How to find neutral leptons of the νMSM?

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Abstract

An extension of the Standard Model by three singlet fermions with masses smaller than the electroweak scale allows to explain simultaneously neutrino oscillations, dark matter and baryon asymmetry of the Universe. We discuss the properties of neutral leptons in this model and the ways they can be searched for in particle physics experiments. We establish, in particular, a lower and an upper bound on the strength of interaction of neutral leptons coming from cosmological considerations and from the data on neutrino oscillations. We analyse the production of neutral leptons in the decays of different mesons and in pp collisions. We study in detail decays of neutral leptons and establish a lower bound on their mass coming from existing experimental data and Big Bang Nucleosynthesis. We argue that the search for a specific missing energy signal in kaon decays would allow to strengthen considerably the bounds on neutral fermion couplings and to find or definitely exclude them below the kaon threshold. To enter into cosmologically interesting parameter range for masses above kaon mass the dedicated searches similar to CERN PS191 experiment would be needed with the use of intensive proton beams. We argue that the use of CNGS, NuMI, T2K or NuTeV beams could allow to search for singlet leptons below charm in a large portion of the parameter space of the νMSM. The search of singlet fermions in the mass interval $2-5$ GeV would require a considerable increase of the intensity of proton accelerators or the detailed analysis of kinematics of more than 10^{10} B-meson decays.

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I. INTRODUCTION

In a search for physics beyond the Standard Model (SM) one can use different types of guidelines. A possible strategy is to attempt to explain the phenomena that cannot be fit to the SM by minimal means, that is by introducing the smallest possible number of new particles without adding any new physical principles (such as supersymmetry or extra dimensions) or new energy scales (like the Grand Unified scale). An example of such a theory is the renormalizable extension of the SM, the νMSM (neutrino Minimal Standard Model) [1, 2], where three *light* singlet right-handed fermions (we will be using also the names neutral fermions, or heavy leptons, or sterile neutrinos interchangeably) are introduced. The leptonic sector of the theory has the same structure as the quark sector, i.e. every left-handed fermion has its right-handed counterpart. This model is consistent with the data on neutrino oscillations, provides a candidate for dark matter particle – the lightest singlet fermion (sterile neutrino), and can explain the baryon asymmetry of the Universe [2]. A further extension of this model by a light singlet scalar field allows to have inflation in the Early Universe [3].

A crucial feature of this theory is the relatively small mass scale of the new neutral leptonic states, which opens a possibility for a direct search of these particles. Let us review shortly the physical applications of the νMSM.

1. Neutrino masses and oscillations. The νMSM contains 18 new parameters in comparison with SM. They are: 3 Majorana masses for singlet fermions, 3 Dirac masses associated with the mixing between left-handed and right-handed neutrinos, 6 mixing angles and 6 CP-violating phases. These parameters can describe any pattern (and in particular the observed one) of masses and mixings of active neutrinos, which is characterized by 9 parameters only (3 active neutrino masses, 3 mixing angles, and 3 CP-violating phases). Inspite of this freedom, the *absolute* scale of active neutrino masses can be established in the νMSM from cosmology and astrophysics of dark matter particles [1, 4–7]: one of the active neutrinos must have a mass smaller than $\mathcal{O}(10^{-5})$ eV. The choice of the small mass scale for singlet fermions leads to the small values of the Yukawa coupling constants, at the level $10^{-6} - 10^{-12}$, which is crucial for explanation of dark matter and baryon asymmetry of the Universe.

2. Dark matter. Though the νMSM does not have any extra stable particle in comparison with the SM, the lightest singlet fermion, N_1, may have a life-time τ_{N_1} greatly exceeding the age of the Universe and thus play a role of a dark matter particle [8–11]. Dark matter sterile neutrino is likely to have a mass in the $\mathcal{O}(10)$ keV region. The arguments leading to the keV mass for dark matter neutrino are related to structure formation and to the problems of missing satellites and cuspy profiles in the Cold Dark Matter cosmological models [12–15]; the keV scale is also favoured by the cosmological considerations of the production of dark matter due to transitions between active and sterile neutrinos [8, 9]; warm DM may help to solve the problem of galactic angular momentum [16]. However, no upper limit on the mass of sterile neutrino exists [3, 17] as this particle can be produced in interactions beyond the νMSM. The radiative decays of N_1 can be potentially observed in different X-ray observations [10, 18], and the stringent limits on the strength of their interaction with active neutrinos [5, 19–27] and their free streaming length at the onset of cosmological structure formation [28–31] already exist. An astrophysical lower bound on their mass is 0.3 keV, following from the analysis of the rotational curves of dwarf spheroidal galaxies [32–34]. The dark matter sterile neutrino can be searched for in particle physics experiments by detailed analysis of the kinematics of β decays of different isotopes [35] and may also have interesting astrophysical applications [36].

3. Baryon asymmetry of the Universe. The baryon (B) and lepton (L) numbers are not conserved in the νMSM. The lepton number is violated by the Majorana neutrino masses, while $B + L$ is broken by the electroweak anomaly. As a result, the sphaleron processes with baryon number non-conservation [37] are in thermal equilibrium for temperatures 100 GeV $< T < 10^{12}$ GeV. As for CP-breaking, the νMSM contains 6 CP-violating phases in the lepton sector and a Kobayashi-Maskawa phase in the quark sector. This makes two of the Sakharov conditions [38] for baryogenesis satisfied. Similarly to the SM, this theory does not have an electroweak phase transition with allowed values for the Higgs mass [39], making impossible the electroweak baryogenesis, associated with the non-equilibrium bubble expansion. However, the νMSM contains extra degrees of freedom - sterile neutrinos - which may be out of thermal equilibrium exactly because their Yukawa couplings to ordinary fermions are very small. The latter fact is a key point for the baryogenesis in the νMSM, ensuring the validity of the third Sakharov condition.

In [40] it was proposed that the baryon asymmetry can be generated through CP-violating sterile neutrino oscillations. For small Majorana masses the total lepton number of the system, defined as the lepton number of active neutrinos plus the total helicity of sterile neutrinos, is conserved and equal to zero during the Universe's evolution. However, because of oscillations the lepton number of active neutrinos becomes different from zero and gets transferred to the baryon number due to rapid sphaleron transitions. Roughly speaking, the resulting baryon asymmetry is equal to the lepton asymmetry at the sphaleron freeze-out.

The kinetics of sterile neutrino oscillations and of the transfers of leptonic number between active and sterile neutrino sectors has been worked out in [2]. The effects to be taken into account include oscillations, creation and destruction of sterile and active neutrinos, coherence in sterile neutrino sector and its lost due to interaction with the medium, dynamical asymmetries in active neutrinos and charged leptons. For masses of sterile neutrinos exceeding ~ 20 GeV the mechanism does not work as the sterile neutrinos equilibrate. The temperature of baryogenesis is right above the electroweak scale.

In [2] it was shown that the νMSM can provide simultaneous solution to the problem of neutrino oscillations, dark matter and baryon asymmetry of the Universe.

4. Inflation. In [3] it was proposed that the νMSM may be extended by a *light* inflaton in order to accommodate inflation. To reduce the number of parameters and to have a common source for the Higgs and sterile neutrino masses the inflaton-νMSM couplings can be taken to be scale invariant at the classical level and the Higgs mass parameter can be set to zero. The mass of the inflaton can be as small as few hundreds MeV, and the coupling of the lightest sterile neutrino to it may serve as an efficient mechanism for the dark matter production.

5. Fine-tunings in the νMSM. The phenomenological success of the νMSM requires a number of fine tunings. In particular, one of the singlet fermion masses should be in the $\mathcal{O}(10)$ keV region to provide a candidate for the dark-matter particle, while two other masses must be much larger but almost degenerate [2, 40] to enhance the CP-violating effects in the sterile neutrino oscillations leading to the baryon asymmetry. In addition, the Yukawa coupling of the dark matter sterile neutrino must be much smaller than the Yukawa couplings of the heavier singlet fermions, to satisfy cosmological and astrophysical constrains [1]. These fine-tunings are "natural" in a sense that they are stable against radiative corrections. Moreover, in [41] was shown that a specific mass-coupling pattern for the singlet fermions, described above, can be a consequence of a lepton number symmetry, slightly broken by the Majorana mass terms and Yukawa coupling constants. At the same time not all 18 new parameters are fixed: the allowed region in parameter space is quite large to yield variety of signatures to be tested with different experiments and methods.

To summarize, none of the *experimental facts*, which are sometimes invoked as the arguments for the existence of the large $\sim 10^{10} - 10^{15}$ GeV intermediate energy scale between the W-boson mass and the Planck mass, really requires it. The smallness of the active neutrino masses may find its explanation in small Yukawa couplings rather than in large energy scale. The dark matter particle, associated usually with some stable SUSY partner of the mass $\mathcal{O}(100)$ GeV or with an axion, can well be a much lighter sterile neutrino, practically stable on the cosmological scales. The thermal leptogenesis [42], working well only at large masses of Majorana fermions, can be replaced by the baryogenesis through light singlet fermion oscillations. The inflation can be associated with the light inflaton field rather than with that with the mass $\sim 10^{13}$ GeV, with the perturbation power spectrum coming from inflaton self-coupling rather than from its mass.

Putting all the physics beyond the Standard Model below the electroweak scale is not harmless, as it can be confronted with experiment at *low* energies (see e.g. [43] for a discussion of neutrinoless double beta-decay in the framework of the νMSM). The aim of this paper is to analyse the possibilities to search for singlet fermions responsible for baryon asymmetry of the Universe in the νMSM. Finding these particles and studying their properties in detail (in particular, CP-violating amplitudes) would allow to compute the *sign* and the *magnitude* of the baryon asymmetry of the Universe theoretically along the lines of [2] and confront this prediction with observations. The existence of the U(1) lepton symmetry provides an argument in favour of $\mathcal{O}(1)$ GeV mass of these singlet leptons [41]. In addition, the structure of their couplings to the particles of the SM is almost fixed by the data on neutrino oscillations. It is interesting to know, therefore, what would be the experimental signatures of the neutral singlet fermions in this mass range and in what kind of experiments they could be found. To answer this question, in this paper we will consider the variant of the νMSM without addition of the inflaton; we will discuss what kind of differences one can expect if the light inflaton is included elsewhere.

Naturally, several distinct strategies can be used for the experimental search of these particles. The first one is related to their production. The singlet fermions participate in all reactions the ordinary neutrinos do with a probability suppressed roughly by a factor $(M_D/M_M)^2$, where M_D and M_M are the Dirac and Majorana masses correspondingly. Since they are massive, the kinematics of, say, two body decays $K^\pm \to \mu^\pm N$, $K^\pm \to e^\pm N$ or three-body decays $K_{L,S} \to \pi^\pm + e^\mp + N$ changes when N is replaced by an ordinary neutrino. Therefore, the study of *kinematics* of rare meson decays can constrain the strength of the coupling of heavy leptons. This strategy has been used in a number of experiments for the search of neutral leptons in the past [44, 45], where the spectra of electrons or muons originating in decays of π- and K-mesons have been studied. The second strategy is to look for the decays of neutral leptons inside a detector [46–49] ("nothing" $\to$ leptons and hadrons). Finally, these two strategies can be unified, so that the production and the decay occurs inside the same detector [50].

Clearly, to find the best way to search for neutral leptons, their decay modes have to be identified and branching ratios must be estimated. A lot of work in this direction has been already done in Refs. [51–54] for the general case; we add new general results for three body meson decays. To analyze the corresponding quantities in the νMSM we will constrain ourselves by the singlet fermion masses below the mass of the beauty mesons, $M_N \lesssim 5$ GeV, considering this mass range as the most plausible because of the reasons presented above. We will use the specific νMSM predictions for the branching ratios.

We arrived at the following conclusions.

(i) The singlet fermions with the masses smaller than M_π are already disfavoured on the basis of existing experimental data of [46, 47] and from the requirement that these particles do not spoil the predictions of the Big Bang Nucleosynthesis (BBN) [55, 56] (s.f. [57]).

(ii) The mass interval $M_\pi < M_N < M_K$ is perfectly allowed from the cosmological and experimental points of view. Moreover, it is not excluded that further constraints on the couplings of singlet fermions can be derived from the reanalysis of the *already existing but never considered from this point of view* experimental data of KLOE collaboration and of the E949 experiment[1]. In addition, the NA48/3 (P326) experiment at CERN would allow to find or to exclude completely singlet fermions with the mass below that of the kaon[2]. The search for the missing energy signal, specific for the experiments mentioned above, can be complemented by the search of decays of neutral fermions, as was done in CERN PS191

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experiment [46, 47]. To this end quite a number of already existing or planned neutrino facilities (related, e.g. to CNGS, MiniBoone, MINOS or T2K), complemented by a near (dedicated) detector (like the one of CERN PS191) can be used[3]. At the same time, the existing setups of the MiniBooNE or MINOS experiments would unlikely allow to probe the cosmologically interesting parameter space of the νMSM for $M_N < 450$ MeV, where strong bounds on the parameters coming from CERN PS191 experiment already exist. However, MiniBooNE and MINOS can possibly improve the existing limits or find neutral fermions in the mass region 450 MeV $< M_N < M_K$, where current bounds are weak (s.f. [57]). The record intensity of the neutrino beam at CNGS and T2K experiment are quite promising for heavy neutrino searches and calls for a detailed study of the possibility of neutral fermions detection at (possible) near detectors.

(iii) For $M_K < M_N < M_D$ the search for the missing energy signal, potentially possible at beauty, charm and τ factories, is unlikely to gain the necessary statistics and is very difficult if not impossible at hadronic machines like LHC[4]. So, the search for decays of neutral fermions is the most effective opportunity. In short, an intensive beam of protons hitting the fixed target, creates, depending on its energy, pions, strange, charmed and beauty mesons that decay and produce heavy neutral leptons. A part of these leptons then decay inside a detector, situated some distance away from the collision point. The dedicated experiments on the basis of the proton beams NuMI or NuTeV at FNAL, CNGS at CERN, or JPARC can touch a very interesting parameter range for $M_N \lesssim 1.8$ GeV.

(iv) Going above D-meson but still below B-meson thresholds is very hard if not impossible with present or planned proton machines or B-factories. To enter into cosmologically interesting parameter space would require the increase of the present intensity of, say, CNGS beam by two orders of magnitude or producing and studying the kinematics of more than 10^{10} B-mesons.

The paper is organized as follows. In Section II we discuss the relevant part of the νMSM Lagrangian and specify the predictions for the couplings of these particles coming from the data on neutrino oscillations and cosmological considerations. In Section III we analyze the present experimental and cosmological limits on the properties of these particles. In

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[3] We thank Francois Vannucci for discussion of this point.

[4] We thank Tasuya Nakada for discussion of this point.

Section IV we analyze the decay modes of singlet fermions. In Section V we consider the production of heavy neutral leptons in decays of K-, D- and B-mesons and of τ-lepton. In Section 6 we analyze the possibilities of their detection in existing and future experiments. We conclude in Section 7.

II. THE LAGRANGIAN AND PARAMETERS OF THE νMSM

For our aim it is more convenient to use the Lagrangian of the νMSM[5] in parameterization of Ref. [41]:

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{MSM}} + \bar{\widetilde{N}}_I i\partial_\mu \gamma^\mu \widetilde{N}_I - F_{\alpha I}\,\bar{L}_\alpha \widetilde{N}_I \tilde{\Phi} - M \bar{\widetilde{N}}_2^{\,c} \widetilde{N}_3 - \frac{\Delta M_{IJ}}{2}\,\bar{\widetilde{N}}_I^{\,c} \widetilde{N}_J + \text{h.c.}\,, \qquad (1)$$

where $\widetilde{N}_I$ are the right-handed singlet leptons (we will keep the notation without tilde for mass eigenstates), $\tilde{\Phi}_i = \epsilon_{ij}\Phi^*_j$, Φ and L_α $(\alpha = e, \mu, \tau)$ are the Higgs and lepton doublets, respectively, F is a matrix of Yukawa coupling constants, M is the common mass of two heavy neutral fermions, ΔM_{IJ} are related to the mass of the lightest sterile neutrino N_1 responsible for dark matter and produce the small splitting of the masses of N_2 and N_3, $\Delta M_{IJ} \ll M$. The Yukawa coupling constants of the dark matter neutrino $|F_{\alpha 1}| \lesssim 10^{-12}$ are strongly bounded by cosmological considerations [1] and by the X-ray observations [5] and can be safely neglected for the present discussion and the sterile neutrino N_1 field can be omitted from the Lagrangian.

