On the capacity of multiple input erasure relay channels: The Non-degraded case

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Abstract

We consider in this paper a network that consists of two senders and two receivers. We further assume that each sender could act as a relay for other communications. All channels connecting these nodes are supposed to be erasure channels where symbols are received correctly (error-free), or lost. This model is realistic for many practical scenarios in the context of wireless and sensor networks.

In a previous works, we have addressed the capacity region of this network under physically degraded hypothesis. The non-degraded case is addressed in this paper. We derive a capacity bound for the proposed network and we show that it can be reached through a practical coding scheme based on MDS codes. We make also a comparison of the achieved rates compared to a simple time sharing of single sender relay channels.

I. Introduction

Wireless networks consist of senders, receivers and intermediate nodes that more or less collaborate to achieve a communications. An important problem in this context consists of finding the best possible nodes collaboration scheme which maximize the transferred information. Information theory aims toward finding the set of transfer rate that are ultimately achievable for any given scenario. Recently network coding [15], [11], [21] has been proposed as a new paradigm to look at the issue of network capacity. Network coding defines a new type of collaboration schemes which consists of mixing the received information through a coding scheme defined for each node and forwarding the encoded version. Cooperative diversity idea presented in [18], [19], [14] is also relevant to the wireless networks. Cooperative diversity is a new form of spatial diversity whereby diversity gains are achieved via the nodes cooperation

In this paper we present a network coding scheme achieving the capacity bound for a scenario consisting of two sender that want to send different information to two receivers. Each sender can act also as a relay for the other communication. We assume here that all channels connecting the four nodes are erasure channels where symbols are received correctly (error-free), or lost. The analyzed scenario is different from the classical multi-user channel as the two communications to a single receiver are not allowed to interfere with each other. Interferences between different simultaneous communications are managed by using separated physical channels or through time scheduling (centralized or distributed using a Medium Access protocol as CSMA/CA). This simplification might allow the establishment of the capacity of relay channels, as shown in [5] for example for the discrete-memoryless relay channel with orthogonal components.

Moreover; from the viewpoint of higher layer where applications stand, wireless networks appear as erasure channels. Sender sends packets that might be received by the destination nodes or be erased because of transmission errors, collisions, or buffer overflows. Under erasure channel hypothesis, simple closed form bound can be derived. In [16] a simple form capacity bound is derived for the single sender-receiver multi-relay channel and under perfect side information hypothesis at the decoder. The side information is provided in the form of the exact erasure pattern over **every link** in the network. The capacity bound is achieved through a random coding scheme, but it seems that the achievability is only valid under degraded hypothesis. In [17] the capacity of a general stationary and ergodic broadcast erasure channel is derived that leads to a simple linear capacity bound. This bound can be achieved optimally through a simple time sharing mechanism called Priority Encoding Technique. In [8] and [7] the capacity of the single relay

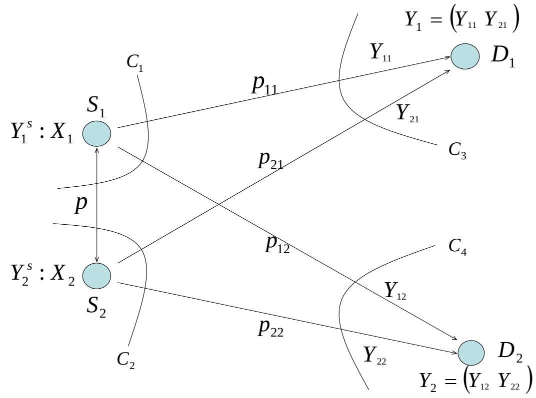

Fig. 1. Multi-sender relay Channel

erasure channel is derived under degraded and non degraded hypothesis. A coding schemes based on a practical MDS code are provided that achieves this capacity without need to any side information. The proposed scheme does not need the exact erasure pattern over every link in the channel, and knowing the average packet loss over links is enough to construct this scheme. The general packet loss matrix of the channel is a statistic parameter of channel, and could be estimated by error modeling schemes [1], [6]. Extension of the results to the more general case of multi-relay erasure channel has been proposed in [9].

