Column Generation for the Split Delivery VRP

Alberto Ceselli, DTI – University of Milan
Giovanni Righini, DTI – University of Milan
Matteo Salani, TRANSP-OR – EPF Lausanne

CG2k8 – Aussois, June 19th 2007

The problem

- The split delivery vehicle routing problem (SDVRP)
- Literature review

A CG scheme

- Formulations
- The pricing problem

Results

- Experiments

The VRP

It is given:

- a fleet of vehicles (K), each having a loading capacity (Q)
- a set of customers (V), each requiring the delivery of goods (d_i)
- a network (G=(V,A))

Decide:

- a route for each vehicle

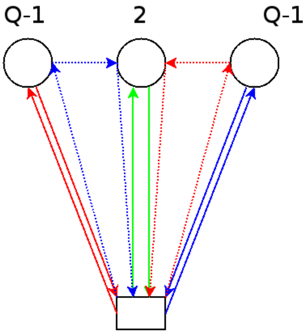
Such that:

- each customer is in a route
- the sum of demands of the customers in each route does not exceed the vehicle capacity
- the total travel distance is minimized

(BCP Fukasawa et al. '06, Book Golden et al eds. '08)

The Split Delivery VRP

SDVRP: each customer can belong to more than one route, and (fractionally) served by more than one vehicle:



potentially yielding a X2 saving.

It is a problem with several applications.

Heuristics

- Frizzel and Griffin ('95): grid network, tight multiple time windows and nonlinear loading costs, contruction and local search, instances with up to 150 customers
- Bompadre Dror Orlin ('98): approximation algorithms
- Archetti Savelsbergh Speranza ('06): tabu search (up to 200 customers)
- Archetti Savelsbergh Speranza ('07): MIP based heuristic (same instances)
- Chen Golden Wasil ('07): construction and MIP heuristic (up to 200 customers)
- Jin Liu Eksioglu ('07): column generation heuristic (good for instances with large customer demands).

Exact methods

- Reduction to VRP (if data is rational in polynomial space and time)
- Dror Laporte Trudeau ('94): arc-based formulation, subtour and connectivity constraints, branching (up to 20 customers to optimality)
- Belenguer Martinez Mota ('00): polyhedral study, model for a relaxation of the problem
- Jin Lin Bowden ('06): two-stage (partitioning–routing), with 7 new classes of valid inequalities (up to 20 customers to optimality)

Column generation

- Gendreau Dejax Feillet Gueguen ('07): SDVRP with TWs
  - Set covering ILP formulation
  - Column generation and hard pricing problem
  - Relaxed model with easier pricing
  - Few instances with up to 50 customers to optimality
- *Desaulniers (CG2k8): SDVRP with TWs*
  - instances with up to 100 customers to optimality

Our contribution

A problem reformulation and CG scheme which:

- yields good lower bounds on the optimal value
- is ‘simple’ to compute
- allows for many VRP strategies to be applied (valid cuts, branching ...)
- ‘nicely’ fits in a branch-and-price-and-cut scheme

SDVRP flow formulation

Flow formulation (Dror Laporte Trudeau '94):
FLP

$$
\begin{aligned}
\min z_{FP} = \sum_{i\in V}\sum_{j\in V} c_{ij} \sum_{k\in K} x_{ijk} & & \\
\text{s.t.} \sum_{k\in K} y_{ik} = 1 & \qquad \forall i \in V & \\
\sum_{i\in V} d_i y_{ik} \leq Q & \qquad \forall k \in K & (1)\\
\sum_{j\in V} x_{ijk} \geq y_{ik} & \qquad \forall i \in V, k \in K & (2)\\
\text{subtour \& VRP const ...} & & (3)\\
x_{ijk} \in \{0,1\}, y_{ik} \geq 0 & \qquad \forall i,j \in V, k \in K & (4)
\end{aligned}
$$

