The Constrained Multinomial Logit: A semi–compensatory choice model

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Introduction

- Traditional logit models assume a compensatory utility function (trade-off between attributes).
- This approach fails to recognize attribute thresholds in consumer behavior.
- A mixed strategy is proposed, using compensatory utilities with cutoff factors that that restrain choices to the available domain.

Contents of presentation

- Discrete choice problem
- Constrained random utility
- Constrained multinomial logit
- Application examples
- Calibration issues
- Conclusions

The constrained discrete choice problem

- Consumer's problem:

$$\max_{\delta_{ni}} \sum_{i \in C} \delta_{ni} U_n(X_i, p_i)$$

$$s.a \quad \sum_{i \in C} \delta_{ni} = 1, \quad \delta_{ni} \in \{0,1\} \quad \forall i \in C$$

- Requires to:
  - Specify a utility function able to include constraints or
  - Specify a predefined set of available alternatives (*C*)

The constrained discrete choice problem

- **Rational behavior:** Max utility s.t. constraints:

$$\max_{\delta_{ni}} \sum_{i \in C} \delta_{ni} U_n(X_i, p_i)$$

$$s.t \quad \sum_{i \in C} \delta_{ni} = 1, \quad \delta_{ni} \in \{0,1\} \quad \forall i \in C$$

$$a_{nk} \leq X_{ik} \leq b_{nk} \quad \forall i \in C, k = \{1, \ldots, K-1\}$$

$$a_{nK} \leq p_i \leq b_{nK} \quad \forall i \in C$$

Approaches

- Non compensatory utility, e.g. elimination by aspects **Tversky, 1972**

- Two stage approach. **Manski, 77** **Swait - BenAkiva, 87** **BenAkiva - Boccara, 95** **Cantillo - Ortúzar, 04**
  - Generate each consumer's feasible choice set.
    Difficulty: large choice sets
  - Heuristic to reduce choice sets **Morikawa, 95**

- One step approach: reduced utility
  Deterministic model: linear penalties included in utility. **Swait, 01**
  Continuous but non-differentiable

  Simulate availability/perception implicitly in the extended utility. Binomial logit **Cascetta - Papola, 01**

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Constrained random utility

$$V_n(Z_i) = V_n^C(Z_i) + \frac{1}{\mu}\ln\phi_{ni}(Z_i) + \varepsilon_{ni}$$

Compensatory (indirect) utility

Utility penalty

Random term

$$\phi_{ni} = \prod_{k=1}^{K} \phi_{nki}^{L}(a_{nk}) \cdot \phi_{nki}^{U}(b_{nk})$$

Constrained random utility

- Lower and upper cutoffs:

$$\phi_{nki}^{L} = \frac{1}{1+\exp\left(\omega_k \left(a_{nk} - Z_{ki} + \rho_k\right)\right)} = \begin{cases} \to 1 & if \quad a_{nk} << Z_{ki} \\ \eta_k & if \quad a_{nk} = Z_{ki} \end{cases}$$

$$\phi_{nki}^{U} = \frac{1}{1+\exp\left(\omega_k \left(Z_{ki} - b_{nk} + \rho_k\right)\right)} = \begin{cases} \to 1 & if \quad b_{nk} >> Z_{ki} \\ \eta_k & if \quad b_{nk} = Z_{ki} \end{cases}$$

