Optical flow and depth from motion for omnidirectional images using a TV-L1 variational framework on graphs

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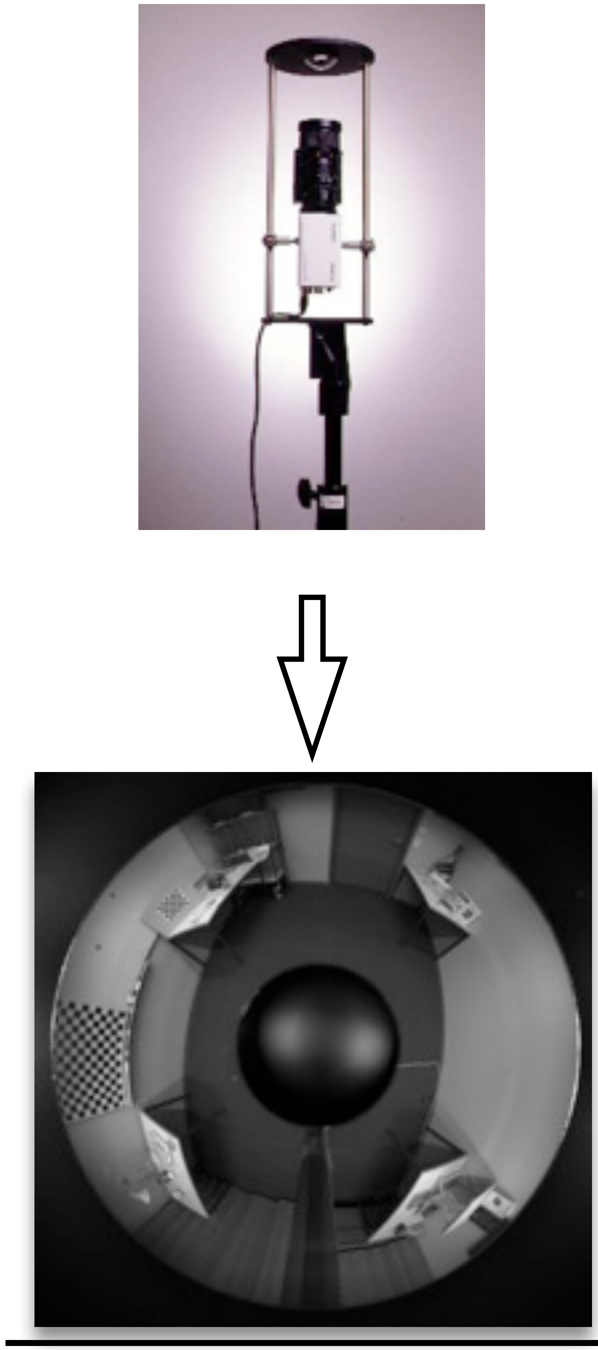
Motivations

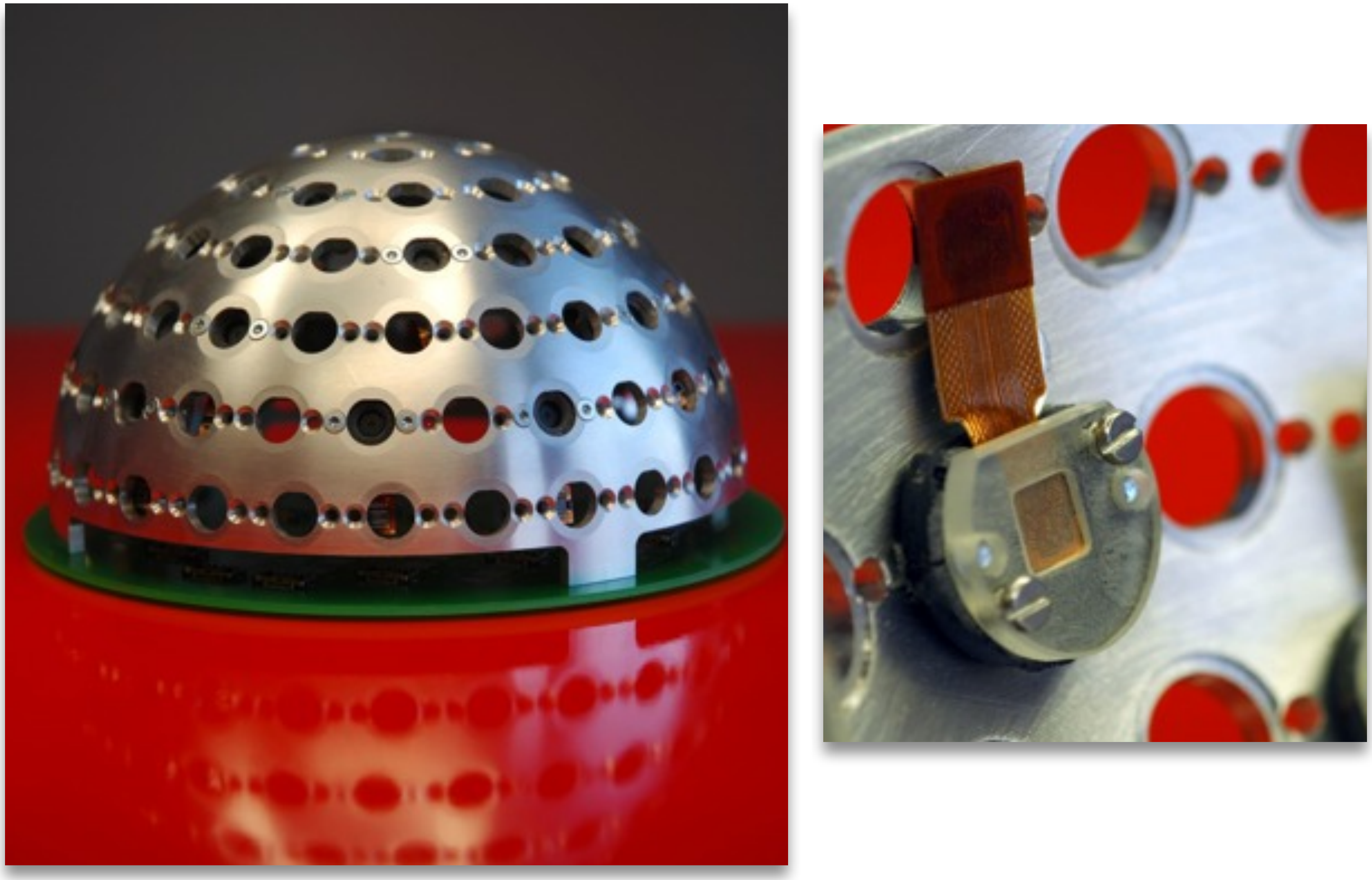
- Optical flow and dense depth map estimation are typical inverse problems in computer vision.
- Variational methods:
  - pro: very good performances
  - cons: quite heavy
- BUT efficient (real time) GPU-based implementation is possible for planar images.
- Omnidirectional vision systems are attractive in many applications (Robotics, 3D reconstruction)
  - They suffer of rather complex distortions

Omnidirectional Vision Systems

Catadioptric camera



Panoptic camera



104 webcams arranged on a hemispherical aluminum board

http://lts2www.epfl.ch/Panoptic

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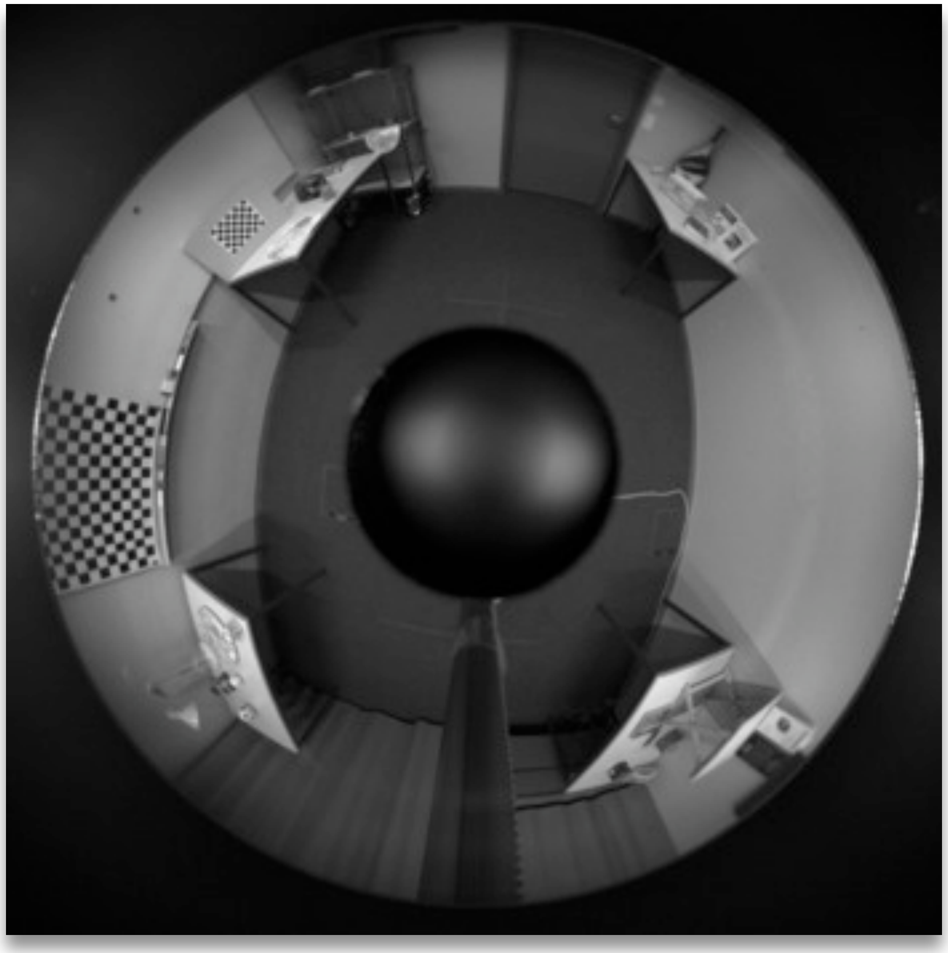
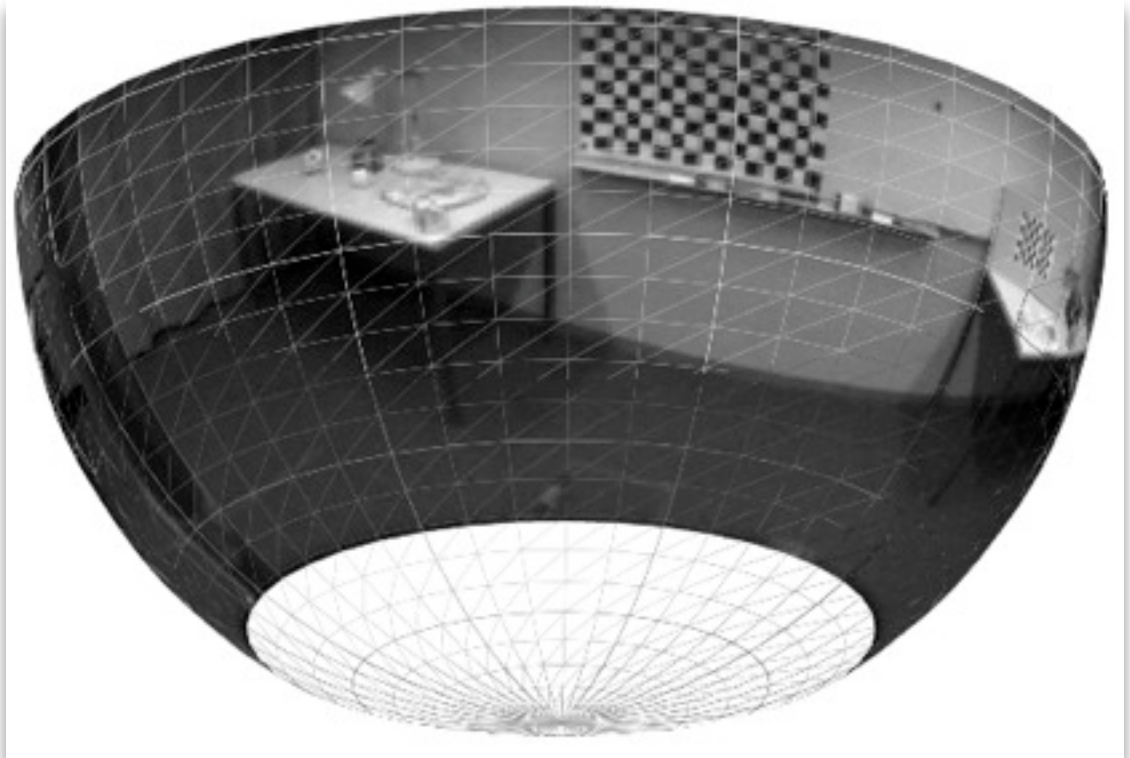
2-Sphere Representation

