\beta \hat{\mathcal{H}}_{\mathrm{SM}})$, $\beta \equiv 1/T$, is the equilibrium SM density matrix at a temperature T with zero chemical potentials, and $|0\rangle$ is the vacuum state for sterile neutrinos. The physical meaning of eq. (4.5) is clear — it describes a system with no sterile neutrinos, while all the SM particles are in thermal equilibrium. It is this expression which was used for computation of DM abundance and for computation of baryon asymmetry in the νMSM in refs. [13, 14, 19, 5, 18].

---

[11]We work in the basis in which $M_{12} = M_{13} = 0$, $M_{23} = M_{32} = M$, $M_1 = M_{11} \ll M$, $M_{22} \sim M_{33} \ll M$.

[12]Yet another mechanism for production of singlet fermions is the decays of Z and W bosons to sterile neutrino and left-handed lepton. Since the rate of this reaction is suppressed not only by the square of the same Yukawa coupling but also by an extra gauge constant, we expect it to be subdominant.

5. Higher dimensional operators

In the first part of the paper we analysed the inflation and reheating in the model with action (2.1). However, one may expect that there are corrections to this action, suppressed by some large energy scale (the Planck mass). We consider the following higher dimensional operators as an addition to the SM or νMSM Lagrangian:

$$\delta\mathcal{L}_{\mathrm{NR}} = \frac{\beta}{M_P}\Phi^\dagger\Phi\bar{N}^cN - \frac{a_6}{M_P^2}(\Phi^\dagger\Phi)^3 + \frac{f_{ab}}{M_P}\bar{L}_a^c\Phi\Phi^\dagger L_b + \cdots + \mathrm{h.c.} \ . \tag{5.1}$$

The natural value of all dimensionless coupling constants is about one.

The following questions arise:

1. Do these operators spoil the picture of inflation discussed above?

2. Does the reheating change?

3. Can the singlet fermions be created due to these operators in substantial amounts?

In this section we analyse these issues. Along the lines of consideration in section 2, we rewrite first these operators in the Einstein frame. For Higgs part $\delta\mathcal{L}_{NR}$ this yields a modification of the potential in (2.2), (2.6). The transformation rule for the Higgs-fermion interaction is readily obtained, if we also make the conformal transformation of all the fermionic fields ψ

$$\psi \to \hat{\psi} = \Omega^{-3/2}\psi \ . \tag{5.2}$$

The kinetic part for the fermions is conformally invariant, while the Yukawa part of the action

$$S_{J,\mathrm{Yukawa}} = \int d^4x\sqrt{-g}\, Y(h)\bar{\psi}\psi \ , \tag{5.3}$$

changes. Here $Y(h)$ describes the generalised Yukawa interaction providing the fermion ψ with mass. It is not important for the present discussion whether this is the Majorana or the Dirac fermion. The corresponding Einstein frame term is

$$S_{E,\mathrm{Yukawa}} = \int d^4x\sqrt{-\hat{g}}\, \frac{Y\left[h\left(\chi\right)\right]}{\Omega(\chi)}\bar{\hat{\psi}}\hat{\psi} \ . \tag{5.4}$$

Specifically, for the Dirac mass this yields the Einstein-frame terms like

$$S_{E,\mathrm{Dirac}} = \int d^4\sqrt{-\hat{g}}\, \frac{m(v)}{v}\frac{h(\chi)}{\Omega(\chi)}\bar{\hat{\psi}}\hat{\psi} \ , \tag{5.5}$$

which we already used in deriving (3.8), (3.9). Note, this mass rescaling is similar to that of massive gauge bosons.

For the Majorana higher dimensional term in (5.1) we get

$$\delta\mathcal{L}_{E,\mathrm{NR,Majorana}} = \frac{\beta}{2M_P}\frac{h(\chi)^2}{\Omega(\chi)}\hat{N}^c\hat{N} \ . \tag{5.6}$$

Note, that at the reheating stage, $(M_P/\xi \ll \chi \ll M_P/\sqrt{\xi})$, we have $\Omega \sim 1$, so the only change is the field substitution in accordance with (2.8).

5.1 Contributions of the higher dimensional operators to inflation

Clearly, the main effect of the higher dimensional operators is expected when the Jordan field h is large, of order M_P. Indeed, at this scale the higher order Higgs field operators may spoil the flatness of the Einstein frame potential (2.9), and the fermion Majorana mass terms can also give sizeable radiative corrections to the potential. This can change the inflationary properties of the model. In particular, inflation may turn to be impossible if the sign of the slope of the potential is changed in the inflationary region. Or, the predictions of the CMB spectral index and tensor-to-scalar ratio may leave the experimentally admitted region. However, the constraints on higher dimensional operators, imposed by the requirements that the inflation is not spoiled, turn out to be rather weak. The reason is that the inflationary potential is only essential at sufficiently small values of the Higgs field, $h \lesssim h_{WMAP} \sim 10 M_P/\sqrt{\xi}$. These values are well below the Planck mass, so that non-ronormalizable contributions are well suppressed. We analyse below in some detail the contribution of operators (5.1) to inflationary potential.

5.1.1 Higgs operators

Let us analyse the following higher order terms added to the Higgs potential:

$$\delta V = \frac{a_n h^n}{2^{n/2} M_P^{n-4}} \, , \tag{5.7}$$

$n = 6, 8, \ldots$. As far as the operators are suppressed by the Plank mass, their effect is mostly important at high values of h. At this scale we have[13] $d\chi/dh \simeq \sqrt{6} M_P/h$, $\Omega^4 \simeq \xi^2 h^4/M_P^4$. The contributions to the slow roll parameters at $N \gtrsim 60$ are

$$\delta\epsilon = \frac{4(n-4)^2}{3\lambda^2} \frac{a_n^2}{2^n} \left(\frac{h}{M_P}\right)^{2n-8} \, , \tag{5.8}$$

$$\delta\eta = \frac{2(n-4)^2}{3\lambda} \frac{a_n}{2^{n/2}} \left(\frac{h}{M_P}\right)^{n-4} \, . \tag{5.9}$$

The main contribution comes from the lowest order power term, h^6. Thus, for the change of the parameters at the normalized-to-WMAP value of the field we have

$$\delta\epsilon \sim 1.7 \times 10^{-5} a_6^2 \, , \tag{5.10}$$

$$\delta\eta \sim 0.005 a_6 \, , \tag{5.11}$$

for $\lambda = 0.25$. This implies the change in the spectral index $\delta n_s \simeq 0.01 a_6 - 0.0001 a_6^2$ and in the tensor to scalar ratio $\delta r \simeq 0.0003 a_6^2$.

To keep the spectral index within 1σ bounds $0.94 < n_s < 0.98$ (at small r), see figure 3, the coefficient for the dimension six operator in the Higgs potential should be $|a_6| \lesssim 3$. Hence, no significant contributions are expected from the higher order operator with natural values of the coefficients of order one.

---

[13]At the end of inflation the values of $d\chi/dh$ and Ω are different, but it only slightly changes the WMAP value for ξ, as far as the contribution from (5.7) are more suppressed for lower h.

5.1.2 Yukawa terms

Let us now analyse the effect of the Yukawa terms for the sterile neutrinos on the inflation. They come from the fermionic loop contributions to the effective potential for the Higgs field. According to (5.4) and (5.6) the mass term for the right handed neutrinos in the Einstein frame has the form

$$\mathcal{L}_{\text{mass}} = \left[\frac{M_I}{2\Omega(h)} + \frac{\beta h^2}{2M_P\Omega(h)} \right] \bar{N}^c N \;,$$

where M_I is the usual Majorana mass term for the sterile neutrinos in νMSM. For large h the first term is suppressed, but the second one (dimension 5 operator) provides the mass, growing with the field. The latter could change significantly the effective potential for the Higgs field. Indeed, the term $yH\bar{L}N$ induces, in the Einstein frame, the usual Dirac lepton mass, the growth of which stops at $h \sim M_P/\sqrt{\xi}$, being suppressed by $\Omega(h)$. This dimension 5 contribution to the mass yields the following contribution to the Higgs effective potential in the inflation region

$$\delta U(h) = -\frac{m_N^4(h)}{32\pi^2} \log \frac{m_N^2(h)}{\mu^2} \simeq -\frac{\beta^4 h^8}{32\pi^2\Omega^4 M_P^4} \log \frac{h^4}{\Omega^2\mu^2} \;. \tag{5.12}$$

For high enough β this changes the sign of the derivative of $U(h)$ at some h, which would make the inflation impossible or limit its duration.[14] Let us calculate the slope of the potential

$$\frac{dU/d\chi}{U} = \frac{U'}{U\chi'} \simeq \frac{4}{h\Omega^2} \left(1 - \frac{h^6\xi^3}{M_P^6} \frac{\beta^4 \log(h^4/\Omega^2\mu^2)}{8\pi^2\lambda\xi^2} \right) \frac{1}{\chi'} \;, \tag{5.13}$$

where $'$ means derivative with respect to h and we neglected the derivative of the logarithm. It is required that $U'(h) > 0$ for at least 60 e-foldings of inflation, i.e. for all h satisfying $h\sqrt{\xi}/M_P \lesssim 10$. The logarithm here, accounting at least for the inflationary epoch, $1 \lesssim h\sqrt{\xi}/M_P \lesssim 10$, is about $\log(100) \sim 5$. This implies the constraint

$$\frac{\beta^4 \log(h^4/\Omega^2\mu^2)}{8\pi^2\lambda\xi^2} < 10^{-6} \;,$$

leading to

$$\beta^2 \lesssim 47 \left(\frac{\lambda}{0.25} \right) \;. \tag{5.14}$$

For smaller Higgs masses the bound is stronger, but always much larger than one. We see, that for rather large value of the dimensionless constant in front of dimension-5 mass operator for the right-handed neutrinos, the inflation is not spoilt. It is also straightforward to check by exact calculation of the spectral index that constraints from the WMAP on n_s lead to essentially the same bound on β.

