Geodesic active fields on the sphere

IMAGE REGISTRATION

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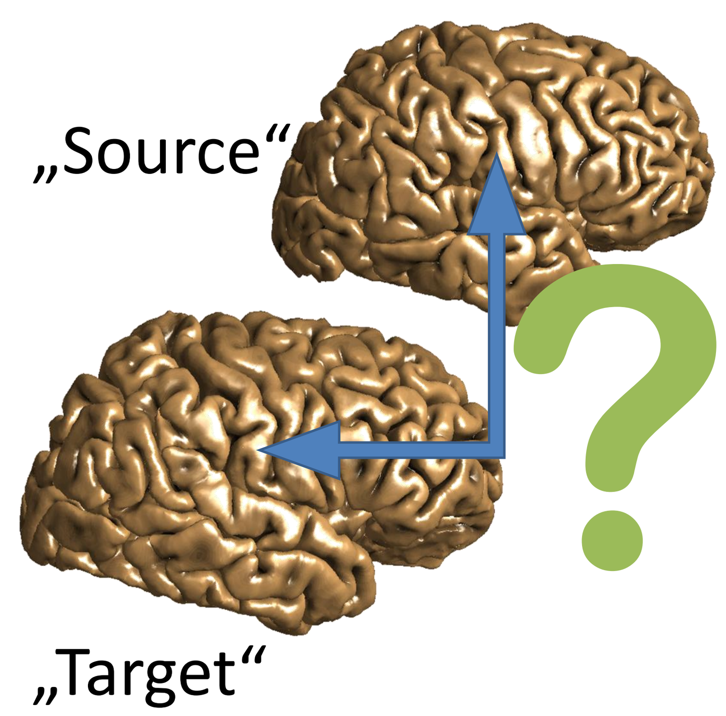
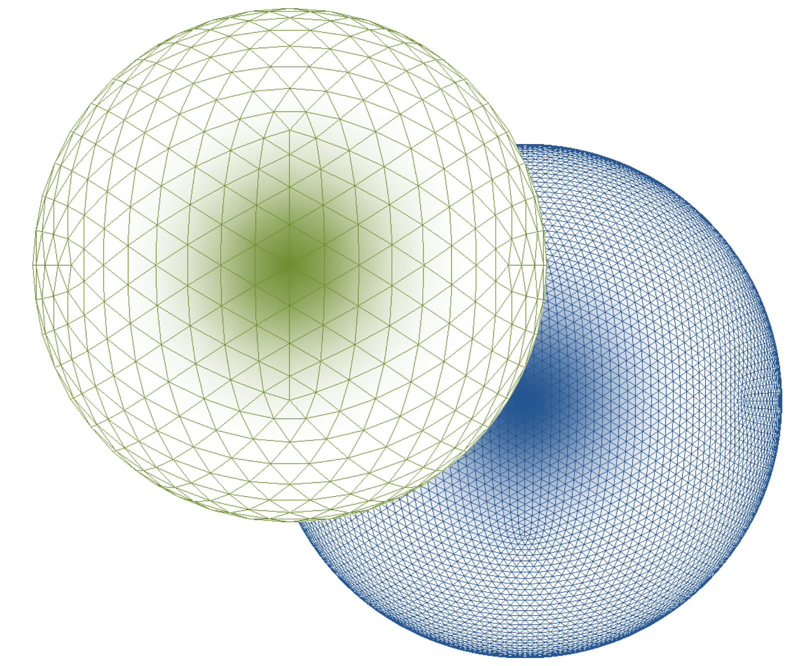
Image Registration
To register cortical maps, it is commonplace to exploit their spherical topology by inflation.



How to comb a porcupine?
The hairy ball theorem states, that no artifact-free, global parametrization of the whole sphere exists.



Diffusive regularization
Impulse response on a coarse and a fine spherical mesh.

Motivation

Image registration is a pre-processing task in many applications, where similar images need to be aligned before they can be compared. Here, we focus on the registration of spherical images, for example spherical maps that are obtained after inflation of the brain cortex. Other applications can be found in omnidirectional vision, such as 3D scene reconstruction and robot navigation. Image registration algorithms usually optimize an additive compromise between an image alignment and a regularization term. In this work, we propose to use geodesic active fields with mutual weighting instead.