$$\rho_k = \frac{1}{\omega_k} \cdot \ln\left(\frac{1-\eta_k}{\eta_k}\right)$$

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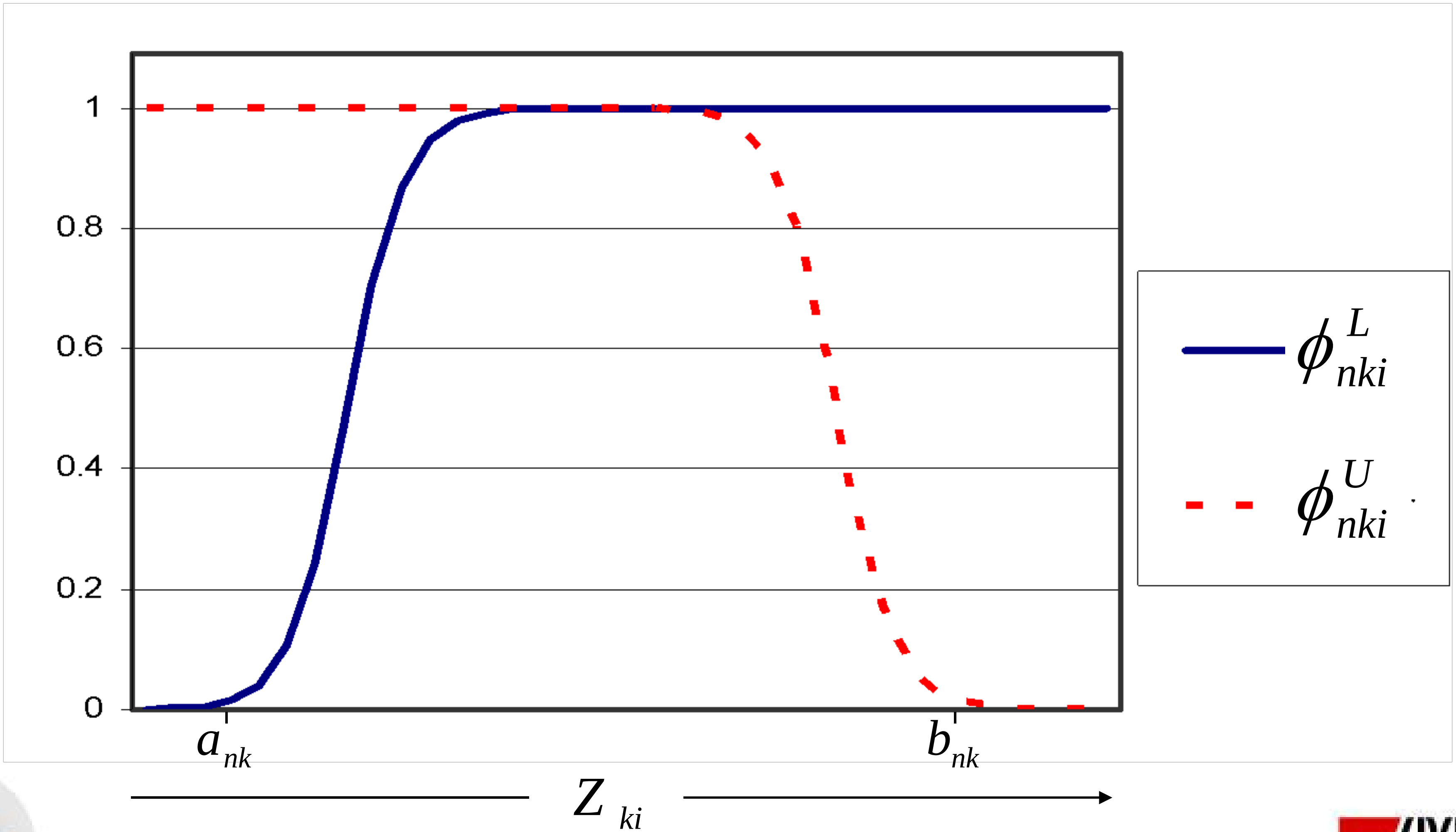
Constrained random utility



Constrained Multinomial Logit

$$V_n(Z_i) = V_n^C(Z_i) + \frac{1}{\mu}\ln\phi_{ni}(Z_i) + \varepsilon_{ni}$$

Gumbel distributed (0,*μ*)

$$P_{ni} = \frac{\phi_{ni}\cdot\exp(\mu V_{ni}^C)}{\sum_{j\in C}\phi_{nj}\cdot\exp(\mu V_{nj}^C)}$$