SDVRP flow formulation

Flow formulation (Dror Laporte Trudeau '94):
FLP

$$\min z_{FP} = \sum_{i\in V}\sum_{j\in V} c_{ij} \sum_{k\in K} x_{ijk}$$

$$\text{s.t.} \sum_{k\in K} y_{ik} = 1 \qquad \forall i \in V$$

$$(x_{ijk}, y_{ik}) \in \Omega_k \qquad \forall\, k \in K$$

LP relaxation and convexification:
$\Omega_k = \text{conv}\{(x_{ijk}, y_{ik}) \mid 0 \le x_{ijk} \le 1, y_{ik} \ge 0, (1), (2), (3)\}$

DW reformulation

For each $k \in K$, given an extreme point r: $(\bar{x}_{ij}^r, \bar{y}_i^r) \in \Omega_k$

$c_r = \sum_{i \in V} \sum_{j \in V} c_{ij} \bar{x}_{ij}^r$

and

$$
\begin{aligned}
x_{ijk} &= \sum_{r \in \Omega_k} \bar{x}_{ij}^r \lambda_r && \forall i, j \in V \\
y_{ik} &= \sum_{r \in \Omega_k} \bar{y}_i^r \lambda_r && \forall i \in V \\
\text{s.t.} \sum_{r \in \Omega_k} \lambda_r &= 1 \\
\lambda_r &\geq 0 && \forall r \in \Omega_k
\end{aligned}
$$

Extended formulation

CCLP

$$\begin{aligned} \min z_{CCLP} = & \sum_{k\in K}\sum_{r\in\Omega_k} c_r\lambda_r \\ \text{s.t.} & \sum_{k\in K}\sum_{r\in\Omega_k} \bar{y}_i^r\lambda_r \geq 1 && \forall i \in V(\pi_i) \\ & \sum_{r\in\Omega_k} \lambda_r \leq 1 && \forall k \in K \\ & \lambda_r \geq 0 && \forall k \in K, r \in \Omega_k \end{aligned} \tag{1}$$

(+ tightening constraints)
observation: there always exists a solution in which only cols with at most 1 fract coordinate are selected (set $\bar{\Omega}$). (Jin et al '07)

Simplifying the pricing

- let be $a_i^r = \lceil \bar{y}_i^r \rceil$
- for each $k \in K$ we define $\tilde{\Omega}_k$ as the set of columns satisfying

$$\sum_{i \in V | a_i^r = 1} d_i - \max_{i \in V | a_i^r = 1} d_i + 1 \leq Q$$

- we observe that $\bar{\Omega}_k \subseteq \tilde{\Omega}_k$
- we substitute each covering constraint (1) as follows

$$\sum_{k \in K} \sum_{r \in \Omega_k} \bar{y}_i^r \lambda_r \geq 1 \quad \forall i \in V \rightarrow$$

$$\sum_{k \in K} \sum_{r \in \tilde{\Omega}_k} a_i^r \lambda_r \geq 1 \quad \forall i \in V \tag{2}$$

- we obtain a relaxation of the master
  (adding more vars and rounding up the lhs of $\geq$ constr.).

Final model

MP

$$\min z_{MP} = \sum_{k \in K} \sum_{r \in \tilde{\Omega}_k} c_r \lambda_r$$

$$\text{s.t.} \sum_{k \in K} \sum_{r \in \tilde{\Omega}_k} a_i^r \lambda_r \geq 1 \qquad \forall i \in V \ (\gamma_i \ )$$

$$\sum_{r \in \tilde{\Omega}_k} \lambda_r \leq 1 \qquad \forall k \in K$$

$$\lambda_r \geq 0 \ \forall k \in K, \ r \in \tilde{\Omega}_k$$

$$\tilde{c}_r = \sum_{i \in V} \sum_{j \in V} c_{ij} \bar{x}_{ij}^r - \sum_{i \in V} \gamma_i \ a_i^r + \ldots$$