However; in all of these work the simplified case of single transmitter has been considered. The more general case of multi-sender multi-receiver has been rarely addressed in the literature, as it is much more difficult to handle. In [10] a collaboration scheme in a simple communication scenario where two senders want to send different informations to two receivers (see Fig. 1) is presented. The cut-set bound is derived and is shown to be achievable under degraded hypothesis by a practical coding scheme. The main idea of the proposed collaboration scheme consists of exchanging enough information between source nodes such that the information in each sender become correlated. In this case, we fall in the context of Slepian-Wolf coding [20] where there are two correlated sources that have to be transferred to a common receiver. We developed a Slepian-Wolf type coding based on MDS code that enables the efficient transfer of the two sources over an erasure channel. It is shown that the proposed scheme achieve the cut-set bound under the degraded hypothesis, *i.e.* we assume that the two senders access to the same information through information exchange.

In this paper, we will analyze the more general case of non-degraded multi-input relay channel. In this case we are not anymore assuming that the two senders can have access to the same information by exchanging information. We present a collaboration scheme that can use incomplete information exchanged between the senders to achieved the capacity bound.

The paper is organized as follow. Section II gives the capacity bound of an erasure multi-sender erasure relay channel. In the section III we show that this bound is achievable through a practical coding scheme. Some practical comments and conclusion will be presented finally.

II. THEORETICAL BOUND

The specific multi-sender relay network that will be studied in this paper is a network composed of two senders ($\mathcal{S}_1$ and $\mathcal{S}_2$) and two receivers ($\mathcal{D}_1$ and $\mathcal{D}_2$) as shown in Fig. 1. The sender $\mathcal{S}_i,\ i = 1, 2$ sends information to the two receivers $\mathcal{D}_j,\ j = 1, 2$. Simultaneously each sender might acts as a relay for the other sender. The Multi-sender relay channel can be described with 8 random variables $X_i,\ i = 1, 2$ representing the symbols sent by the sender, $Y_{ij},\ i, j = 1, 2$ representing symbols received from each sender by each receiver and Y_i^s representing the symbol received by sender i from the other sender. The conditional probability density function $p\left(y_{ij}, y_i^s\ i, j=1,2 | x_i\right)\ i=1,2)2$ defines the multi-sender relay channel. This last function gives the probability that when x_i is sent by $\mathcal{S}_i,\ i=1,2$, (y_{i1}, y_{i2}) are received at $\mathcal{D}_i$ and y_i^s is received at sender i. We further define $Y_j = (Y_{1j}, Y_{2j},\ j=1,2)$ as the total information received at $\mathcal{D}_j$. This description assumes that each receiver is linked to the senders through two separated channels. The separation of the two channels might be achieved by using different physical channel or by using time scheduling. We further assume that information send by a sender might be received by all receivers as well as the other sender.

The considered multi-sender relay channel consists of 2 separate erasure broadcast channel (as defined in [17]) $(X_i; Y_i^s, Y_{i1}, Y_{i2}),\ i=1,2$ and two erasure relay channels (as defined in [8]) $(X_1; Y_2^s : X_2; Y_{12}, Y_{22})$ and $(X_2; Y_1^s : X_1; Y_{11}, Y_{21})$. The loss probability between $\mathcal{S}_i$ and $\mathcal{D}_j$ is defined as p_{ij} and the loss probability between the two sender is supposed to be equal to p in the two direction (as shown in Fig. 1). This last assumption is for clarity sake, however the results might be extended straightforwardly to take in account possible asymmetry between the two senders. No memoryless assumption on the loss statistics over the channel is needed and all the results given here are valid for all stationary and ergodic erasure channels.