Geometric regularization

Geodesic active fields (GAF) for image registration embed the deformation field in a higher dimensional space:

$$X : (x, y) \to (x, y, u, v)$$

A metric tensor h_{ij} is defined, and pulled back as $g_{\mu\nu}$ into the initial domain:

$$\begin{cases} h_{ij} = \mathrm{diag}(1, 1, \beta^2, \beta^2) \\ g_{\mu\nu} = \partial_\mu X^i \partial_\nu X^j h_{ij} \end{cases}$$

The determinant of the metric tensor, g, is a measure of the irregularity of the embedded deformation field.

Weighting function

To incorporate the matching quality of the images, we include a weighting function f,

e.g. $f = 1 + \alpha\,(F(\mathbf{x}) - M(\mathbf{x} + \mathbf{u}))^2$

where F and M are source and target image, and define the weighted Polyakov energy:

$$E = \int f \sqrt{g}\,\mathrm{d}\mathbf{x}$$

Minimizing this energy drives the deformation field toward minimal hyper-area, while being attracted by a deformation field that brings the two images into registration.

Spherical specificities

Because the sphere is not a Riemannian manifold, the GAF framework can not be applied directly. Instead, we replace the global embedding of the whole image domain by many local embeddings at each vertex of the spherical mesh, introducing a local coordinate chart at each node.
The deformation field is modeled using local tangent vectors. Gradient and curvature computations are discretized on the one-ring patch around a vertex, using parallel transport on the spherical geodesic.
For image resampling, we use triangle walk on the mesh and barycentric interpolation.

Conclusions and Outlook

The Geodesic active fields framework offers many advantages over standard techniques such as Demons. The GAF energy is purely geometric and parametrization invariant. It applies to any Riemannian image domain. Geometric regularization allows for more sophisticated smoothing than Gaussian diffusion. The multiplicative link between data-term and regularization renders the smoothing data-dependent and spatially-adaptive, which is valuable e.g. in presence of spatially varying noise. Future work will focus on fast numerical schemes and application to real cortical feature maps.

References

B. Fischl, M. I. Sereno, and A. M. Dale, „Cortical surface-based analysis II: Inflation, flattening, and a surface based coordinate system," *Neuroimage*, 9(2):195-207, 1999.

N. Sochen, R. Kimmel, and R. Malladi, „A general framework for low level vision," *IEEE Trans. Image Process.*, 7(3):310-318, 1998.