---

[14]Of course, one could imagine starting inflation exactly from the top of the potential, where the derivative of $U(h)$ is zero. However, this corresponds to a highly tuned situation, keeping in mind that the change of the derivative is due to interplay between tree level term and radiative corrections to the effective potential.

5.2 Sterile neutrino production

As we have seen in section 3.5, during preheating the sterile neutrinos are produced very slowly by the renormalizable dimension 4 operators. Let us estimate the contribution of the dimension-5 operators (5.1) to sterile neutrino production during and after preheating. We will separately analyse the production in the thermal bath *after* reheating by the annihilation process $hh \to NN$, and *during* reheating by the decay of the inflaton-Higgs condensate.

5.2.1 Thermal production

Let us start with the study of neutrino production in the primordial plasma. In this section we consider the neutrino production *after* reheating of the Universe, at $T \sim T_r$ (the higher temperatures are more essential due to the suppression of the relevant operators by the Planck mass). Here the electroweak symmetry is restored, and the production of sterile neutrinos goes through annihilation of the Higgs bosons (4 degrees of freedom, corresponding to the unbroken phase of the SM) due to coupling (5.1).

The cross section of this process is (neglecting the neutrino mass)

$$\sigma_{hh \to NN} = \frac{\beta^2}{8\pi M_P^2} \ . \tag{5.15}$$

In the absence of other sources of neutrino production, the interaction (5.6) contributes to the r.h.s. of the Boltzmann equation for sterile neutrino density n_N

$$\frac{d}{dt}(a^3 n_N) = a^3 4\sigma_{hh \to NN} n_h^2 \ . \tag{5.16}$$

Here we took into account the annihilation of all four modes, n_h stands for the density of each scalar degree of freedom.

This equation can be easily integrated accounting for the fact that at $T < T_r$ the Universe is at the radiation dominated stage:[15]

$$a \propto \sqrt{t} \ , \qquad n_h = \frac{\zeta(3)}{\pi^2} T^3 \ , \tag{5.17}$$

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{\pi^2 g_*}{90}} \frac{T^2}{M_P} = \frac{1}{2t} \ , \qquad s = g_* \frac{4\pi^2}{90} T^3 \ , \tag{5.18}$$

The solution of equation (5.16) gives the neutrino density-to-entropy ratio,

$$\Delta_N \equiv \frac{n_N}{s} = \frac{135\zeta^2(3)\sqrt{10}}{4\pi^8 g_*^{3/2}} \frac{\beta^2}{M_P} (T_r - T) \ ,$$

where $g_* = 106.75$ is the total number of degrees of freedom of the SM. We also set the initial abundance to zero. We will compute it in the next chapter, dealing with sterile neutrino production during preheating.

---

[15]We suppose that at the temperature T_r the Universe is already in a fully thermalised state. Though this is not exactly the case, we expect that this assumption can only overestimate the number of produced neutrinos, since the non-equilibrium Higgs spectra are more enhanced in the infrared region in comparison with the thermal one.

Putting the numbers, we get at low temperatures $T \ll T_r$:

$$\Delta_N = 1.5 \times 10^{-5} \beta^2 \frac{T_r}{M_P} \ . \tag{5.19}$$

With the use of this relation we can answer the question whether the primordial thermal production of the lightest practically stable sterile neutrinos can substantially contribute to the DM abundance. The neutrino-to-entropy ratio remains intact, so at the current moment we have

$$s_0 = \frac{n_{N,0}}{\Delta_N} = \frac{n_{B,0}}{\Delta_B} \ ,$$

where $\Delta_B = 0.87 \times 10^{-10}$ is the baryon-to-entropy ratio and $n_{B,0}$ is the present baryon number density. Therefore, we can write for the sterile neutrino abundance Ω_N

$$\frac{\Omega_N}{\Omega_{DM}} = \frac{\Omega_B}{\Omega_{DM}} \frac{m_N}{m_p} \frac{\Delta_N}{\Delta_B} \ ,$$

where m_N and m_p are the sterile neutrino and proton masses, respectively, $\Omega_B = 0.046$ and $\Omega_{DM} = 0.23$ are the baryon and DM abundances [34]. Hence,

$$\frac{\Omega_N}{\Omega_{DM}} = \frac{M_N}{27\,\text{keV}} \cdot \frac{\beta^2 T_r}{M_P} \ . \tag{5.20}$$

So, for the maximal allowed β (5.14) and for the reheat temperature in the range (3.13), we conclude that the neutrino mass, required to provide the proper DM abundance, should be in the range

$$M_N = \left(\frac{0.25}{\lambda}\right) \left[13 \left(\frac{0.25}{\lambda}\right)^{1/4} \text{MeV} \div 42\,\text{MeV} \right] \ . \tag{5.21}$$

If the dimension 5 operator is present with the "natural" coefficient $\beta \sim 1$, then the mass of the (long living) sterile neutrino should not exceed

$$M_N < 600\,\text{MeV} \left(\frac{0.25}{\lambda}\right)^{1/4} \ , \tag{5.22}$$

in order not to overproduce the DM (the upper bound from (3.13) is used for the estimate).

5.2.2 Production during preheating

In this section we consider the neutrino production in the early Universe right after inflation got terminated. The production during this period happens due to the effective interaction with the Einstein frame field χ

$$\mathcal{L} = \left(\frac{\beta}{\sqrt{6}\xi} |\chi| \bar{N}^c N + \text{h.c.} \right) \ . \tag{5.23}$$

We will see that this mechanism produces more sterile neutrinos, than the thermal production discussed above.

To find particle production due to the time-dependent fermion mass one has to study the Dirac equation following from (5.23). However, to simplify the discussion, we proceed

as in section 3.5 and replace $|\chi|$ by χ. Then the rate of fermion production coincides, to the lowest order in Yukawa coupling, with the decay rate of a collection of scalar particles with certain mass and number density. An analysis performed in Appendix F shows that the number density of produced fermions is not affected by this replacement, though the spectrum changes.

We can write the Boltzmann equation as

$$\frac{d}{dt}(a^3 n_N) = a^3 \Gamma_{\chi \to NN} n_\chi \, ,$$

where we replaced the oscillating source by an ensemble of free scalar particles with the number density $n_\chi = \frac{\omega}{2} X^2$ and particle decay width $\Gamma_{\chi \to NN} = \frac{\beta^2}{6\xi^2} \frac{\omega}{8\pi}$.

Then with background (3.4), (3.5) one gets (the early-time contribution is negligible):

$$n_N(X) = \frac{\beta^2 \sqrt{\lambda}}{48\sqrt{2}\pi\xi} X_{\text{cr}}^2 X \, .$$

Dividing this by the entropy (3.31) we get

$$\Delta_N = \frac{\beta^2}{32\pi\sqrt{\pi}} \left(\frac{30}{\lambda g_*}\right)^{1/4} \frac{1}{\xi} \sqrt{\frac{X_{\text{cr}}}{X}} = \frac{\beta^2 \sqrt{10}}{32\pi^2 \sqrt{g_*}\xi^2} \frac{M_P}{T_r}$$

$$= 1.8 \times 10^{-12} \left(\frac{0.25}{\lambda}\right) \frac{\beta^2 M_P}{T_r} \, . \tag{5.24}$$

This is larger than contribution from thermal generation (5.19), for the reheating temperature in the range (3.13).

Proceeding analogously to the previous subsection we get

$$\frac{\Omega_N}{\Omega_{DM}} = \beta^2 \frac{M_N}{2.2 \times 10^8 \, \text{keV}} \left(\frac{0.25}{\lambda}\right) \frac{M_P}{T_r} \, . \tag{5.25}$$

So, for the maximal allowed β given by (5.14) and for the reheat temperature in the range (3.13), we conclude that the neutrino mass, required to provide proper DM abundance, should be in the range

$$M_N = 65 \, \text{keV} \div 210 \left(\frac{\lambda}{0.25}\right)^{1/4} \text{keV} \, . \tag{5.26}$$

If $\beta \sim 1$, that is what is naturally expected, we have

$$M_N < 3 \left(\frac{0.25}{\lambda}\right) \, \text{MeV} \, .$$

Let us summarize the results obtained above. If a more complete, than the νMSM, theory leads to higher-order non-renormalizable operators characterised by a "natural" constant $\beta \sim 1$, then the mass of the DM sterile neutrino must not exceed few MeV.[16] Otherwise, it will be produced in amounts enough to overclose the Universe. Sterile neutrinos, produced at reheating, can only play the role of CDM, since their mass must exceed

---

[16]For smaller β this limit scales as $M_N \propto 1/\beta^2$.

65 keV. This requirement comes from the inflationary upper limit on β (5.14). Finally, if the sterile neutrino has a mass in $\mathcal{O}(10)$ keV region and thus plays a role of WDM candidate, the thermal primordial production, discussed in this section, plays no role.

The higher dimensional operators, of course, produce also heavier singlet fermions of the νMSM. In section 6 we analyse whether this has any influence on the low-temperature baryogenesis due to singlet fermion oscillations.

6. Higher dimensional operators and baryon asymmetry

It is shown in section 5 that the abundance of DM sterile neutrino, created at reheating due to higher dimensional interactions, cannot exceed

$$\Delta_N \sim 10^{-5} \ . \tag{6.1}$$

Hence, the "primordial" (related to inflation) creation of DM is not effective for light sterile neutrinos, and may play a role only if $M_N \geq 65$ keV. Interestingly, this number is only somewhat larger than an upper limit on the mass of DM sterile neutrino produced resonantly due to lepton asymmetry generated in the νMSM [18, 19], $M_1 \lesssim 50$ keV. In other words, the initial condition (4.5) is certainly valid for sufficiently light singlet fermion (mass below 65 keV), which could play a role of WDM.