Every single viewpoint optical system admits a unique mapping on a 2-sphere

Omnidirectional image

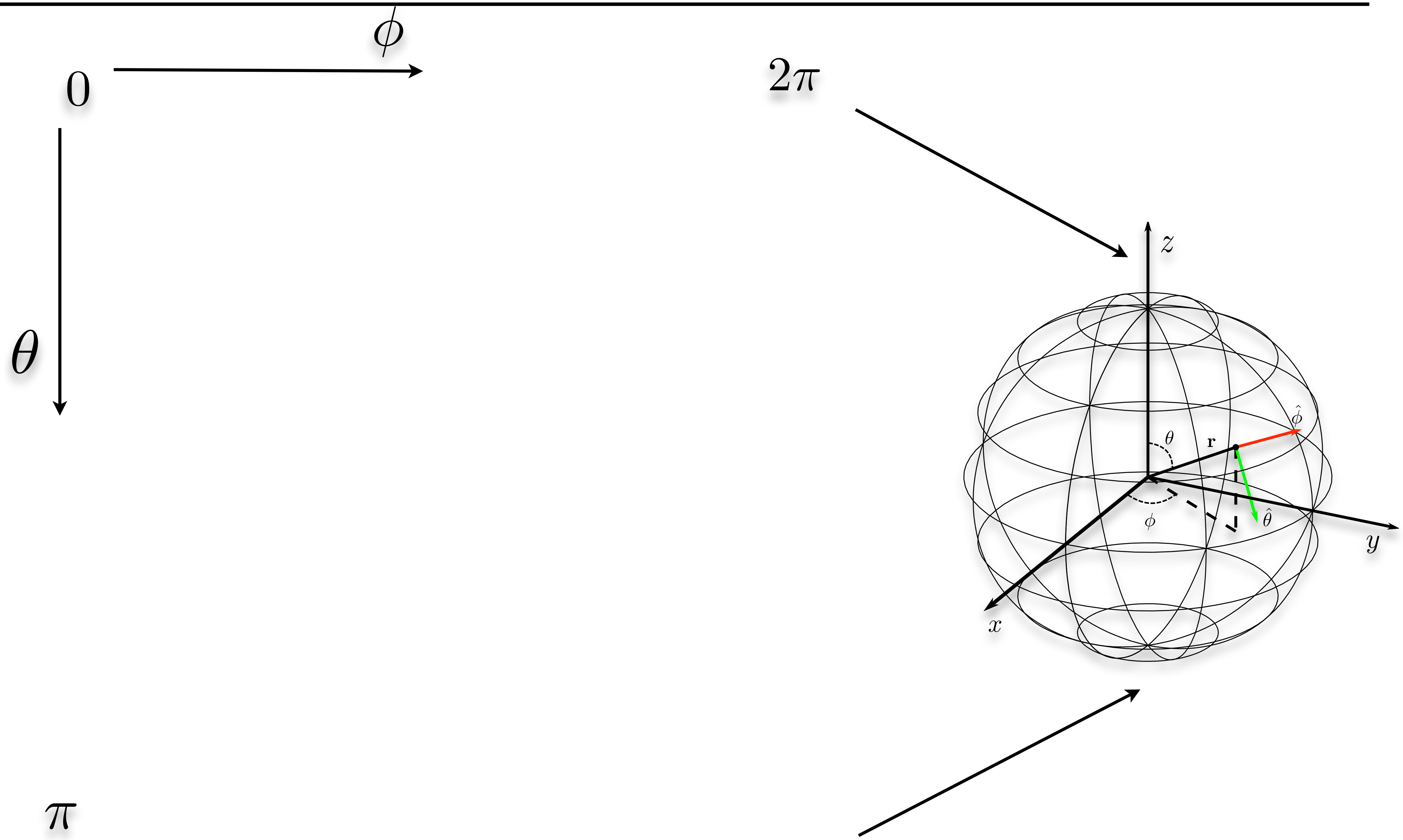


Projection on 2-Sphere



Reference: Baker and Nayar. A theory of single-viewpoint catadioptric image formation. Int J Comput Vis (1999) vol. 35 (2) pp. 175-196

Synthetic Spherical Video Sequence



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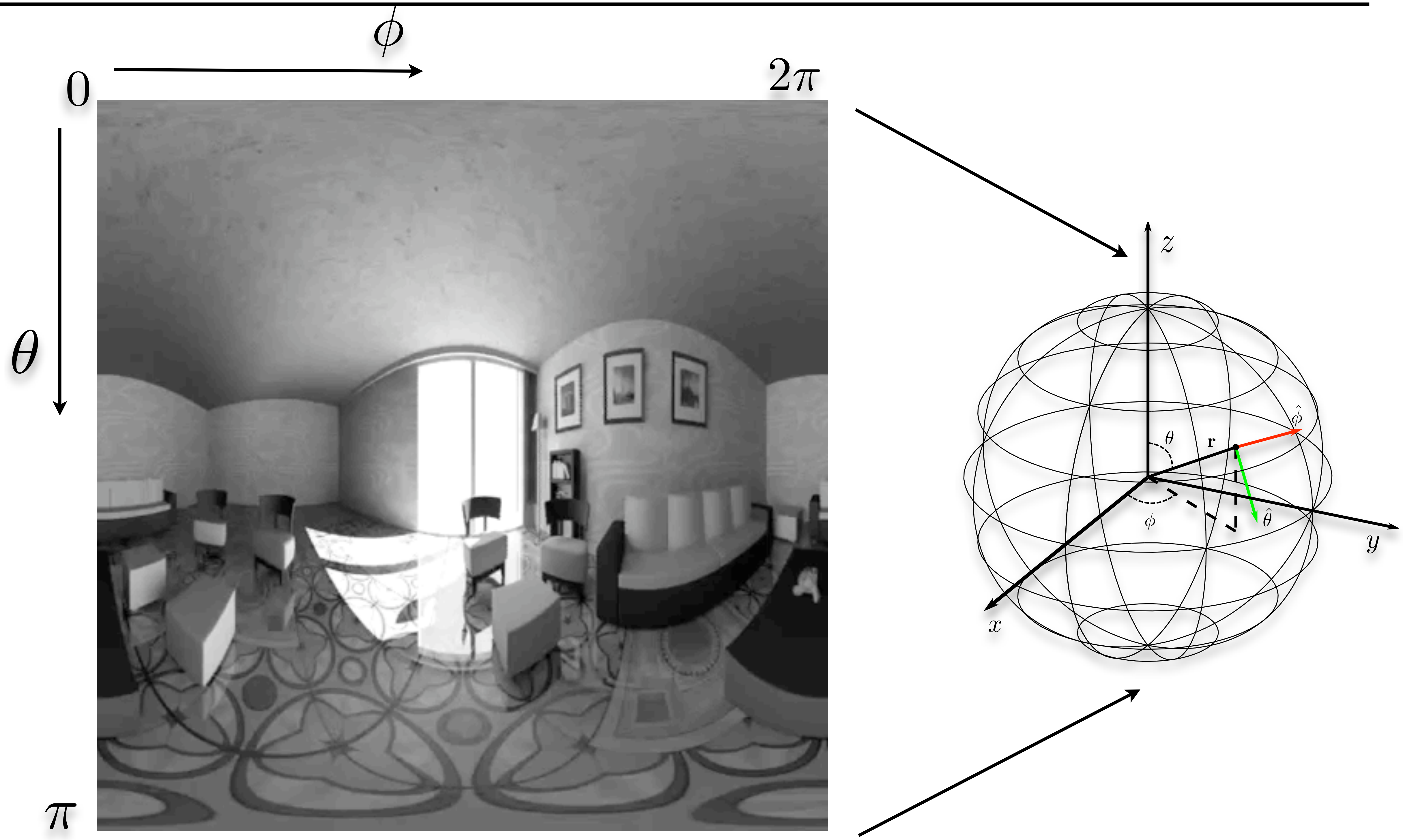