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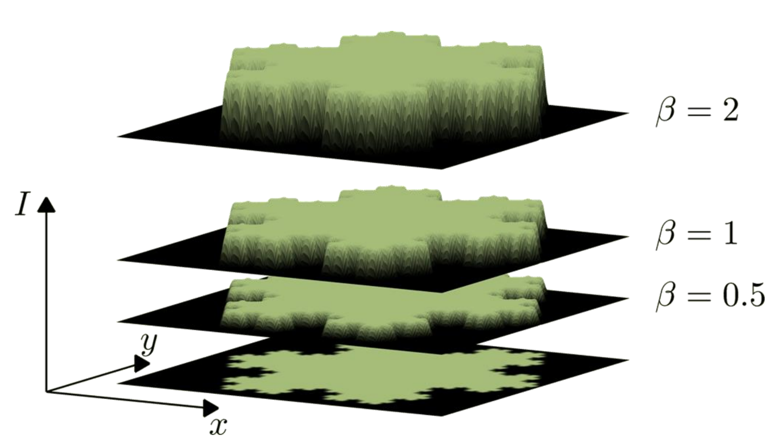
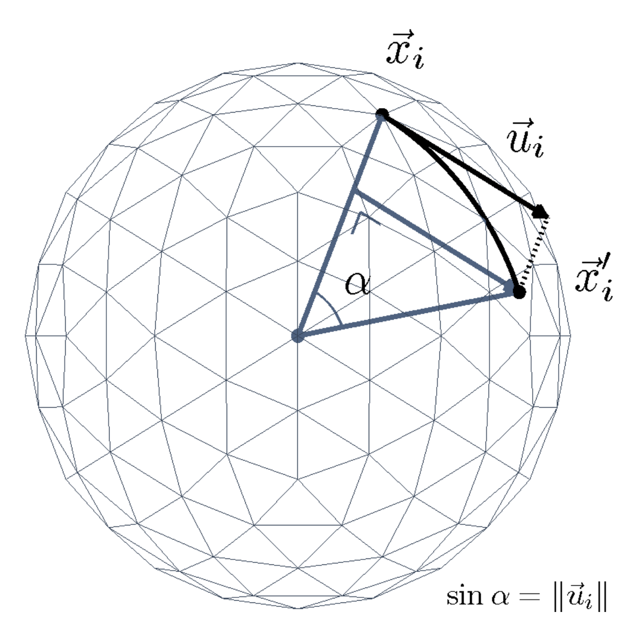
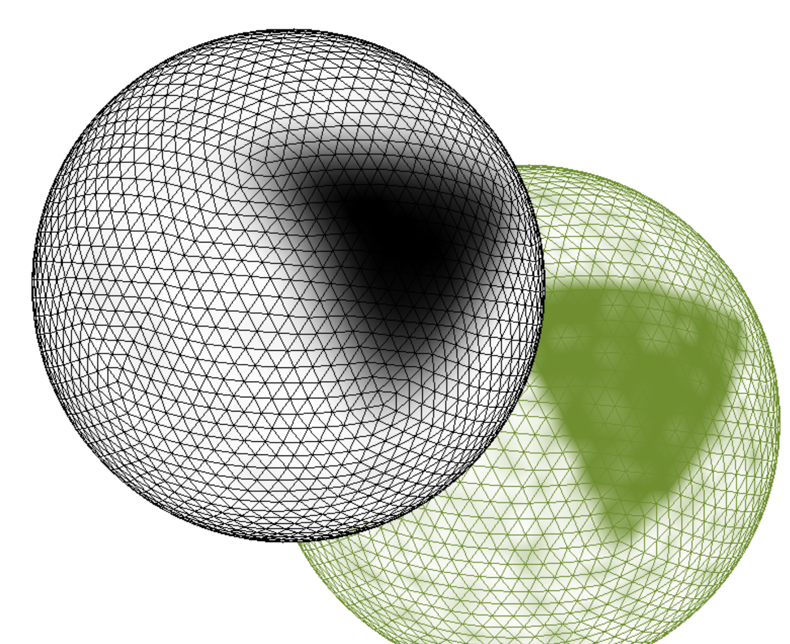
Beltrami embedding
A graylevel image is embedded in 3D. Minimizing the Polyakov action smoothens the image.



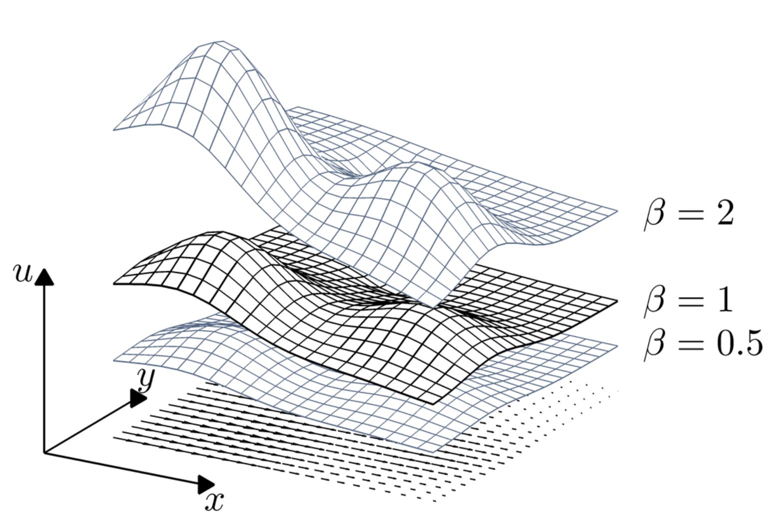
Deformation model
The individual displacement of a mesh vertex from $\mathbf{x}_i$ to $\mathbf{x}'_i$ is encoded by a tangent vector $\mathbf{u}_i$.