Other singlet fermions can be produced due to the same type of higher dimensional interactions (note that for the abundance computation the magnitude of the Majorana mass plays no role), and their abundance is bounded from above by (6.1). This number is much smaller than one, meaning that the heavier singlet fermions are practically absent at the beginning of the hot Big Bang. Still, in order to proof that the density matrix (4.5) can be used as an initial condition, one must show that the CP asymmetries in distribution of singlet fermions do not exceed the baryon asymmetry of the Universe $\sim 10^{-10}$.

It is not difficult to see that this is indeed the case. To this end consider the most general form of the leptonic part of the Lagrangian, taking into account the higher dimensional operators of dimensionality 5 as well:

$$\mathcal{L}_{CP} = \frac{\beta_{IJ}}{M_P}\Phi^\dagger\Phi\bar{N}_I^c N_J + \frac{f_{\alpha\beta}}{M_P}\bar{L}_\alpha^c\tilde{\Phi}\tilde{\Phi}^\dagger L_\beta - y_{\alpha I}\,\bar{L}_\alpha N_I\tilde{\Phi} + g_{\alpha\beta}\bar{L}_\alpha E_\beta\Phi + \text{h.c.} \ , \tag{6.2}$$

where E_β are the right-handed charged leptons. To get an amplitude of CP-violating effects, one may consider the imaginary parts of re-parametrisation invariant products of Yukawa couplings, which can be considered as a generalization of Jarlskog invariant for the Kobayashi-Maskawa quark mixing to this case (see also [46, 47]). These invariants can be written as traces in flavour space of the products of β_{IJ}, $f_{\alpha\beta}$, $y_{\alpha I}$ and $g_{\alpha\beta}$ (no contraction between Greek and Latin indexes).

The fermion production due to Higgs oscillations and Higgs scattering appears first to the second order in these couplings. As we have seen in section 5 the leading effect comes from the first term in (6.2). Clearly, there is no CP-violation in this order. To the fourth order in coupling constants the CP-violating effects appear through the CP-violating trace $\text{Tr}[y^\dagger \beta y f]$. Since from the flatness of potential $|f| \lesssim \beta \lesssim 6$, and because $y_{\alpha I}$ are strongly bounded from above by (4.2,4.4), there is a suppression of the asymmetry at least by 10 orders of magnitude in comparison with (6.1). Going to higher orders makes the situation

even worse. To conclude, the initial conditions for the Big Bang are correctly described by eq. (4.5), even if higher dimensional operators are included in the νMSM, and thus the baryogenesis is a low-temperature phenomenon, having nothing to do with inflation or Planck scale physics.

7. Conclusions

In this paper we analysed in detail the evolution of the Universe in the scenario where the Higgs boson of the SM plays a role of the inflaton. The history of the Universe can be divided into three stages. The first one is inflation. Here the non-minimal coupling of Higgs to gravity makes the effective scalar potential flat, the Universe expands exponentially and the necessary spectrum of perturbation is generated. This stage finishes roughly at $h \sim M_P$. During the second stage the Universe expands as under matter domination. The Higgs field oscillates in the nearly quadratic potential for $M_P/\xi < h < M_P$, and the particle production is not effective. When h reaches the critical value $h \simeq M_P/\xi$ the energy stored in Higgs zero mode is transferred rapidly in other degrees of the SM, producing the hot Big Bang with temperature $T_r \approx 10^{14}\,\text{GeV}$. After this time the Universe is dominated by radiation.

We have shown that at the onset of the radiation dominated epoch the densities of all CP-odd operators in the SM can be put to zero and demonstrated the for the case of the νMSM the concentrations of the singlet fermions are negligible at T_r.

We also considered an extension of the SM and νMSM adding to them higher dimensional operators suppressed by the Planck scale. We analysed the constraints on these operators coming from the condition to have successful inflation. We demonstrated that the concentrations of the singlet fermions at T_r can be safely put to zero, provided the mass of DM sterile neutrino does not exceed $100\,\text{keV}$. This means that in this case the production of baryon asymmetry and of dark matter must occur at small temperatures (about and below the electroweak scale) by essentially the same mechanism, as was described in [18, 19]. The properties of singlet fermions can be almost unambiguously fixed by different cosmological considerations [18, 19].

We found that the presence of higher-dimensional operators provides a new mechanism for primordial production of DM sterile neutrino. This mechanism is effective in models with sufficiently heavy sterile neutrinos, $M_N \gtrsim 100\,\text{keV}$. No presently available astrophysical constraints (in particular, those associated with X-rays) can exclude this possibility, since production occurs even if DM sterile neutrino Yukawa couplings are identically equal to zero. However, if the solution of the short scale difficulties of the CDM scenario [48, 49, 50, 51, 52, 53, 54] is to be given by the WDM, this region of the parameter space should be discarded.[17] At the same time, there are no reasons to expect that these operators are suppressed by the scale exceeding the Planck one (i.e. it is unlikely that $\beta < 1$). Therefore, the models with DM sterile neutrinos heavier than $3\,\text{MeV}$ are generally disfavoured due to problems with dark matter overproduction. These arguments provide an extra justification of sub MeV mass of the lightest singlet fermion within the νMSM.

---

[17]See, however, ref. [55], where it is shown that heavy sterile neutrinos could be WDM for other types of production mechanisms.

Note added

Some time after our paper was posted at arXiv the article [56] devoted to the same subject appeared. Most of the conclusions of [56] are similar to ours. In particular, the authors of [56] used a similar formalism to analyze the transfer of energy from the Higgs field oscillations to gauge bosons. A detailed analysis of differences and similarities of these works goes beyond the scope of the present paper. We would like just to mention that some (not inessential for physical consequences) differences in numerics are presumably due to the fact that annihilation of created gauge bosons was not accounted for in [56], leading to the different rate of transfer of energy to relativistic particles at later stages of reheating.

Acknowledgments

A. W boson production

A.1 Semiclassic approach

The discussion in this section closely follows [44]. At first approximation a creation of W bosons can be regarded as a creation of particles with mass (3.8) varying with the amplitude of the Higgs field (3.4). This approximation breaks when important amount of energy is transferred from the inflaton zero mode (3.7). If at this moment the energy is in the relativistic modes this corresponds to the moment T_r of transition to the radiation dominated expansion.

To solve the equation (3.14) we rescale the variables by

$$\Phi_{\mathbf{k}} = a^{3/2} \phi_{\mathbf{k}} . \tag{A.1}$$

This leads to the equation for an oscillator with varying frequency

$$\ddot{\Phi}_{\mathbf{k}} + k_0^2(t) \Phi_{\mathbf{k}} = 0 , \tag{A.2}$$

$$k_0^2(t) = \frac{\mathbf{k}^2}{a^2} + m_W^2(t) + \Delta , \tag{A.3}$$

where $\Delta \equiv -\frac{3}{4} \left(\frac{\dot{a}}{a} \right) - \frac{3}{2} \frac{\ddot{a}}{a}$ is always small and can be neglected. The initial conditions, corresponding to vacuum oscillations, are

$$\Phi_{\mathbf{k}}(t) = \frac{\mathrm{e}^{-ik_0 t}}{\sqrt{2k_0}} . \tag{A.4}$$

Equation (A.2) can be solved in the adiabatic approximation when $\dot{k}_0 \ll k_0^2$. At $\mathbf{k} = 0$ this condition is equivalent (up to a change of the scale factor a, which is negligible in our case)

to $\dot{m}_W \ll m_W^2$. This is true for

$$|t - t_j| \gg \left(\frac{\sqrt{6}\xi}{2g^2 M_P X \omega}\right)^{1/3} = \frac{1}{4^{1/3}K} , \tag{A.5}$$

where t_j are moments when inflaton crosses zero, so that $m^2(t_j) = 0$, and K is the natural scaling parameter defined in (A.12). In these regions adiabatic solution is

$$\Phi_{\mathbf{k}} = \frac{\alpha_{\mathbf{k}}^j}{\sqrt{2k_0}} e^{-i \int_0^t k_0 dt} + \frac{\beta_{\mathbf{k}}^j}{\sqrt{2k_0}} e^{+i \int_0^t k_0 dt} , \tag{A.6}$$

where parameters $\alpha_{\mathbf{k}}^j$, $\beta_{\mathbf{k}}^j$ remain constant within $t_{j-1} < t < t_j$. At the moments t_j the coefficients get changed by the Bogolubov transformation

$$\begin{pmatrix} \alpha_{\mathbf{k}}^{j+1} e^{-i\theta_{\mathbf{k}}^j} \\ \beta_{\mathbf{k}}^{j+1} e^{+i\theta_{\mathbf{k}}^j} \end{pmatrix} = \begin{pmatrix} 1/D_{\mathbf{k}} & R_{\mathbf{k}}^*/D_{\mathbf{k}}^* \\ R_{\mathbf{k}}/D_{\mathbf{k}} & 1/D_{\mathbf{k}}^* \end{pmatrix} \begin{pmatrix} \alpha_{\mathbf{k}}^j e^{-i\theta_{\mathbf{k}}^j} \\ \beta_{\mathbf{k}}^j e^{+i\theta_{\mathbf{k}}^j} \end{pmatrix} , \tag{A.7}$$

where $R_{\mathbf{k}}$ and $D_{\mathbf{k}}$ are the "reflection" and "transition" coefficients for each interval $t_{j-1} < t < t_j$ (they obey the equality $|R_\kappa|^2 + |D_\kappa|^2 = 1$), and