First Frame

I_0

Second Frame I_1

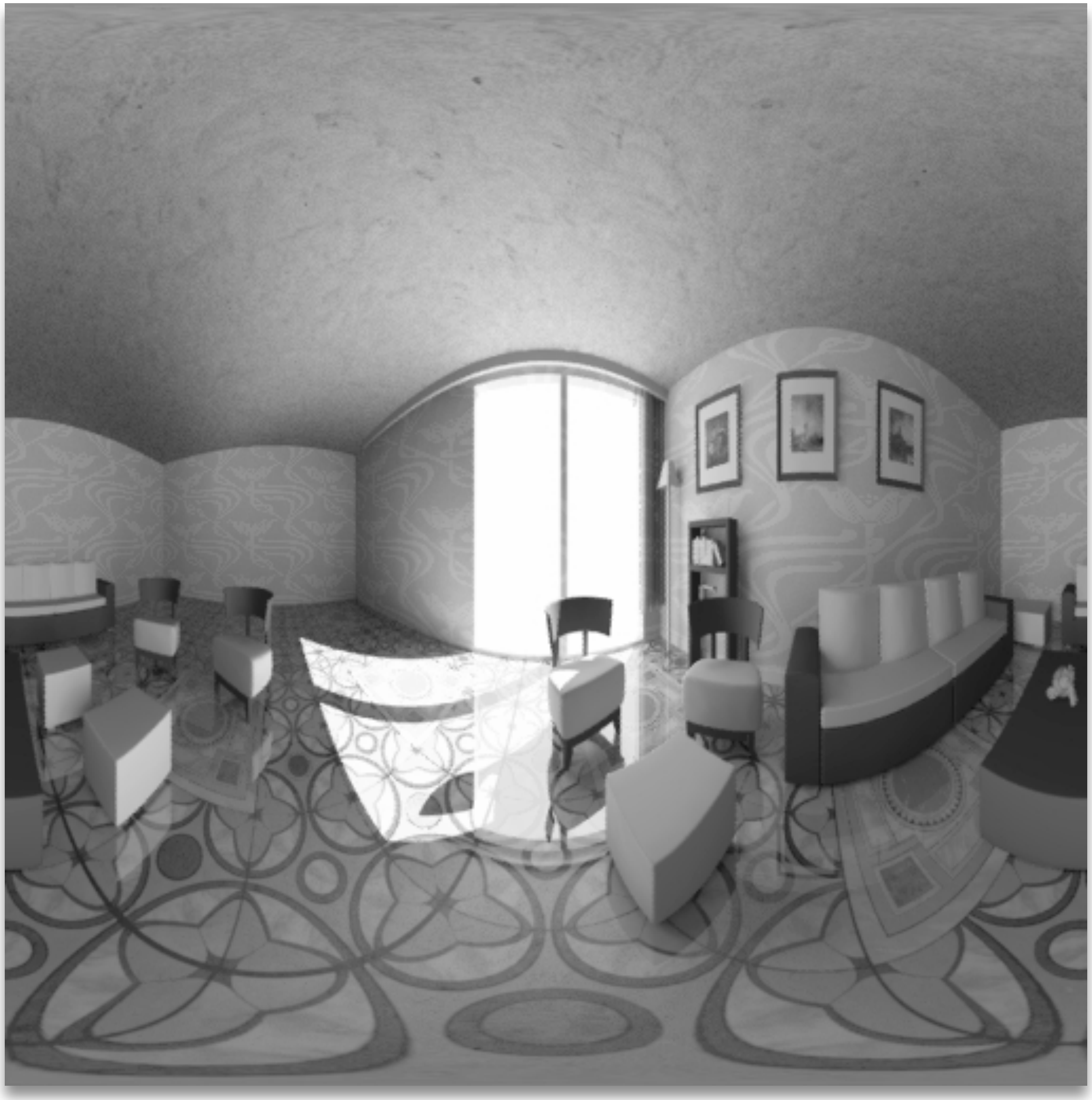

Image Residual $|I_1 - I_0|$

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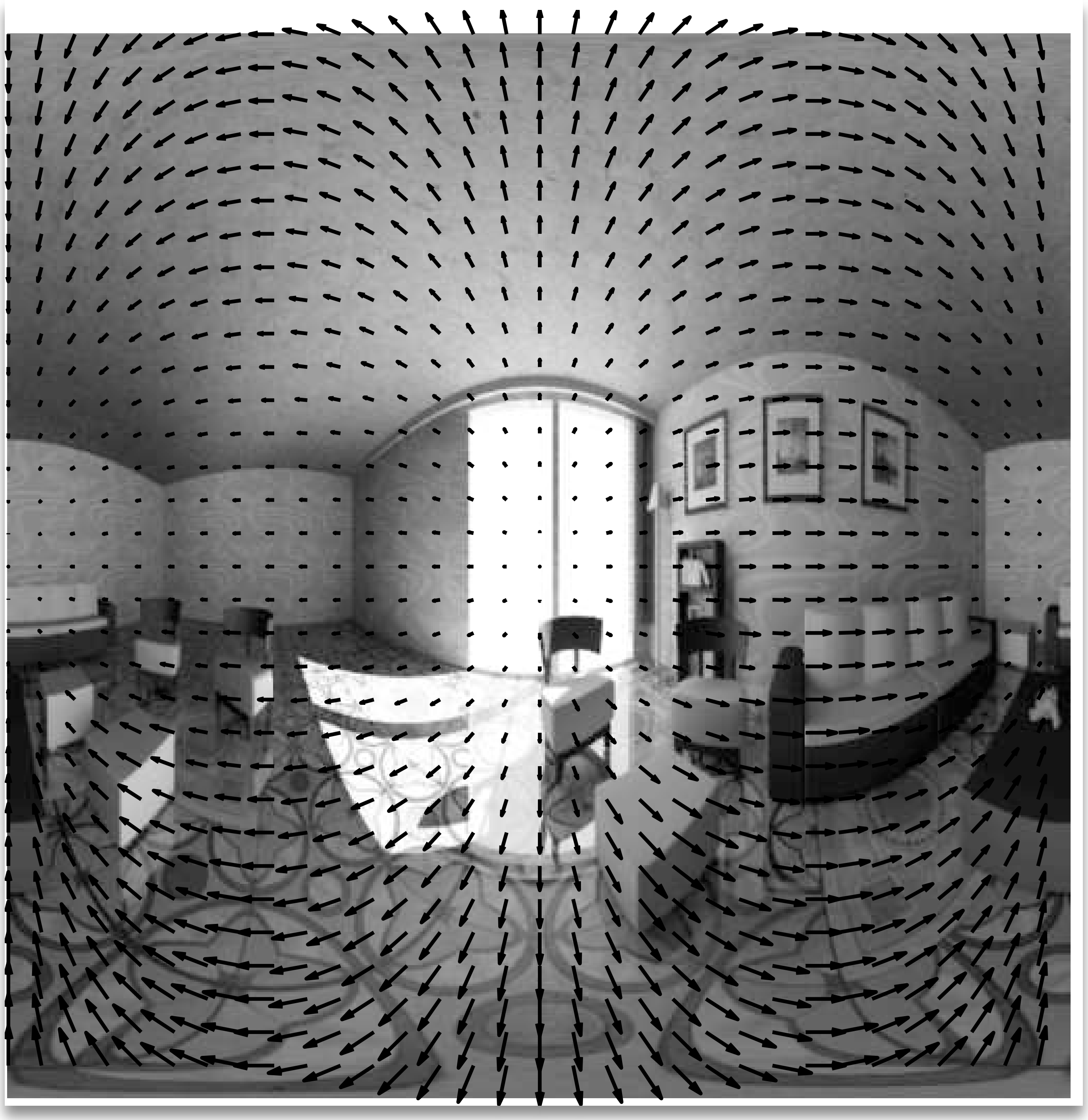
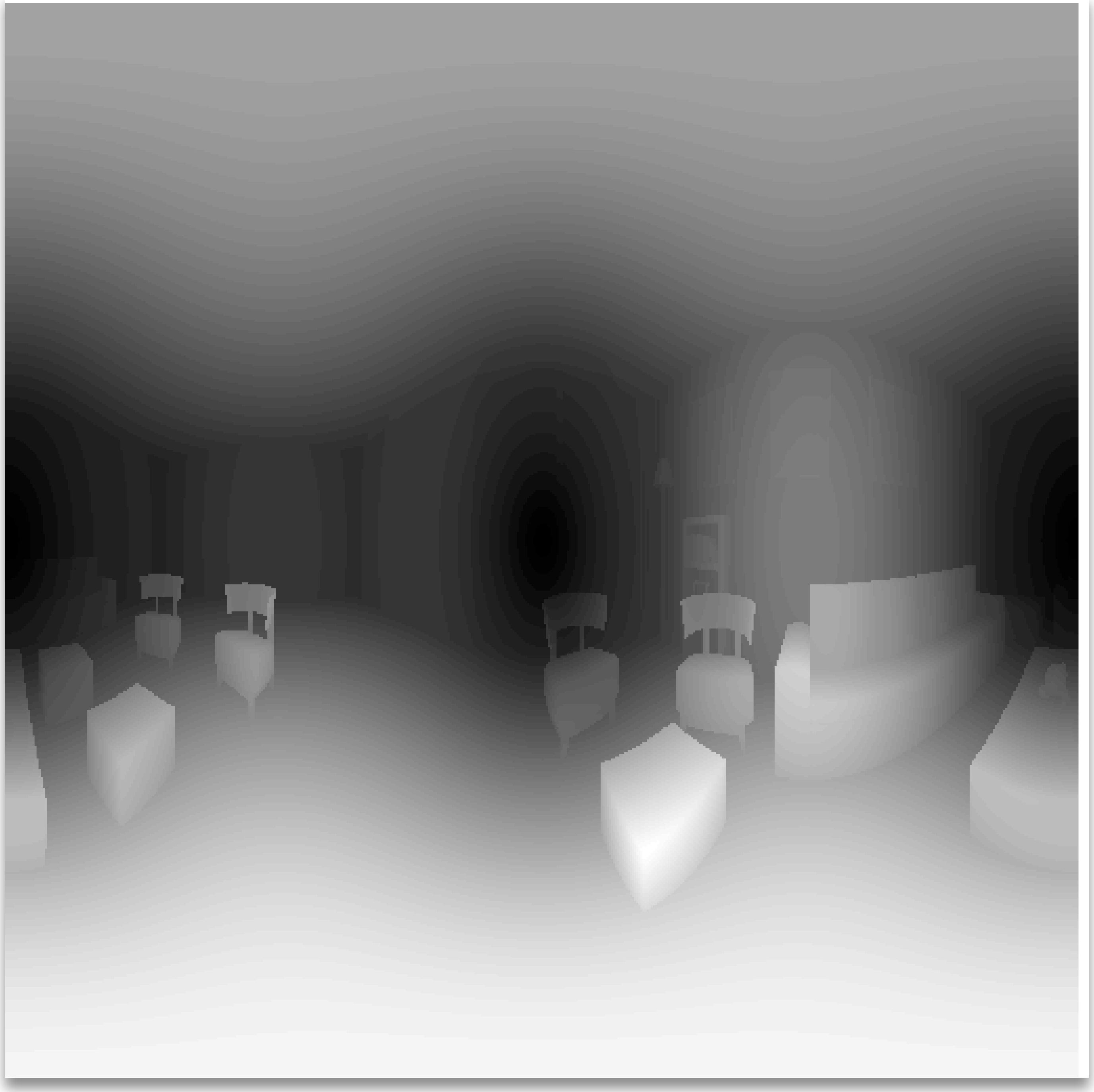
Optical Flow Field