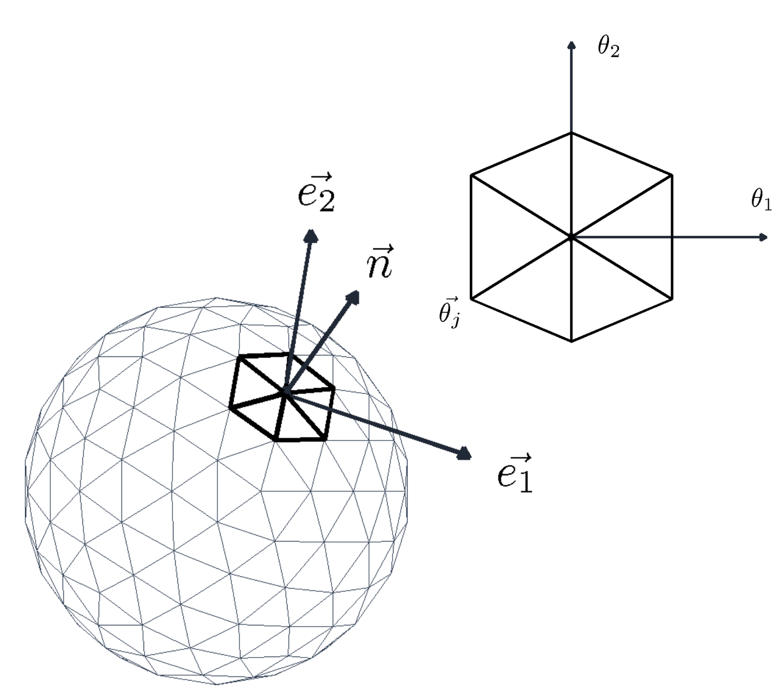


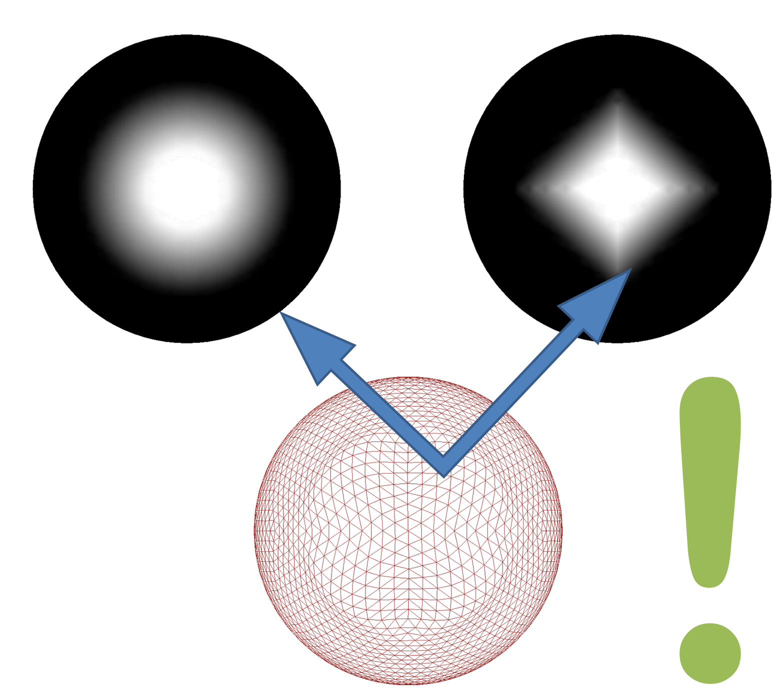
Anisotropic regularization
The degree of anisotropy in the regularization can be controlled by the aspect ratio β.

Geodesic active fields
The Polyakov energy is a measure of deformation field regularity. Weighting drives the minimization toward registration.



Local coordinates
For each vertex, a local coordinate chart is constructed based on two tangent vectors and the normal. Surrounding deformation vectors are mapped through parallel transport.





Registration of synthetic images
A disc and a square painted on a spherical mesh are successfully registered.