In the limit $\Delta M_{IJ} \to 0$, $F_{\alpha 2} \to 0$ the Lagrangian (1) has a global U(1) lepton symmetry [41]. In this paper we will assume that the breaking of this symmetry is small not only in the

[5] Of course, this Lagrangian is not new and is usually used for the explanation of the small values of neutrino masses via the see-saw mechanism [58]. The see-saw scenario assumes that the Yukawa coupling constants of the singlet fermions are of the order of the similar couplings of the charged leptons or quarks and that the Majorana masses of singlet fermions are of the order of the Grand Unified scale. The theory with this choice of parameters can also explain the baryon asymmetry of the Universe but does not give a candidate for a dark matter particle. Another suggestion is to fix the Majorana masses of sterile neutrinos in $1-10$ eV energy range (eV see-saw) [59] to accommodate the LSND anomaly [60]. This type of theory, however, cannot explain dark matter and baryon asymmetry of the universe. Also, the MiniBooNE experiment [61] did not confirm the LSND result. The νMSM paradigm is to determine the Lagrangian parameters from available observations, i.e. from requirement that it should explain neutrino oscillations, dark matter and baryon asymmetry of the universe in a unified way. This leads to the singlet fermion Majorana masses *smaller* than the electroweak scale, in the contrast with the see-saw choice of [58], but much larger than few eV, as in the eV see-saw of [59].

mass sector (which is required for baryogenesis and explanation of dark matter), but also in the Yukawa sector, $|F_{\alpha 3}| \ll |F_{\beta 2}|$. For the case when $|F_{\alpha 3}| \sim |F_{\beta 2}|$ our general conclusions remain the same, but the branching ratios for different reactions can change. In this work we also neglect all CP-violating effects, which go away if the lepton number symmetry is exact.

To characterize the measure of the $U(1)_L$ symmetry breaking, we introduce a small parameter $\epsilon = F_3/F \ll 1$, where $F_i^2 = [F^\dagger F]_{ii}$, and $F_2 \equiv F$. As was shown in [41], there is a lower bound on ϵ coming from the baryon asymmetry of the Universe, $\epsilon \gtrsim 0.6 \cdot \kappa \cdot 10^{-4}(M/\mathrm{GeV})$, where $\kappa = 1(2)$ for the case of normal(inverted) hierarchy in active neutrino sector.

The mass eigenstates ($N_{2,3}$ without tilde) are related to $\widetilde{N}_{2,3}$ by the unitary transformation,

$$\widetilde{N} = U_R N \,, \tag{2}$$

where the 2×2 matrix U_R has the form

$$U_R \simeq \frac{e^{i\phi_0}}{\sqrt{2}} \begin{pmatrix} e^{i\phi_1} & e^{i\phi_2} \\ -e^{-i\phi_2} & e^{-i\phi_1} \end{pmatrix} \,, \tag{3}$$

where the phases ϕ_k can be expressed through the elements of ΔM_{IJ}, the explicit form of which is irrelevant for us.

As a result, for $\epsilon \ll 1$ the interaction of the mass eigenstates N_2 and N_3 has a particular simple form,

$$L_N \simeq -\frac{1}{\sqrt{2}} f_\alpha \bar{L}_\alpha (N_2 + N_3)\tilde{\Phi} - \frac{M_2}{2}\bar{N_2^c} N_2 - \frac{M_3}{2}\bar{N_3^c} N_3 + \mathrm{h.c.} \,, \tag{4}$$

where $f_\alpha = |F_{\alpha 2}|$. The masses M_2 and M_3 must be almost the same (baryogenesis constraint), $\Delta M^2 = |M_2^2 - M_3^2| \lesssim 10^{-5} M^2$ [2, 40, 41]. The baryon asymmetry generation occurs most effectively if $\Delta M^2 \simeq (2\ \mathrm{keV})^2$, but smaller and larger degeneracy works well also.

The fact that two heavy fermions are almost degenerate in mass may be important for analysis of the experimental constraints. In decays of different mesons or τ-lepton a coherent combination $(N_2 + N_3)$ will be created, while in a detector of size l situated on the distance L from the creation point an admixture of the $(N_2 - N_3)$ state with the probability (in the relativistic limit) $P \sim \sin^2\phi$ will appear (E is the energy of the neutral fermion, $\phi = \Delta M^2 L/(4E)$). For $\phi l/L \gg 1$ coherence effects are not essential and the description of the process in terms of N_2 and N_3 is completely adequate, while if $\phi l/L \sim 1$ the coherence effects are important, and order ϵ terms describing the interactions of $(N_2 - N_3)$ with the particles of the SM must be included. Numerically, if $\Delta M^2 \gtrsim (2\text{ keV})^2$, $l \sim 10$ m, and $E < 100$ GeV, then $\phi l/L \gtrsim 10^3$, and $N_2 \leftrightarrow N_3$ oscillations can be safely neglected. Only this case will be considered in what follows.

As it was demonstrated in [41], the coupling constants f_α can be expressed through the elements of the active neutrino mass matrix M_ν. To present the corresponding relations, we parameterize M_ν following Ref. [62]:

$$M_\nu = V^* \cdot \text{diag}(m_1, m_2 e^{2i\delta_1}, m_3 e^{2i\delta_2}) \cdot V^\dagger \ , \tag{5}$$

with $V = R(\theta_{23})\text{diag}(1, \mathrm{e}^{\mathrm{i}\delta_3}, 1)\mathrm{R}(\theta_{13})\mathrm{R}(\theta_{12})$ the active neutrino mixing matrix [63], and choose for normal hierarchy $m_1 < m_2 < m_3$ and for inverted hierarchy $m_3 < m_1 < m_2$. All active neutrino masses are taken to be positive. As was shown in [1, 4–6], the one of the active neutrino masses must be much smaller than the solar mass difference, $m_{\text{sol}} = \sqrt{\Delta m^2_{\text{sol}}} \simeq 0.01$ eV, so that other active neutrino masses are simply equal to $m_{\text{atm}} = \sqrt{\Delta m^2_{\text{atm}}} \simeq 0.05$ eV and to m_{sol} for the case of normal hierarchy and to m_{atm} with a mass splitting $\delta m = m^2_{\text{sol}}/2m_{\text{atm}}$ for the case of inverted hierarchy.

The coupling F is given by [41]:

$$F^2 \simeq \kappa \frac{m_{\text{atm}} M}{2v^2 \epsilon} \ , \tag{6}$$

where $v = 174$ GeV is the vev of the Higgs field and $\kappa \simeq 1(2)$ for the case of normal (inverted) hierarchy.

The ratios of the Yukawa couplings f_α can be expressed through the elements of the active neutrino mixing matrix [41]. A simple expression can be derived for the case $\theta_{13} = 0$, $\theta_{23} = \pi/4$, which is in agreement with the experimental data. For normal hierarchy there are possibilities:

$$f_e^2 : f_\mu^2 : f_\tau^2 \approx \frac{m_2}{m_3} \sin^2\theta_{12} |1 \pm x|^2 : \frac{1}{2}|1 - x^2|^2 : \frac{1}{2}|1 \pm x|^4 \ , \tag{7}$$

where $x = ie^{i(\delta_1 - \delta_2 - \delta_3)} \sqrt{\frac{m_2}{m_3}} \cos\theta_{12}$, and all combinations of signs are admitted. For a numerical estimate one can take [62] $\sin^2\theta_{12} \simeq 0.3$, leading to $x \simeq 0.35ie^{i(\delta_1 - \delta_2 - \delta_3)}$ and to $f_e^2/(f_\mu^2 + f_\tau^2) \sim 0.05$. In other words, the coupling of the singlet fermion to the leptons of the first generation is suppressed, whereas the couplings to the second and third generations are close to each other.

For the case of inverted hierarchy two out of four solutions are almost degenerate and one has [41]:

$$f_e^2 : f_\mu^2 : f_\tau^2 \approx \frac{1+p}{1-p} : \frac{1}{2} : \frac{1}{2} \ , \tag{8}$$

where $p = \pm \sin\delta_1 \sin(2\theta_{12})$. Taking the same value of θ_{12} as before, we arrive at $f_e^2/(f_\mu^2 + f_\tau^2) \sim (0.04 - 25)$, depending on the value of unknown CP-violating phase δ_1. The couplings of $N_{2,3}$ to μ and τ generations are almost identical, but the coupling to electron and its neutrino can be enhanced or suppressed considerably. The corrections to relations (7,8) are of the order of $\mathcal{O}(\epsilon)$ and for $\epsilon \sim 1$ the ratios of the coupling constant can be quite different from those in eqns. (7,8).

The relations (6,7,8) form a basis for our analysis of experimental signatures of heavy neutral leptons. In most of the works the strength of the coupling of a neutral lepton X to charged or neutral currents of flavour α is characterized by quantities $U_{\alpha X}$ and $V_{\alpha X}$. In the case of the νMSM there are two neutral leptons with almost identical couplings (if $\epsilon \ll 1$), so that

$$|U_{\alpha 1}| = |V_{\alpha 1}| = |U_{\alpha 2}| = |V_{\alpha 2}| \equiv |U_\alpha| \ . \tag{9}$$

The overall strength of the coupling is given by

$$U^2 \equiv \sum_\alpha |U_\alpha|^2 = \frac{F^2 v^2}{2M^2} \ , \tag{10}$$

whereas the relations between different flavours follow from (7,8).

As it was found in [41](see also [2, 40]), for successful baryogenesis the constant F must be small enough, $F \lesssim 1.2 \times 10^{-6}$, otherwise N_2 and N_3 come to thermal equilibrium above the electroweak scale and the baryon asymmetry is erased. This leads to the upper bound

$$U^2 < 2\kappa \times 10^{-8} \left(\frac{\text{GeV}}{M}\right)^2 \ . \tag{11}$$

It is the smallness of the required strength of coupling which makes the search for neutral leptons of the νMSM be a very challenging problem, especially for large M.

In the framework of the νMSM, a lower bound on U can be derived as well. The maximal value of the parameter ϵ, characterizing the breaking of the U(1) leptonic symmetry is $\epsilon = 1$. This results in

$$U^2 > 1.3\kappa \times 10^{-11} \left(\frac{\text{GeV}}{M}\right) \ . \tag{12}$$

Further cosmological constraints on the couplings of heavy sterile neutrinos are coming from BBN. The cosmological production rate of these particles peaks roughly at the temperature [55] $T_{peak} \sim 10\,(M/\text{GeV})^{1/3}$ GeV and for $U^2 > 2\times 10^{-13}\,(\text{GeV}/M)$ they were in thermal equilibrium in some region of temperatures around T_{peak}. This is always true, since in the νMSM the constraint (12) is required to be valid. We will see below that the BBN constraints are in fact stronger than those of (12) for relatively small fermion masses $M_N < 1$ GeV. On the basis of inequalities (11,12) and limits from BBN, the νMSM can be probed (either confirmed or ruled out) in particle physics experiments.

The relations (7,8) still allow a lot of freedom in relations between Yukawa couplings to different leptonic flavours, since the Majorana CP-violating phases in the active neutrino mass matrix are not known. Therefore, to present quantitative predictions we will consider three sets of Yukawa couplings corresponding to three "extreme hierarchies", when value of Yukawa constants f_α, f_β are taken to be as small as possible compared to another one f_γ, $\alpha \neq \beta \neq \gamma$, which thus mostly determines the overall strength of mixing U^2. In what follows we will refer to these sets as benchmark models I, II and III with ratios of coupling constants which can be read off from eqs. (7), (8):

$$\begin{aligned}
\text{model I}\ &: \quad f_e^2 : f_\mu^2 : f_\tau^2 \approx 52:1:1\,, \quad \kappa = 1\,,\\
\text{model II}\ &: \quad f_e^2 : f_\mu^2 : f_\tau^2 \approx 1:16:3.8\,, \quad \kappa = 2\,,\\
\text{model III}\ &: \quad f_e^2 : f_\mu^2 : f_\tau^2 \approx 0.061:1:4.3\,, \quad \kappa = 2\,.
\end{aligned}$$

Let us explain how these numbers were obtained. For the model I we simply increase in a maximal way the value of the coupling constant to electron, choosing the appropriate combination of signs in eq. (8). In case of model III the coupling of N to the third generation of leptons is stronger than to the others. This could only happen if the hierarchy of active neutrino masses is normal, see eq. (7). Choosing real and positive x one can see that the maximum value of the ratio $|f_\tau/f_\mu|^2$ is given by

$$\frac{|f_\tau|^2}{|f_\mu|^2} \simeq \left(\frac{1+x}{1-x}\right)^2 . \tag{13}$$

As reference point we choose the central values of parameters of neutrino mixing (see, e.g. [62]), that gives $x \approx 0.35$. This means that the ratio (13) can be as large as 4.3 (varying the parameters of the active neutrino mixing matrix within their error bars one arrives at a bit larger number). By the same type of reasoning the maximal values of the ratio $|f_\tau/f_e|^2$ is given by

$$\frac{|f_\tau|^2}{|f_e|^2} \simeq \left(\frac{m_2}{2m_3}\sin^2\theta_{12}\cdot\left(\frac{1-x}{|1+x|^2}\right)^2\right)^{-1} \simeq 71\ . \quad (14)$$

Similar considerations provide values of Yukawa couplings in model II.

These benchmark models are choosen to show the variety of quantitative predictions within originally 18-dimensional parameter space of νMSM, constrainted already by cosmology, astrophysics, and observations of neutrino oscillatuions. For a given process, they should be confined between numbers given for benchmark models for $\epsilon \ll 1$. A special study should be undertaken to outline the actual range of νMSM predictions in case of $\epsilon \sim 1$, when relations (7) and (8) become invalid.

III. LABORATORY AND BBN CONSTRAINTS ON THE PROPERTIES OF HEAVY LEPTONS

The aim of this section is to discuss whether the past experiments devoted to the search for neutral leptons have entered into cosmologically interesting parameter range defined by eqns. (11,12). In addition, we will consider the Big Bang Nucleosynthesis constraints on the properties of heavy leptons in the νMSM.

The analysis of the published works of different collaborations reveals that for the mass of the neutral lepton $M > 450$ MeV none of the past or existing experiments enter into interesting for νMSM region defined by eq. (11). The NuTeV upper limit on the mixing is at most 10^{-7} in the region $M \simeq 2$ GeV [48], whereas the NOMAD [49] and L3 LEP experiment [50] give much weaker constraints. Note that the eqns. (11,12) give at $M = 2$ GeV: $6\cdot 10^{-12} < U^2/\kappa < 5\cdot 10^{-9}$.

The best constraints in the small mass region, $M < 450$ MeV are coming from the CERN PS191 experiment [46, 47], giving[6] roughly $|U_{e,\mu}|^2 \lesssim 10^{-9}$ in the region 250 MeV $< M <$ 450

[6] *The most recent published results* of CERN SPS experiment [47] contain the exclusion plots up to 400 MeV. In a previous publication, [46], the limit on U_e^2, though not as strong as in [47], was presented up to 450 MeV. We became aware of PhD Thesis of J.-M. Levy [64] (we thank F. Vannucci for providing us a copy of this manuscript) which contains the experimental exclusion plots for U_e^2 and $|U_eU_\mu|$ up to 450 MeV. We use these unpublished results in our work. If the results of [64] are ignored, our plots should be modified accordingly in the region 400 MeV $< M_N <$ 450 MeV, and phenomenologically viable region

MeV (the NuTeV limit in this mass range is some two orders of magnitude weaker). These numbers are already in the region (11) and thus provide non-trivial limits on the parameters of the νMSM. Moreover, as it will be seen immediately, the considerations coming from BBN allow to establish a number of *lower* bounds on the couplings of neutral leptons which decrease considerably the admitted window for the couplings and masses of the neutral leptons.

The successful predictions of the BBN are not spoiled provided the life-time of sterile neutrinos is short enough. Then neutrinos decay before the onset of the BBN and the products of their decays thermalize. This question has been studied in [55] and we will use the results of their general analysis for the case of Models I-III described in Section II.

First, we note that [55] considered the case of one sterile neutrino of Dirac type, whereas we have two Majorana sterile neutrinos[7]. This means that we have exactly the same number of degrees of freedom and that the constraints of [55], expressed in terms of *lifetime* of sterile neutrino are applicable to our case.

Ref. [55] studied in detail only the mass range 10 MeV $< M_N <$ 140 MeV, for higher masses these authors argued that the life-time τ_N of the heavy lepton must be smaller than 0.1 s to definitely avoid any situation when heavy lepton decay products could change the standard BBN pattern of light element abundances. We note in passing that it would be extremely interesting to repeat the computation of [55] for $M_N >$ 140 MeV in order to have a robust BBN constraints in this mass range; meanwhile we will just require (conservatively) that $\tau_N <$ 0.1 s for neutral fermions heavier than π-meson.