Ground Truth

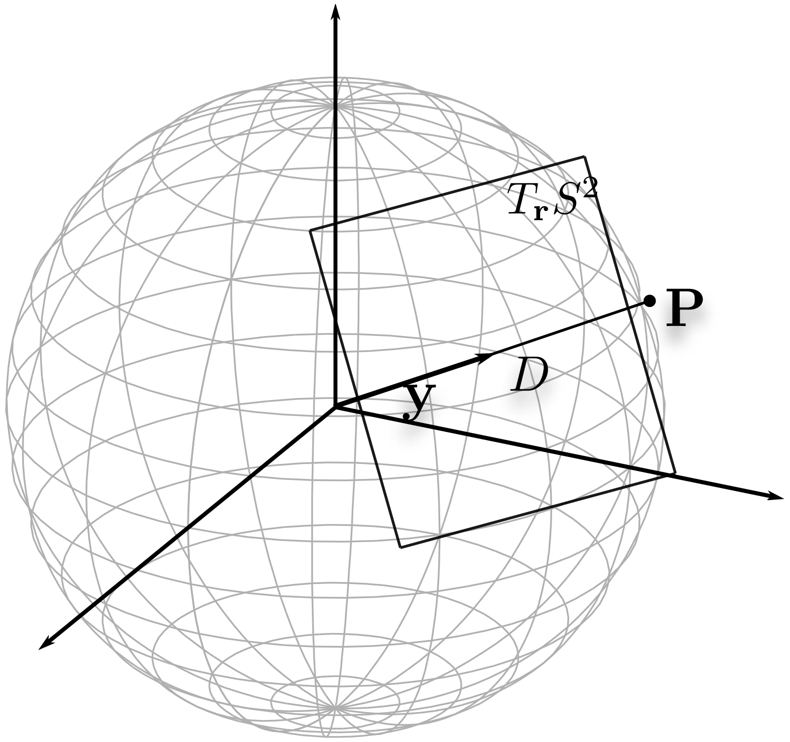


Optical Flow Field - module

Ground Truth



Spherical Optical Flow



$$\mathbf{y} \in S^2$$

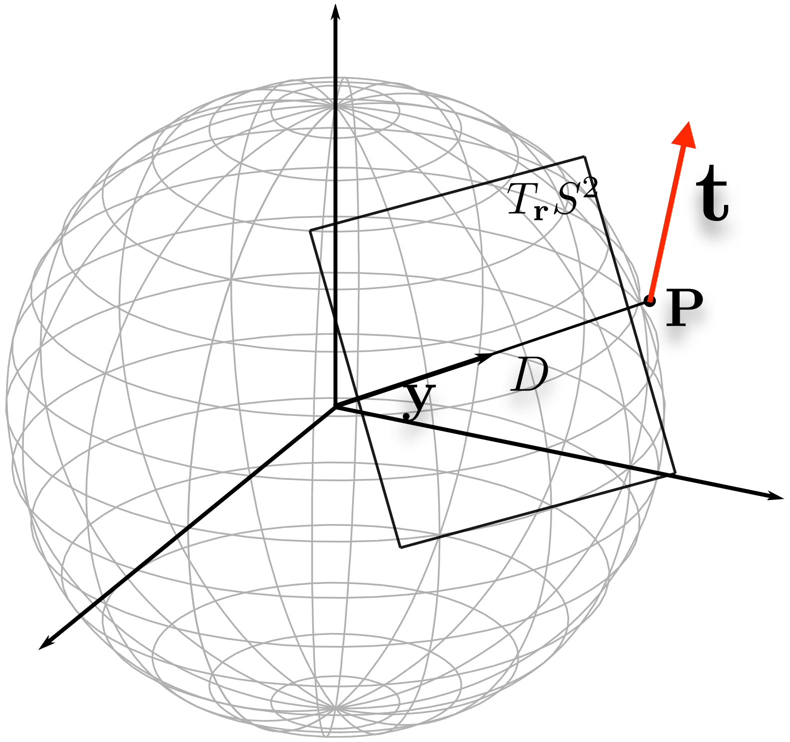
$$\mathbf{u} = D^{-1}\mathbf{t}_s$$

$\mathbf{u}(\mathbf{y})$ optical flow

$D(\mathbf{y})$ depth map

$$I : \mathbb{R}^3 \longrightarrow \mathbb{R}$$ (radial function)

Spherical Optical Flow



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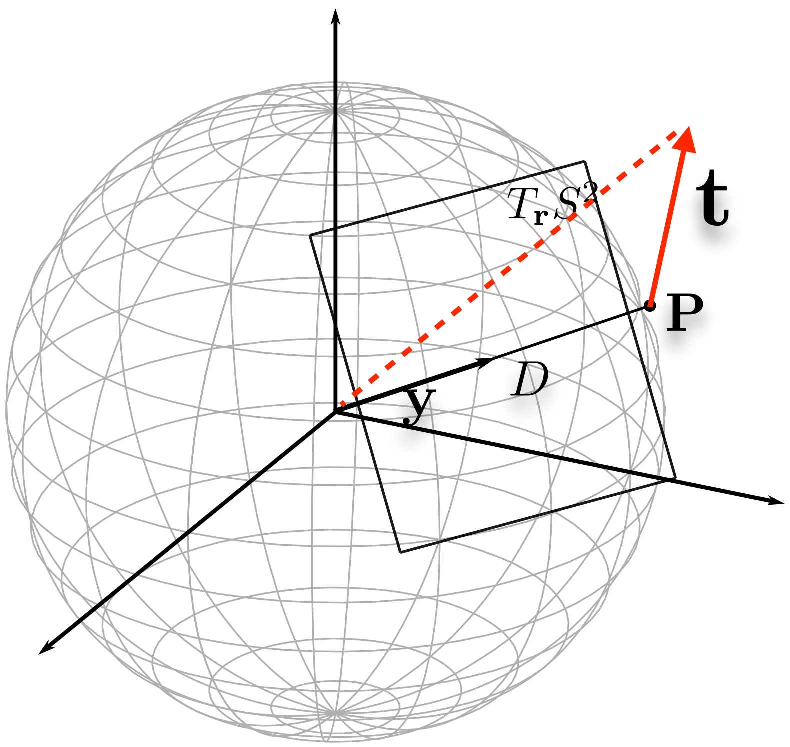

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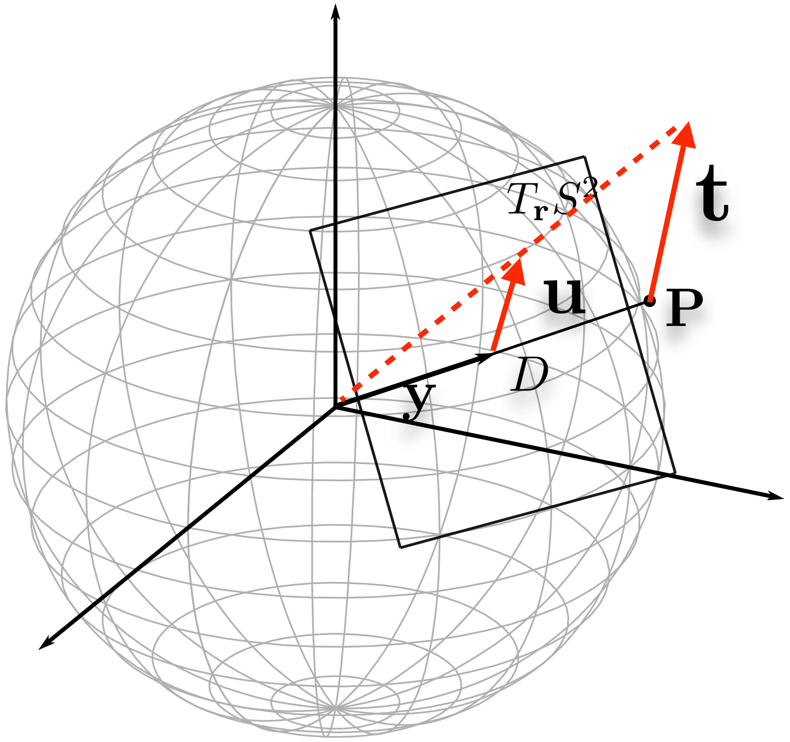

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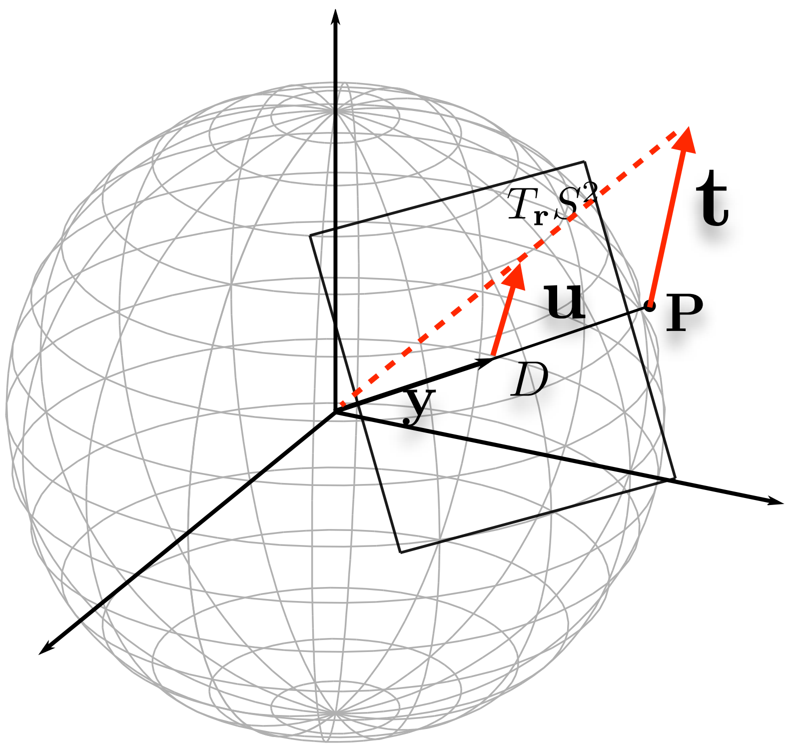

