

Modeling dominated choice alternatives using the constrained multinomial logit

F. Martinez, University of Chile, CL; E. Cascetta, F. Pagliara, University of Naples Federico II, IT, M. Bierlaire, EPFL Lausanne, CH, K. W. Axhausen, ETH, Zurich, CH

Speaker: Michel Bierlaire

`transp-or.epfl.ch`

Transport and Mobility Laboratory
Ecole Polytechnique Fédérale de Lausanne, Switzerland



Outline

- Motivation
- Concept of cutoffs (Constrained logit model)
- Concept of dominance
- Using dominance in the Constrained Logit Model
- Preliminary results
- Perspectives

Motivation

- Discrete choice models.
- Concept of utility based on trade-offs.
- Attributes threshold generally not accounted for.
- Dominated alternatives may not even be considered in the choice set.
- How do we model that?

Motivation

- Manski (1977): individual-based choice-set based on deterministic constraints
- Swait and Ben-Akiva (1987): random constraints
- Swait (2001), Martinez et al. (2008): Attribute cutoffs
- Cascetta and Papola (2005), Cascetta et al. (2007): implicit perception, dominance values

Idea: combine cutoffs and dominance

Cutoffs

Optimization problem of rational consumer n :

$$\max_{\delta_{ni}} \sum_{i \in \mathcal{C}} \delta_{ni} U_{in}(X_i)$$

subject to

$$\sum_{i \in \mathcal{C}} \delta_{ni} = 1, \quad \delta_{ni} \in \{0, 1\}, \forall i \in \mathcal{C}$$

But attributes are meaningful only within some bounds

$$l_{nk} \leq X_{ik} \leq u_{nk} \quad \forall i \in \mathcal{C}, \forall k$$

Cutoffs

Idea: relax the constraint in a probabilistic way

Example: constraint $\ell \leq X$

$$\begin{aligned}V_{\text{not considered}} &= \ell + \varepsilon_1 \\V_{\text{considered}} &= X + \varepsilon_2\end{aligned}$$

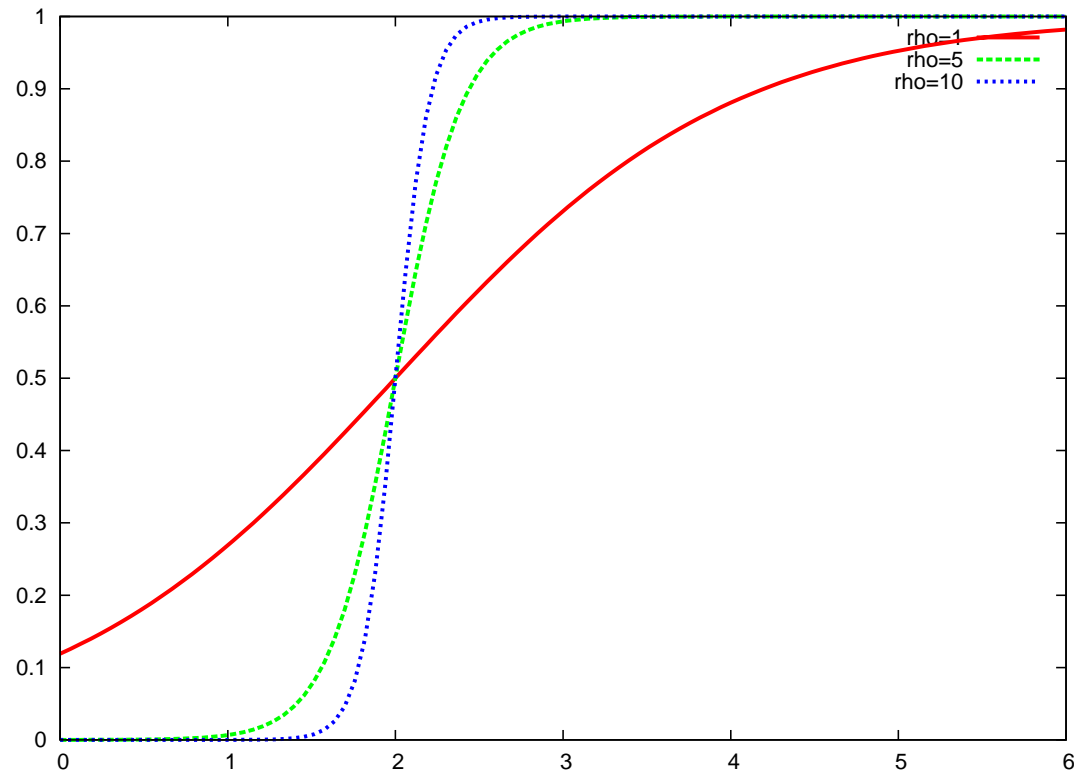
$$P(\text{considered}) = \frac{e^{\rho X}}{e^{\rho X} + e^{\rho \ell}} = \frac{1}{1 + e^{\rho(\ell - X)}}$$