$$\theta_{\mathbf{k}}^j \equiv \int_0^{t_j} k_0 dt . \tag{A.8}$$

To find the coefficients $R_{\mathbf{k}}$ and $D_{\mathbf{k}}$ for each interval we need to solve exactly eq. (A.2) in the vicinities of the moments when $m_W \simeq 0$ (and where the adiabatic approximation is inapplicable) and match this solution with (A.6) in the intermediate region of the field amplitude. Obviously, exact solution is impossible, but we can approximate $m_W^2(t)$ near zero as const $\cdot|t - t_j|$. This approximation to the potential is good enough at

$$|t - t_j| \ll \frac{\sqrt{6}}{\omega} . \tag{A.9}$$

The regions (A.5) and (A.9) intersect for

$$X > \frac{\lambda}{48\sqrt{6}\pi\alpha_W} X_{\mathrm{cr}} , \tag{A.10}$$

which covers all the possibly interesting reheating period. At the same time, one can estimate, that the effects of "smoothing" of the $|t - t_j|$ are also insignificant up to approximately X_{cr}. Thus, we can match the solution of the linear equation and adiabatic solution. To solve the "linearised" equation we rescale the variables as

$$\kappa = \frac{|\mathbf{k}|}{Ka} , \quad \tau = K(t - t_j) \tag{A.11}$$

with

$$K \equiv \left[\frac{g^2 M_P^2}{6\xi^2}\sqrt{\frac{\lambda}{2}}X(t_j)\right]^{1/3} = \left[\omega\frac{g^2}{2\sqrt{6}}\frac{M_P X(t_j)}{\xi}\right]^{1/3}. \tag{A.12}$$

Then, for small τ eq. (A.2) takes the form

$$\frac{d^2\Phi_{\mathbf{k}}}{d\tau^2} + (\kappa^2 + |\tau|)\Phi_{\mathbf{k}} = 0 \ , \tag{A.13}$$

which can be readily solved analytically in terms of the Airy functions. Matching the solution with the asymptotic form (A.6) at $t = t_j$ one gets (see Appendix D for details)

$$D_{\mathbf{k}} = e^{2i\left(\frac{2}{3}\kappa^3 + \frac{\pi}{4}\right)} \frac{i\left[\operatorname{Ai}'(-\kappa^2)\operatorname{Bi}(-\kappa^2) - \operatorname{Ai}(-\kappa^2)\operatorname{Bi}'(-\kappa^2)\right]}{(\operatorname{Bi}(-\kappa^2) + i\operatorname{Ai}(-\kappa^2))(\operatorname{Bi}'(-\kappa^2) + i\operatorname{Ai}'(-\kappa^2))} \ , \tag{A.14}$$

$$R_{\mathbf{k}} = e^{2i\left(\frac{2}{3}\kappa^3 + \frac{\pi}{4}\right)} \frac{-\operatorname{Ai}'(-\kappa^2)\operatorname{Ai}(-\kappa^2) - \operatorname{Bi}(-\kappa^2)\operatorname{Bi}'(-\kappa^2)}{(\operatorname{Bi}(-\kappa^2) + i\operatorname{Ai}(-\kappa^2))(\operatorname{Bi}'(-\kappa^2) + i\operatorname{Ai}'(-\kappa^2))} \ . \tag{A.15}$$

Thus, we can calculate the occupation number $n_{\mathbf{k}}^j \equiv |\beta_{\mathbf{k}}^j|^2$ at the moment t_j

$$n_{\mathbf{k}}^{j+1} = \frac{|R_{\mathbf{k}}|^2}{|D_{\mathbf{k}}|^2} + \frac{1 + |R_{\mathbf{k}}|^2}{|D_{\mathbf{k}}|^2}n_{\mathbf{k}}^j + 2\sqrt{1 + n_{\mathbf{k}}^j}\sqrt{n_{\mathbf{k}}^j}\frac{|R_{\mathbf{k}}|}{|D_{\mathbf{k}}|^2}\cos(\theta_{\text{tot}}^j) \ , \tag{A.16}$$

$$\theta_{\text{tot}}^j = -2\theta_{\mathbf{k}}^j - 2\left(\frac{2}{3}\kappa^3 + \frac{\pi}{4}\right) + \arg\alpha_{\mathbf{k}}^j - \arg\beta_{\mathbf{k}}^j \ . \tag{A.17}$$
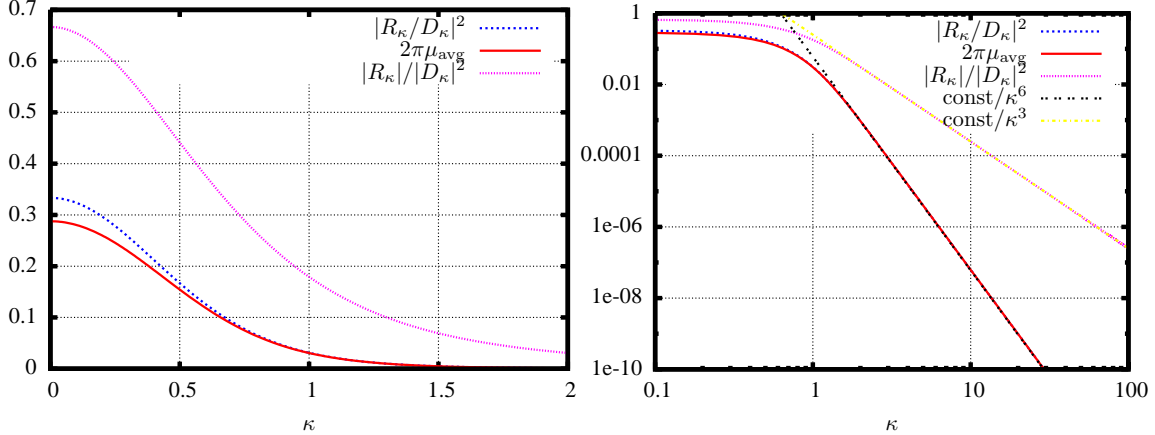

Figure 5: Particle creation coefficients for (3.16), and the effective resonance exponent parameter (A.23). The right plot is in the log-log scale, to show κ^{-6} and κ^{-3} behaviour at large momenta.

The total particle number density for $t_j < t < t_{j+1}$ is given by

$$n(t_j < t < t_{j+1}) = \int \frac{d^3\mathbf{k}}{(2\pi a)^3}n_{\mathbf{k}}^j \ . \tag{A.18}$$

A.2 Non-resonance production

In this case, we estimate the production of the particles simply by the first term in (A.16) (only one degree of freedom of W boson is accounted for)

$$\frac{dn}{dt} = \frac{\omega}{\pi}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{|R_{\mathbf{k}}|^2}{|D_{\mathbf{k}}|^2} \approx \frac{A}{2\pi^3}\omega K^3 \ , \tag{A.19}$$

with the numeric coefficient $A = 0.0455$. The spectrum of the created particles is presented in figure 5. It has cutoff at $|\mathbf{k}|/a \sim K$ and power law tail $|\mathbf{k}|^{-6}$. Thus, at $X > X_{\text{cr}}$ the produced particles are *nonrelativistic,* as far as $K < \sqrt{\langle m_W^2 \rangle}$. So, to study the transition to the radiation domination one should analyse further the W boson decays into relativistic particles, which is done in the next subsection.

Let us also note, that taking into account three polarizations of the vector bosons and their different types is made as follows

$$n_{W^+} = n_{W^-} = 3n \;, \quad n_Z = \frac{3}{\cos^2 \theta_W} n \;, \tag{A.20}$$

where θ_W is the weak mixing angle, and $n_{W^\pm}$, n_Z are densities for W$^\pm$ and Z-bosons, respectively.

A.3 Stochastic resonance

If the occupation numbers $n_{\mathbf{k}}$ exceed one, then the first term in (A.16) can be neglected, and the last two terms, proportional to $n_{\mathbf{k}}$, yield the resonance — an exponentially rapid particle creation. If $n_{\mathbf{k}} \gg 1$ and the shift in the phase, $\Delta\theta$, is large, then we can neglect the first term in (A.16) and write approximately

$$n_{\mathbf{k}}^{j+1} \simeq n_{\mathbf{k}}^j e^{2\pi \mu_{\mathbf{k}\,\text{avg}}^j} \;. \tag{A.21}$$

The average growth exponent $\mu_{\mathbf{k}\,\text{avg}}$ is obtained by averaging of the exponent over the random phase

$$\mu_{\mathbf{k}\,\text{avg}}^j \equiv \mu_{\text{avg}} \left(\frac{|\mathbf{k}|}{K(t_j)a(t_j)} \right) = \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{1}{2\pi} \log \left(\frac{1+|R_{\mathbf{k}}|^2}{|D_{\mathbf{k}}|^2} + 2\frac{|R_{\mathbf{k}}|}{|D_{\mathbf{k}}|^2} \cos(\theta) \right) \;, \tag{A.22}$$

The integral can be found exactly, leading to

$$\mu_{\text{avg}}(\kappa) = \frac{-\log(|D_{\mathbf{k}}|^2)}{2\pi} \;, \tag{A.23}$$

which is presented in figure 5.

If we neglect the expansion of the Universe, then the time derivative of the total particle number can be estimated as

$$\frac{dn}{dt} = \int \frac{d^3\mathbf{k}}{(2\pi)^3 a^3} n_{\mathbf{k}} 2\omega \mu_{\text{avg}} \left(\frac{|\mathbf{k}|}{Ka} \right) \sim 2\omega Bn \;, \tag{A.24}$$

where the numerical coefficient $B \sim \mu_{\text{avg}}(0) \simeq 0.045$. The exact expression depends on the exact spectrum of the generated particles, and can be omitted at our level of precision. Note, that the same equation, describing the exponential creation, is true without any change for $n_{W^\pm}$ and n_Z. The difference appears in the pre-exponential behaviour only.