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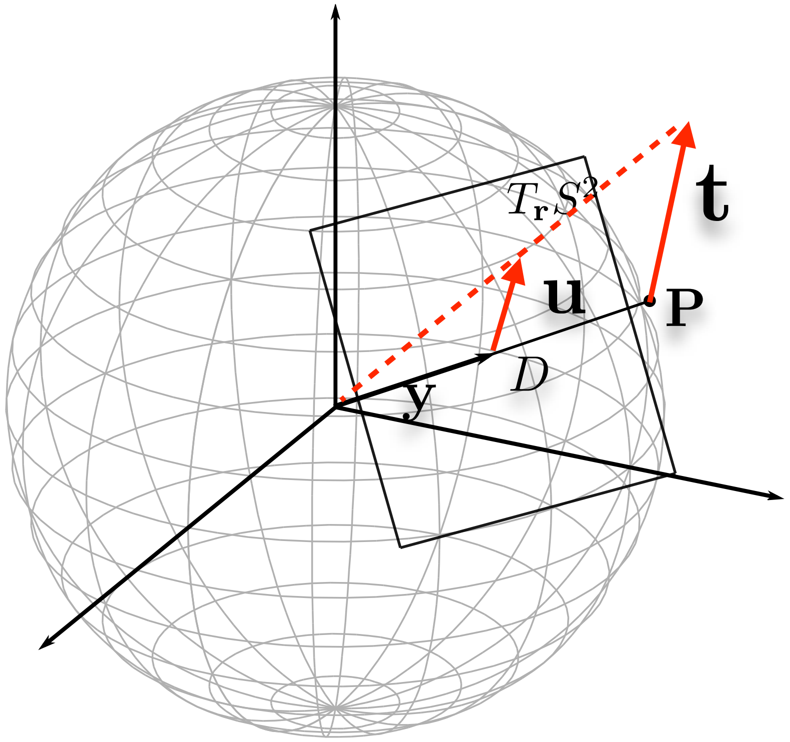
Brightness consistency

$$I_0(\mathbf{y}) - I_1(\mathbf{y} + \mathbf{u}) = 0$$

Spherical Optical Flow



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Linear approximation

$$I_0(\mathbf{y}) - (\nabla_s I_1(\mathbf{y}))^T \mathbf{u} - I_1(\mathbf{y}) = 0$$

TV-L1 Inverse Problem

$$J = \boxed{\int_{\Omega} \psi(\nabla u^i)\, \mathrm{d}\Omega} + \lambda \boxed{\int_{\Omega} |\rho(I_0, I_1, \mathbf{u})|}\, \mathrm{d}\Omega$$

TV Regularization

L1 norm fidelity term, robust to outliers

$$\psi(\nabla_s u^i) = \sum_i |\nabla_s u^i|, \qquad i \in \{1, 2\}$$

$$\rho(\mathbf{u}) = I_1(\mathbf{y}) + (\nabla_s I_1(\mathbf{y}))^T \mathbf{u} - I_0(\mathbf{x})$$

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Functional Splitting

$$J = \int_\Omega \psi(\nabla u^i) + \frac{1}{2\theta}|\mathbf{u} - \mathbf{v}|^2 + \lambda\rho(\mathbf{v})\mathrm{d}\Omega$$

v is an auxiliary variable close to **u**

Two Step Algorithm

$$J = \int_\Omega \psi(\nabla u^i) + \frac{1}{2\theta}|\mathbf{u} - \mathbf{v}|^2 + \lambda\rho(\mathbf{v})\mathrm{d}\Omega$$

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1. Fix **u** and solve $$\min_{\mathbf{v}} \left\{ \int_\Omega \frac{1}{2\theta}|\mathbf{u} - \mathbf{v}|^2 + \lambda\rho(\mathbf{v})\mathrm{d}\Omega \right\}$$

"Easy" to solve, solution can be found pointwise by a soft thresholding scheme

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Two Step Algorithm

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"Easy" to solve, solution can be found pointwise by a soft thresholding scheme

2. Fix **v** and solve $$\min_{\mathbf{u}} \left\{ \int_\Omega \psi(\nabla_s u^i) + \frac{1}{2\theta}|\mathbf{u} - \mathbf{v}|^2 \mathrm{d}\Omega \right\}$$

- Classical TV denoising problem
- We need an **efficient discretization scheme**

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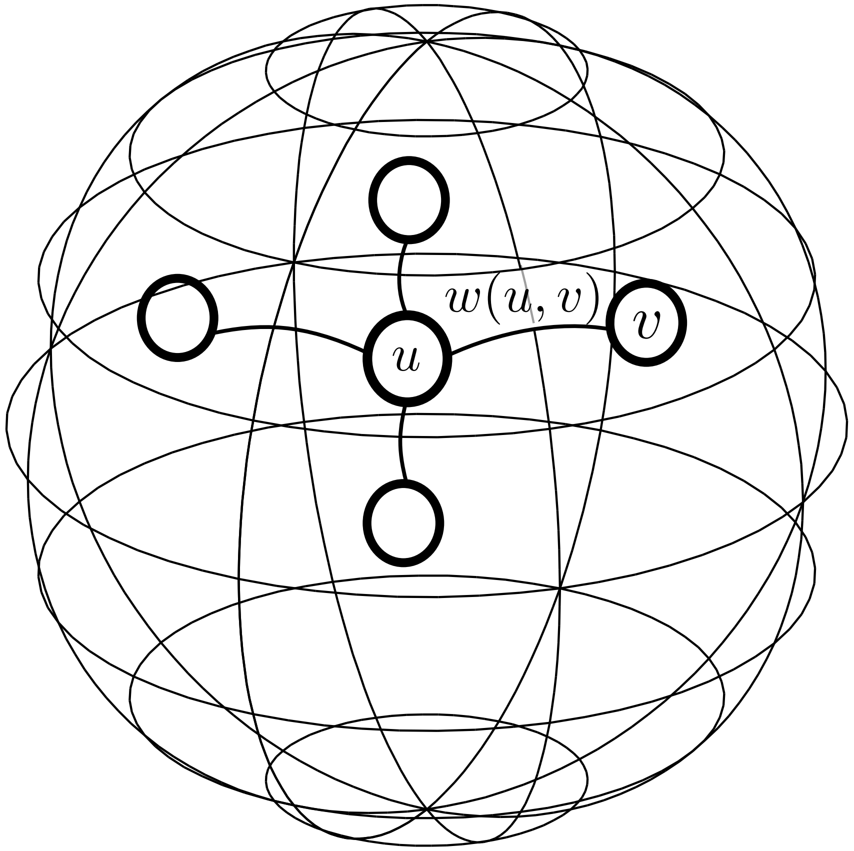
Graph Discretization

$$\Gamma = (V, E, w) \qquad w : E \mapsto \mathbb{R} \qquad w(u,v) = w(v,u) > 0$$

- Spherical geometry embedded in connectivity.
- Weights are decreasing with the geodesic distance.
- Can handle irregular sampling grid.



Graph differential geometry

Gradient (value on edges)

$$(\nabla^w f)(u,v) = \sqrt{\frac{w(u,v)}{d(u)}} f(u) - \sqrt{\frac{w(u,v)}{d(v)}} f(v)$$

Divergence at vertex u

$$(div^w F)(u) = \sum_{u \sim v} \sqrt{\frac{w(u,v)}{d(v)}} (F(v,u) - F(u,v))$$

Local variation at vertex v

$$\|\nabla^w f\| = \sqrt{\sum_{u \sim v} \left[\left(\nabla^w f \right) (u,v) \right]^2}$$

Degree at vertex v

$$d(v) = \sum_{u \sim v} w(u,v)$$

Reference: Zhou and Scholkopf. Regularization on discrete spaces. Lect Notes Comput Sc (2005) vol. 3663 pp. 361-368

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Graph Regularization

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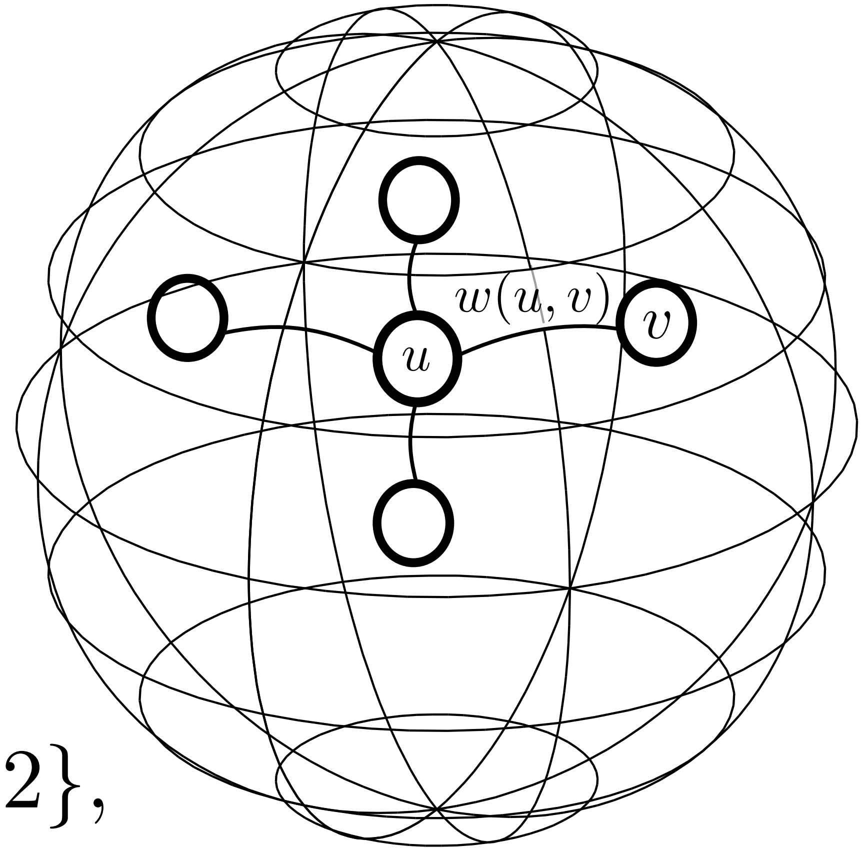

Discrete TV subproblem

$$\min_{u^i} \left\{ ||u^i||_{TV^w} + \frac{1}{2\theta} ||u^i - v^i||^2 \right\} \qquad i \in \{1,2\}$$

Graph based Chambolle iterations

$$v^i = u^i - \theta div^w \mathbf{p}_i \qquad i \in \{1,2\},$$

$$\mathbf{p}_i^{n+1} = \frac{\mathbf{p}_i^n + \tau \nabla^w (div^w \mathbf{p}_i^n - u_i/\theta)}{1 + \tau |\nabla^w (div \mathbf{p}_i^n - u_i/\theta)|} \qquad i \in \{1,2\}$$

References: Chambolle. An algorithm for total variation minimization and applications. J Math Imaging Vis (2004) vol. 20 (1-2) pp. 89-97
Peyre et al. Non-local Regularization of Inverse Problems. Computer Vision-Eccv (2008)