Let's suppose that the total rate of information between $\mathcal{S}_i$ and $\mathcal{D}_j$ to be defined as R_{ij}. As said before we have two broadcast channels in the multi-sender scenario described in Fig. 1. The total rate R_{ij} could be splitted in two components : a private information rate R_{ij}^p which is the rate of information being sent from i only to j and a common information rate R_{ij}^c which is the rate that will be decodable jointly by the two receivers. In this paper we assume as in [12], [17] a degraded message-set to be sent over each broadcast channel, *i.e.* one receiver receives the private and common information and the other receives only the common information. Let's therefore suppose that the private information rate sent by $\mathcal{S}_i$ is equal to R_i^p and that the common information rate is R_i^c. In other terms, let us suppose that for example $\mathcal{D}_1$ have to receive private and common information sent by $\mathcal{S}_1$ and $\mathcal{D}_2$ have to receive only the common information. We have therefore $R_{11} = R_1^p + R_1^c$ and $R_{12} = R_1^c$. However, it is shown in [17] that one cannot do better than time-sharing for broadcast erasure channels. Meaning that there is a trade-off between the rate of private and common information. Larger private information rate means lower common rate and therefore lower reception rate for the receiver receiving only common information. This means that it is sound to suppose that all information are broadcasted as common information and private information to each receivers are sent through time sharing. Therefore, we will assume that there is only common information to be broadcasted by the senders to all receivers with rate R_i^*, i being the index of the sender.

Theorem 1 (Capacity region bound) *Under the hypothesis that X_1 and X_2 being independent the capacity region of multiple-Input relay channel in Fig. 1 is bounded by :*

$$\begin{cases} R_1^* \le I(X_1; Y_2^s, Y_{11}, Y_{12}) \\ R_2^* \le I(X_2; Y_1^s, Y_{21}, Y_{22}) \\ R_1^* + R_2^* \le I(X_1; Y_{11}) + I(X_2; Y_{21}) \\ R_1^* + R_2^* \le I(X_1; Y_{12}) + I(X_2; Y_{22}) \end{cases}$$

Moreover, in the special case of degraded channel when (Y_{11}, Y_{12}) are physically degraded versions of Y_2^s (resp. (Y_{11}, Y_{12}) are physically degraded versions of Y_1^s) the two first terms of the bound are remplaced by : $R_1^* \le I(X_1; Y_2^s)$ and $R_2^* \le I(X_2; Y_1^s)$ as shown in [10].

Proof. See the proof in appendix.

However; as we say before we are only considering the transfer of common information, *i.e.* the information sent by $\mathcal{S}_1$ (resp. $\mathcal{S}_2$) should be send to $\mathcal{D}_1$ and $\mathcal{D}_2$. Moreover, $\mathcal{D}_1$ and $\mathcal{D}_2$ do not participate in the relaying process. Under such a situation the two first bound of the capacity bound are changed to :

$$R_1^* \le \min\{I(X_1; Y_2^s, Y_{11}), I(X_1; Y_2^s, Y_{12})\}$$
$$R_2^* \le \min\{I(X_2; Y_1^s, Y_{21}), I(X_2; Y_1^s, Y_{22})\}$$

The capacity bound might be simplified thanks to the erasure nature of channels and the Shearer theorem :

Theorem 2 (Shearer Theorem [4]) *Let X^n be a collection of n random variables and Z^n be a collection of n boolean random variable, such that for each i, $1 \leq i \leq n$, $E\{Z_i\} = 1 - \tilde{C}$. If $X^n(Z^n)$ is a sub-collection containing the i^th random variable X_i if $Z_i = 1$. Then $E\{H(X^n(Z^n))\} \geq \tilde{C} H(X^n)$ $\square$.*

The theorem can be extended to conditional entropy as well. It can be shown thanks to this theorem that the capacity of a stationary and ergodic point to point erasure channel with an erasure process Z have a very simple form[17], $\tilde{C} = 1 - E\{Z_i\}$, where $E\{Z_i\} = 1 - \tilde{C}$ is the average erasure probability on the channel. Using this theorem the capacity bound is simplified to :

Theorem 3 *The capacity region bound over a multiple-Input erasure relay channel is bounded as :*

$$\begin{cases} R_1^* \leq \min\{(1 - p \cdot p_{11}), (1 - p \cdot p_{12})\} \\ R_2^* \leq \min\{(1 - p \cdot p_{21}), (1 - p \cdot p_{22})\} \\ R_1^* + R_2^* \leq (1 - p_{11}) + (1 - p_{21}) \\ R_1^* + R_2^* \leq (1 - p_{12}) + (1 - p_{22}) \end{cases} \quad (1)$$

In the special case of degraded channel, the first two terms of the bound are degraded to : $R_1^* \leq (1 - p)$ and $R_2^* \leq (1 - p)$.