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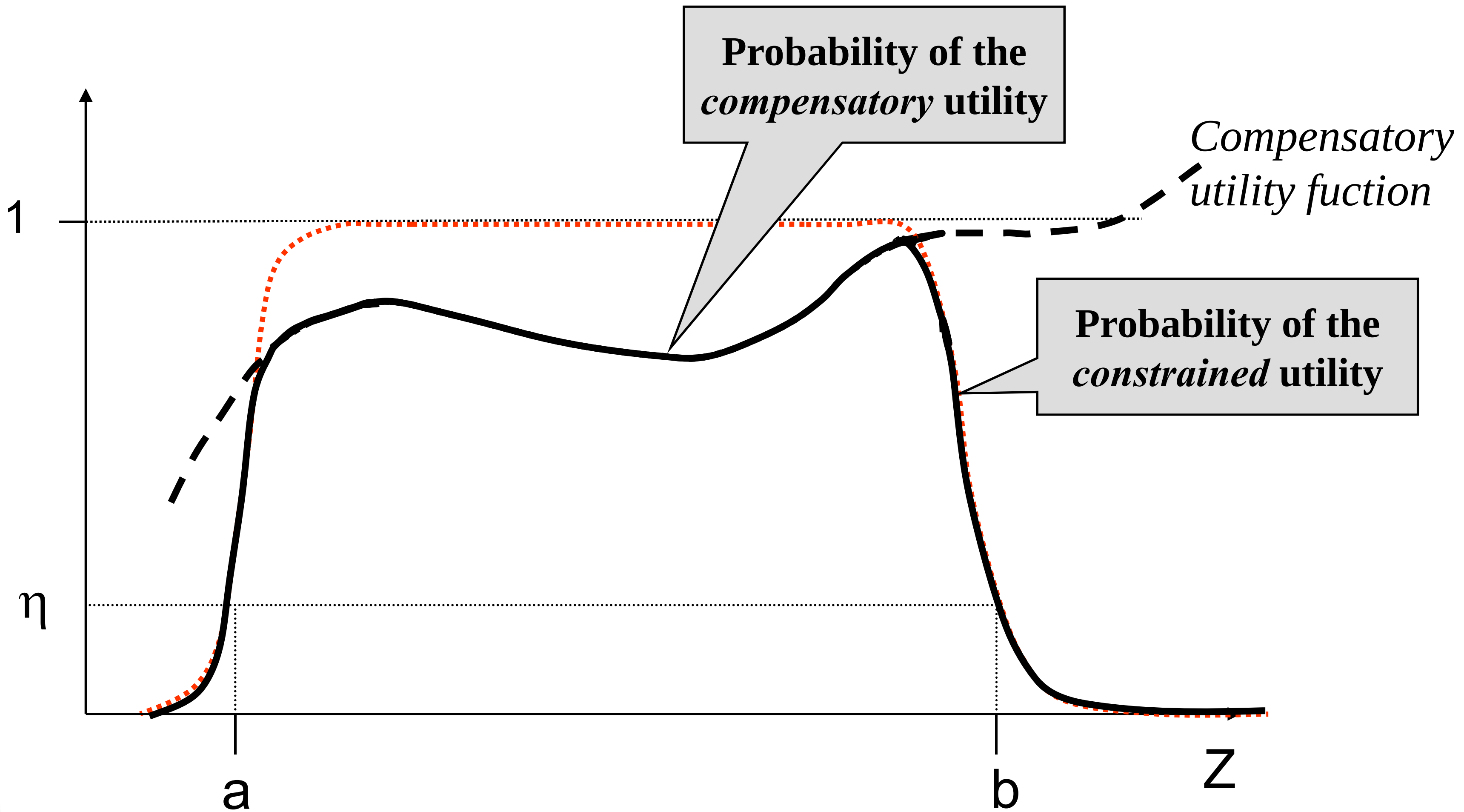
Constrained Multinomial Logit