Example: constraint $X \leq u$

$$P(\text{considered}) = \frac{e^{-\rho X}}{e^{-\rho X} + e^{-\rho u}} = \frac{1}{1 + e^{\rho(X - u)}}$$

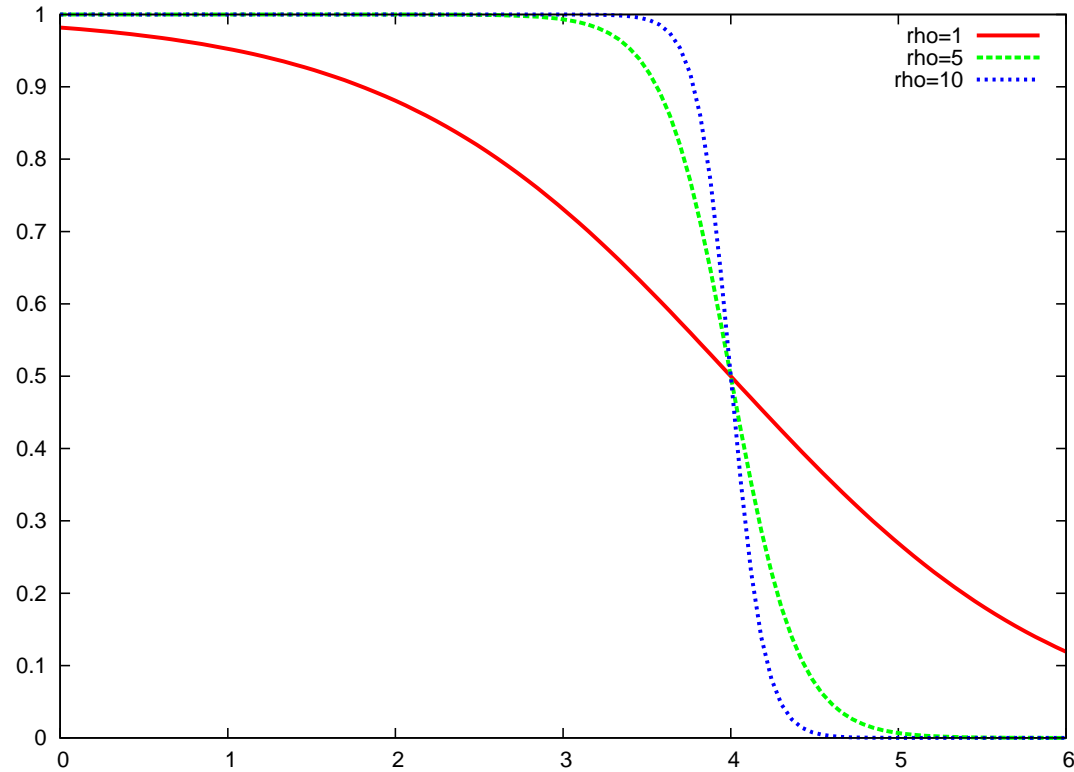
Cutoffs

Example: $2 \leq X$