The first two bounds in this theorem, are constraints bounding the rate available for collaboration between the two senders. The two last bounds are bounding the amount of information coming in the receiver. In the next section, we will provide a coding scheme achieving the given bound.

III. Achievability and Coding Method

In this section we propose a coding scheme which attains the capacity bound shown in theorem 3. The proposed coding scheme is a combination of basic techniques as Slepian-Wolf partitioning, cooperative coding for relay channel and block markov decoding. We first describe the coding scheme for the degraded case and then we extend it to the non-degraded hypothesis.

Let's describe the characteristics of the class of Maximal Distance Separable (MDS) codes [22] that will be used thereafter. Let's suppose a systematic (n, k) MDS code taking k information symbols and generating n encoded symbols. Now this code has the property that the initial k information symbols can be retrieved from any combination of k encoded symbols out of the n symbols constituting the block, *i.e* the MDS code can retrieve up to $n - k$ erasure in a block of n packets. A MDS code can achieve the capacity of a stationary and ergodic point to point erasure channel asymptotically with a block size $n \rightarrow \infty$ if its rate R is less than the capacity of the channel. Maximal Distance Separable (MDS) code leads to sphere packing codes for erasure channels.

A. Coding scheme description: Degraded case

We assume $L + 1$ block of transmission. We also consider at each sender $\mathcal{S}_i$, L blocks of data each containing k_i information symbols (packets) $B^i = \{s_1^i, \ldots, s_{k_i}^i\}$. Let's suppose that at the end of block l we have been able to decode at $\mathcal{S}_1$, the l^{th} block of the message sent by $\mathcal{S}_2$ ($B_l^2 = (s_{l \cdot k_2}^2, s_{l \cdot k_2 + 1}^2, \ldots, s_{l \cdot k_2 + k_2 - 1}^2)$). We will validate this hypothesis further. Moreover, let's assume that the $(l+1)^{\text{th}}$ block of message sent by $\mathcal{S}_1$ is $B_{l+1}^1 = (s_{(l+1) \cdot k_1}^1, s_{(l+1) \cdot k_1 + 1}^2, \ldots, s_{(l+1) \cdot k_1 + k_1 - 1}^2)$.

Now let's define an MDS code with an encoding matrix $G_{(k_1 + k_2) \times n}^1$. It generates the n encoded packets of the $(l+1)^{\text{th}}$ block from the $k_1 + k_2$ given packets consisting of the k_1 packets of the $(l+1)^{\text{th}}$ block of message of $\mathcal{S}_1$ and k_2 packets of the $(l)^{\text{th}}$ block of message send by $\mathcal{S}_2$ and received in the previous block, *i.e.* $X_{l+1}^1 = [B_{l+1}^1 \ B_l^2] \times G^1$.

$$Y_0^L = [B_1^0\ B_2^0\ B_1^1\ B_2^1 \dots B_1^L\ B_2^L] \begin{bmatrix} G_{11} & G_{21} & 0 & 0 & \dots & 0 & 0 \\ G_{12} & G_{22} & G_{11} & G_{21} & \dots & 0 & 0 \\ 0 & 0 & G_{12} & G_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & G_{11} & G_{21} \\ 0 & 0 & 0 & 0 & \dots & G_{12} & G_{22} \end{bmatrix} \quad (2)$$