Constrained Multinomial Logit

- Properties:
  - Preserves the closed logit formula
- Represents a joint logit model
  - Modeling compensatory choice
  - Modeling constraint violation
- Applications:
  - **Real estate supply**: planning regulations
  - **Consumers**: income and time budgets, attribute perception, externalities and agglomeration economies
  - **Transport**: congestion

Application example 1

- Land-use model (Martinez et al, 2008):

$$P_{n/i} = \frac{\phi_{ni} \cdot \exp(B_{ni})}{\sum_{g \in C} \phi_{gi} \cdot \exp(B_{gi})}$$

$$B_{ni} = \alpha_n + \sum_k \beta_{nk} x_{ik}$$

$$\phi_{ni} = \frac{1}{1 + \exp\left(\omega \left(\delta_n - I_i\right)\right)}$$

Average income in zone *i*

To estimate:

$$\delta_n = a_n + \rho_n$$

$$\omega, \beta_{nk}$$

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Application example 1

Calibration of cutoff parameters for a land use model

Parameter	Income group	MNL $\phi = 1$	CMNL $\phi \neq 1$
α_n	2	−3.329	−0.238**
	3	−8.130	−2.272**
	4	−14.228	−4.781
	5	−24.808	−10.257
ln(floor space)	2	−1.840	−0.012**
	3	−0.078*	0.493**
	4	−0.857	1.022
	5	0.346**	2.502
ln(zone income)	2	0.723*	
	3	1.075	
	4	2.013	
	5	2.442	
Accessibility	1	0.283**	0.492**
	2	1.636	1.692
	3	2.262	2.295
	4	3.926	3.966
	5	3.125*	3.377
δ_n	1		−63.769**
	2		−23.953
	3		−18.290
	4		−11.601**
	5		−0.069**
ω			0.242
Log-likelihood		−3.313	−3.316
Nr observations		600	600

Note: Estimates without asterix are significant (t-test>1.96), except when indicated by * (1.7<test-t<1.96) and by ** (t-test<1.7).

Application example 1

- Similar log-likelihood
- Cutoff parameters were possible to identify
- Constants are lower in the CMNL (behavior explained by cutoffs)
- Different forecasting results when constrained attribute changes significantly

Application example 2

- Land-use model (MUSSA 2008)

Number of bidding households

$$P_{n/i} = \frac{\phi_{ni} \cdot H_n \cdot \exp(B_{ni})}{\sum_{g \in C} \phi_{gi} \cdot H_g \cdot \exp(B_{gi})}$$

$$\phi_{ni} = \frac{1}{1 + \left(\frac{1-\eta}{\eta}\right) \exp(\omega (a_n - I_i))}$$

