Two-stage column generation

A novel framework

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Outline

Motivation

Optimization of container terminal operations

Methodology

Two-stage column generation

Container terminals: Overview

Container terminals: Quayside

Tactical Berth Allocation with QCs Assignment

G. Giallombardo, L. Moccia, M. Salani & I. Vacca
Proceedings of the European Transport Conference, October 2008.

Problem description

- *Tactical Berth Allocation Plan* (TBAP): assignment and scheduling of ships to berths, according to time windows for both berths and ships;
- *Quay-Cranes Assignment*: a QC profile (number of QCs per shift, ex. 332) is assigned to each ship.

Issues

- the chosen profile determines the ship's handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

Objective

- maximize the value of profiles and minimize yard-related costs.

Tactical Berth Allocation with QCs Assignment

MILP formulation

- compact decision variables: scheduling (x_{ij}^k), profile assignment (λ_{ip})
- precedence constraints, capacity constraints, time windows constraints

Column generation approach

- Dantzig-Wolfe (extensive) reformulation
- we associate sequences of ships to berths $\longrightarrow$ extensive decision variables z_s^k
- ESPPRC pricing subproblem

Complexity of the pricing subproblem

- the handling time of each ship depends on the profile assigned to the ship;
- one node for each ship, for each profile, for each time step;
- the associated network is huge $\longrightarrow$ solving ESPPRC is impractical!

Two-stage column generation

M. Salani & I. Vacca
Proceedings of the Swiss Transport Research Conference, October 2008.

Context

Dantzig-Wolfe (DW) reformulation of combinatorial problems.

Motivation

Many problems exhibit a *compact* formulation with many variables (possibly an exponential number) which results in an unmanageable associated pricing problem, when the extensive formulation is obtained through DW.

Similar problems, in addition to TBAP:

- VRP with Discrete Split Delivery
- Field Technician Scheduling Problem
- Routing helicopters for crew exchanges on off-shore locations

Two-stage column generation

Novel idea

Develop a framework in which a combinatorial problem is solved starting from a partial compact formulation, with the same approach used in column generation (CG) for the restricted extensive formulation, obtaining a partial restricted master problem.

Algorithm 1: Two-stage column generation

Input: partial compact formulation with a subset of compact variables (λ_p)

repeat

  repeat

    generate extensive variables (z_s^k) for partial master problem **(CG1)**

  until *optimal partial master problem* ;

  generate compact variables (λ_p) for partial compact formulation **(CG2)**

until *optimal master problem* ;

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Two-stage column generation

Advantages

- the pricing problem is easier to solve;
- possibly many sub-optimal compact variables are left out from the formulation;

Drawbacks

- we don't obtain a valid lower bound from (CG1).

Possible solution to LB computation

- add some ad-hoc artificial variables to the partial compact formulation;
- in TBAP, for instance, we add artificial super-optimal profiles by combining variables λ_p not yet in the partial compact formulation.

$\Longrightarrow$ **Consistent methodology, although such bounds may be weak.**

Two-stage column generation: reduced costs

Input: partial compact formulation with a subset of compact variables (λ_p)

repeat

  repeat

    generate extensive variables (z_s^k) for partial master problem **(CG1)**

  until *optimal partial master problem* ;

  generate compact variables (λ_p) for partial compact formulation **(CG2)**

until *optimal master problem* ;

- In **(CG1)** standard column generation applies: the dual optimal vector π is known at every iteration and thus reduced costs $\tilde{c} = [c - \pi A]$ of variables z_s^k can be directly estimated.
- In **(CG2)** we need to know the reduced costs of variables λ_p in order to decide which variables are profitable to be added to the partial compact formulation, if any. Unfortunately, we do not have any *direct* information available.

Reduced costs of compact variables

- Walker (1969): method which can be applied when the pricing problem is a pure linear program.
- Poggi de Aragao & Uchoa (2003): coupling constraints in the master problem formulation.
- Irnich (2007): reduced costs estimation based on paths (not directly applicable to our two-stage framework).
- Salani & V. (2007): reduced costs estimation obtained through complementary slackness conditions, applicable to general compact formulations.

Sub-optimal variable detection

Given IP $= \{\min c^T x : Ax \geq b, x \in \mathbb{Z}_+^n\}$ with upper bound UB, let π be a feasible solution to the dual of the linear programming relaxation of IP.

Theorem (Nemhauser & Wolsey, 1988)
If the reduced cost of a non-negative integer variable exceeds a given optimality gap, the variable must be zero in any optimal integer solution. In other words:

$$\tilde{c}_e = (c - \pi A)_e > UB - \pi b \implies x_e = 0 \tag{1}$$

Theorem (Irnich et al., 2007)
If the minimum reduced cost of all path variables of a DW master problem containing arc (i, j) exceeds a given optimality gap, no path that contains arc (i, j) can be used in an optimal solution. Hence, the arc (i, j) can be eliminated. In other words:

$$\min_{p \in \mathcal{F}_{ij}^{st}} \tilde{c}_p(\pi) > UB - \pi b \implies x_{ij} = 0 \tag{2}$$

where $\mathcal{F}_{ij}^{st}$ is the set of feasible $s - t$ paths containing arc (i, j).

Sub-optimal variable detection

Let the restricted master problem (MP) be defined on the whole set of profiles P and let the partial restricted master problem (PMP) be defined on a subset of profiles $P' \subset P$.

Let UB be an upper bound for both MP and PMP, let LB be a lower bound for MP and LB' be a lower bound for PMP, with $LB' \geq LB$.

Let π be a feasible dual solution to MP and π' be a feasible dual solution to PMP.

$\Longrightarrow$ Given the reduced cost $\tilde{c}_s$ of sequence s and a profile p, we define the quantities:

$$lb_p = LB + \min_{s \in \mathcal{F}_p} \tilde{c}_s \quad (3)$$

$$lb'_p = LB' + \min_{s \in \mathcal{F}'_p} \tilde{c}_s \quad (4)$$

where:

- $\mathcal{F}_p = \{$ (subset of) feasible sequences s induced by P such that $\lambda_p = 1\}$
- $\mathcal{F}'_p = \{$ (subset of) feasible sequences s induced by P' such that $\lambda_p = 1\}$

Sub-optimal variable detection

Variable elimination rule

$$lb_p > UB \implies \lambda_p = 0 \text{ in optimal MP } \quad \text{(over } P \text{)}$$

$$lb'_p > UB \implies \lambda_p = 0 \text{ in optimal PMP (over } P'\text{)}$$

Question

Can variable elimination in PMP be extended to MP?

Conjecture

The result cannot be extended straightforward... but we are working on additional sub-optimality conditions which would allow the extension.

Conclusion & future work

Main contribution

- a novel framework to tackle problems with a combinatorial number of compact formulation variables.

Ongoing work

- computational tests;
- improve lower bounds;
- sub-optimal variable detection.