Typical momentum of the produced particles is again K, so they are non-relativistic, and analysis of their conversion into light relativistic particle is needed. It proceeds via annihilation (scattering) and is studied in detail in Appendix B.2.

B. W boson decays and scatterings: energy transfer into relativistic particles

B.1 W boson decays

Now let us analyse whether some processes, like decay or scattering of the W bosons, may destroy the resonance picture described in Appendix A.1. These processes may destroy the resonance behaviour by taking the bosons out of the resonance region in two ways: either by transferring the energy to other particles, or by changing the boson momenta and taking it out of the resonance region.

Changing momentum could be expected in $WW \to WW$ scatterings. However, as the typical W boson momentum is smaller than their mass $K < \langle m_W \rangle$, one can not achieve in scatterings momenta larger than K, and this process (though rather effective) can be safely neglected.

So, the two remaining processes are decay and annihilation of the W bosons, which transfer the energy to the relativistic (light) particles, and, depending on their rate, may also prevent the development of parametric resonance.

We start with the analysis of the decay process of the gauge bosons created at the moment t_j. The (average) decay width of the SM gauge boson is given by (3.20). We also estimate the W boson mass as the averaged value over inflaton oscillations,

$$\langle m_W^2 \rangle = \frac{g^2}{2\sqrt{6}} \frac{M_P \langle |\chi| \rangle}{\xi} = 2\alpha_W X_{\mathrm{cr}} X \;, \tag{B.1}$$

where $\langle |\chi| \rangle = \frac{2X(t)}{\pi}$. If the decay is faster than the exponent of the stochastic parametric resonance $2\omega B$ (see (3.19)), then the parametric resonance never settles, and creation is dominated by the first term in (A.16), see section A.2. The inequality $\Gamma > 2\omega B$ leads to (3.21). So, for the period before (3.21) the production happens only due to the first term in (3.16). Let us check, that the energy in the decay products of the W bosons remains small for this period.

During the time period (3.21), when the decay is fast, the creation of the particles is non-resonant (3.18). We can write the approximate Boltzmann equation for this period

$$\frac{d}{dt} \left(a^3 n \right) = a^3 \left(\frac{A}{2\pi^3} \omega K^3 - \Gamma n \right) \;. \tag{B.2}$$

The solution to this equation in the semi-stationary regime, corresponding to vanishing time derivative in the left-hand side, is

$$n_{\mathrm{decay}} \simeq \frac{A\omega K^3}{2\pi^3 \Gamma} \;. \tag{B.3}$$

The semi-stationary approximation $|\dot{n}|$, $3Hn \ll \Gamma n$, is valid for $X < \frac{4\alpha_W}{\lambda}(0.8\alpha_W)^2 \xi^2 X_{\mathrm{cr}} \approx 0.7 \times 10^5 X_{\mathrm{cr}}$, that is always after the end of inflation.

We can also check, that the occupation numbers $n_{\mathbf{k}}$ are really much smaller than one and we are in the non-resonant regime (3.18). As far as the typical physical momenta of the W bosons are of the order of K we have

$$n_{\mathbf{k}} \sim \frac{n_{\mathrm{decay}}}{K^3} = \frac{\sqrt{X_{\mathrm{cr}}} \, A \sqrt{\lambda}}{4\,\pi^3 \sqrt{\alpha_W} \, (0.8\alpha_W) \sqrt{X}} \approx 0.06 \left(\frac{\lambda}{0.25} \right)^{1/2} \sqrt{\frac{X_{\mathrm{cr}}}{X}} \;, \tag{B.4}$$

which is much smaller than one for all interesting X.

The energy during this stage is converted to the SM particles produced in W boson decays. They are light and relativistic, and typical energy transferred to them in each decay is of the order $\sqrt{\langle m_W^2 \rangle}$, as far as the W bosons are non-relativistic. Thus, we can write the Boltzmann equation in the expanding Universe for the energy density in the relativistic SM particles as

$$\frac{d}{dt}\left(a^4 \rho\right) \simeq a^4 \left(6 + \frac{3}{\cos^3 \theta_W}\right) \sqrt{\langle m_W^2 \rangle} n_{\text{decay}} \Gamma \tag{B.5}$$

$$\simeq a^4 \left(6 + \frac{3}{\cos^3 \theta_W}\right) \sqrt{\langle m_W^2 \rangle} \frac{A \omega K^3}{2\pi^3} \ ,$$

where the coefficient $\left(6 + \frac{3}{\cos^3 \theta_W}\right)$ accounts both for the different number and masses of created $W^\pm$- and Z-bosons (see eq. (A.20)). The solution is saturated by late time and reads

$$\rho = \left(6 + \frac{3}{\cos^3 \theta_W}\right) \sqrt{\langle m_W^2 \rangle} \frac{A \omega K^3}{2\pi^3} \frac{6}{13} t \ . \tag{B.6}$$

This reaches the inflaton energy density $\rho \sim \rho_{\text{inf}}$ (3.7) at

$$X \simeq \left(6 + \frac{3}{\cos^3 \theta_W}\right)^{2/3} \frac{4 \cdot 3^{\frac{2}{3}} \xi^{\frac{2}{3}} X_{\text{cr}} A^{\frac{2}{3}} \alpha_W}{13^{\frac{2}{3}} \pi^{\frac{4}{3}} \lambda^{\frac{1}{3}}} \approx 5.8 X_{\text{cr}} \ , \tag{B.7}$$

that is much later than the end of the non-resonant creation period (3.21). We conclude, that the energy drain by W boson decays is irrelevant during the non-resonant inflaton decay.

B.2 W bosons annihilation

Self scattering of the W bosons, like $WW \to WW$ is of little interest for us, as far as it does not take the bosons out of the stochastic resonance zone (the bosons are non-relativistic, so after scattering they retain their small momenta).

It is easy to check, that for the W boson number density (B.3) saturated by the boson decays (discussed in Appendix B.1), during the period (3.21) of non-resonant inflaton decay the annihilation to fermions is negligible, $\sigma n_{\text{decay}}^2 < \Gamma n_{\text{decay}}$ (relation formally holds for $X > 0.1 X_{\text{cr}}$).

The scattering proceeds much more actively at larger particle densities, so the relevant production mechanism is given by the stochastic resonance (3.19). Thus, we can approximate the effective Boltzmann equation for the W boson particle number as

$$\frac{d}{dt}\left(a^3 n_W\right) = a^3 \left(2B\omega n_W - \sigma n_W^2\right) \ , \tag{B.8}$$

where $n_W = n_{W^+} = n_{W^-}$. Its approximate solution (3.23) is obtained, again, by setting the derivative in the left-hand side to zero. This is true for $d(a^3 n_{\text{scatter}})/dt \ll a^3 2B\omega n_{\text{scatter}}$, that is for $X \ll 4B\xi X_{\text{cr}} \approx 4 \times 10^3 X_{\text{cr}}$, while we are interested in much smaller X. We should check of course, that the particle density is not to small, to allow for stochastic resonance to work. Again, for the typical occupation number (up to some numerical factor) we get

$$n_{\mathbf{k}} \sim \frac{n_{\text{scatter}}}{K^3} = \frac{2B}{5\pi^2 \alpha_W^2} \approx 3.4 \ . \tag{B.9}$$

This is larger, than one, thus we may hope that exponential creation is already a reasonable approximation for (A.16). The energy drain is obtain if we recall that the W bosons are nonrelativistic, so each scattering provides the energy transfer of $2\sqrt{\langle m_W^2 \rangle}$, leading to (3.24). The solution of (3.24) is also saturated at late times, so (if the initial time is small) we have

$$\rho = \frac{96\,\xi\,X_{\mathrm{cr}}^{\frac{7}{2}}\,B^2\,\sqrt{X}\,\sqrt{\lambda}}{65\,\pi\,\sqrt{\alpha_W}} \approx 73\left(\frac{\lambda}{0.25}\right)\sqrt{X_{\mathrm{cr}}^7 X}\ . \tag{B.10}$$

One can see, that it reaches inflaton energy density $\rho \sim \rho_{\mathrm{inf}}$ (3.7) at

$$X \sim \frac{16\,6^{\frac{2}{3}}\,\xi^{\frac{2}{3}}\,B^{\frac{4}{3}}}{65^{\frac{2}{3}}\,\pi^{\frac{2}{3}}\,\alpha_W^{\frac{1}{3}}\,\lambda^{\frac{1}{3}}}X_{\mathrm{cr}} \approx 110 X_{\mathrm{cr}}\ . \tag{B.11}$$

This is earlier, than the end of the non-resonant production region (3.21). Taking into account Z bosons makes this process even more active. This means, that after the moment (3.21), the parametric resonance starts, and due to higher concentration of the gauge bosons, the energy is rapidly transferred into relativistic SM fermions via gauge boson annihilation. So, we expect that the transfer of the energy to the relativistic modes via the gauge bosons completes by approximately (3.21).

C. Non-resonant Higgs production on the nonlinearities of the potential for small χ

One needs to analyse the production of particles by (3.14), but with the mass

$$m^2\,(t) = \begin{cases} \omega^2 & \text{for } X\cos[\omega t] > \frac{\omega}{\sqrt{3\lambda}}\ , \\ 3\lambda X^2 \cos^2[\omega t] & \text{for } X\cos[\omega t] < \frac{\omega}{\sqrt{3\lambda}}\ . \end{cases} \tag{C.1}$$

One way to find the generation by this source is to use the method described in section A.1. The adiabatic approximation holds while the mass does not change, and close to the moments t_j the problem can be solved after replacing the cosine with the quadratic function. Alternatively, if the number of generated particles is small, a simpler perturbative approach can be used. We use the perturbative approach here.