For the masses in the interval 10 MeV $< M <$ 140 MeV the constraint on the mixing angle, based on a fit to numerical BBN computations [55], reads

$$U_{I\beta}^2 > \frac{1}{2}\left(s_{1,\beta}\,(M/\mathrm{MeV})_\beta^\alpha + s_{2,\beta}\right) \tag{15}$$

with $s_{1,e} = 140.4$, $s_{1,\mu} = s_{1,\tau} = 568.4$, $s_{2,e} = -1.05\cdot 10^{-5}$, $s_{2,\mu} = s_{2,\tau} = -5.17\cdot 10^{-6}$, $\alpha_e = -3.070$ and $\alpha_\mu = \alpha_\tau = -3.549$ (we took a conservative bound equivalent to adding one extra neutrino species, as explained in [55]); the limits (15) are valid in the models where sterile neutrino mix predominantly with only one active flavor. Here we took into account

expands.

[7] The concentration of the dark matter sterile neutrinos is well below the equilibrium one so that its existence may be safely neglected at this time.

that in Ref. [55] neutrinos of Dirac type have been considered, while we discuss neutrino of Majorana type, hence the total width contains an extra factor 2 in comparison with the Dirac case and the constraint of U^2 is in fact 2 times weaker than that of [55]. The limits (15) can be converted into limits on the mixing U^2 for the models I-III.

To consider higher masses we computed the life-time of heavy leptons (the details of computation can be found in Section IV) and required that it exceeds 0.1 s, to make a conservative exclusion plot. The most important decay channels for $M_N < M_K$ are the two-body semileptonic ones $N \to \pi^0 \nu$, $N \to \pi^\pm e^\mp$, $N \to \pi^\pm \mu^\mp$.

For various patterns of neutrino mixing we present the experimental and BBN constraints in Fig. 1. Note that in extracting the limits on mixing from [46, 47] (this experiment presented 90% confidence level exclusion plot) we also take into account that there are two degenerate neutrinos in the νMSM, and that the constraints in [46, 47] are given for Dirac type sterile neutrinos. For the same value of the mixing angles, the same number of sterile neutrino helicity states are created in both Dirac and Majorana cases, but in the former case only half of states contribute to each decay channel. Hence, the constraints on $|U_e|^2$, $|U_e||U_\mu|$ and $|U_\mu|^2$ are in fact by a factor 2 stronger, since the number of decay events is proportional to $|U|^4$.

One can see that depending on the type of the neutrino mass hierarchy and specific branching ratios in the benchmark models I-III the phenomenologically allowed region of parameter space can be reduced or enlarged. Moreover, the masses below the π meson mass are excluded in most cases[8] but still there are models where small regions of the parameter-space above the pion mass are perfectly allowed[9]. We would also like to stress that the branching ratios for $\epsilon \sim 1$ can be quite different from (7,8) leading to extra uncertainties.

Above pion mass, the BBN limits are down to two order of magnitude below the direct limits form CERN PS191 experiment, thus one-two orders of magnitude improvement is required to either confirm or disprove the νMSM with sterile neutrinos lighter than 450 MeV. For the three benchmark models we transfered these limits to the upper limits on overall

[8] For the νMSM with light inflaton BBN bounds are weaker and masses below pion are certainly allowed [41].

[9] Note that our exclusion plot is different from that of Ref. [57], where the coupling of sterile neutrino to τ generation was not considered. Moreover, eq. (3.1) of this paper contains a factor 4 error. In addition, the formula (21) of [56] for the probability of $N \to \pi^0 \nu$ decay is not correct, see discussion in Section IV.

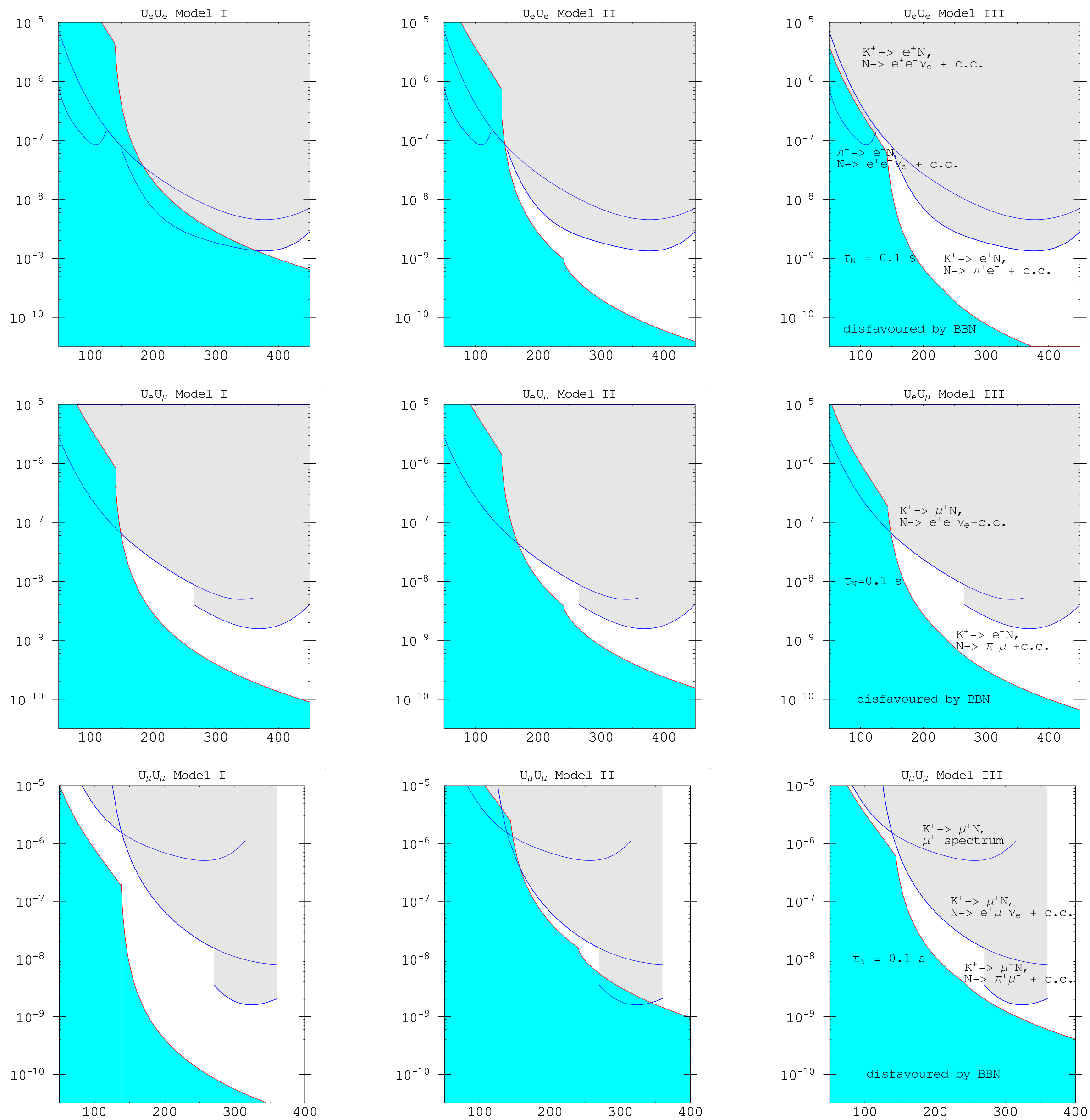

FIG. 1: Limits on $|U_e|^2$, $|U_e||U_\mu|$ and $|U_\mu|^2$ for three benchmark models (I-III from left to right) from BBN (lower bound) and from direct searches in the CERN PS191 experiment (upper bound). Blank regions are phenomenologically allowed.

mixing U^2 and neutrino lifetime and plotted them in Fig. 4.

The improvement required to test the νMSM with sterile neutrinos lighter than 450 MeV can be done with either new kaon experiments, such as one planned in JPARC, or special analysis of the available data on kaon decays collected in Brookhaven and Frascati. In particular, E787/E949 Collaboration reported limit on $K^+ \to \pi^+ X$ decay with X being hypothetical long-lived neutral particle [65]. With statistics of thousand of billions charged kaons, available in this experiment, one can expect to either prove or completely rule out νMSM with sterile neutrinos lighter than 450 MeV. The same conclusion is true for the third stage of CERN NA48 experiment.

In the next two Sections we discuss the decays and production of neutral fermions for a mass range up to 5 GeV, to understand the requirements to possible future experiments that could allow to enter into interesting parameter space for neutral fermion masses above 400 MeV.

IV. DECAYS OF HEAVY NEUTRAL LEPTONS

Heavy neutral leptons we consider ($M_N \gtrsim 10$ MeV) are unstable, since decay channels to light active leptons, $N \to \bar{\nu}_\alpha \nu_\alpha \nu_\beta$, $N \to e^+e^-\nu_\alpha$ are open; the modes like $N_{2,3} \to N_1 + \ldots$ are strongly suppressed. Hereafter charge conjugated modes are also accounted resulting in double rates for Majorana neutrinos as compared to Dirac case. For heavier leptons more decay modes are relevant,

$$N \to \mu e \nu,\ \pi^0\nu,\ \pi e,\ \mu^+\mu^-\nu,\ \pi\mu,\ Ke,\ K\mu\ , \eta\nu, \rho\nu, \ldots$$

Decays of sterile neutrinos have been exhaustedly studied in literature. For convenience we present explicit formulae for relevant decay rates in Appendix A. Most of them (but not all) can be be obtained straightforwardly by making use of the formulae for Dirac neutrinos presented in Ref. [54], which we found to be correct.

Neutrino decays branching ratios for benchmark models I-III and $M_\pi < M_N < 2$ GeV are plotted in Figs. 2, 3. For heavier neutrino many-hadron final states become important, and one can use spectator quarks to calculate the corresponding branching ratios. Below 2 GeV the contribution of these modes to total neutrino width is less than 10%. Neutrino lifetime is constrained by limits (11), (12) on overall strength of mixing. The results for models I, II and III are presented in Fig. 4a: in phenomenologically viable models neutrino lifetime is confined by corresponding solid (upper limits) and dashed (lower limits) lines. The horizontal solid line indicates the order-of-magnitude upper limit on neutrino life time, $\tau_N < 0.1$ c, which guarantees that the results of standard BBN remain intact [55] for $M_N \gtrsim 140$ MeV. In a given model the range of neutrino mass, where the corresponding solid

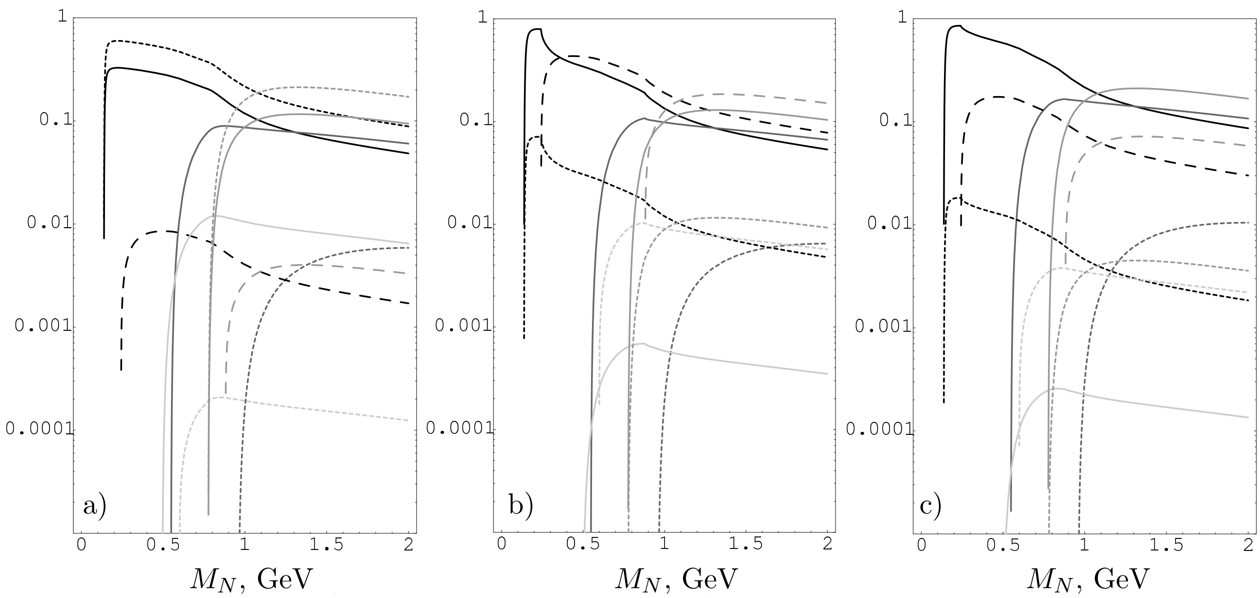

FIG. 2: Branching ratios of neutrino two-body decays $N_I \to XY$ as functions of neutrino mass M_N for models with the same hierarchy in mixing as in models: a) I, b) II, c) III; different lines correspond to different modes: $\pi\nu$ (solid black), πe (short-dashed black), $\pi\mu$ (long-dashed black), Ke (solid light gray), $\eta\nu$ (solid dark gray), $\eta'\nu$ (short-dashed dark gray), $K\mu$ (dashed light gray), $\rho\nu$ (solid gray), ρe (short-dashed gray), $\rho\mu$ (long-dashed gray).

line(s) is(are) above the corresponding dashed one(s) is disfavoured.

These limits imply limits on overall mixing U^2 plotted in Fig.4b: in phenomenologically viable models mixing U^2 is confined by corresponding solid and dashed lines. One can see that the constraint from BBN is stronger than the see-saw constraint (12) for $M \lesssim 1$ GeV. However, it is worth noting that the limit $\tau_N < 0.1$ s may happen to be too conservative and can presumably be relaxed to some extent provided careful study of processes in primordial plasma in BBN epoch. In what follows, for the three benchmark models we give upper and lower limits on various neutrino rates. For a given neutrino mass these limits are saturated respectively by the tightest among upper limits and tightest among lower limits on neutrino mixing, presented in Fig. 4b. Only these tightest limits are used below.

Note in passing that as we already mentioned the νMSM predictions beyond benchmark models could deviate to some extent from a naive interplay between benchmark numbers. At the same time for any set of parameters the presence of both upper and lower bounds on neutrino rates is a general feature of νMSM, which allows it to be falsified.

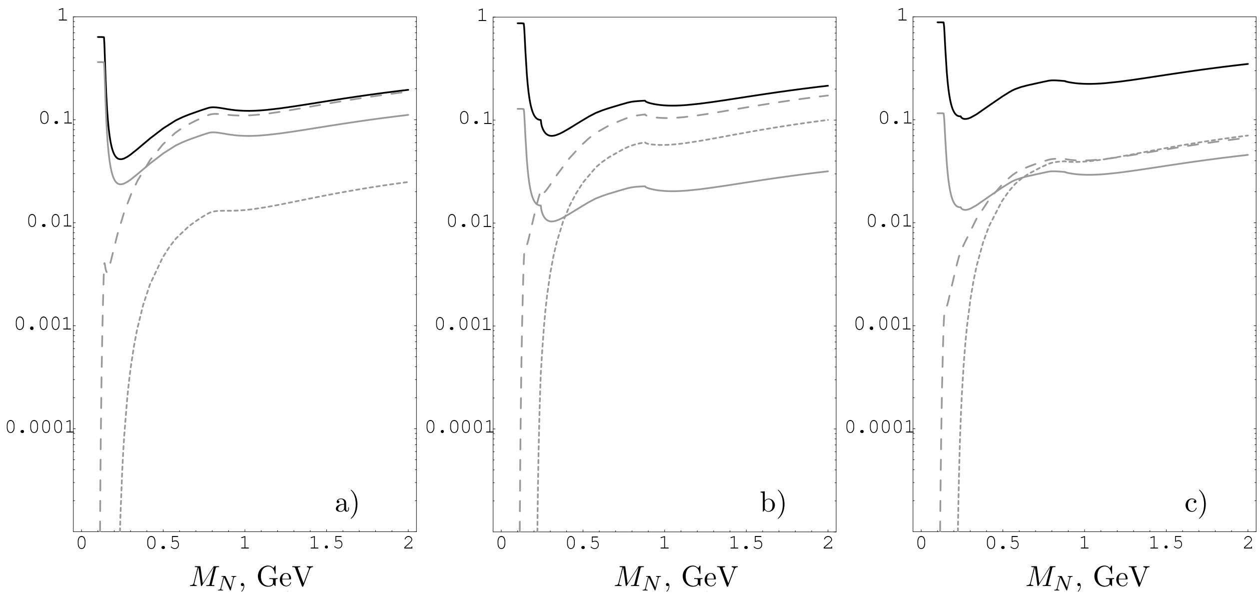

FIG. 3: Branching ratios of neutrino three-body decays $N_I \to ABC$ as functions of neutrino mass M_N for models with the same hierarchy in mixing as in models: a) I, b) II, c) III; different lines correspond to different modes: $\bar{\nu}\nu\nu$ (sum over all invisible modes, solid black), νe^+e^- (solid gray), $\nu e\mu$ (sum over two modes, long-dashed gray), $\nu\mu^+\mu^-$ (short-dashed gray).