Final model

MP

$$\min z_{MP} = \sum_{k\in K} \sum_{r\in\tilde{\Omega}_k} c_r \lambda_r$$

$$\text{s.t.} \quad \sum_{r\in\tilde{\Omega}_k} a_i^r \lambda_r \geq y_{ik} \qquad \forall k \in K,\ \forall i \in V\ (\gamma_{ik})$$

$$\sum_{r\in\tilde{\Omega}_k} \lambda_r \leq 1 \qquad \forall k \in K$$

$$\sum_{k\in K} y_{ik} = 1 \qquad \forall i \in V$$

$$\lambda_r \geq 0\ \forall k \in K,\ r \in \tilde{\Omega}_k \qquad y_{ik} \geq 0\ \forall i \in V,\ k \in K$$

$$\tilde{c}_r = \sum_{i\in V} \sum_{j\in V} c_{ij}\bar{x}_{ij}^r - \sum_{i\in V} \gamma_{ik} a_i^r + \ldots$$

Final model

MP

$$\min z_{MP} = \sum_{k \in K} \sum_{r \in \tilde{\Omega}_k} c_r \lambda_r$$

$$\text{s.t.} \quad \sum_{r \in \tilde{\Omega}_k} a_i^r \lambda_r \geq y_{ik} \qquad \forall k \in K,\ \forall i \in V\ (\gamma_{ik})$$

$$\sum_{r \in \tilde{\Omega}_k} \lambda_r \leq 1 \qquad \forall k \in K$$

$$\sum_{k \in K} y_{ik} = 1 \qquad \forall i \in V$$

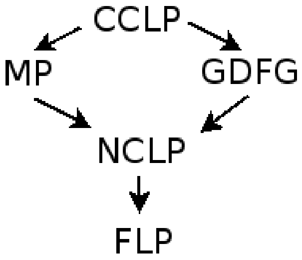
$$\sum_{i \in V} d_i y_{ik} \leq Q \qquad \forall k \in K$$

$$\lambda_r \geq 0\ \forall k \in K,\ r \in \tilde{\Omega}_k \qquad y_{ik} \geq 0\ \forall i \in V,\ k \in K$$

$$\tilde{c}_r = \sum_{i \in V} \sum_{j \in V} c_{ij} \bar{x}_{ij}^r - \sum_{i \in V} \gamma_{ik} a_i^r + \ldots$$

Quality of the bound

- FLP: three-index flow based formulation
- CCLP: DW reformulation of FLP
- MP: our formulation
- GDFG: Gendreau et al formulation
- NCLP: DW reformulation of FLP leaving the capacity constraints in the master problem



The pricing problem (PP)

The PP is a resource constrained elementary shortest path problem

- labels contain both
  D: the total demand of the visited customers
  d_{sc}: the demand of the potential split customer
- during extension, the capacity constraint can still be respected if $D + d_j - \max(d_j, d_{sc}) + 1 \leq Q$.
- label S' can dominate label S'' only if
  $D' \leq D''$
  $D' - d'_{sc} \leq D'' - d''_{sc}$

Pricing problem - implementation

- bounded bi-directional DP
- Decremental State Space relaxation with smart core initialization (RS '07)
- Set U of unreachable customers (Feillet '04)
- Greedy pricer
- Heuristic DP pricer (relaxed domination criteria + Fractional Knapsack Bounding)
- involved multiple pricing policy (tackle symmetries)

Computational results

We implemented the CG scheme in C using GLPK 4.16 as LP solver, subset of Solomon instances (23 r- and 4 c- instances with TWs)

	GDFG	MP
avg dual. gap	1.34%	1.64%
avg CPU time(s)	3.81	16.2
inst. with best bound	9	11
inst. with no dual. gap	7	11

Additional remarks

- Effect of stabilization (using GLPK interior point method for LPs):
  50% iterations reduction (but much longer LP solution times).
- Heuristics: only integrality checking.
- Branching: only naïve branching implemented,
  some instances with up to 50 customers solved to optimality.

Additional remarks

- Effect of stabilization (using GLPK interior point method for LPs):
  50% iterations reduction (but much longer LP solution times).
- Heuristics: only integrality checking.
- Branching: only naïve branching implemented,
  some instances with up to 50 customers solved to optimality.

Many thanks for your attention :o) Comments :?!