Let us first neglect the expansion of the Universe during several oscillations of the inflaton field. In this case the number of particle of the mass ω generated by the quadratic potential

$$\mathcal{L}_{\mathrm{int}} = \frac{m^2\,(t) - \omega^2}{2}\,(\delta\chi)^2\ ,$$

is given by

$$n_{\mathbf{k}}\,(t) = \left|\frac{1}{2k_0}\int_0^t dt\,\left(m^2\,(t) - \omega^2\right)\mathrm{e}^{2ik_0 t}\right|^2\ ,$$

where $k_0^2 = k^2 + \omega^2$. The integral is equal to (for the moment of time around $t \sim t_l = \frac{2\pi}{\omega}l$, $l = 1, 2, \ldots$; the integral value changes while the inflaton field crosses zero, but the exact form is not important)

$$n_k\,(t_l) = \frac{1}{16}\frac{\sin^2\left(2l\frac{\pi k_0}{\omega}\right)}{\sin^2\left(\frac{\pi k_0}{\omega}\right)}L^2\ ,$$

where

$$L = \frac{3\lambda X^2}{2} \cdot \frac{\omega}{k_0} \left\{ \frac{1}{k_0 + \omega} \sin\left[4\pi\left(\frac{k_0}{\omega} + 1\right)\epsilon\right] - \frac{1}{k_0 - \omega} \sin\left[4\pi\left(\frac{k_0}{\omega} - 1\right)\epsilon\right] \right\} , \quad \text{(C.2)}$$

and the parameter ϵ is defined from the equation

$$\sin\left[2\pi\epsilon\right] \equiv \frac{\omega}{\sqrt{3\lambda}X} . \quad \text{(C.3)}$$

At large times, $t \gg \omega^{-1}$, using the equality $\lim_{t \to \infty} \frac{\sin^2 xt}{\pi x^2 t} = \delta(x)$ one gets

$$\frac{n_k(t)}{t} \simeq \frac{\omega^2}{4\pi^2} \pi \sum_{l=1}^{\infty} L^2 \delta(k_0 - \omega l) .$$

Thus, integration over momenta gives a convergent sum

$$\frac{n(t)}{t} = \frac{9\lambda^2 X^4}{2^5\pi^3} \sum_{l=2}^{\infty} \frac{1}{l^2}\sqrt{1 - \frac{1}{l^2}} \cdot \left(\frac{1}{l+1}\sin\left[4\pi(l+1)\epsilon\right] - \frac{1}{l-1}\sin\left[4\pi(l-1)\epsilon\right]\right)^2 . \quad \text{(C.4)}$$

The sum is saturated for $4\pi n\epsilon \simeq 1$. This implies the typical energy of the produced particles,

$$E \sim \frac{\omega}{4\pi\epsilon} \sim \frac{1}{2}\sqrt{3\lambda}X ,$$

which is larger, than ω, so the particles are relativistic.

Using formulas from Appendix E we get the following estimates for the production rate

$$\dot{n} \simeq \frac{n(t)}{t} \simeq \frac{4\pi\omega^4}{15\pi^3} \frac{\omega}{\sqrt{3\lambda}X} ,$$

and for the energy flux

$$\dot{\rho} \simeq \frac{\rho(t)}{t} \simeq \frac{\omega^5}{2\pi^3} .$$

Reintroducing the expansion of the Universe in the usual way by changing $\dot{n} \to \frac{d(a^3 n)}{a^3 dt}$, $\dot{\rho} \to \frac{d(a^4\rho)}{a^4 dt}$, we have the number and energy densities at late time

$$n = \frac{4\pi\omega^4}{15\pi^3}\frac{\omega}{4}\frac{t}{\sqrt{3\lambda}X(t)} , \quad \text{(C.5)}$$

$$\rho = \frac{3}{11}\frac{\omega^5}{2\pi^3}t . \quad \text{(C.6)}$$

D. Tunnelling through a $-|x|$ barrier

The solution of eq. (A.13) is given by the Airy functions for negative and positive times:

$$\Phi_{\mathbf{k}}(\tau < 0) = A_- \operatorname{Ai}(\tau - \kappa^2) + B_- \operatorname{Bi}(\tau - \kappa^2) , \quad \text{(D.1)}$$

$$\Phi_{\mathbf{k}}(\tau > 0) = A_+ \operatorname{Ai}(-\tau - \kappa^2) + B_+ \operatorname{Bi}(-\tau - \kappa^2) . \quad \text{(D.2)}$$

The coefficients should be determined by the matching conditions at $\tau = 0$

$$\Phi_{\mathbf{k}}(0-) = \Phi_{\mathbf{k}}(0+) , \qquad\qquad \Phi'_{\mathbf{k}}(0-) = \Phi'_{\mathbf{k}}(0+) . \qquad (D.3)$$

It is comfortable, however, firstly to match the coefficients $A_\pm$, $B_\pm$ with $\alpha = \alpha_{\mathbf{k}}^j e^{-i\theta_{\mathbf{k}}^j}$, $\beta = \beta_{\mathbf{k}}^j e^{+i\theta_{\mathbf{k}}^j}$, $\alpha' = \alpha_{\mathbf{k}}^{j+1} e^{-i\theta_{\mathbf{k}}^j}$, and $\beta' = \beta_{\mathbf{k}}^{j+1} e^{i\theta_{\mathbf{k}}^j}$ from the expansion (A.6). The asymptotic expansions of the Airy functions are

$$\mathrm{Ai}(-x) = \frac{1}{\sqrt{\pi} x^{1/4}} \sin\left(\frac{2}{3} x^{3/2} + \frac{\pi}{4}\right) , \qquad (D.4)$$

$$\mathrm{Bi}(-x) = \frac{1}{\sqrt{\pi} x^{1/4}} \cos\left(\frac{2}{3} x^{3/2} + \frac{\pi}{4}\right) . \qquad (D.5)$$

Then, the solution matched with (A.6) at large τ (for matching one should use in (A.6) only linear part of the mass, $m_W(t) \simeq \mathrm{const} \cdot |t - t_j|$)

$$\Phi_{\mathbf{k}}(\tau < 0) = \sqrt{\frac{\pi}{2}} \left[\alpha e^{-i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})} + \beta e^{i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})}\right] \mathrm{Bi}(\tau - \kappa^2) + \qquad (D.6)$$

$$+ i\sqrt{\frac{\pi}{2}} \left[\alpha e^{-i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})} - \beta e^{i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})}\right] \mathrm{Ai}(\tau - \kappa^2) ,$$

$$\Phi_{\mathbf{k}}(\tau > 0) = \sqrt{\frac{\pi}{2}} \left[\alpha' e^{i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})} + \beta' e^{-i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})}\right] \mathrm{Bi}(-\tau - \kappa^2) + \qquad (D.7)$$

$$+ i\sqrt{\frac{\pi}{2}} \left[-\alpha' e^{i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})} + \beta' e^{-i(\frac{2}{3}\kappa^3 + \frac{\pi}{4})}\right] \mathrm{Ai}(-\tau - \kappa^2) .$$

Using the linear relations between α, β, α', β' from condition at zero (D.3), and the definition of $R_{\mathbf{k}}$, $D_{\mathbf{k}}$ from (A.7), which is

$$\alpha' = \alpha \frac{1}{D_{\mathbf{k}}} + \beta \frac{R_{\mathbf{k}}^*}{D_{\mathbf{k}}^*} , \qquad (D.8)$$

$$\beta' = \alpha \frac{R_{\mathbf{k}}}{D_{\mathbf{k}}} + \beta \frac{1}{D_{\mathbf{k}}^*} , \qquad (D.9)$$

one gets the expressions (A.14), (A.15).

E. Useful sums

In Appendix C we obtain the following sums

$$S_1 = \sum_{l=2}^{\infty} \frac{1}{l^2} \sqrt{1 - \frac{1}{l^2}} \cdot \left(\frac{1}{l+1} \sin\left[4\pi (l+1) \epsilon\right] - \frac{1}{l-1} \sin\left[4\pi (l-1) \epsilon\right]\right)^2 , \qquad (E.1)$$

$$S_2 = \sum_{l=2}^{\infty} \frac{1}{l} \sqrt{1 - \frac{1}{l^2}} \cdot \left(\frac{1}{l+1} \sin\left[4\pi (l+1) \epsilon\right] - \frac{1}{l-1} \sin\left[4\pi (l-1) \epsilon\right]\right)^2 , \qquad (E.2)$$

where ϵ is a small dimensionless parameter, $\epsilon \ll 1$. These sums are saturated at $l \sim 1/4\pi\epsilon$, hence to get the leading order results in ϵ one can replace these sums with corresponding

integrals by introducing a new variable u as $u = 4\pi l\epsilon$, $du = 4\pi\epsilon$. Then to the leading order in ϵ one arrives at

$$S_1 = \int_0^\infty \epsilon^5 \frac{du}{u^6} \left(2u\cos u - 2\sin u\right)^2 = \frac{4\pi}{15}\left(4\pi\epsilon\right)^5 \ , \tag{E.3}$$

$$S_2 = \int_0^\infty \epsilon^4 \frac{du}{u^5} \left(2u\cos u - 2\sin u\right)^2 = \left(4\pi\epsilon\right)^4 \ . \tag{E.4}$$

F. Nonresonant particle production with h and $|h|$ sources

Here we compare production of fermions by h and $|h|$ sources and conclude, that the corresponding production rates are the same, though the spectra differ.