V. PRODUCTION OF HEAVY NEUTRAL LEPTONS

In high-energy experiments the most powerful sources of heavy neutral leptons are the kinematically allowed weak decays of mesons (and baryons) created in beam-beam and beam-target collisions. Obviously, the relevant hadrons are those which are stable with respect to strong and electromagnetic decays.

The spectrum of outgoing heavy neutral leptons N in a given experiment is determined mostly by the spectrum of produced hadrons H subsequently decaying into heavy leptons. Since relevant hadrons contain one heavy quark Q, differential cross section of their direct production $d\sigma_H^{dir}$ can be estimated by use of the factorization theorem

$$\frac{d\sigma_H^{dir}}{dp_{H,L}dp_{H,T}^2} = \int_0^1 dz \cdot \delta\left(p_Q - zp_H\right) \cdot D_{H,Q}(z) \cdot \frac{d\sigma_Q^{dir}}{dp_{Q,L}dp_{Q,T}^2} \;, \tag{16}$$

where $d\sigma_Q^{dir}$ is differential cross section of direct Q-quark production[10], $p_{H,L}$, $p_{H,T}$ and $p_{Q,L}$, $p_{Q,T}$ are longitudinal and transverse spatial momenta of hadron H and heavy quark Q,

[10] We assume non-polarized beam(s) and target and hence axial symmetry.

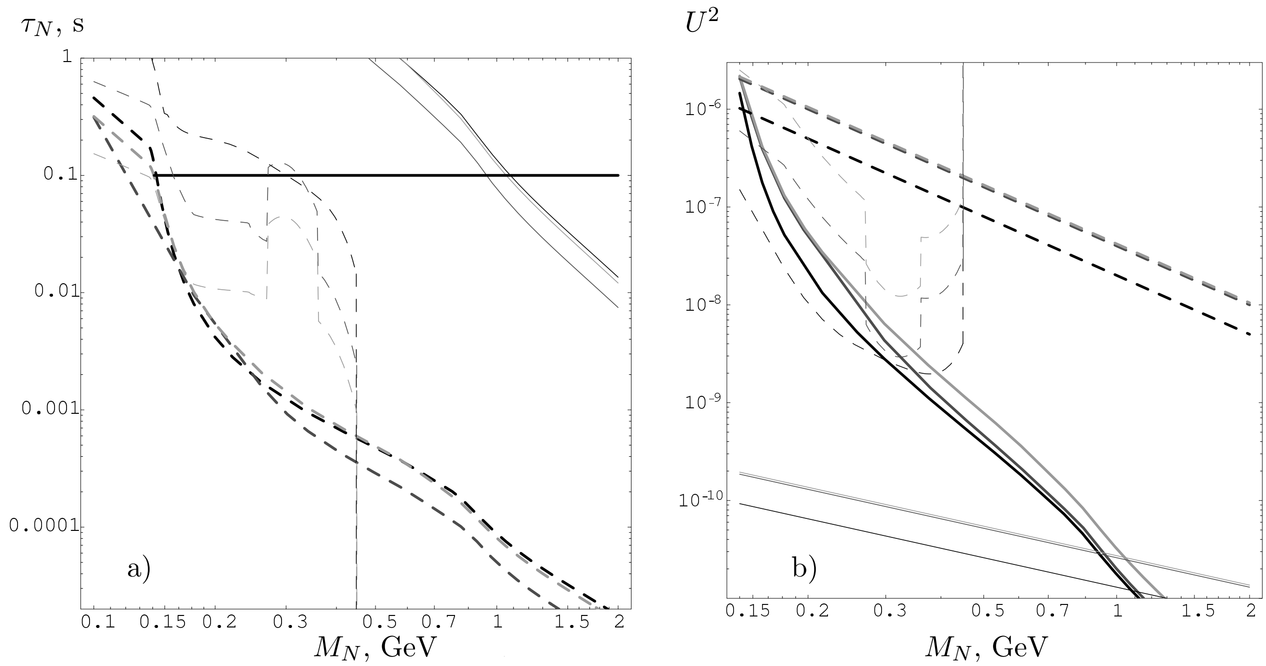

FIG. 4: a) Upper (solid lines) and lower limits (dashed lines) on neutrino lifetime in models I (black), II (dark gray) and III (light gray); a horizontal thick solid black line indicates the upper limit from BBN, $\tau_N < 0.1$ c, suggested in Ref.[55], thin solid lines are limits from eq. (12), thick dashed lines refer to eq. (11), thin dashed lines correspond to limits from direct searches for sterile neutrinos discussed in Section III; charge-conjugated modes are accounted. b) Lower (solid lines) and upper limits (dashed lines) on overall mixing U^2 in models I (black), II (dark gray) and III (light gray). Thin solid lines and thick dashed lines depict limits from eqs. (12) and (11), respectively. Thick solid lines indicate lower limits from order-of-magnitude BBN bound on neutrino lifetime, $\tau_N < 0.1$ c for $M_N > 140$ MeV, thin dashed lines refer to limits from direct searches for sterile neutrinos discussed in Section III.

respectively; zp_H is a part of hadron momentum carried by heavy quark and a fragmentation function $D_{H,Q}(z)$ describes the details of hadronization. The differential cross section entering the integrand in eq.(16) can be calculated within perturbative QCD, while function $D_{H,Q}(z)$ comprises non-perturbative information. There are several approximations to $D_{H,Q}(z)$ in literature, e.g. commonly used in high energy physics generator PYTHIA adopts modified Lund fragmentation function [66]

$$D(z) \propto \frac{(1-z)^a}{z^{1+b\cdot m_Q^2}} \cdot \mathrm{e}^{-\frac{b}{z}\cdot\left(M_H^2+p_{H,T}^2\right)}$$

with default parameters $a = 0.3$ and $b = 0.58$ GeV^{-2}.

The rate of hadron production depends on the intensity of collisions. The distribution of total number of directly produced hadrons dN_H^{dir} reads

$$\frac{dN_H^{dir}}{dp_{H,L}dp_{H,T}^2} = \frac{d\sigma_H^{dir}}{dp_{H,L}dp_{H,T}^2} \cdot \mathcal{L}_{acc} \ ,$$

where $\mathcal{L}_{acc}$ is an integrated luminosity of a given experiment and we neglect tiny imprints of real bunch structure on outgoing hadronic spectra. Note that we are interested in hadrons stable with respect to strong and electromagnetic decays, thus apart of direct production they emerge due to strong and electromagnetic decays of other hadrons, which give indirect contribution dN_H^{ind}. The distribution of the total number of produced hadrons dN_H is a sum of both contributions,

$$\frac{dN_H}{dp_{H,L}dp_{H,T}^2} = \frac{dN_H^{dir}}{dp_{H,L}dp_{H,T}^2} + \frac{dN_H^{ind}}{dp_{H,L}dp_{H,T}^2} \ .$$

Produced hadrons stable with respect to strong and electromagnetic decays travel distances of about $\beta_H \cdot \tau_H \cdot \gamma_H$ (β_H, τ_H and γ_H are speed, lifetime and boost factor of a given hadron) and then decay weakly, producing some amount of heavy neutral leptons. In the hadron rest frame the spatial momentum of heavy lepton p_N can be correlated with the hadron total spin. Consequently, in the laboratory frame there can be additional to Lorenz boost contribution to correlations between p_N and p_H. This contribution is smearing with growth of statistics and can be also neglected if typical γ-factor of hadrons is large, $\gamma_H = E_H/M_H \gg 1$. Hence, in the laboratory frame, the distribution of heavy leptons over spatial momentum is given by

$$\begin{aligned} \frac{dN_N}{dp_{N,L}dp_{N,T}^2} = \sum_H \tau_H \cdot \int \frac{dB_H\left(H \to N + \ldots\right)}{dE_N} \cdot dE_N \\ \times \int d^3\mathbf{n}_\gamma \cdot \delta\left(\mathbf{p_N} - \mathbf{p_H} - \mathbf{n}_\gamma \cdot \sqrt{E_N^2 - M_N^2}\right) \cdot \frac{dN_H}{dp_{H,L}dp_{H,T}^2} \ , \end{aligned} \tag{17}$$

where we integrate over unit sphere boosted to laboratory frame and sum up all contributions from all relevant hadrons; $dB_H\left(H \to N + \ldots\right)$ is a differential inclusive branching ratio of hadron H into heavy neutrino. These branching ratios can be straightforwardly obtained for each hadron with help of the standard technique used to calculate weak decays in the framework of the SM. Indeed, in both models (MSM and νMSM) neutrinos are produced mostly via virtual W-boson (charged current): the only difference is that in νMSM neutrinos are massive. For heavy neutrinos this results[11] in enhancement of pure leptonic decay modes which are strongly suppressed in the SM by charged lepton masses.

The heavier the quark the lower its production rate; hence, a class of the lightest kinematically allowed hadrons saturates heavy neutrino production. As we explained in Section III, neutrinos in phenomenologically viable νMSM are likely to be heavier than pion. If neutrino N_I is lighter than kaon, the dominant source of neutrinos is decaying kaons,

$$K^{\pm} \to l_{\alpha}^{\pm} N_I \ , \tag{18}$$

$$K_L \to \pi^{\mp} l_{\alpha}^{\pm} N_I \ . \tag{19}$$

The two-body decays (18) have been already studied in literature (see, e.g., Refs. [52, 53]). For convenience, the differential branching ratio is presented in Appendix B. Contribution of three-body decays (19) to neutrino production is suppressed by phase volume factor; as the largest impact they give a few per cent at $M_N \simeq M_{\pi}$; the corresponding differential branching ratio is presented in Appendix B.

In models with neutrino heavier than kaon but lighter than charmed hadrons, decays of those latter dominate neutrino production. The largest partial width to heavy neutrinos is exhibited by D_s-meson which leptonic decays $D_s \to l_{\alpha} N_I$ are not suppressed by CKM mixing angles as compared to similar decays of D-mesons, $D^{\pm} \to l_{\alpha}^{\pm} N_I$. Semileptonic three-body decay modes

$$D_s \to \eta^{(\prime)} l_{\alpha} N_I \ , \quad D \to K l_{\alpha} N_I \ , \tag{20}$$

$$D_s \to \phi l_{\alpha} N_I \ , \quad D \to K^* l_{\alpha} N_I \tag{21}$$

are unsuppressed by CKM-mixing as well, and are sub-dominant in general. For sufficiently light neutrinos, $M_K \lesssim M_N \lesssim 700$ MeV, D-meson semileptonic decay modes give contribution comparable to $D_s \to l_{\alpha} N_I$ at $M_K \lesssim M_N \lesssim 700$ MeV, because the D-meson total production dominates over D_s-production in hadronic collisions. Differential branching ratios of the leptonic decays and the semileptonic decays (20) of charmed mesons are provided by general formulae in Appendix B, where the expression of differential branching ratio to vector mesons $V = \phi, K^*$ (21) is also presented. Both the rest of kinematically allowed

[11] Also, in models with heavy neutrinos values of hadronic form factors governing semileptonic width are changed in accordance with shift in virtuality of W-boson.

three-body decay modes and four-body decay modes, e.g. $D \to K\pi l_\alpha N_I$, are strongly suppressed by either CKM mixing or phase volume factor and can be neglected. The largest contribution from charmed baryons comes from the decay $\Lambda_c \to \Lambda l_\alpha N$ and is negligibly small for heavy neutrino production.

In models where neutrino masses are within the range 2 GeV $\lesssim M_N \lesssim$ 5 GeV, neutrinos are produced mostly in decays of beauty mesons. These are also mostly leptonic and semileptonic decays, which branching ratios are described by general formulae presented in Appendix B. As compared to D-meson decays, B-meson decays into heavy neutrinos are strongly suppressed by off-diagonal entries of CKM matrix. For neutrinos lighter than about 2.5 GeV semileptonic modes to charm mesons, e.g. $B \to D^{(*)} l N_I$, dominate over leptonic mode $B \to l N_I$ because of both larger CKM-mixing, $|V_{bc}| \gg |V_{bu}|$, and larger values of hadronic form factors,[12] $f_B/M_B \ll f_+, f_0$. $B_c \to l N_I$ is more promising, but B_c-production in hadron collisions is suppressed. For heavier neutrinos leptonic modes dominate. The baryon contribution is subdominant at any M_N.

Note, that additional, but always subdominant, contribution to heavy neutrino production comes from decays of τ-leptons (if kinematically allowed), which emerge as results of decays of D_s- and B-mesons.

The total number of produced heavy leptons N_N is given by the integration of eq. (17) over $\mathbf{n}_\gamma$ and E_N. For order-of-magnitude estimates one can use the following simple approximation,

$$N_N = \sum_H N_H \cdot \mathrm{Br}\left(H \to N \ldots\right) \ ,$$

with N_H being a total number of produced hadrons H, which in turn can be estimated as

$$N_H = N_Q \cdot \mathrm{Br}\left(Q \to H\right) \ ,$$

where N_Q is a total number of produced heavy quarks Q and $\mathrm{Br}\left(Q \to H\right)$ is a relative weight of the channel $Q \to H$ in Q-quark hadronization. For strange meson the reasonable estimate is $\mathrm{Br}\left(s \to K^-\right) = \mathrm{Br}\left(s \to K_L\right)$. Following Ref. [67] we set $\mathrm{Br}\left(c \to D^+\right) = 0.4 \cdot \mathrm{Br}\left(c \to D_0\right)$ and assuming $\mathrm{Br}\left(c \to D_s\right) = \mathrm{Br}\left(c \to \Lambda_c\right)$ obtain for relevant hadrons

$$\mathrm{Br}\left(c \to D^+\right) = 0.2 \ , \quad \mathrm{Br}\left(c \to D_0\right) = 0.5 \ , \quad \mathrm{Br}\left(c \to D_s\right) = 0.15 \ .$$

[12] This is a consequence of strong overlapping between quark wave functions in the meson required to produce virtual W-boson in case of leptonic decay.

For beauty mesons we use [68]

$$\mathrm{Br}\left(b \to B^+\right) = \mathrm{Br}\left(b \to B^0\right) = 0.4\ , \quad \mathrm{Br}\left(b \to B_s\right) = 0.1\ .$$

For each heavy quark Q the dominant contribution to heavy neutral lepton production comes from leptonic and semileptonic decays of mesons. The limits on branching ratios for relevant decays are plotted in Figs. 5-15 as function of neutrino mass for three benchmark

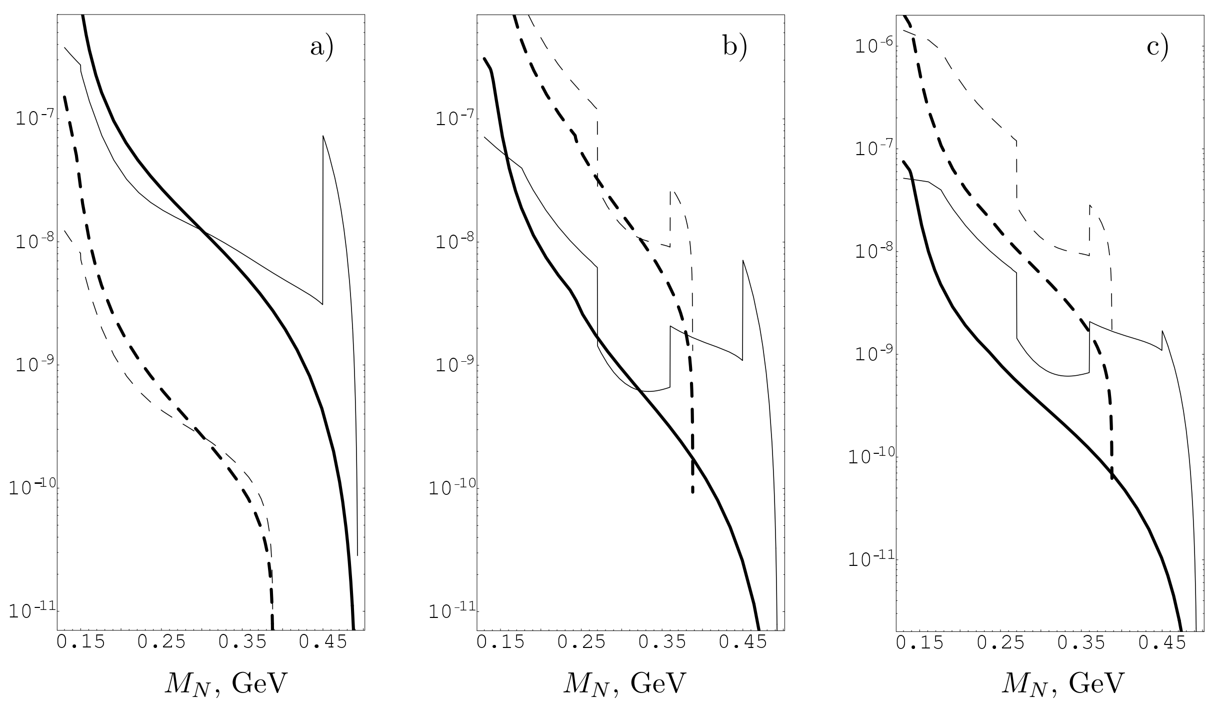

FIG. 5: Branching ratios of decays $K \to eN_I$ (solid lines) and $K \to \mu N_I$ (dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_K$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively.

models. Within νMSM the interesting branching ratios are confined between corresponding thin (upper limit) and thick (lower limit) lines: inside these regions all limits on U^2 plotted in Fig. 4 are fulfilled, in a given model the neutrino mass region, where the corresponding thin line is below the corresponding thick line, is disfavoured. Rate doubling due to heavy neutrino degeneracy is taken into account.