Sender $\mathcal{S}_2$ can easily decode the block $(l+1)$ broadcasted by $\mathcal{S}_1$, if it has received enough packets over the erasure channel connecting $\mathcal{S}_1$ to $\mathcal{S}_2$. As it have in memory the k_2 values in B_l^2, this happen if $n(1-p) > k_1$, *i.e.* asymptotically if the rate of the MDS code used at $\mathcal{S}_1$ is less $(1-p)$. The rate of the MDS code is equal to $R_1^* = \frac{k_1}{n}$ as out of the $k_1 + k_2$ symbols used at the encoder input, k_2 of them are redundant (have been sent before over the channel). The decoded block B_{l+1}^1 is to be used in the next transmission block combined with block B_{l+2}^1 constructing X_{l+2}^2. As the initiation block we can use an all-zero block $B_0^j,\ j = 1, 2$ know to everybody in the network. Sender $\mathcal{S}_2$ encodes its proper information and mixes them with the information received from $\mathcal{S}_1$ using exactly the same mechanism but with a different encoding matrix G^2. With the same arguments we have $R_2^* = \frac{k_2}{n} \le (1-p)$. We will choose the encoding matrix G^1 and G^2 such that $[G^1\ |G^2]$ defines an MDS code, *i.e.* each sub-matrix of $[G^1\ |G^2]$ is invertible.

Now let's see what happen at the receiver side when senders use such a coding schemes. At each receiver we will receive some packets coming from each sender. Asymptotically with large n, in each block of transmission containing, each receiver $\mathcal{D}_j$ will receive $n(1-p_{1j}) + n(1-p_{2j})$ packets. In transmission block l, the packets received from the senders are a combination of B_l^1, B_l^2, B_{l-1}^1 and B_{l-1}^2. Clearly the decoding can not be done using only the information received in block l. The L-block Markov decoding technique can be applied. The idea is to do the decoding after reception of L blocks. Let suppose that the encoding matrix $[G^1\ G^2]$ is rewritten as:

$$[G^1\ G^2] = \begin{bmatrix} G_{11} & G_{21} \\ G_{12} & G_{22} \end{bmatrix}$$

where G_{ij} is a $k \times n$ matrix with the MDS property.

After receiving L blocks $\mathcal{D}_j$ we will receive a sub-sequence (because of erased symbols) $Y_0^L(Z_{ij}^{2Ln})$, where Z_{ij}^{2Ln} is the loss process observed over erasure channel going from $\mathcal{S}_i$ to $\mathcal{D}_j$ during the $2Ln$ transmissions of L blocks. Y_0^L is vector of length $2nL$ symbols obtained as in Eq. 2. The obtained L-block code of rate $R^* = \frac{(k_1+k_2)L}{2nL}$ (as the first block $[B_1^0, B_2^0]$ is known) is still an MDS code code as every sub-matrix of the encoding matrix will be invertible. Asymptotically, we will receive at each receiver $nL(1-p_{1j}) + nL(1-p_{2j})$ packets. The MDS code can be decoded if $nL(1-p_{1j}) + nL(1-p_{2j}) > (k_1+k_2)(L+1)$, in other term if $\frac{L+1}{L}(R_1^* + R_2^*) < (1-p_{1j}) + (1-p_{2j})$. We can see therefore that the proposed coding scheme achieves the capacity bound under degraded hypothesis when $L \to \infty$.

B. Coding scheme description: Non Degraded case

In this section we will present a coding schemes applicable without the degraded assumption. The main difference between this situation and the degraded case is that sender $\mathcal{S}_i$ is not supposed to know all information sent by the other sender. In the other term, Y_{11} and Y_{12} might have some relevant information that have not been received by $\mathcal{S}_2$, where under the degraded assumption all information available at $\mathcal{D}_1$ and $\mathcal{D}_2$ are also available at $\mathcal{S}_2$.