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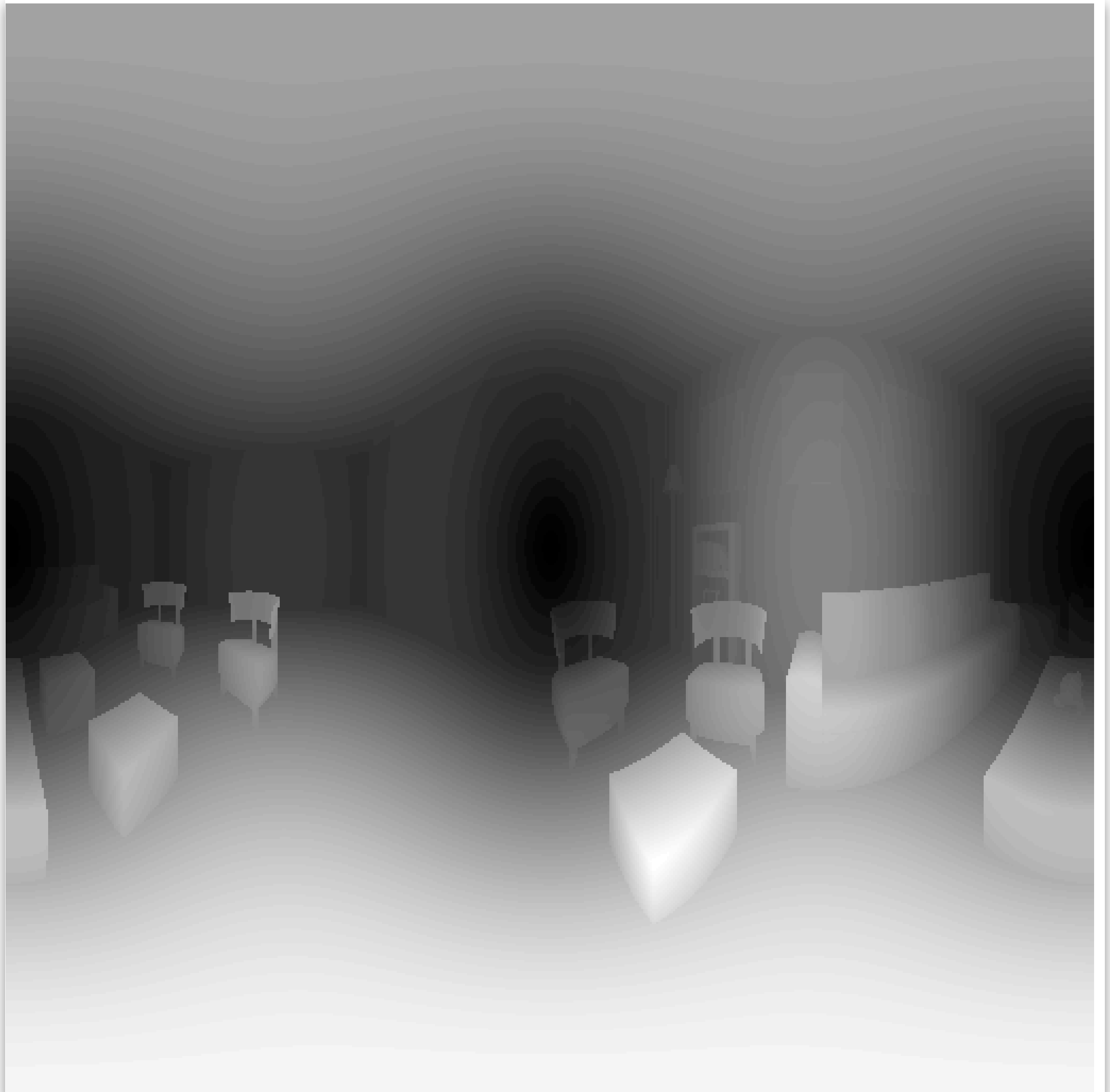
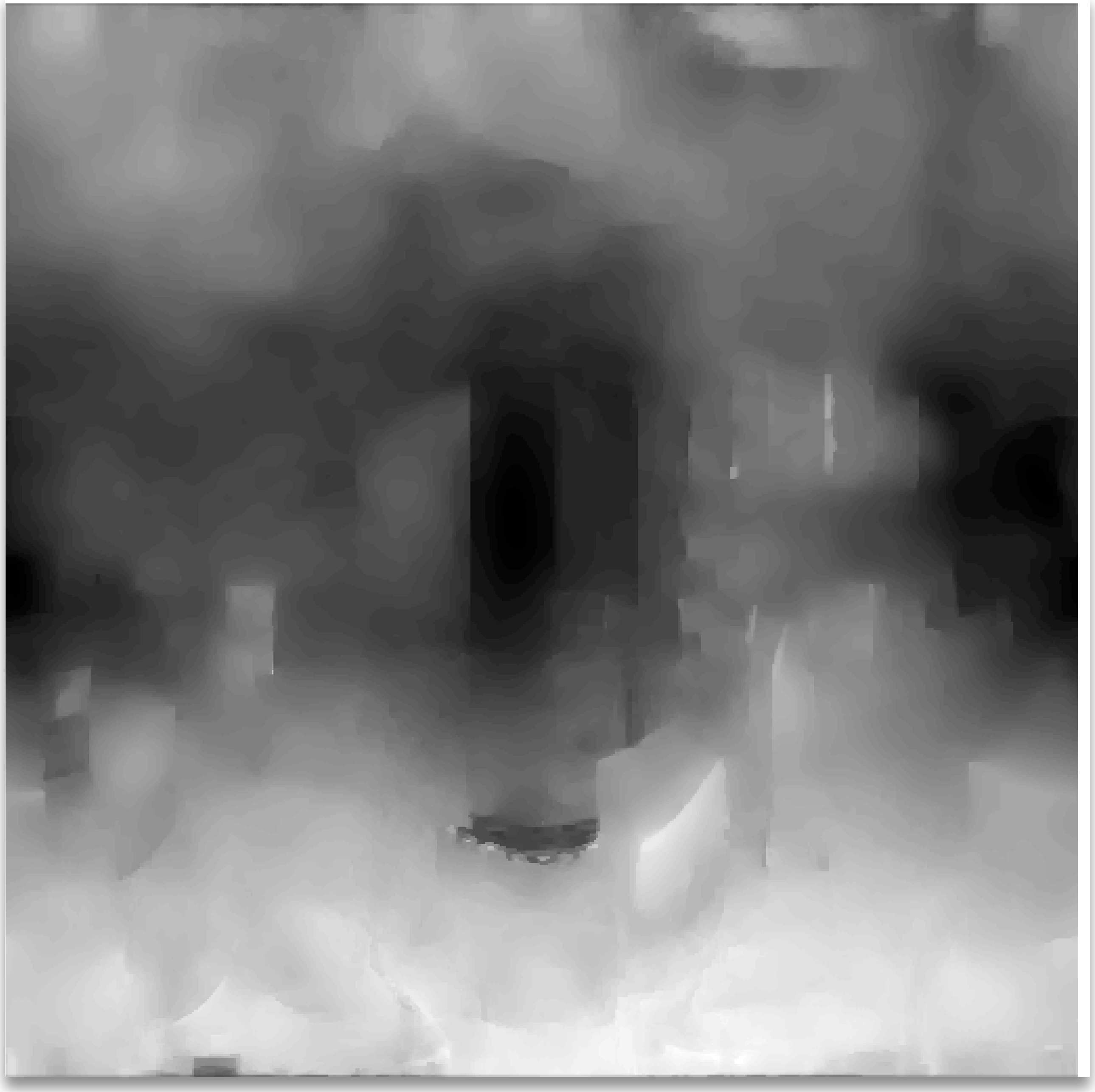
Optical Flow Results - our solution

Ground Truth

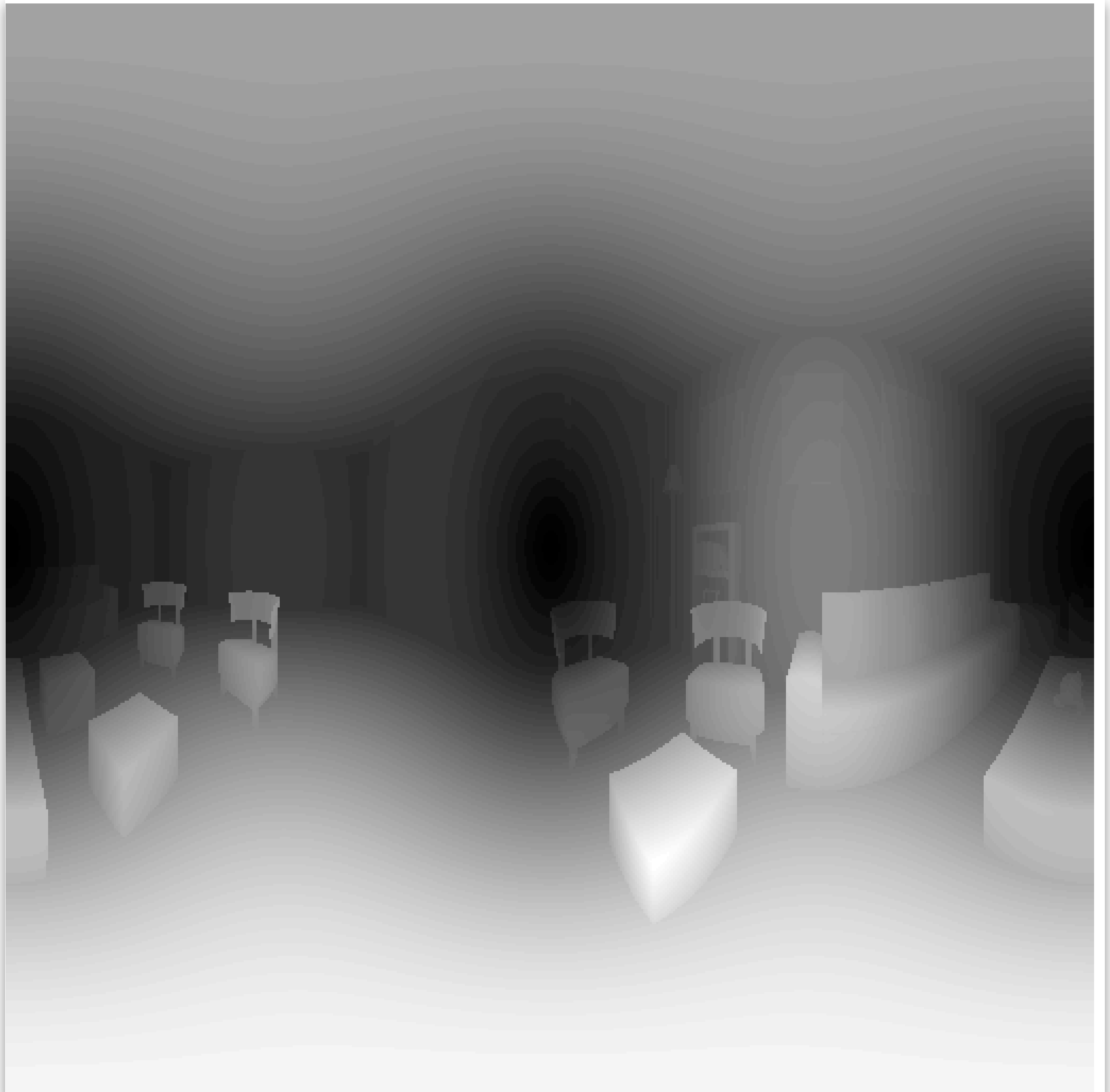


Estimated optical flow (module)