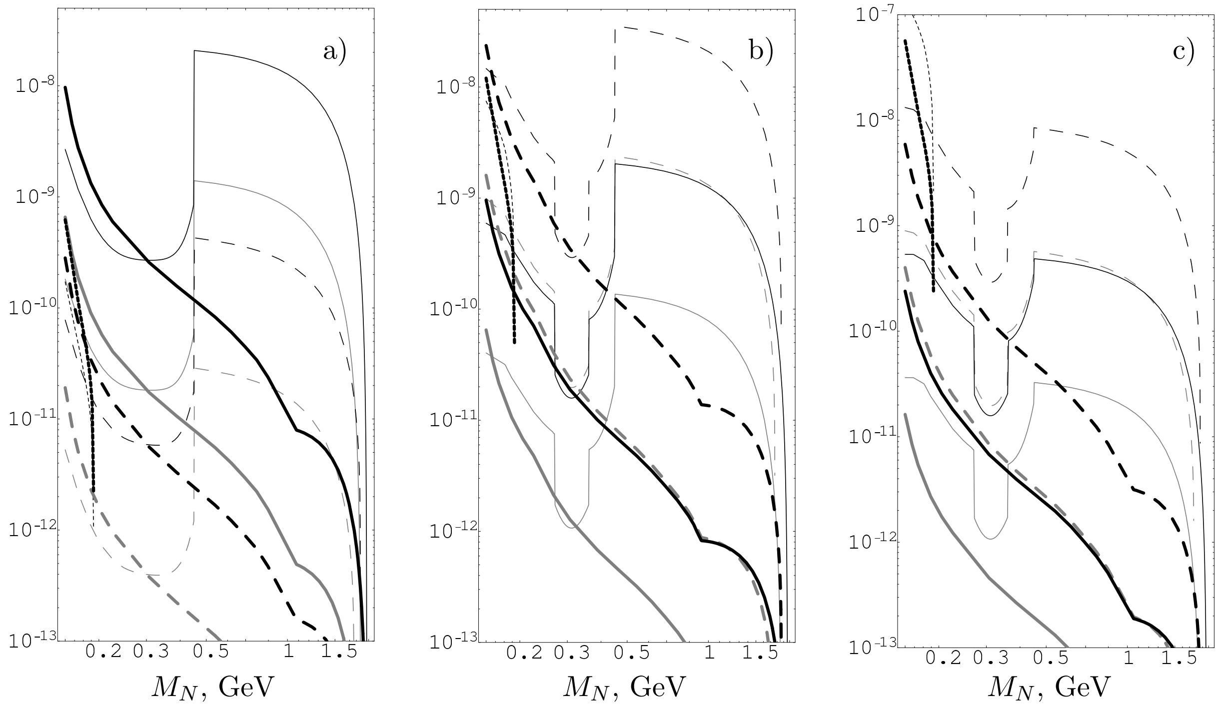

FIG. 6: Branching ratios of decays $D \to eN_I$ (gray solid lines), $D \to \mu N_I$ (gray long-dashed lines), $D_s \to eN_I$ (black solid lines), $D_s \to \mu N_I$ (black long-dashed lines) and $D_s \to \tau N_I$ (black short-dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively.

The two-body decays can be searched for to probe νMSM: produced charged leptons are monochromatic with spatial momenta

$$|\mathbf{p_l}| = \sqrt{\left(\frac{M_H^2 + M_N^2 - M_l^2}{2M_H}\right)^2 - M_N^2} \; .$$

The positions of these peaks in charged lepton spectra and their heights are correlated obviously for different modes and mesons. These features is a very clean signature of heavy leptons. From the plots in Fig. 6 one concludes that statistics of billions charmed hadrons is needed to probe νMSM with neutrino of masses 0.5 GeV $\lesssim M_N \lesssim$ 2 GeV. In models with lighter neutrinos kaon decays are important and required statistics is smaller. Contrary, in models with heavier neutrinos statistics has to be larger and it is a challenging task for future

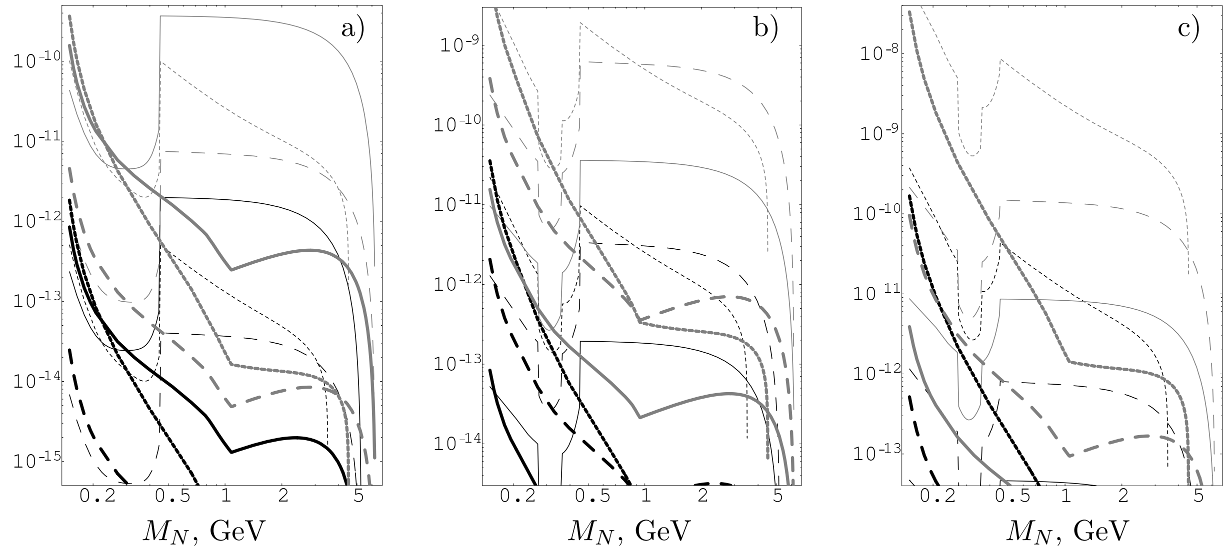

FIG. 7: Branching ratios of decays $B \to eN_I$ (black solid lines), $B \to \mu N_I$ (black long-dashed lines), $B \to \tau N_I$ (black short-dashed lines), $B_c \to eN_I$ (gray solid lines), $B_c \to \mu N_I$ (gray long-dashed lines) and $B_c \to \tau N_I$ (gray short-dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_B$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively.

B-factories. Note that the set of phenomenologically interesting models where neutrinos are produced in kaon decays can be examined completely, as it requires billions of kaons and collected World statistics is much larger.

Semileptonic decays also contribute to heavy lepton production, but spectra of outgoing leptons and mesons are not monoenergetic, making this process be less promising probe of νMSM heavy neutrinos.

To illustrate the relative weight of different mesons in total neutrino production we plot in Fig. 16 the quantity

$$\xi_Q \equiv \sum_H \xi_{Q,H} \;, \qquad \xi_{Q,H} \equiv \mathrm{Br}\,(Q \to H) \cdot \mathrm{Br}\,(H \to N \ldots)$$

(where all considered above leptonic and semileptonic decays of strange, charmed and beauty mesons are taken into account, $Q = s, c, b$) within relevant ranges of neutrino masses M_N.

With a reasonable estimate of strange, charm and beauty cross sections at large ener-

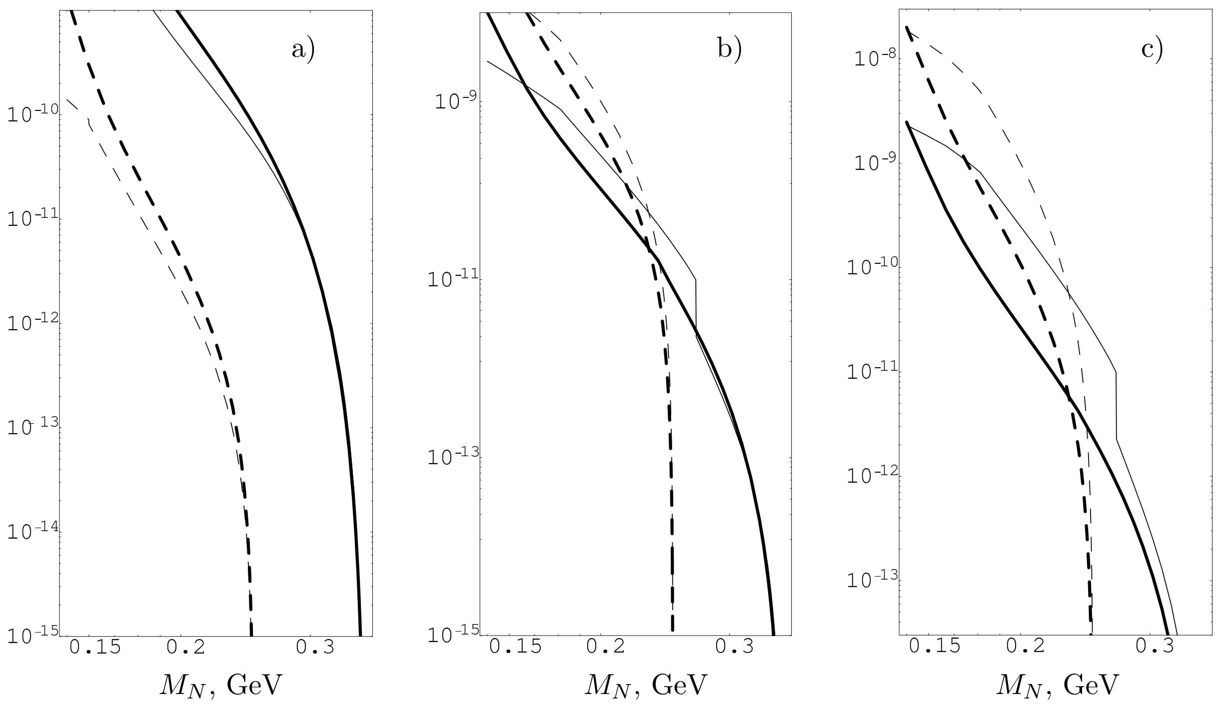

FIG. 8: Branching ratios of semileptonic decays $K \to \pi e N_I$ (solid lines) and $K \to \pi\mu N_I$ (dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_K$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors are taken from Refs. [68].

gies [67]

$$\sigma_{pp\to s} \sim 1/7 \cdot \sigma_{pp}^{total} \;, \quad \sigma_{pp\to c} \sim 10^{-3} \cdot \sigma_{pp}^{total} \;, \quad \sigma_{pp\to b} \sim 10^{-5} \cdot \sigma_{pp}^{total} \;,$$

one concludes that to produce a few neutrinos lighter than kaon, 10^7-10^{10} collisions is required, while for heavier neutrinos the statistics should be four orders of magnitude (0.5 GeV $\lesssim M_N \lesssim$ 2 GeV) or even eight orders of magnitude (2 GeV $\lesssim M_N \lesssim$ 4 GeV) larger.

Note in passing that in our considerations baryon decays as well as decays with more than three particles in a final state have been neglected. These additional contributions to neutrino production are expected to be insignificant.

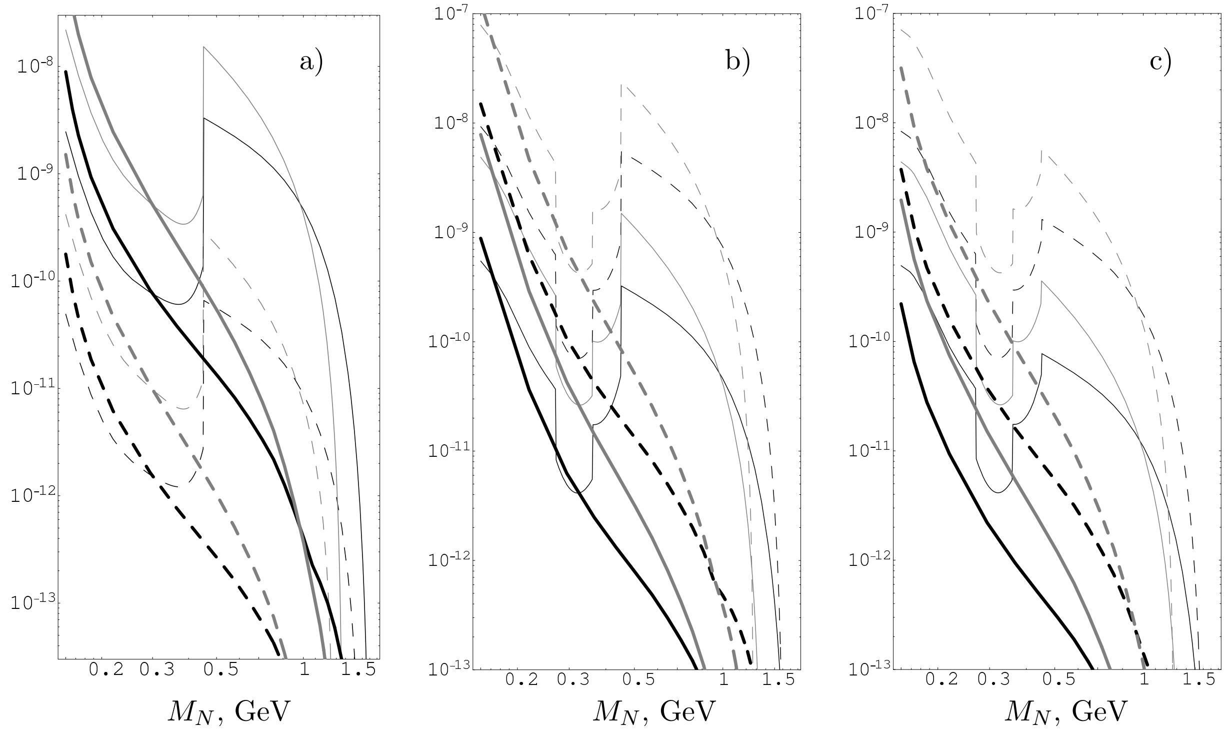

FIG. 9: Branching ratios of semileptonic decays $D \to \pi e N_I$ (black solid lines), $D \to \pi \mu N_I$ (black dashed lines), $D \to K e N_I$ (gray solid lines) and $D \to K \mu N_I$ (gray dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors are taken from Refs. [69].

VI. PROSPECTS FOR FUTURE EXPERIMENTS

Generally, there are two types of processes where heavy neutrinos can be searched for: neutrino production hadron decays and neutrino decays into SM particles.