Let's suppose that $\mathcal{S}_1$ use a MDS code with block size n. Asymptotically with large n, $n(1-p)$ (resp. $n(1-p_{11})$ and $n(1-p_{12})$) packets are received at $\mathcal{S}_2$ (resp. $\mathcal{D}_1$ and $\mathcal{D}_2$). Out of these packets

$n(1-p)(1-p_{11})(1-p_{12})$ are received at the three receiver $\mathcal{S}_2$, $\mathcal{D}_1$ and $\mathcal{D}_2$, and $np_{11}p_{12}(1-p)$ (resp. $npp_{12}(1-p_{11})$ and $npp_{11}(1-p_{12})$) packets are received only at $\mathcal{S}_2$ (resp. D_1 and $\mathcal{D}_2$). $\mathcal{S}_2$ has to forward enough packets to eliminate ambiguity at the receivers. However, $\mathcal{S}_2$, $\mathcal{D}_1$ and $\mathcal{D}_2$ are not aware of the packets they have respectively received.

Let's suppose that at the end of block l, $\mathcal{S}_1$ has received $k_2^* = n(1-p)$ packets of the codeword X_l^2 sent by $\mathcal{S}_2$. We call these packets B_l^{1*}. Let's also assume that the $(l+1)^{\text{th}}$ block of message sent by $\mathcal{S}_1$ is B_{l+1}^1. The symbols sent by $\mathcal{S}_1$ over block $(l+1)$ are defined as $X_{l+1}^1 = [B_{l+1}^1 \; B_l^{2*}] \times G^1$. In block $l+1$, $k_1^* = n(1-p)$ packets of the codeword X_{l+1}^1 received at S_2 (B_{l+1}^{1*}) would be used in the next transmission block to combined with B_{l+2}^1 and construct X_{l+2}^2.

The first constraint on this coding scheme only a proportion p_{11} (resp. p_{12}) of the packets received at $\mathcal{S}_2$ are independent from those received at $\mathcal{D}_1$ (resp. $\mathcal{D}_2$). Nevertheless, the total number of independent packets received at $\mathcal{S}_2$ and $\mathcal{D}_1$ (resp. $\mathcal{D}_2$) must be larger than k_1 to guarantee that B_{l+1}^1 could be decoded at the receiver nodes, *i.e.* $n(1-p_{11}p) > k_1$ and $n(1-p_{12}p) > k_1$. The same arguments can be also applied when $\mathcal{S}_2$ is sender. This results to the first two terms of the bound presented in theorem3.

In transmission block l, the packets received at the receivers are a combination of B_l^1, B_l^2, B_{l-1}^{1*} and B_{l-1}^{2*} where B_{l-1}^{1*} (resp. B_{l-1}^{2*}) is also a combination of B_{l-1}^1 and B_{l-2}^{2*} (resp. B_{l-1}^2 and B_{l-2}^{1*}). If we consider each information symbol as an unknown variable, the decoding process in the receiver consists of resolving a linear equation system with $L(k_1+k_2)$ unknow variables s_j^i, $i=1,2$ $j \in [1,(L+1)k_i]$. As the encoded packets sent by $\mathcal{S}_1$ and $\mathcal{S}_2$ are independent from each others by construction, if the number of received packets received at the receivers (leading to an independent equation) are more than the number of unknown variables, the equation could be solved and the initial packets can be decoded. Therefore, if $nL(1-p_{1j}) + nL(1-p_{2j}) > (k_1+k_2)(L+1)$, $j=1,2$ the decoding process can be successful in each receiver. This prove the achievability of the capacity bound.

IV. Practical comments

The proposed coding scheme is original from several perspectives. It provides a practical and simple way of doing Slepian-Wolf coding in the context of erasure channels. In place of sending the information independently from $\mathcal{S}_1$ and $\mathcal{S}_2$, we send an index obtained by mixing information coming from the two senders. By doing this we reduce the amount of information to be sent by each sender and we reach a collaboration gain. Moreover, in this setting all symbols received over the multicast channel are useful to decode the final information.