Cutoffs

Example: $X \leq 4$



Cutoffs

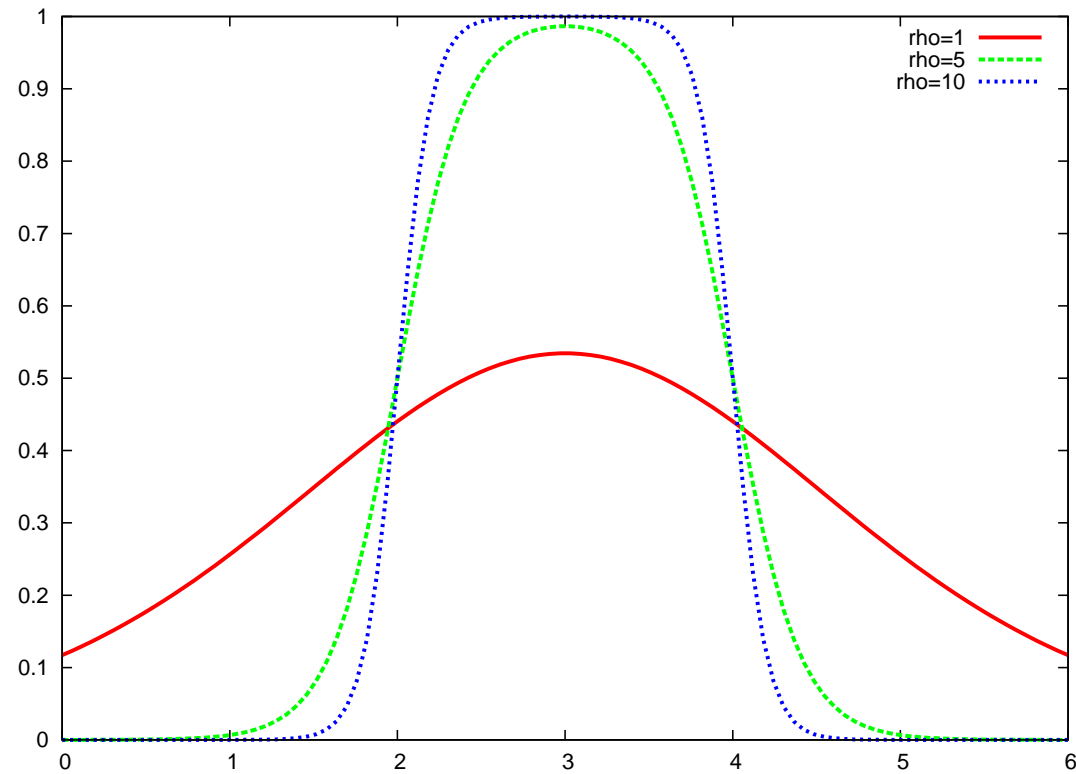
Constraint $l \leq X \leq u$

$$P(\text{considered}) = \frac{1}{1 + e^{\rho(l-X)}} \frac{1}{1 + e^{\rho(X-u)}}$$