In Section V we presented plots with hadron branching ratios to neutrinos in the frameworks of the three benchmark models. ¿From these plots one can conclude that statistics expected at proposed Super B-factories give a chance to explore νMSM with neutrinos lighter than about 1 GeV and probe some part of parameter space, if neutrino masses are in 1-2 GeV range. For heavier neutrinos typical branching ratios become too small, so even with large number of available hadrons actually small uncertainties in prediction of background

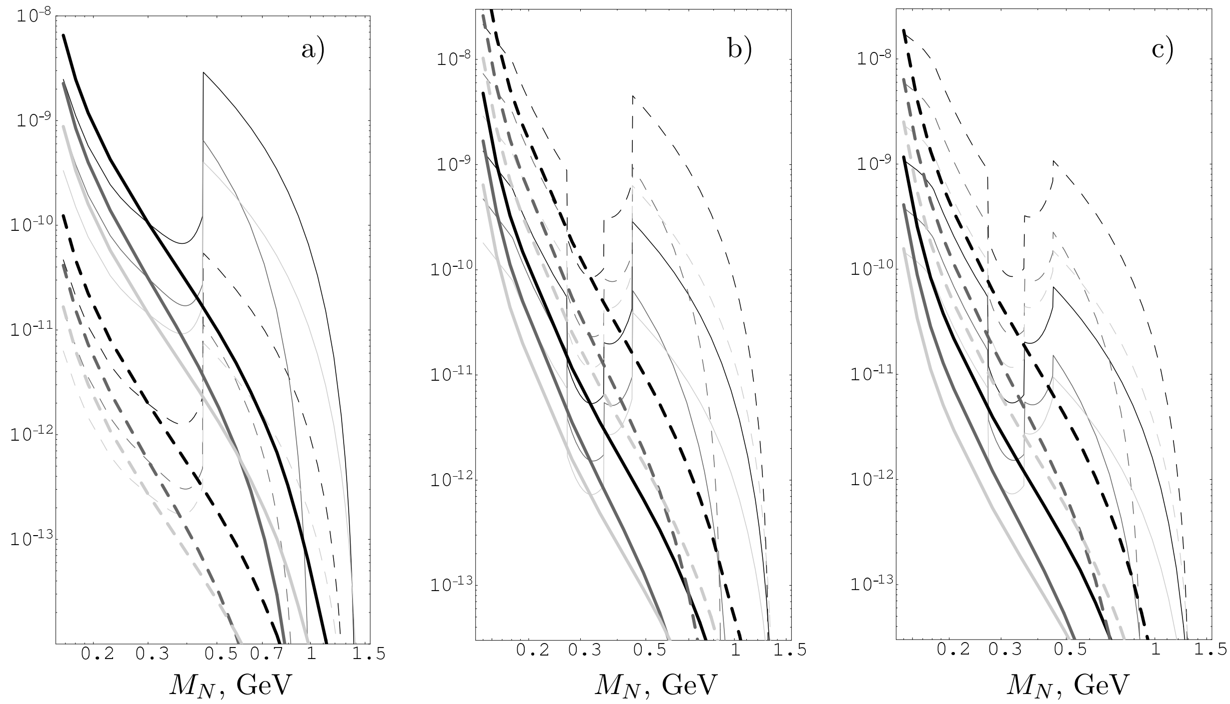

FIG. 10: Branching ratios of semileptonic decays $D_s \to XlN_I$, $X = \eta, \eta', K$ (black, dark gray, light gray lines), $l = e, \mu$ (solid and dashed lines), as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors are taken from Ref. [70].

can make any searches insensitive.

For heavier neutrinos the most promising experiments are beam-target experiments with high intensity of a beam and high energy of incident protons. Heavy neutrinos from decays of numerous secondary hadrons will travel some distance and then decay into SM particles with branching ratios discussed in Section IV. With lifetime in the range $10^{-1} \div 10^{-5}$ s neutrino covers a distance in exceed of one kilometer, so a detector aimed at searches for neutrino decay signatures should be placed at an appropriate small distance from the target to avoid decrease of statistics due to neutrino beam divergence. In what follows we consider the experimental setup with appropriately thin target, assuming that produced in beam-target collision hadrons decay freely without further interaction inside the target. So, this

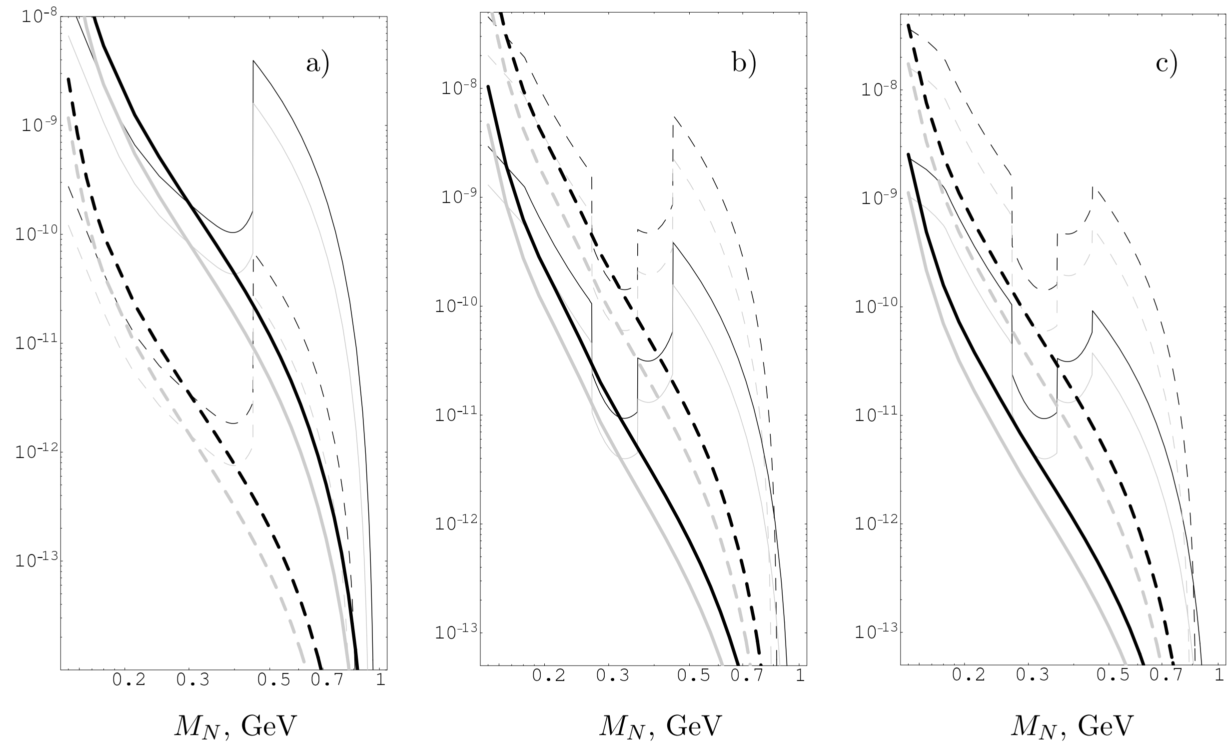

FIG. 11: Branching ratios of semileptonic decays $D \to K^* e N_I$ (black solid lines), $D \to K^* \mu N_I$ (black dashed lines), $D_s \to \phi e N_I$ (gray solid lines) and $D_s \to \phi \mu N_I$ (gray dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors are taken from Refs. [70, 71].

is not a classical beam-dump setup. For classical beam-dump experiment secondary kaons interact in material before decay, that change their contribution to production of neutrinos with $M_N < M_K$, which estimate requires additional study. Heavier neutrinos are produced mostly by D- and B-mesons, which even in beam-dump setup decay before interaction. *Hence, for $M_N > M_K$ our results obtained below are valid for beam-dump experiment as well.*

The total number of neutrinos produced by N_{POT} incident upon a target protons with energy E is given by

$$N_N(E) = \sum_{Q=u,d,s,\dots} \xi_Q \cdot \frac{\sigma_{pA\to Q}(E)}{\sigma_{pA}^{total}(E)} \cdot N_{POT}(E) \cdot M_{pp}(E) \; ,$$

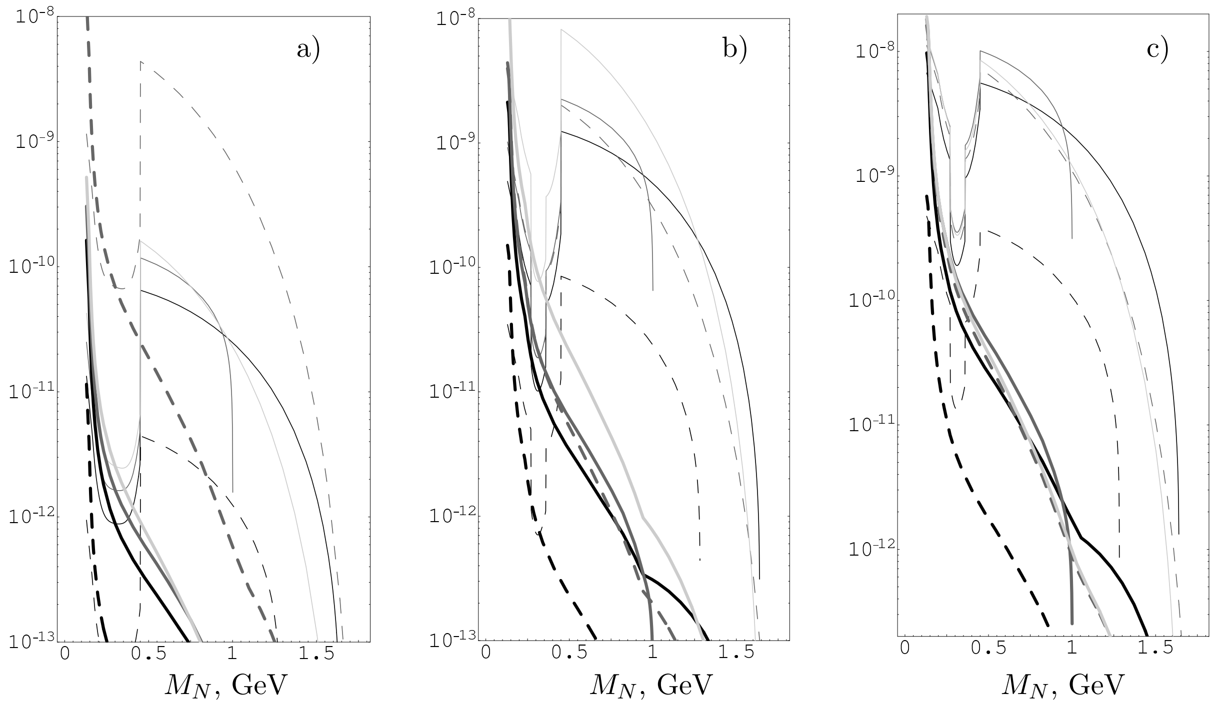

FIG. 12: Branching ratios of decays $\tau \to \pi N$ (black solid lines), $\tau \to KN$ (black dashed lines), $\tau \to \rho N$ (dark gray solid lines), $\tau \to \nu e N_I$ (sum over all active neutrino species, dark gray dashed lines), $\tau \to \nu\mu N_I$ (sum over all active neutrino species, light gray solid lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_\tau$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively.

where A refers to the target material and $M_{pp}(E)$ is a total multiplicity (average number of secondary particles in proton-proton collision). Here we suppose that all beam protons interact once in the target; the account of finite thickness of the target is straightforward and results in effective decrease in N_{POT}. Assuming as a reasonable approximation at large E

$$\frac{\sigma_{pA\to Q}(E)}{\sigma_{pA}^{total}(E)} \approx \frac{\sigma_{pp\to Q}(E)}{\sigma_{pp}^{total}(E)} \equiv \chi_Q(E) \,,$$

we arrived at

$$N_N(E) = \sum_{Q=s,c,b} \xi_Q \cdot \chi_Q(E) \cdot N_{POT}(E) \cdot M_{pp}(E) \,.$$

Below we present the numerical estimates for four high energy beams available today or will be available in the nearest future: CNGS, NuMi, JPARC (T2K setup) and TeVatron

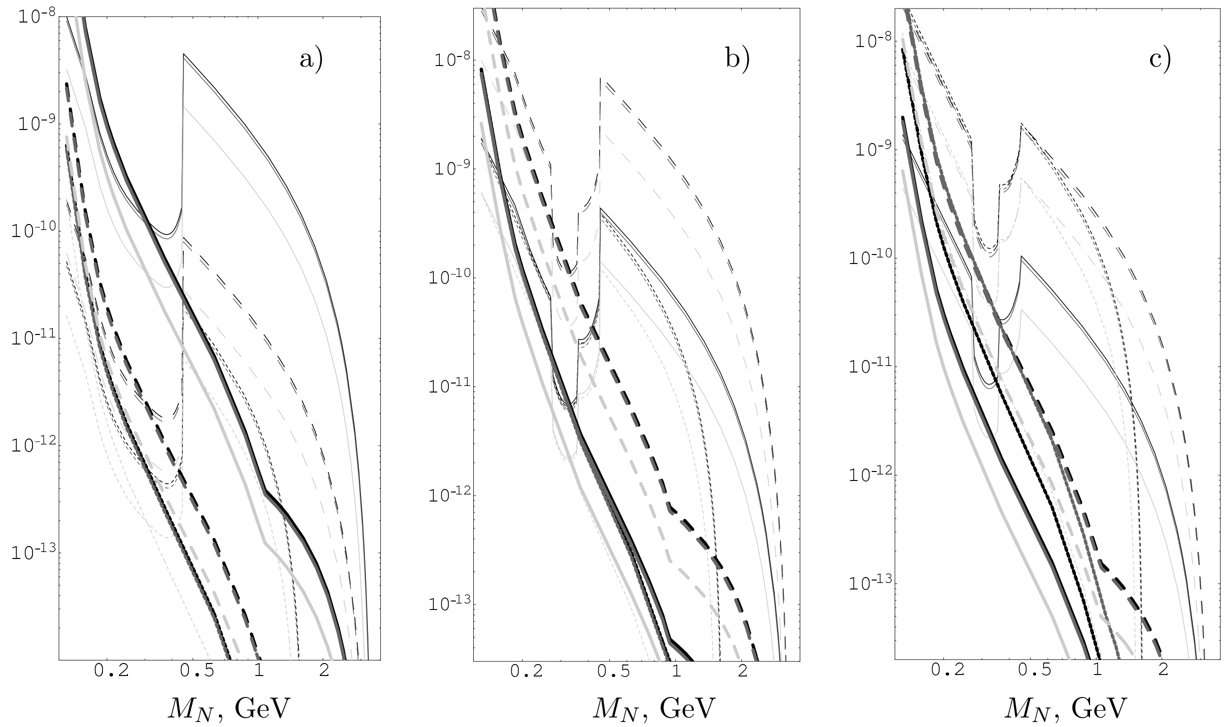

FIG. 13: Branching ratios of semileptonic decays $B \to DlN_I$ (black lines), $B_s \to D_s lN_I$ (dark gray lines) and $B_c \to \eta_c lN_I$ (light gray lines), $l = e, \mu, \tau$ (solid, long dashed and short dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_B$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors for B_s-meson decays are taken to be equal to the form factors for B-meson decays from Ref. [71], for B_c-meson decays we adopted form factors from Ref. [72].

(NuTeV setup). The relevant parameters of these beams are presented in Table I. To

Experiment	E, GeV	N_{POT}, 10^{19}	M_{pp} [68]	χ_s [66]	χ_c [67]	χ_b [67]	$\langle p_L^K \rangle$, GeV	$\langle p_L^D \rangle$, GeV	$\langle p_L^B \rangle$, GeV
CNGS [73]	400	4.5	13	1/7	$0.45 \cdot 10^{-3}$	$3 \cdot 10^{-8}$	44	58	58
NuMi [74]	120	5	11	1/7	$1 \cdot 10^{-4}$	10^{-10}	24	24	24
T2K [75]	50	100	7	1/7	$1 \cdot 10^{-5}$	10^{-12}	8.5	10	10
NuTeV [76]	800	1	15	1/7	$1 \cdot 10^{-3}$	$2 \cdot 10^{-7}$	68	82	82

TABLE I: Adopted values of relevant for heavy neutrino production parameters of several experiments.