As far as the number of the created particles is small, $n_{\mathbf{k}} \ll 1$, we can use the perturbation theory. Then, the perturbation

$$\hat{H}_{int} \equiv \int d^3\mathbf{x}\, m\left(t\right) \bar{\Psi}\Psi$$

leads to the number density at the moment t

$$n_k\left(t\right) = \int_0^t m\left(t'\right) e^{2ik_0 t'} dt' \int_0^t m\left(t''\right) e^{-2ik_0 t''} dt'' \ .$$

and total particle number

$$n(t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} n_k(t) \ .$$

Here we calculate the particle density side by side for two different sources

$$m(t) = m\sin(\omega t) \ , \tag{F.1}$$

$$m(t) = |m\sin(\omega t)| \ , \tag{F.2}$$

and massless fields $\bar{\Psi}$, Ψ, so $k_0 = k$. One can check that for $n = 0, 1, \ldots$

$$\int_{\frac{2\pi n}{\omega}}^{\frac{2\pi\left(n+\frac{1}{2}\right)}{\omega}} dt \sin\left(\omega t\right) e^{2ikt} = -\frac{2\omega}{4k^2 - \omega^2} \cdot e^{i\frac{\pi k}{\omega}} \cos\left(\frac{k\pi}{\omega}\right) \cdot e^{i\frac{4\pi kn}{\omega}} \ , \tag{F.3}$$

$$\int_{\frac{2\pi\left(n+\frac{1}{2}\right)}{\omega}}^{\frac{2\pi(n+1)}{\omega}} dt \sin\left(\omega t\right) e^{2ikt} = -\frac{2\omega}{4k^2 - \omega^2} \cdot e^{3i\frac{\pi k}{\omega}} \cos\left(\frac{k\pi}{\omega}\right) \cdot e^{i\frac{4\pi kn}{\omega}} \ . \tag{F.4}$$

Hence for the full $(n+1)$th period

$$\int_{\frac{2\pi n}{\omega}}^{\frac{2\pi(n+1)}{\omega}} dt \sin\left(\omega t\right) e^{2ikt} = \frac{4i\omega}{4k^2 - \omega^2} \cdot e^{i\frac{2\pi k}{\omega}} \cos\left(\frac{k\pi}{\omega}\right) \cdot \sin\left(\frac{k\pi}{\omega}\right) e^{i\frac{4\pi kn}{\omega}} \ , \tag{F.5}$$

$$\int_{\frac{2\pi n}{\omega}}^{\frac{2\pi(n+1)}{\omega}} dt \left|\sin\left(\omega t\right)\right| e^{2ikt} = \frac{-4\omega}{4k^2 - \omega^2} \cdot e^{i\frac{2\pi k}{\omega}} \cos^2\left(\frac{k\pi}{\omega}\right) \cdot e^{i\frac{4\pi kn}{\omega}} \ , \tag{F.6}$$

and summing over $N + 1$ periods one arrives at

$$\int_0^{\frac{2\pi(N+1)}{\omega}} dt \, \sin(\omega t) \, e^{2ik_0 t} = \frac{4i\omega}{4k^2 - \omega^2} \cdot e^{i\frac{2\pi k(N+1)}{\omega}} \cdot \cos\left(\frac{k\pi}{\omega}\right) \cdot \sin\left(\frac{k\pi}{\omega}\right) \cdot \frac{\sin\left(\frac{2\pi k(N+1)}{\omega}\right)}{\sin\left(\frac{2\pi k}{\omega}\right)} \, ,$$

(F.7)

$$\int_0^{\frac{2\pi(N+1)}{\omega}} dt \, |\sin(\omega t)| \, e^{2ik_0 t} = \frac{4i\omega}{4k^2 - \omega^2} \cdot e^{i\frac{2\pi k(N+1)}{\omega}} \cdot \cos^2\left(\frac{k\pi}{\omega}\right) \cdot \frac{\sin\left(\frac{2\pi k(N+1)}{\omega}\right)}{\sin\left(\frac{2\pi k}{\omega}\right)} \, .$$

(F.8)

Hence the number of produced particles for $N + 1$ periods of oscillations is

$$n\left(t = \frac{2\pi(N+1)}{\omega}\right) = \frac{m^2}{2\pi^2} \int dk \frac{16\omega^2 k^2}{(4k^2 - \omega^2)^2} \cos^2\left(\frac{k\pi}{\omega}\right) \cdot \sin^2\left(\frac{k\pi}{\omega}\right) \frac{\sin^2\left(\frac{2\pi k(N+1)}{\omega}\right)}{\sin^2\left(\frac{2\pi k}{\omega}\right)} \, ,$$

(F.9)

$$n\left(t = \frac{2\pi(N+1)}{\omega}\right) = \frac{m^2}{2\pi^2} \int dk \frac{16\omega^2 k^2}{(4k^2 - \omega^2)^2} \cos^4\left(\frac{k\pi}{\omega}\right) \cdot \frac{\sin^2\left(\frac{2\pi k(N+1)}{\omega}\right)}{\sin^2\left(\frac{2\pi k}{\omega}\right)} \, .$$

(F.10)

Assuming that a tiny amount of particles is produced per each period one can turn to continuous variable T in these expressions. To obtain the particle production rate we are interested in linear in T contribution. It comes from poles in the integrands. Having this in mind and making use of the relation

$$\lim_{t\to\infty} \frac{\sin^2 \alpha t}{\pi t \alpha^2} = \delta(\alpha) \, ,$$

(F.11)

one proceeds with calculations. For the source (F.1) one makes use of the identity

$$\sin^2\left(\frac{2\pi k(N+1)}{\omega}\right) = \sin^2\left(\left(k - \frac{\omega}{2}\right)t\right) \, ,$$

while for the source (F.2):

$$\sin^2\left(\frac{2\pi k(N+1)}{\omega}\right) = \sin^2(kT) \, .$$

Then for the first source one obtains

$$n(t) = \frac{m^2}{2\pi^2} \int \frac{dk \, k^2}{\left(k + \frac{\omega}{2}\right)^2} \frac{\omega^2}{4} \frac{\sin^2(Tk)}{\left(k - \frac{\omega}{2}\right)^2}$$

and with help of (F.11)

$$n(t) = t \frac{m^2}{2\pi^2} \int \frac{dk \, \pi k^2}{\left(k + \frac{\omega}{2}\right)^2} \frac{\omega^2}{4} \delta\left(k - \frac{\omega}{2}\right) \, .$$

This is the monochromatic spectrum. The number of produced particles is

$$n(t) = t \cdot \frac{m^2 \omega^2}{32\pi} \, .$$

For the second source (F.2) one has

$$n\left(t\right) = \frac{m^2}{2\pi^2} \int \frac{dk \ k^2}{\left(k + \frac{\omega}{2}\right)^2} \frac{\omega^2}{4} \frac{\cos^2\left(\frac{k\pi}{\omega}\right)}{\left(k - \frac{\omega}{2}\right)^2} \frac{\sin^2\left(Tk\right)}{\sin^2\left(\frac{k\pi}{\omega}\right)} \ .$$

Here the double-pole at $k = \omega/2$ is cancelled by the double-zero from cosine squared. But there are a lot of poles due to sine in the denominator. In this case the useful variant of (F.11) is

$$\lim_{t\to\infty} \frac{\sin^2\left(tk\right)}{\pi t \sin^2\left(\frac{k\pi}{\omega}\right)} = \frac{\omega^2}{\pi^2}\delta\left(k\right) \ .$$

It gives for the spectra

$$n\left(t\right) = t\frac{m^2}{2\pi^2} \cdot \frac{\omega^2}{\pi} \int \frac{dk \ k^2}{\left(k + \frac{\omega}{2}\right)^2} \frac{\omega^2}{4} \frac{\sum_n \delta\left(k - \omega n\right)}{\left(k - \frac{\omega}{2}\right)^2}$$

Finally, integrating over momenta and summing up the series

$$\sum_{n=0}^{n=\infty} \frac{n^2}{\left(n^2 - \frac{1}{4}\right)^2} = \frac{\pi^2}{4}$$

one gets

$$n\left(t\right) = t \cdot \frac{m^2\omega^2}{32\pi} \ ,$$

the same answer as for the first source (F.1).

References

[1] A. Linde, *Inflationary cosmology, Lect. Notes Phys.* **738** (2008) 1–54, [arXiv:0705.0164], [SPIRES].

[2] S. Raby, *SUSY GUT Model Building, Eur. Phys. J.* **C59** (2008) 223–247, [arXiv:0807.4921], [SPIRES].

[3] V. A. Rubakov, *Large and infinite extra dimensions: An introduction, Phys. Usp.* **44** (2001) 871–893, [hep-ph/0104152], [SPRIES].

[4] T. Asaka, S. Blanchet, and M. Shaposhnikov, *The νMSM, dark matter and neutrino masses, Phys. Lett.* **B631** (2005) 151–156, [hep-ph/0503065], [SPRIES].

[5] T. Asaka and M. Shaposhnikov, *The νMSM, dark matter and baryon asymmetry of the universe, Phys. Lett.* **B620** (2005) 17–26, [hep-ph/0505013], [SPRIES].

[6] M. Shaposhnikov and D. Zenhausern, *Quantum scale invariance, cosmological constant and hierarchy problem, Phys. Lett.* **B671** (2009) 162–166, [arXiv:0809.3406], [SPIRES].

[7] M. E. Shaposhnikov and I. I. Tkachev, *Quantum scale invariance on the lattice*, arXiv:0811.1967, [SPIRES].

[8] M. Shaposhnikov and D. Zenhausern, *Scale invariance, unimodular gravity and dark energy, Phys. Lett.* **B671** (2009) 187–192, [arXiv:0809.3395], [SPIRES].