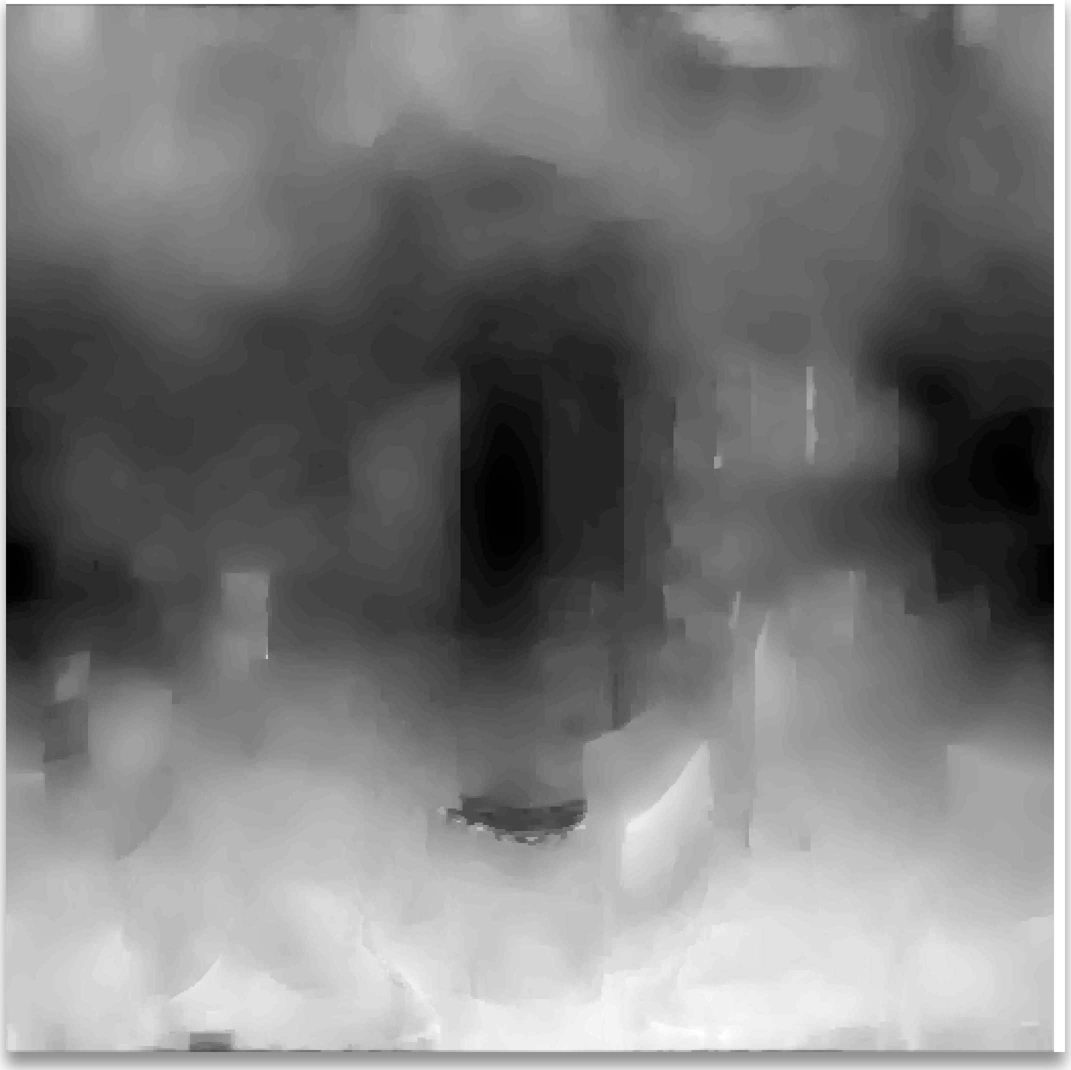
Optical Flow Results - planar technique

Ground Truth



Estimated optical flow (module)

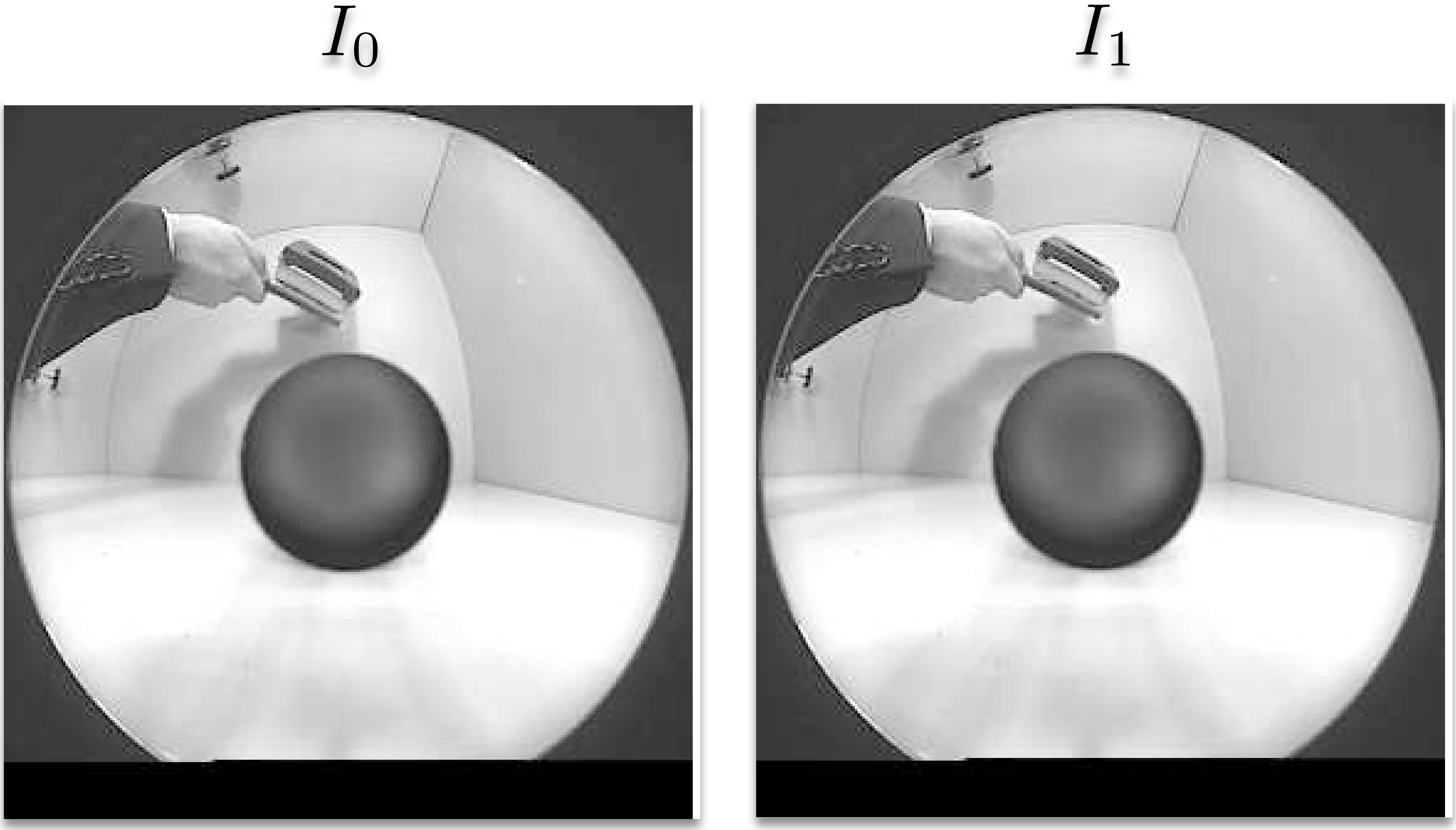
Planar-TVL1



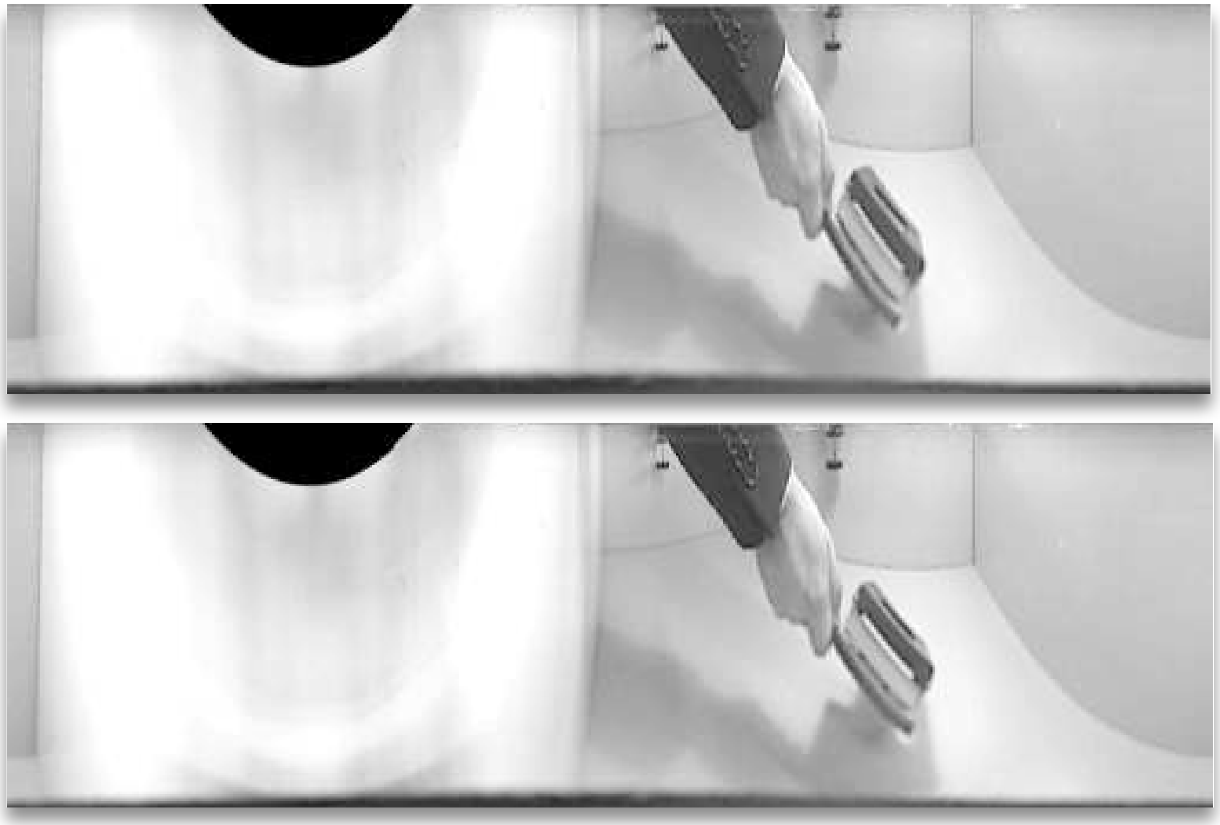
GrH-TVL1

Error	Module (SSE)	Angle (AAE)
Planar-TVL1	9.5354	0.2839
GrH-TVL1	**2.02**	**0.1509**