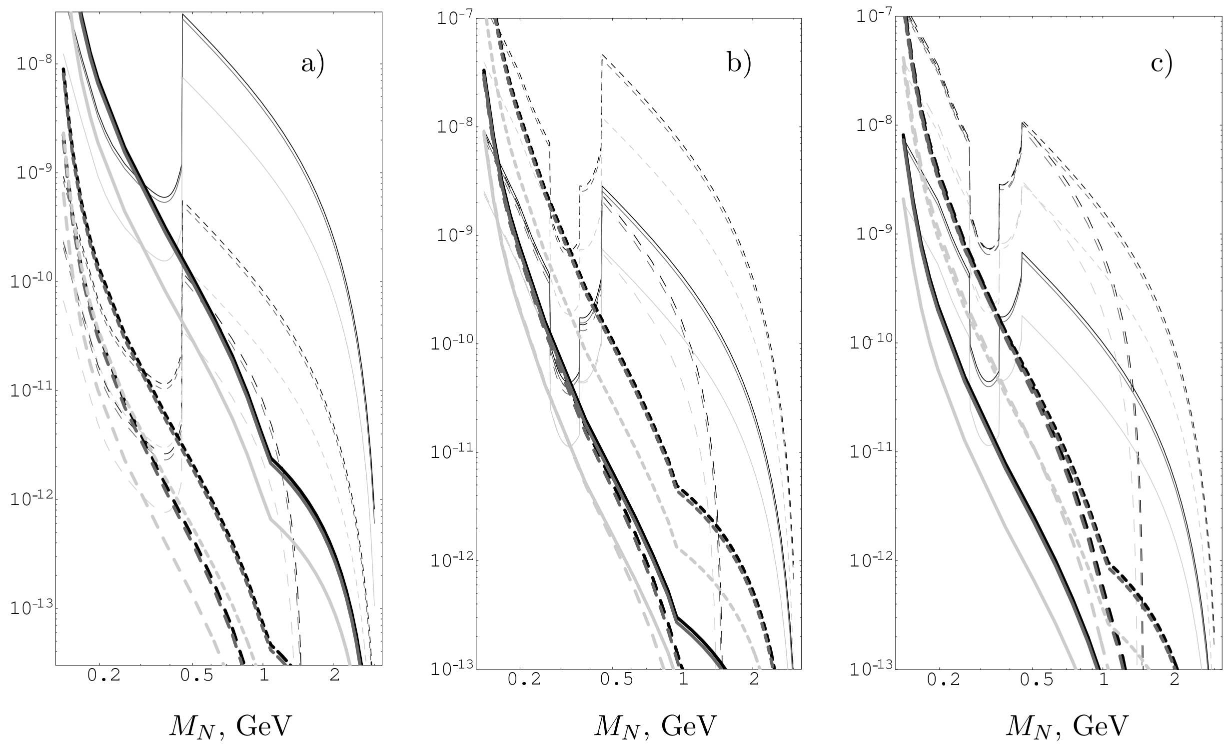

FIG. 14: Branching ratios of semileptonic decays $B \to D^* l N_I$ (black lines), $B_s \to D_s^* l N_I$ (dark gray lines) and $B_c \to J/\psi l N_I$ (light gray lines), $l = e, \mu, \tau$ (solid, short dashed and long dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III. In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors for B_s-meson decays are taken to be equal to the form factors for B-meson decays from Ref. [71], for B_c-meson decays we adopted form factors from Ref. [72].

estimate the mean longitudinal momenta $\langle p_{H,L} \rangle$ of D- and B-mesons we make use of the parameterization

$$\frac{d\sigma}{dx_F} \propto (1 - x_F)^c \ , \quad x_F \equiv \frac{p_{H,L}}{p_{H,L}^{max}} \ ,$$

with $c = 7.7$ for $E = 800$ GeV [77, 78], $c = 4.9$ for $E = 400$ GeV [79] and $c = 3$ for $E = 120$ GeV and $E = 50$ GeV as an factor-of-two estimate. In case of kaons we use the estimate

$$\langle p_{K,L} \rangle = \frac{1}{2} \left(\langle p_{D,L} \rangle + \frac{E}{M_{pp}} \right) \ ,$$

As we show in Section V, the dominant contribution to the total neutrino production in

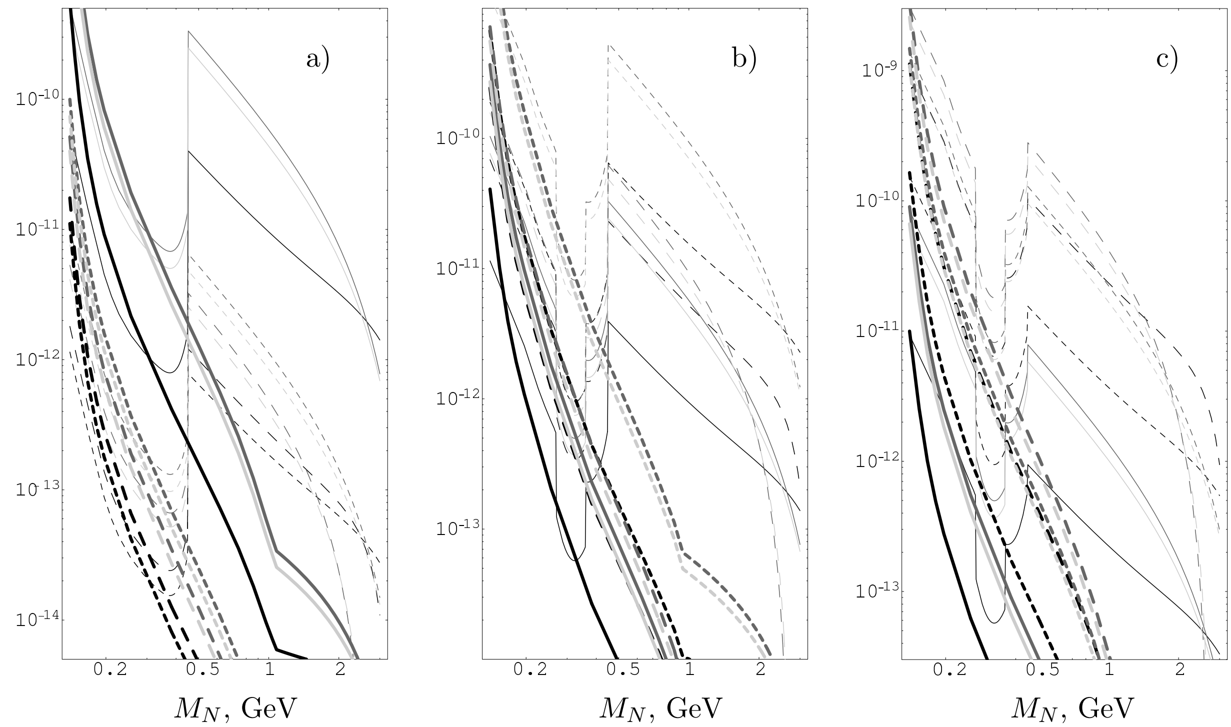

FIG. 15: Branching ratios of semileptonic decays $B \to \pi l N_I$ (black lines), $B \to \rho l N_I$ (dark gray lines) and $B_s \to K^* l N_I$ (light gray lines), $l = e, \mu, \tau$ (solid, long dashed and short dashed lines) as functions of heavy neutrino mass M_N in models: a) I, b) II, c) III.In a phenomenologically viable model and heavy neutrino mass within $M_\pi \lesssim M_N \lesssim M_D$, the branching ratios are confined between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively; form factors are taken from Refs. [70, 71].

collisions come mostly from two-body hadron decays. Thus, with neutrino longitudinal momentum uniformly distributed in hadron rest frame one gets for average neutrino momentum in laboratory frame

$$\langle p_{N,L} \rangle_H = \frac{1}{2} \langle p_{H,L} \rangle \cdot \left(1 + \frac{M_N^2}{M_H^2} \right) \ .$$

If neutrino decay length exceeds detector length Δl, the total number of neutrino decays inside the fiducial volume is

$$N_N^{decays} = N_N(E) \cdot \frac{\Delta l}{\tau_N} \cdot \sum_H \frac{M_N}{\langle p_{N,L} \rangle_H} \cdot \epsilon_N^H$$

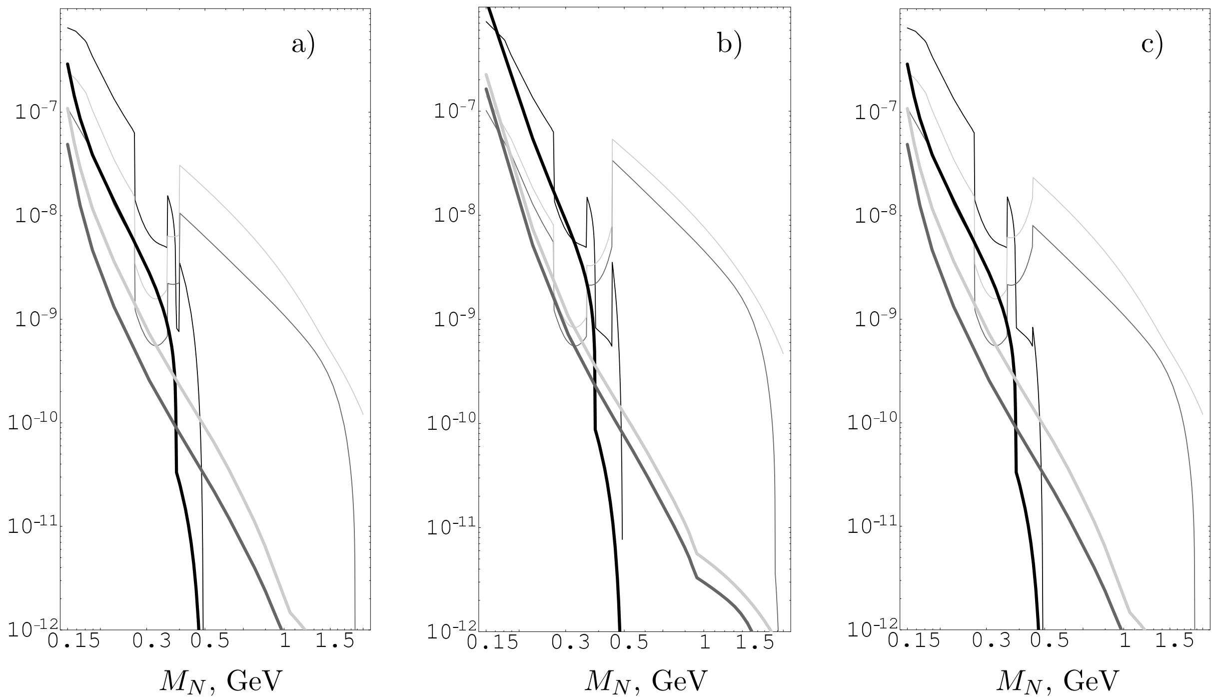

FIG. 16: Inclusive heavy lepton production by strange (black lines), charm (dark gray lines) and beauty (light gray lines) hadrons in models: a) I, b) II and c) III; within νMSM the interesting rates are between corresponding thin and thick lines which show upper and lower limits on U^2 from Fig. 4b, respectively.

with ϵ_N^H being a relative contribution of a given hadron H to total neutrino production,

$$\epsilon_N^H = \frac{N_H(E) \cdot \mathrm{Br}\,(H \to N \ldots)}{N_N(E)} ,$$

where the number of produced hadrons of a type H is estimated as

$$N_H(E) = N_{POT}(E) \cdot M_{pp}(E) \cdot \chi_Q(E) \cdot \mathrm{Br}\,(Q \to H) \ .$$

Finally we obtain for the total number of neutrino decays inside the detector

$$N_N^{decays} = N_{POT} \cdot M_{pp} \cdot \frac{\Delta l}{\tau_N} \cdot \sum_{Q,H} \chi_Q \cdot \xi_{Q,H} \cdot \frac{M_N}{\langle p_L^N \rangle_H} \ .$$

For the four available beams with parameters presented in Table I and $\Delta l = 5$m the quantitative predictions are given in Fig. 17 for the three benchmark models *with account of all experimental and theoretical constraints;* both lower and upper bounds scale with mixing

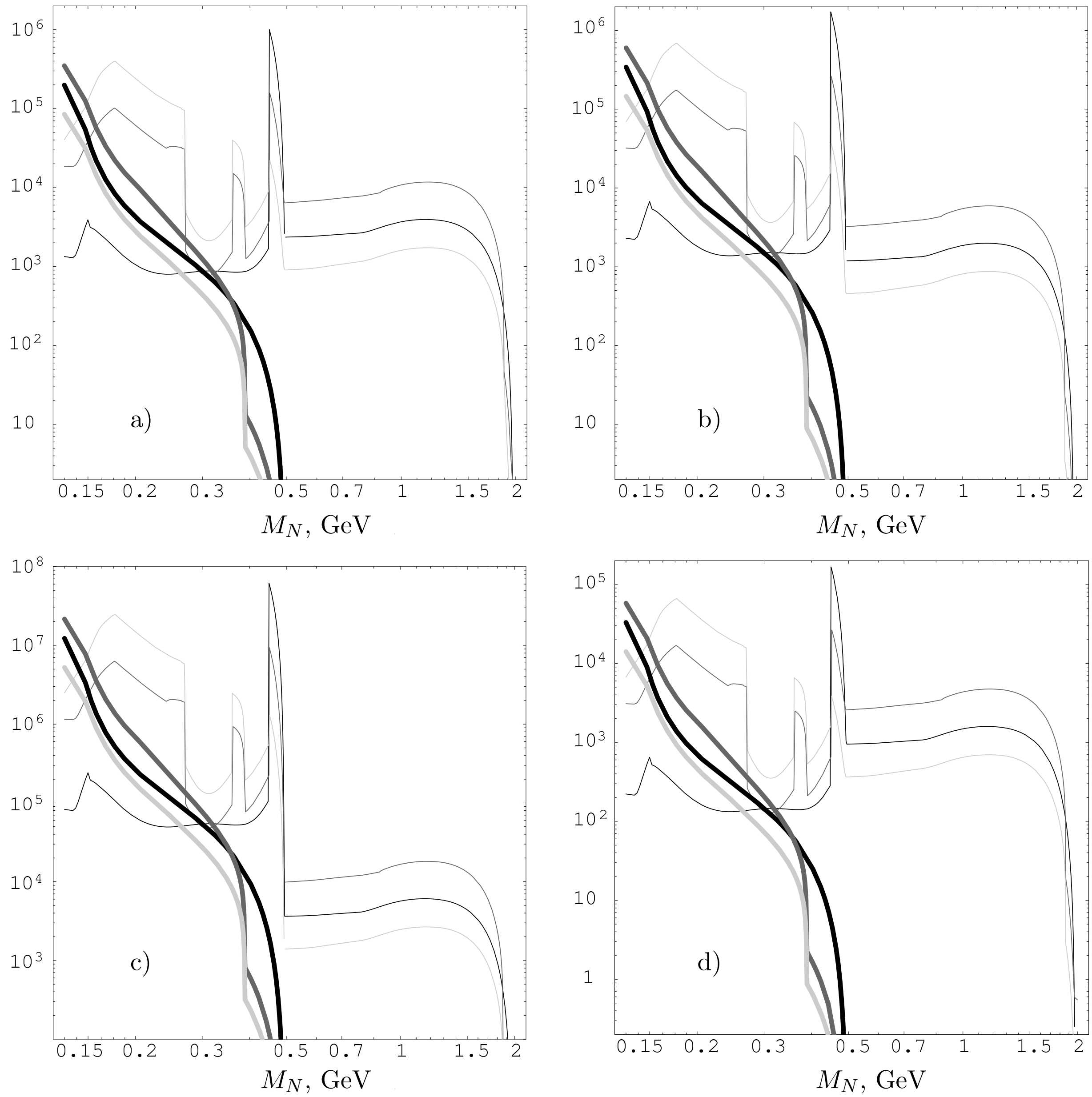

FIG. 17: Number of sterile neutrino decays within 5m-length fiducial volume for a) CNGS, b) NuMI, c) T2K, d) NuTeV beams as a function of sterile neutrino mass M_N. Black, dark gray and light gray lines refer to benchmark models I, II and III, respectively; in phenomenologically viable models the number of decay events are confined by corresponding thin (upper limits) and thick (lower limits) lines.

as U^4. One has to multiply these numbers by the value of the corresponding branching ratio (see plots in Section IV), if interested in a particular neutrino decay mode.

Note in passing that in these estimates we neglected neutrino beam spreading due to nonzero average transverse momentum $\langle p_{N,T}\rangle$. With a detector of width Δl_T placed at a distance l from a target this is justified if

$$\zeta \equiv \frac{\langle p_T^N\rangle_H}{\langle p_L^N\rangle_H} \cdot \frac{l}{\Delta l_T} \sim \frac{M_H}{\langle p_L^N\rangle_H} \cdot \frac{l}{\Delta l_T} \lesssim 1 \; .$$

Otherwise the predictions for neutrino signal *have to be corrected by a suppression factor* of order $1/\zeta^2$. Similar suppression factor should be accounted for in case of off-axis detector. Likewise, as the next approximation to N_N^{decays} one has to consider the realistic distributions of hadron and neutrino momenta instead of their average values. Finally, in case of real neutrino experiment, additional suppression can emerge due to focusing system. For a given experiment with fixed geometry and detector not tuned to searches for heavy neutrinos, these suppression factors can be obtained after dedicated studies; they can be as large as one-two orders of magnitude, depending on the neutrino mass. This studies are beyond the scope of this work.