Up to now we have considered the case where all the sent information are common information. Now if one want to send private information he might use a simple time sharing scheme. Let's suppose that we want to send with a rate R_{ij} from $\mathcal{S}_i$ to $\mathcal{D}_j$. The cut-set bound become equal to

$$\begin{cases} R_{11}+R_{12} \leq \min\{(1-p\cdot p_{11}),(1-p\cdot p_{12})\} \\ R_{21}+R_{22} \leq \min\{(1-p\cdot p_{21}),(1-p\cdot p_{22})\} \\ R_{11}+R_{21} \leq (1-p_{11})+(1-p_{21}) \\ R_{12}+R_{22} \leq (1-p_{12})+(1-p_{22})\} \end{cases}$$

Under this situation it would be possible to design a time-sharing mechanism based on the proposed coding scheme, sending information from $\mathcal{S}_i$ to $\mathcal{D}_j$ a proportion of time equal to $\frac{R_{ij}}{R_{1j}+R_{2j}}$. This time sharing mechanism achieves the cut-set bound, meaning that the previous coding scheme can be applied even in the case that private information is to be send to the received.

It is noteworthy mentioning that the decoding process of MDS code has a $\mathcal{O}(n)\log n$ complexity. This can be reduced to a linear complexity through use of Tornado codes that are almost-MDS [13]. The main drawback of the method is large decoding delay at receiver that comes from the fact that we have to wait till reception of L block before decoding. However, there is a trade-off between rate efficiency and delay, through the choice of L; larger L Leads to larger rates but larger delays and *vice-versa*.

As the last point, let's compare the obtained capacity region with the scenario that two relay channels, $(\mathcal{S}_1, \mathcal{S}_2, \mathcal{D}_j),\ j = 1,2$ and $(\mathcal{S}_2, \mathcal{S}_1, \mathcal{D}_j),\ j = 1,2$, use a time-sharing to transmit over channel. Let us α being the proportion of time one uses the relay channel. Now the achievable rate is governed by the capacity region of the degraded erasure relay channel [7]. Using this capacity bound we have :

$$\begin{cases} R_1^* \le \alpha \min\{(1-pp_{11}), (1-pp_{12})\} \\ R_2^* \le (1-\alpha) \min\{(1-pp_{21}), (1-pp_{22})\} \\ R_1^* \le (1-\alpha) \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22})\} \\ R_2^* \le \alpha \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22})\} \end{cases}$$

Adding the two last bounds obtained in scenario 2, we have :

$$R_1^* + R_2^* \le \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22})\}$$

Therefore; using the time-sharing between single-input erasure relay channels the rate available for collaboration between the two senders is lower than proposed coding scheme.

V. Conclusion

We have presented here a capacity region for the non-degraded multi-sender erasure relay channel. We first derive the capacity bound for the proposed scenario, and then we propose a coding scheme based on MDS code achieving the capacity of this channel. This coding scheme provides a practical and simple way of doing Slepian-Wolf coding in the context of erasure channels. We also show that this coding scheme can achieve the rate higher than a simple time sharing of single sender relay channels.

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APPENDIX

Let's W_1 and W_2 the messages sent by $\mathcal{S}_1$ and $\mathcal{S}_2$. Let's further assume that they are independent and chosen randomly (uniformly) over the sets of integers $\mathcal{W}_1 = \{1, 2, .., 2^{nR_1^*}\}$ and $\mathcal{W}_2 = \{1, 2, .., 2^{nR_2^*}\}$. The rate R_1^* can then bound as :

$$\begin{aligned}
nR_1^* &= H(W_1) \\
&\overset{(a)}{=} H(W_1|W_2) \\
&= I(W_1; Y_2^{sn}, Y_{11}^n, Y_{12}^n|W_2) + H(W_1|Y_2^{sn}, Y_{11}^n \\
&\quad , Y_{12}^n, W_2) \\
&\overset{(b)}{\leq} I(W_1; Y_2^{sn}, Y_{11}^n, Y_{12}^n|W_2) + n\epsilon_n \\
&\overset{(c)}{=} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}|Y_2^{si-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_2) \\
&\quad -H(Y_{2i}^s, Y_{11i}, Y_{12i}|Y_2^{si-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_1, W_2) \\
&\quad +n\epsilon_n \\
&\overset{(d)}{\leq} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad Y_2^{si-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_1, W_2, X_{1i}, X_{2i}) + n\epsilon_n \\
&\overset{(e)}{\leq} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad X_{1i}, X_{2i}) + n\epsilon_n \\
&\overset{(f)}{=} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad X_{1i}) + n\epsilon_n \\
&= \textstyle\sum_{i=1}^n I(X_{1i}; Y_{si}^2, Y_{11i}, Y_{12i}) + n\epsilon_n \\
&= \textstyle\sum_{i=1}^n I(X_{1q}; Y_{sq}^2, Y_{11q}, Y_{12q}|Q = i) + n\epsilon_n \\
&= nI(X_{1Q}; Y_{sQ}^2, Y_{11Q}, Y_{12Q}|Q) + n\epsilon_n \\
&\overset{(g)}{=} nI(X_1; Y_2^s, Y_{11}, Y_{12}|Q) + n\epsilon_n \\
&\overset{(h)}{\leq} nI(X_1; Y_2^s, Y_{11}, Y_{12}) + n\epsilon_n
\end{aligned}$$