We denote this quantity by $\phi_n(X)$

Cutoffs

Example: $2 \leq X \leq 4$



Cutoffs

The utility function now becomes

$$V_i = \sum_k \beta_k X_{ik} + \sum_{k^*} \frac{1}{\rho} \ln \phi_n(X_{ik^*})$$

where k^* ranges only on constrained attributes. Note that

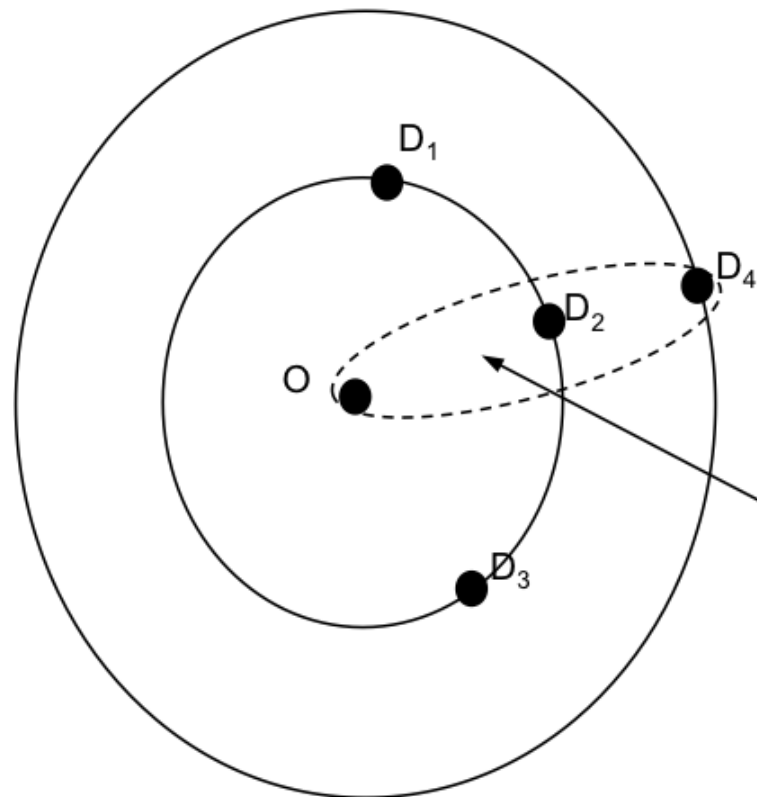
$$\begin{aligned} \ln \phi(X) &= -\ln(1 + e^{\rho(\ell - X)}) - \ln(1 + e^{\rho(X - u)}) \\ &= -\ln(1 + e^{\rho\ell} e^{-\rho X}) - \ln(1 + e^{\rho X} e^{-\rho u}) \end{aligned}$$

Can be estimated, although it is difficult

Dominance

- Destination choice (origin o)
- Dominance variables: reflect the spatial position and hierarchies of alternatives
- Dominance rules:
 - Weak dominance: Alternative d dominates alternative d^* if
 1. $A_d > A_{d^*}$ (attractivity attribute)
 2. $c_{od} < c_{od^*}$ (generalized transportation cost)
 - Strong dominance: d strongly dominates d^* if it weakly dominates it and is along the path to reach d^* from o

Dominance



$$WP_1 = WP_2 = WP_3 = WP_4$$

$$C_{OD1} = C_{OD2} = C_{OD3} < C_{OD4}$$

D_1, D_2, D_3 dominate **WEAKLY** D_4

D_2 **STRONGLY** dominates D_4

area of possible zones
STRONGLY dominating D_4

Dominance

Examples of dominance variables for destination d .

Consider 3 conditions:

- (a) d^* has average price lower than d
- (b) $\text{dist}(o, d^*) < \text{dist}(o, d)$
- (c) Strong rule: $\text{dist}(o, d^*) + \text{dist}(d^*, d) < \text{dist}(o, d)$

Strong global dominance variable nbr of d^* verifying (a), (b) and (c).

Weak global dominance variable nbr of d^* verifying (a) and (b)

Weak spatial dominance variable nbr of d^* verifying (b)

Strong spatial dominance variable nbr of d^* verifying (b) and (c).

Dominance

Dominance variables are introduced directly in the utility function of an MNL model (Cascetta and Papola, 2005):

$$U_d = \sum_k \beta_k X_{dk} + \sum_j \gamma_j Y_{dj}$$

Dominance within CML

Idea: alternatives with a high dominance variable are not considered

Constraint:

$$Y_{dj} \leq u$$

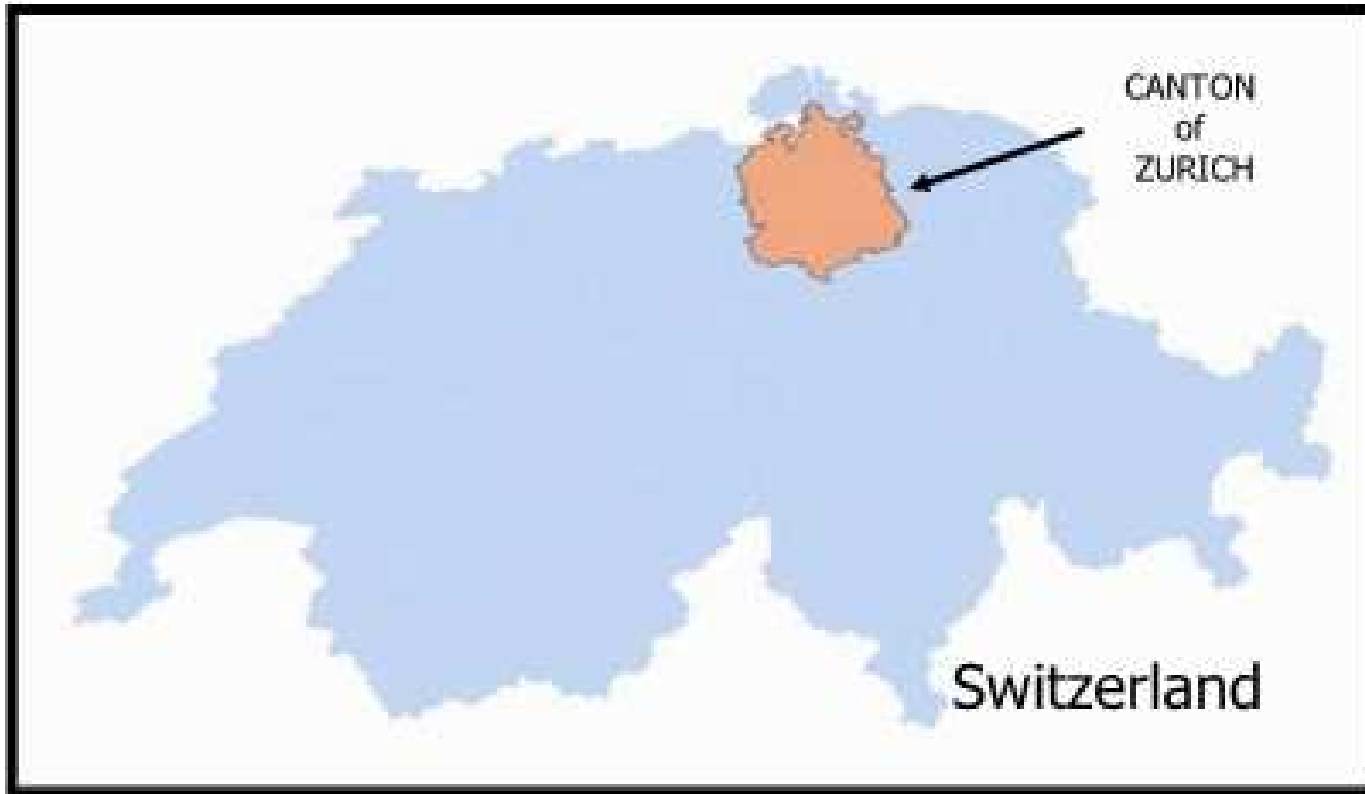
Problem: what is a reasonable threshold u ?

Let's use the cutoffs:

$$\ln \phi(Y_{dj}) = -\ln(1 + e^{\rho Y_{dj}} e^{-\rho u}) = -\ln(1 + \bar{u} e^{\rho Y_{dj}})$$

We try to estimate \bar{u}

Case study: canton Zürich



Residential location choice

Model specification:

$Price_d$	average land price of zone d
$LnStock_d$	log of the housing stock in zone d
$Logsum_{od}^{LM}$	logsum of the mode choice model for work purpose (low-medium income)
$Logsum_{od}^H$	logsum of the mode choice model for work purpose (high income)
$LnWorkPlacesServ_d$	log of the workplaces in services (retail, leisure, services, incl. education and health) in d . Measure of quality of services.

MNL

Number of observations = 657

$$\begin{aligned} \mathcal{L}(0) &= -3419.032 \\ \mathcal{L}(\hat{\beta}) &= -53.971 \\ -2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6730.123 \\ \rho^2 &= 0.984 \\ \bar{\rho}^2 &= 0.983 \end{aligned}$$

Variable number	Description	Coeff. estimate	Robust		
			Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	Logsum _{od} ^H	15.3	2.85	5.36	0.00
2	Logsum _{od} ^{LM}	16.6	2.97	5.58	0.00
3	Price _d	-0.00160	0.000221	-7.24	0.00
4	LnStock _d	1.12	0.102	10.92	0.00
5	LnWorkPlacesServ _d	0.187	0.180	1.04	0.30