$$\rho = \frac{1}{\omega} \cdot \ln\left(\frac{1-\eta}{\eta}\right)$$

To estimate

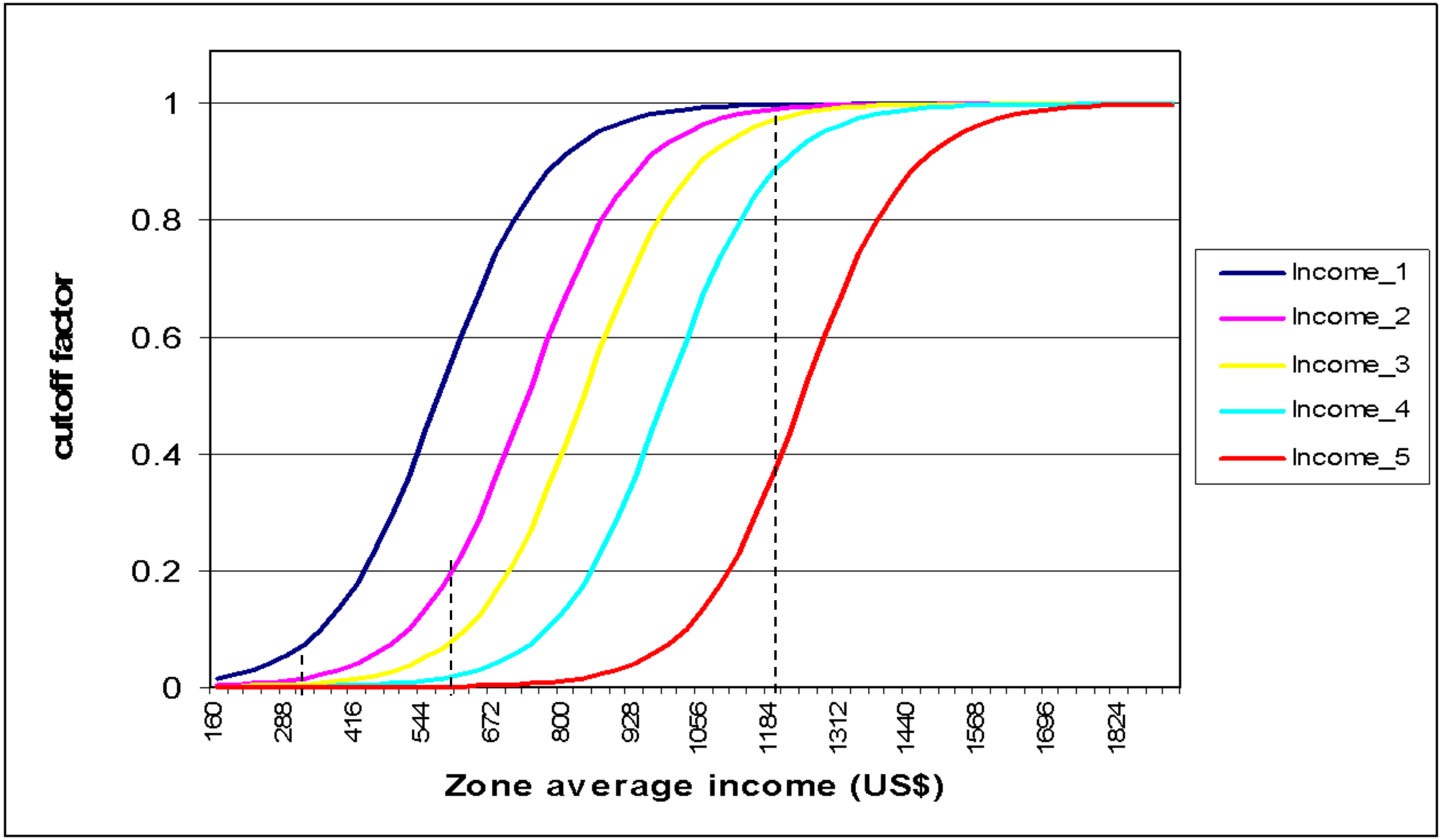
Application example 2

Parameter	Income level				
	1	2	3	4	5
constant	0	0.658	-1.492	-3.799	-10.828
1_appartment	0	-0.118	2.842	-0.102	-0.186
ln(floor space)	0	-0.234	0.356	1.164	0.047
% non_res_surface	0.816	2.519	0.444	2.559	0
accessibility	1.071	1.288	1.736	2.875	2.570
a_n	3.542	8.630	11.879	16.610	24.553
ω	0.3239	0.3239	0.3239	0.3239	0.3239

Min tolerated average zone-income (US$)	113	276	380	531	786

Income levels (US$)	< 296	296 – 593	593 – 1186	1186 – 2371	> 2371

Application example 2



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Calibration issues

- Explicit exogenous constraints (budget, capacity) are useful when forecasting demand.
- **Problems:**
  - Every observation complying with restrictions.
  - Correlation between parameters in the cutoff and the compensatory utility function.

Conclusions

- The CMNL enhances the discrete choice models by imposing a realistic domain avoiding the choice set generation.
- Preserves the closed logit formula.
- Allows to include multiple constraints
- It can be used to model both endogenous and exogenous constraints.
- Requires further research on calibration methods

Questions?