where (a) follows from the independence of W_1 and W_2, (b) follows from Fano's inequality, (c) from the chain rule and definition of mutual information, (d) from the fact that removing conditioning increase the first term and conditioning reduces the second term, (e) from the fact that $Y_i = (Y_{si}^2, Y_{11i}, Y_{12i})$ depends only on the current symbol X_{1i} and X_{1i} [2] by the memoryless property of the channel, (f) from the fact that Y_i is the received vector message if X_{1i} is sent over the channel. Also based on the definition of relay channel [3] X_{2i} only depends on the past received symbols and W_2. Therefore at the transmission time i, the received vector Y_i only depends on X_{1i} and conditionally is independent from X_{2i} (also note that X_{2i} send over a channel different from X_{1i}), (g) by defining $X_1 \triangleq X_{1Q}$, $Y_2^s \triangleq Y_{2Q}^s$, $Y_{11} \triangleq Y_{11Q}$ and $Y_{12} \triangleq Y_{12Q}$ as the new random variable where $Q \rightarrow X_1 \rightarrow (Y_s^2, Y_{11}, Y_{12})$ for $|\mathcal{Q}| \leq \min\{|\mathcal{X}_1|, |\mathcal{Y}_s^2|, |\mathcal{Y}_{11}|, |\mathcal{Y}_{12}|\}$, and (h) from the Markov chain properties.

With the same argument we can show that $R_2^* \leq I(X_2; Y_1^s, Y_{21}, Y_{22})$ which leads us to the two first terms of the capacity bound of theorem1.

At the receiver side we use the cut-set bound defined in [2] and we have :

$$\begin{cases} C_3 : & R_1^* + R_2^* \leq I(X_1, X_2; Y_{11}, Y_{21}) \\ C_4 : & R_1^* + R_2^* \leq I(X_1, X_2; Y_{12}, Y_{22}) \end{cases} \tag{3}$$

As said before, the nodes transmit over the physically separated channel. From the point of view of $\mathcal{D}_1$ (resp. $\mathcal{D}_2$) the channel can be modeled by two point to point channel $\mathcal{S}_1 - \mathcal{D}_1$ and $\mathcal{S}_2 - \mathcal{D}_1$ (resp. $\mathcal{S}_1 - \mathcal{D}_2$ and $\mathcal{S}_2 - \mathcal{D}_2$). Under this hypothesis the maximum of $R_1^* + R_2^*$ achieve if $\mathcal{S}_1$ and $\mathcal{S}_2$ send independent codeword over these two independent channel. This lead to the maximum of $I(X_1, X_2; Y_{11}, Y_{21})$ being equal to $I(X_1; Y_{11}) + I(X_2; Y_{21})$ and maximum of $I(X_1, X_2; Y_{12}, Y_{22})$ being equal to $I(X_1; Y_{12}) + I(X_2; Y_{22})$. In other term, the collaboration between the sender and the relay reduces to ensuring that the variable sent by $\mathcal{S}_1$, X_1, and $\mathcal{S}_2$, X_2, are independent from each other but are still complementary to enable a maximal rate at receiver. This leads to the two last terms of the capacity bound of theorem 1$\Box$.