[9] F. Bezrukov, *νMSM predictions for neutrinoless double beta decay, Phys. Rev.* **D72** (2005) 071303, [hep-ph/0505247], [SPRIES].

[10] A. Boyarsky, A. Neronov, O. Ruchayskiy, and M. Shaposhnikov, *The masses of active neutrinos in the νMSM from X-ray astronomy*, *JETP Lett.* **83** (2006) 133–135, [hep-ph/0601098], [SPRIES].

[11] T. Asaka, M. Shaposhnikov, and A. Kusenko, *Opening a new window for warm dark matter*, *Phys. Lett.* **B638** (2006) 401–406, [hep-ph/0602150], [SPRIES].

[12] M. Shaposhnikov, *A possible symmetry of the νMSM*, *Nucl. Phys.* **B763** (2007) 49–59, [hep-ph/0605047], [SPRIES].

[13] T. Asaka, M. Laine, and M. Shaposhnikov, *On the hadronic contribution to sterile neutrino production*, *JHEP* **06** (2006) 053, [hep-ph/0605209], [SPRIES].

[14] T. Asaka, M. Laine, and M. Shaposhnikov, *Lightest sterile neutrino abundance within the νMSM*, *JHEP* **0701** (2007) 091, [hep-ph/0612182], [SPRIES].

[15] F. Bezrukov and M. Shaposhnikov, *Searching for dark matter sterile neutrino in laboratory*, *Phys. Rev.* **D75** (2007) 053005, [hep-ph/0611352], [SPRIES].

[16] D. Gorbunov and M. Shaposhnikov, *How to find neutral leptons of the νMSM?*, *JHEP* **10** (2007) 015, [arXiv:0705.1729], [SPIRES].

[17] M. Shaposhnikov, *Is there a new physics between electroweak and planck scales?*, arXiv:0708.3550, [SPIRES].

[18] M. Shaposhnikov, *The νMSM, leptonic asymmetries, and properties of singlet fermions*, *JHEP* **08** (2008) 008, [arXiv:0804.4542], [SPIRES].

[19] M. Laine and M. Shaposhnikov, *Sterile neutrino dark matter as a consequence of νMSM-induced lepton asymmetry*, *JCAP* **0806** (2008) 031, [arXiv:0804.4543], [SPIRES].

[20] K. A. Meissner and H. Nicolai, *Conformal symmetry and the standard model*, *Phys. Lett.* **B648** (2007) 312–317, [hep-th/0612165], [SPRIES].

[21] F. L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, *Phys. Lett.* **B659** (2008) 703–706, [arXiv:0710.3755], [SPIRES].

[22] E. K. Akhmedov, V. A. Rubakov, and A. Y. Smirnov, *Baryogenesis via neutrino oscillations*, *Phys. Rev. Lett.* **81** (1998) 1359–1362, [hep-ph/9803255], [SPIRES].

[23] S. Dodelson and L. M. Widrow, *Sterile-neutrinos as dark matter*, *Phys. Rev. Lett.* **72** (1994) 17–20, [hep-ph/9303287], [SPRIES].

[24] X.-D. Shi and G. M. Fuller, *A new dark matter candidate: Non-thermal sterile neutrinos*, *Phys. Rev. Lett.* **82** (1999) 2832–2835, [astro-ph/9810076], [SPRIES].

[25] A. D. Dolgov and S. H. Hansen, *Massive sterile neutrinos as warm dark matter*, *Astropart. Phys.* **16** (2002) 339–344, [hep-ph/0009083], [SPRIES].

[26] K. Abazajian, G. M. Fuller, and M. Patel, *Sterile neutrino hot, warm, and cold dark matter*, *Phys. Rev.* **D64** (2001) 023501, [astro-ph/0101524], [SPRIES].

[27] K. Abazajian, *Production and evolution of perturbations of sterile neutrino dark matter*, *Phys. Rev.* **D73** (2006) 063506, [astro-ph/0511630], [SPRIES].

[28] F. L. Bezrukov, *The Standard model Higgs as the inflaton*, arXiv:0805.2236, [SPIRES].

[29] D. I. Kaiser, *Primordial spectral indices from generalized einstein theories*, *Phys. Rev.* **D52** (1995) 4295–4306, [astro-ph/9408044], [SPRIES].

[30] S. Tsujikawa and H. Yajima, *New constraints on multi-field inflation with nonminimal coupling*, *Phys.Rev.* **D 62** (2000) 123512, [`hep-ph/0007351`], [SPRIES].

[31] S. Tsujikawa and B. Gumjudpai, *Density perturbations in generalized einstein scenarios and constraints on nonminimal couplings from the cosmic microwave background*, *Phys. Rev.* **D69** (2004) 123523, [`astro-ph/0402185`], [SPRIES].

[32] N. Makino and M. Sasaki, *The density perturbation in the chaotic inflation with nonminimal coupling*, *Prog. Theor. Phys.* **86** (1991) 103–118, [SPIRES].

[33] R. Fakir, S. Habib, and W. Unruh, *Cosmological density perturbations with modified gravity*, *Astrophys. J.* **394** (1992) 396, [SPIRES].

[34] **WMAP** Collaboration, E. Komatsu *et. al.*, *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Cosmological Interpretation*, *Astrophys. J. Suppl.* **180** (2009) 330–376, [`arXiv:0803.0547`], [SPIRES].

[35] A. R. Liddle and D. H. Lyth, *The cold dark matter density perturbation*, *Phys. Rept.* **231** (1993) 1–105, [`astro-ph/9303019`], [SPRIES].

[36] A. O. Barvinsky, A. Y. Kamenshchik, and A. A. Starobinsky, *Inflation scenario via the Standard Model Higgs boson and LHC*, *JCAP* **0811** (2008) 021, [`arXiv:0809.2104`], [SPIRES].

[37] F. L. Bezrukov, A. Magnin, and M. Shaposhnikov, *Standard Model Higgs boson mass from inflation*, *Phys. Lett.* **B675** (2009) 88–92, [`arXiv:0812.4950`], [SPIRES].

[38] A. De Simone, M. P. Hertzberg, and F. Wilczek, *Running Inflation in the Standard Model*, `arXiv:0812.4946`, [SPIRES].

[39] F. Bezrukov and M. Shaposhnikov, *Standard Model Higgs boson mass from inflation: two loop analysis*, `arXiv:0904.1537`, [SPIRES].

[40] A. O. Barvinsky, A. Y. Kamenshchik, C. Kiefer, A. A. Starobinsky, and C. Steinwachs, *Asymptotic freedom in inflationary cosmology with a non- minimally coupled Higgs field*, `arXiv:0904.1698`, [SPIRES].

[41] S. Sibiryakov private communication.

[42] C. P. Burgess, H. M. Lee, and M. Trott, *Power-counting and the Validity of the Classical Approximation During Inflation*, `arXiv:0902.4465`, [SPIRES].

[43] J. L. F. Barbon and J. R. Espinosa, *On the Naturalness of Higgs Inflation*, `arXiv:0903.0355`, [SPIRES].

[44] L. Kofman, A. D. Linde, and A. A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev.* **D56** (1997) 3258–3295, [`hep-ph/9704452`], [SPRIES].

[45] C. Jarlskog, *Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Violation*, *Phys. Rev. Lett.* **55** (1985) 1039, [SPIRES].

[46] M. E. Shaposhnikov, *Baryon Asymmetry of the Universe in Standard Electroweak Theory*, *Nucl. Phys.* **B287** (1987) 757–775, [SPIRES].

[47] M. E. Shaposhnikov, *Structure of the High Temperature Gauge Ground State and Electroweak Production of the Baryon Asymmetry*, *Nucl. Phys.* **B299** (1988) 797, [SPIRES].

[48] B. Moore *et. al.*, *Dark matter substructure within galactic halos*, *Astrophys. J.* **524** (1999) L19–L22, [SPIRES].

[49] A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, *Where are the missing galactic satellites?*, *Astrophys. J.* **522** (1999) 82–92, [`astro-ph/9901240`], [SPRIES].

[50] J. Sommer-Larsen and A. Dolgov, *Formation of Disk Galaxies: Warm Dark Matter and the Angular Momentum problem*, *Astrophys. J.* **551** (2001) 608–623, [`astro-ph/9912166`], [SPRIES].

[51] P. Bode, J. P. Ostriker, and N. Turok, *Halo formation in warm dark matter models*, *Astrophys. J.* **556** (2001) 93–107, [`astro-ph/0010389`], [SPRIES].

[52] T. Goerdt, B. Moore, J. I. Read, J. Stadel, and M. Zemp, *Does the fornax dwarf spheroidal have a central cusp or core?*, *Mon. Not. Roy. Astron. Soc.* **368** (2006) 1073–1077, [`astro-ph/0601404`], [SPRIES].

[53] G. Gilmore *et. al.*, *Observed Properties of Dark Matter: dynamical studies of dSph galaxies*, *Nucl. Phys. Proc. Suppl.* **173** (2007) 15–18, [`astro-ph/0608528`], [SPRIES].

[54] L. E. Strigari *et. al.*, *A common mass scale for satellite galaxies of the Milky Way*, *Nature* **454** (2008) 1096–1097, [`arXiv:0808.3772`], [SPIRES].

[55] D. Gorbunov, A. Khmelnitsky, and V. Rubakov, *Constraining sterile neutrino dark matter by phase-space density observations*, *JCAP* **0810** (2008) 041, [`arXiv:0808.3910`], [SPIRES].

[56] J. Garcia-Bellido, D. G. Figueroa, and J. Rubio, *Preheating in the Standard Model with the Higgs-Inflaton coupled to gravity*, `arXiv:0812.4624`, [SPIRES].