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Sheafhom: software for sparse integer matrices. (English summary)

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Homology groups and sheafs of abelian varieties can be computed using computer software through extracting information from sparse-integer matrices. The matrices are constructed from topological specification linked to the homology to be computed.

Sheafhom is a free software package for large-scale computations in the category of $\mathbb{Z}$-modules. Its aim is to compute in any principal ideal domain. The problems in algebraic topology and geometry to be handled consist in solving large sparse systems of linear equations over $\mathbb{Z}$. The initial data consist of a sparse matrix which is brought into a canonical form using the Smith decomposition algorithm. Any algorithm addressing such problems should take advantage of the sparsity of the initial data through ensuring sparsity of the intermediate results.

One key difficulty with the integers (as with similar floating point problems, e.g., Hessenberg reduction and Smith-normal forms) is the numerical instability problem, which has a tendency of being even more serious with integer arithmetic, because of the rapidity with which the number of bits available are exhausted.

The software package proposes an algorithm that handles both critical issues, namely keeping the intermediate results as sparse as possible (avoiding fill in) and ensuring numerical stability. The algorithm uses heuristics in finding the best suitable pivotal element so as to limit both effects during an elementary phase of the Smith decomposition.

The strategy retained uses a triangular Markowitz sort for choosing the pivot (so as to limit fill in) combined with the Lenstra-Lenstra-Lovasz (LLL) algorithm (so as to limit integer explosion).

As for the Markovitz part, the idea is to sort the columns (resp. lines) so that those with a smaller pivotal element appear on the left (resp. on the top). The columns (resp. rows) are searched from left to right (resp. top to bottom) and the pivotal element is found to be the one with minimal absolute value.

As for the LLL part, integer explosion is controlled by modifying columns (resp. rows) so as to become as short and as orthogonal as possible, instead of clearing one column (resp. one row) after the other.

The paper advocates the usage of the Common LISP language for coding the algorithm because of its implementation of arbitrary-precision integers and its object-oriented character. Although part of the binding operations are done at runtime, the compiled code generated is very efficient at the machine level. Sheafhom uses the philosophy of allowing one to change the base ring while keeping efficiency high through the introduction of various low-level data types such as 'sparse-elt', a data structure holding an index and a value of the underlying ring, and 'sparse-v', a sparse vector given as a singly-linked list whose elements are 'sparse-elt'.

The paper illustrates the efficiency of the algorithm proposed with a tutorial example of finding at the free group of the letters A through Z modulo equivalence of all words occurring in a simplified

English dictionary.

The code grew out of regular updates performed over the years since 1993 until the version of the paper, namely Sheafhom 2.1, was released in March, 2005.

Reviewed by *Philippe A. Müllhaupt*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.