MNL

- Very high ρ^2 : 0.98
- Correct signs
- Significant parameters, except the level of services

Next model:

- Include the strong spatial dominance variable (based only on distance, not on price)
- Simple linear specification

$$V_d = \dots + \beta \text{dom}_d$$

Linear dominance

Number of observations = 657

$$\begin{aligned} \mathcal{L}(0) &= -3419.032 \\ \mathcal{L}(\hat{\beta}) &= -47.055 \\ -2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6743.955 \\ \rho^2 &= 0.986 \\ \bar{\rho}^2 &= 0.984 \end{aligned}$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	dom _{<i>d</i>}	-0.0859	0.0120	-7.17	0.00
2	Logsum ^H _{<i>od</i>}	16.1	2.62	6.16	0.00
3	Logsum ^{LM} _{<i>od</i>}	17.1	2.76	6.20	0.00
4	Price _{<i>d</i>}	-0.00245	0.000313	-7.82	0.00
5	LnStock _{<i>d</i>}	1.20	0.133	9.01	0.00
6	LnWorkPlacesServ _{<i>d</i>}	-0.172	0.198	-0.87	0.39

Linear dominance

- Significantly better fit: $-2(-53.971 - 47.055) = 202.052$
- Correct signs
- Significant parameters, except the level of services

Next model: cutoff

$$\begin{aligned} V_d &= \dots - \ln(1 + \bar{u} \exp(\rho \text{dom}_d)) \\ &= \dots - \ln(1 + 1000 \exp(\rho \text{dom}_d)) \end{aligned}$$

Notes:

- the estimation of \bar{u} failed; its value continuously increased
- in the final model, the value $\bar{u} = 1000$ was used.

Cutoff

Number of observations = 657

$$\begin{aligned} \mathcal{L}(0) &= -3419.032 \\ \mathcal{L}(\hat{\beta}) &= -47.057 \\ -2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6743.952 \\ \rho^2 &= 0.986 \\ \bar{\rho}^2 &= 0.984 \end{aligned}$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	Logsum _{od} ^H	16.1	2.62	6.16	0.00
2	Logsum _{od} ^{LM}	17.1	2.76	6.20	0.00
3	Price _d	-0.00245	0.000313	-7.82	0.00
4	LnStock _d	1.20	0.133	9.01	0.00
5	LnWorkPlacesServ _d	-0.172	0.198	-0.87	0.39
6	ρ	0.0859	0.0120	7.17	0.00

Cutoff

- Same improvement than the linear specification
- Actually, the model is almost linear, due to the high value of \bar{u}
- Question: can we accept a linear specification?
- We test it using a Box-Cox transform.

$$V_d = \dots + \beta \frac{\text{dom}_d^\lambda - 1}{\lambda}$$

Box-Cox test

Number of observations = 657

$$\begin{aligned} \mathcal{L}(0) &= -3419.032 \\ \mathcal{L}(\hat{\beta}) &= -43.120 \\ -2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6751.826 \\ \rho^2 &= 0.987 \\ \bar{\rho}^2 &= 0.985 \end{aligned}$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	dom _{<i>d</i>}	-0.579	0.0539	-10.74	0.00
2	Logsum ^H _{<i>od</i>}	16.9	2.66	6.36	0.00
3	Logsum ^{LM} _{<i>od</i>}	18.0	2.68	6.72	0.00
4	Price _{<i>d</i>}	-0.00292	0.000324	-9.00	0.00
5	LnStock _{<i>d</i>}	1.42	0.175	8.10	0.00
6	LnWorkPlacesServ _{<i>d</i>}	-0.328	0.257	-1.28	0.20
7	λ	0.434	0.0388	11.19	0.00

Box-Cox test

- λ is significantly different from 1.0 (t -test = 14.6)
- λ is significantly different from 0.0 (t -test = 11.2)
- The linear specification is rejected

Conclusions

- Main idea: combination of two concepts: cutoffs and dominance
- First estimation results produces large values for the variance of the cutoff, so that it is basically equivalent to the linear model
- But... the linear specification is clearly rejected by a formal test.
- Next steps:
 - Consider new dominance rules, more consistent with the use of cutoffs
 - Investigate other data sets