VII. CONCLUSIONS

A pair of relatively light and almost degenerate Majorana leptons is an essential ingredient of the νMSM. In this model these particles are responsible for observed pattern of neutrino masses and mixings and for baryon asymmetry of the Universe. In the present paper we analysed the properties of the singlet fermions of the νMSM, which can be used for their experimental search. In particular, we discussed their production in decays of different mesons and in pp collisions. We studied the decays of singlet fermions in a wide range of their masses and other parameters, consistent with cosmological considerations.

In the νMSM the strength of the coupling of singlet fermions to ordinary leptons is bounded both from *above* by cosmology (baryon asymmetry of the Universe) and from *below* by neutrino oscillation experiments. In addition, a lower bound on the strength of interaction comes from Big Bang Nucleosynthesis. We analysed the latter constraint in some detail and showed that it is stronger than the one coming from neutrino experiments for masses smaller than 1 GeV.

The only particle physics searches for neutral fermions that were able to enter into the cosmologically interesting region of masses and couplings of neutral fermions, described in this paper, is the CERN PS191 experiment [46, 47]. It was performed some 20 years ago and since then no improvements of the bounds were made. This experiment, together with the BBN considerations, indicates that the masses of neutral leptons should be larger than the pion mass, though definite exclusion of the region below π-meson would require more theoretical (BBN analysis) and experimental work. Quite interestingly, with an order of magnitude improvement of the bounds on the strength of interaction the whole region of masses below kaon mass can be scanned and the neutral leptons could be either ruled out or found in this mass range. It looks likely that the significant improvement of the results of [46, 47] can be achieved by the reanalysis of the *existing* data of KLOE and of E949 experiments, and that the third stage of NA48 would allow to settle down the question.

The search for the neutral fermions that are heavier than K-mesons would require dedicated experiments similar to the CERN PS191 experiment, but with higher intensity and higher energy proton beams. We argued that the use of the CNGS, NuMI, T2K or NuTeV beams can allow to touch the interesting range of mixings for the lepton mass below charm, whereas for going above charm much more intensive accelerators would be necessary.

In conclusion, it is quite possible that the already existing machines can be used for the search of the physics beyond the Minimal Standard Model, responsible for neutrino oscillations, dark matter and baryon asymmetry of the Universe. Clearly, to study the CP-violation in interactions of neutral leptons more statistics would be required, calling for the intensity (rather than energy) increase of the proton beams.

Acknowledgements.

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APPENDIX A: STERILE NEUTRINO DECAYS

Formulae for most decay rates presented here can be obtained straightforwardly by making use of the formulae for Dirac neutrinos from Ref. [54]; we checked them and found to be correct. Formulae for decay rates into fixed final states are identical in Dirac and Majorana cases. Decays $N \to \pi^0\nu$ and $N \to \rho\nu$ have not been considered there. Our result for decay $N \to \pi^0\nu$ differs from the estimate used in Ref.[55] by an additional phase volume factor and by a factor 1/2. Hence, the neutrino life-time used in Ref. [55] to get BBN limits should be multiplied by $2/(1 - M_\pi^2/M_N^2)$, and corresponding limit on neutrino mixing angle should be divided by this factor.

Two-body decay modes are

$$\Gamma\left(N \to \pi^0\nu_\alpha\right) = \frac{|U_\alpha|^2}{32\pi} G_F^2 f_\pi^2 M_N^3 \cdot \left(1 - \frac{M_\pi^2}{M_N^2}\right)^2 ,$$

$$\Gamma\left(N \to H^+ l_\alpha^-\right) = \frac{|U_\alpha|^2}{16\pi} G_F^2 |V_H|^2 f_H^2 M_N^3 \cdot \left(\left(1 - \frac{M_l^2}{M_N^2}\right)^2 - \frac{M_H^2}{M_N^2}\left(1 + \frac{M_l^2}{M_N^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{(M_H - M_l)^2}{M_N^2}\right)\left(1 - \frac{(M_H + M_l)^2}{M_N^2}\right)} ,$$

$$\Gamma\left(N \to \eta\nu_\alpha\right) = \frac{|U_\alpha|^2}{32\pi} G_F^2 f_\eta^2 M_N^3 \cdot \left(1 - \frac{M_\eta^2}{M_N^2}\right)^2 ,$$

$$\Gamma\left(N \to \eta'\nu_\alpha\right) = \frac{|U_\alpha|^2}{32\pi} G_F^2 f_{\eta'}^2 M_N^3 \cdot \left(1 - \frac{M_{\eta'}^2}{M_N^2}\right)^2 ,$$

$$\Gamma\left(N \to \rho^+ l_\alpha^-\right) = \frac{|U_\alpha|^2}{8\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 |V_{ud}|^2 M_N^3 \cdot \left(\left(1 - \frac{M_l^2}{M_N^2}\right)^2 + \frac{M_\rho^2}{M_N^2}\left(1 + \frac{M_l^2 - 2M_\rho^2}{M_N^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{(M_\rho - M_l)^2}{M_N^2}\right)\left(1 - \frac{(M_\rho + M_l)^2}{M_N^2}\right)} ,$$

$$\Gamma\left(N \to \rho^0\nu_\alpha\right) = \frac{|U_\alpha|^2}{16\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 M_N^3 \cdot \left(1 + 2\frac{M_\rho^2}{M_N^2}\right) \cdot \left(1 - \frac{M_\rho^2}{M_N^2}\right)^2 ,$$

where G_F is Fermi coupling constant, $f_\eta = 1.2 f_\pi$, $f_{\eta'} = -0.45 f_\pi$, $g_\rho = 0.102$ GeV^2 [68]; hereafter and for CKM matrix elements we use values from Ref. [68], while for meson decay constants we used most recent values from Refs. [68, 80].

H	π^+	K^+	D^+	D_s	B^+	B_s	B_c
f_H, MeV	130	159.8	222.6	280.1	190	230	480
V_H	V_{ud}	V_{us}	V_{cd}	V_{cs}	V_{ub}	V_{us}	V_{cb}

Three body decay modes read

$$\Gamma\left(N \to \sum_{\alpha,\beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot \sum_\alpha |U_\alpha|^2 \,,$$

$$\Gamma\left(N \to l^-_{\alpha\neq\beta} l^+_\beta \nu_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot |U_\alpha|^2 \left(1 - 8x_l^2 + 8x_l^6 - x_l^8 - 12x_l^4 \log x_l^2\right) \,, \quad x_l = \frac{\max\left[M_{l_\alpha},\ M_{l_\beta}\right]}{M_N} \,,$$

$$\Gamma\left(N \to \nu_\alpha l^+_\beta l^-_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot |U_\alpha|^2 \cdot \Bigg[(C_1 \cdot (1-\delta_{\alpha\beta}) + C_3 \cdot \delta_{\alpha\beta}) \Big(\left(1 - 14x_l^2 - 2x_l^4 - 12x_l^6\right)\sqrt{1-4x_l^2}$$
$$+ 12x_l^4\left(x_l^4 - 1\right) L\Big) + 4\left(C_2 \cdot (1-\delta_{\alpha\beta}) + C_4 \cdot \delta_{\alpha\beta}\right)\Big(x_l^2\left(2 + 10x_l^2 - 12x_l^4\right)\sqrt{1-4x_l^2}$$
$$+ 6x_l^4\left(1 - 2x_l^2 + 2x_l^4\right) L\Big)\Bigg] \,,$$

with

$$L = \log\left[\frac{1 - 3x_l^2 - \left(1 - x_l^2\right)\sqrt{1-4x_l^2}}{x_l^2\left(1 + \sqrt{1-4x_l^2}\right)}\right] \,, \qquad x_l \equiv \frac{M_l}{M_N} \,,$$

and

$$C_1 = \frac{1}{4}\left(1 - 4\sin^2\theta_w + 8\sin^4\theta_w\right) \,, \qquad C_2 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w - 1\right) \,,$$
$$C_3 = \frac{1}{4}\left(1 + 4\sin^2\theta_w + 8\sin^4\theta_w\right) \,, \qquad C_4 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w + 1\right) \,.$$

The Majorana neutrino total decay rate is a sum of all rates presented above multiplied by a factor of 2, which accounts for charge-conjugated decay modes.

APPENDIX B: DECAYS INTO STERILE NEUTRINO

Differential branching ratio of pseudoscalar meson leptonic decays into sterile neutrinos reads

$$\frac{d\mathrm{Br}\left(H^+ \to l_\alpha^+ N\right)}{dE_N} = \tau_H \cdot \frac{G_F^2 f_H^2 M_H M_N^2}{8\pi} |V_H|^2 |U_\alpha|^2 \cdot \left(1 - \frac{M_N^2}{M_H^2} + 2\frac{M_l^2}{M_H^2} + \frac{M_l^2}{M_N^2}\left(1 - \frac{M_l^2}{M_H^2}\right)\right)$$
$$\times \sqrt{\left(1 + \frac{M_N^2}{M_H^2} - \frac{M_l^2}{M_H^2}\right)^2 - 4\frac{M_N^2}{M_H^2}} \cdot \delta\left(E_N - \frac{M_H^2 - M_l^2 + M_N^2}{2M_H}\right) \,, \qquad \text{(B1)}$$

where τ_H is the meson life-time [68].

For differential branching ratios of pseudoscalar meson semileptonic decays one has

$$\frac{d\text{Br}\left(H\to H'l_\alpha^+N\right)}{dE_N}=\tau_H\cdot|U_\alpha|^2\cdot\frac{|V_{HH'}|^2G_F^2}{64\pi^3M_H^2}\times\int dq^2\Bigg(f_-^2(q^2)\cdot\left(q^2\left(M_N^2+M_l^2\right)-\left(M_N^2-M_l^2\right)^2\right)$$
$$+2f_+(q^2)f_-(q^2)\left(M_N^2\left(2M_H^2-2M_{H'}^2-4E_NM_H-M_l^2+M_N^2+q^2\right)+M_l^2\left(4E_NM_H+M_l^2-M_N^2-q^2\right)\right)$$
$$f_+^2(q^2)\Big(\left(4E_NM_K+M_l^2-M_N^2-q^2\right)\left(2M_K^2-2M_\pi^2-4E_NM_K-M_l^2+M_N^2+q^2\right)$$
$$-\left(2M_K^2+2M_\pi^2-q^2\right)\left(q^2-M_N^2-M_l^2\right)\Big)\Bigg)\,, \tag{B2}$$

where $q^2=(p_l+p_N)^2$ is momentum of leptonic pair, $V_{HH'}$ is corresponding entry of CKM matrix and $f_+(q^2),\ f_-(q^2)$ are dimensionless hadronic form factors [68] can be found in literature.

For three-body decays into vector mesons V one obtains

$$\frac{d\text{Br}\left(H\to Vl_\alpha N\right)}{dE_N}=\tau_H\cdot|U_\alpha|^2\cdot\frac{|V_{HV}|^2G_F^2}{32\pi^3M_H}\times\int dq^2\Bigg(\frac{f_2^2}{2}\left(q^2-M_N^2-M_l^2+\omega^2\frac{\Omega^2-\omega^2}{M_V^2}\right)$$
$$+\frac{f_5^2}{2}\left(M_N^2+M_l^2\right)\left(q^2-M_N^2+M_l^2\right)\left(\frac{\Omega^4}{4M_V^2}-q^2\right)+2f_3^2M_V^2\left(\frac{\Omega^4}{4M_V^2}-q^2\right)\left(M_N^2+M_l^2-q^2+\omega^2\frac{\Omega^2-\omega^2}{M_V^2}\right)$$
$$+2f_3f_5\left(M_N^2\omega^2+\left(\Omega^2-\omega^2\right)M_l^2\right)\left(\frac{\Omega^4}{4M_V^2}-q^2\right)+2f_1f_2\left(q^2\left(2\omega^2-\Omega^2\right)+\Omega^2\left(M_N^2-M_l^2\right)\right)$$
$$+\frac{f_2f_5}{2}\left(\omega^2\frac{\Omega^2}{M_V^2}\left(M_N^2-M_l^2\right)+\frac{\Omega^4}{M_V^2}M_l^2+2\left(M_N^2-M_l^2\right)^2-2q^2\left(M_N^2+M_l^2\right)\right)$$
$$+f_2f_3\left(\Omega^2\omega^2\frac{\Omega^2-\omega^2}{M_V^2}+2\omega^2\left(M_l^2-M_N^2\right)+\Omega^2\left(M_N^2-M_l^2-q^2\right)\right)$$
$$+f_1^2\left(\Omega^4\left(q^2-M_N^2+M_l^2\right)-2M_V^2\left(q^4-\left(M_N^2-M_l^2\right)^2\right)+2\omega^2\Omega^2\left(M_N^2-q^2-M_l^2\right)+2\omega^4q^2\right)\Bigg)\,, \tag{B3}$$

where $\omega^2=M_H^2-M_V^2+M_N^2-M_l^2-2M_HE_N$ and $\Omega^2=M_H^2-M_V^2-q^2$; form factors $f_i(q^2)$ can be expressed via standard axial form factors $A_0(q^2),A_1(q^2),A_2(q^2)$ and vector form factor $V(q^2)$ as

$$f_1=\frac{V}{M_H+M_V}\,,\quad f_2=\left(M_H+M_V\right)\cdot A_1\,,\quad f_3=-\frac{A_2}{M_H+M_V}\,,$$
$$f_4=\left(M_V\left(2A_0-A_1-A_2\right)+M_H\left(A_2-A_1\right)\right)\cdot\frac{1}{q^2}\,,\quad f_5=f_3+f_4\,,$$

which can be found in literature.

For two-body decays of τ-lepton into heavy neutrino and meson we obtain

$$\frac{d\mathrm{Br}\left(\tau \to HN\right)}{dE_N} = \tau_\tau \cdot \frac{|U_\tau|^2}{16\pi} G_F^2 |V_H|^2 f_H^2 M_\tau^3 \cdot \left(\left(1 - \frac{M_N^2}{M_\tau^2}\right)^2 - \frac{M_H^2}{M_\tau^2}\left(1 + \frac{M_N^2}{M_\tau^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{\left(M_H - M_N\right)^2}{M_\tau^2}\right)\left(1 - \frac{\left(M_H + M_N\right)^2}{M_\tau^2}\right)} \cdot \delta\left(E_N - \frac{M_\tau^2 - M_H^2 + M_N^2}{2M_\tau}\right) ,$$
$$\frac{d\mathrm{Br}\left(\tau \to \rho N\right)}{dE_N} = \tau_\tau \cdot \frac{|U_\tau|^2}{8\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 |V_{ud}|^2 M_\tau^3 \cdot \left(\left(1 - \frac{M_N^2}{M_\tau^2}\right)^2 + \frac{M_\rho^2}{M_\tau^2}\left(1 + \frac{M_N^2 - 2M_\rho^2}{M_\tau^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{\left(M_\rho - M_N\right)^2}{M_\tau^2}\right)\left(1 - \frac{\left(M_\rho + M_N\right)^2}{M_\tau^2}\right)} \cdot \delta\left(E_N - \frac{M_\tau^2 - M_\rho^2 + M_N^2}{2M_\tau}\right) ,$$

where τ_τ is τ-lepton life-time. For three-body decays of τ-lepton one has

$$\frac{d\mathrm{Br}\left(\tau \to \nu_\tau l_\alpha N\right)}{dE_N} = \tau_\tau \cdot \frac{|U_\alpha|^2}{2\pi^3} G_F^2 M_\tau^2 \cdot E_N \left(1 + \frac{M_N^2 - M_l^2}{M_\tau^2} - 2\frac{E_N}{M_\tau}\right)\left(1 - \frac{M_l^2}{M_\tau^2 + M_N^2 - 2E_N M_\tau}\right)\sqrt{E_N^2 - M_N^2} ,$$
$$\frac{d\mathrm{Br}\left(\tau \to \bar{\nu}_\alpha l_\alpha N\right)}{dE_N} = \tau_\tau \cdot \frac{|U_\tau|^2}{4\pi^3} G_F^2 M_\tau^2 \left(1 - \frac{M_l^2}{M_\tau^2 + M_N^2 - 2E_N M_\tau}\right)^2 \sqrt{E_N^2 - M_N^2}$$
$$\times \left(\left(M_\tau - E_N\right)\left(1 - \frac{M_N^2 + M_l^2}{M_\tau^2}\right) - \left(1 - \frac{M_l^2}{M_\tau^2 + M_N^2 - 2E_N M_\tau}\right)\left(\frac{\left(M_\tau - E_N\right)^2}{M_\tau} + \frac{E_N^2 - M_N^2}{3M_\tau}\right)\right) .$$

Note that omitted here charge-conjugated processes also contribute to Majorana neutrino production.

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