Catadioptric Video Sequence


I_0 I_1

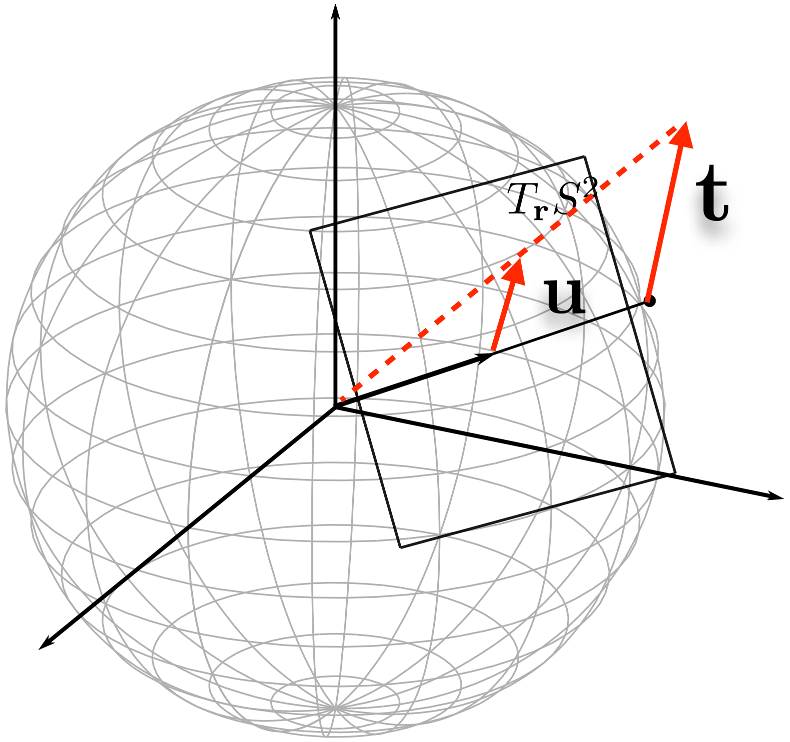
Catadioptric Video Sequence



Initial Image residual

Image residual after motion compensation

A new inverse problem: Depth Estimation



$$\mathbf{y} \in S^2$$

$$\mathbf{u} = D^{-1}\mathbf{t}_s$$

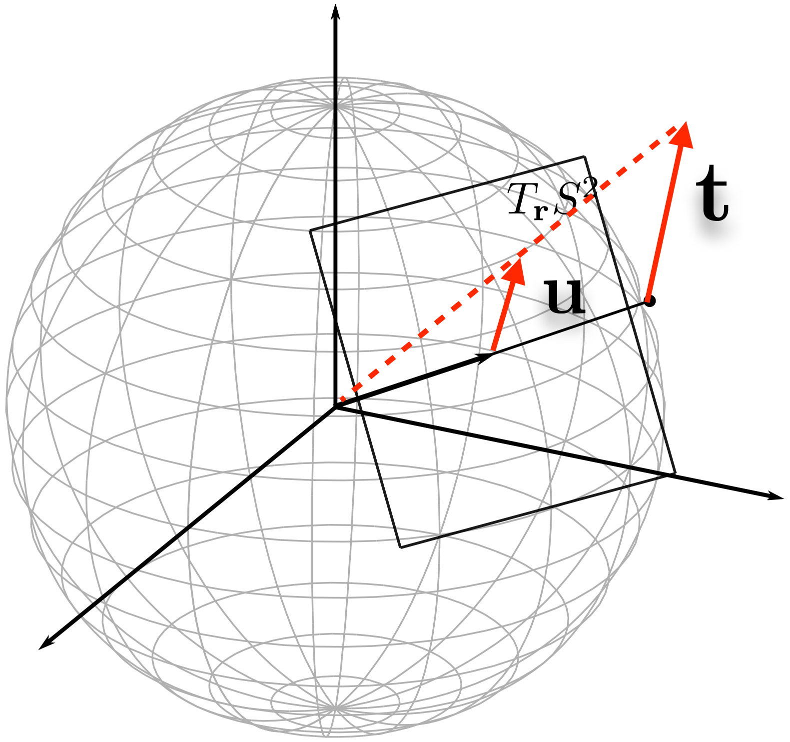
$\mathbf{u}(\mathbf{y})$ optical flow

$D(\mathbf{y})$ depth map

$$Z = D^{-1}$$

optimization variable

A new inverse problem: Depth Estimation



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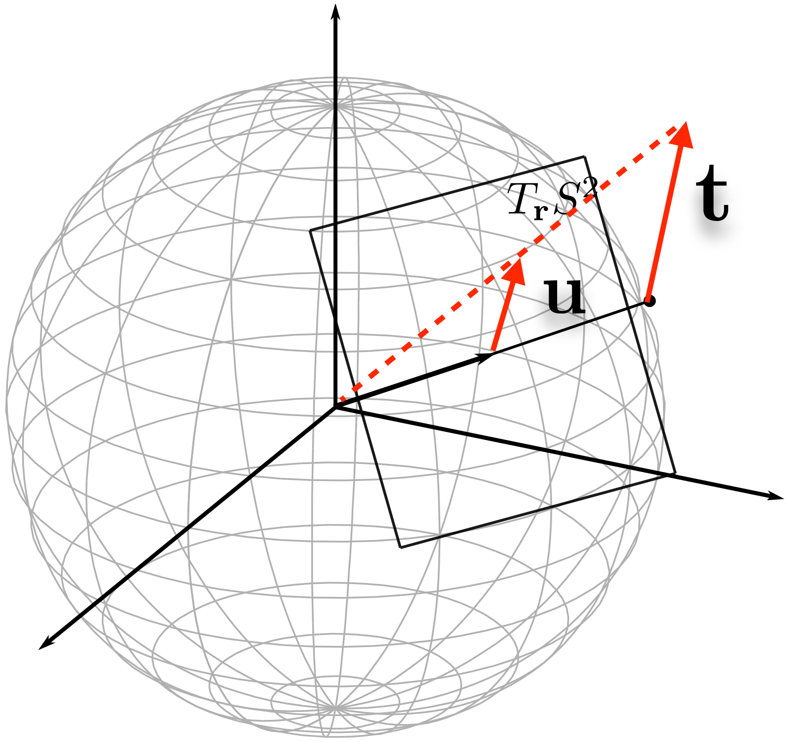
$Z = D^{-1}$ optimization variable

$$I_1(\mathbf{y}) + Z((\nabla I_1)^T \mathbf{t}_s) - I_0(\mathbf{y}) = 0$$

A new inverse problem: Depth Estimation



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$$Z = D^{-1}$$

optimization variable

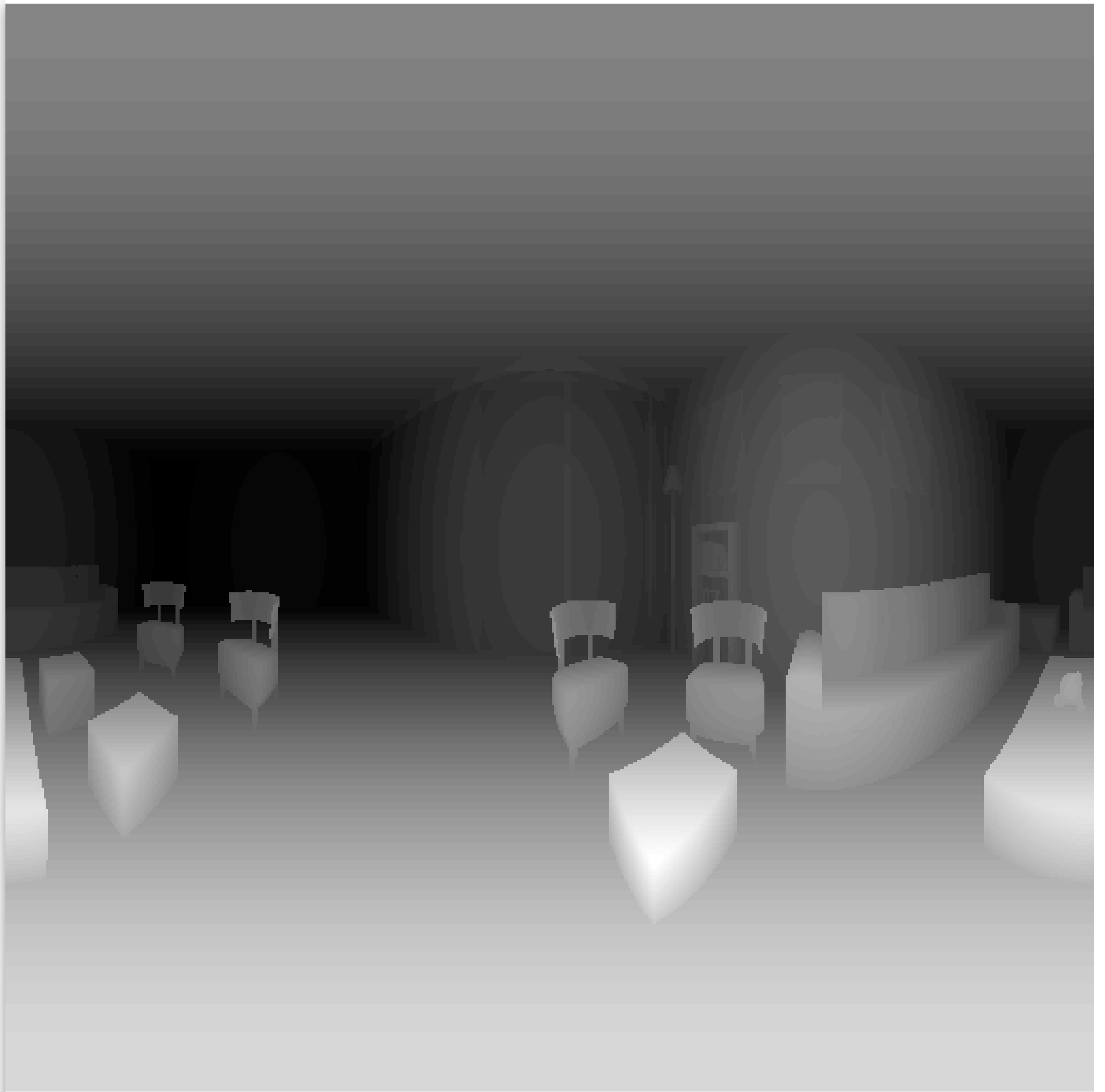
$$I_1(\mathbf{y}) + Z((\nabla I_1)^T \mathbf{t}_s) - I_0(\mathbf{y}) = 0$$

$$J \quad = \quad \int_\Omega |\nabla Z| + \frac{1}{2\theta}(Z - K)^2 + \lambda |I_1(\mathbf{y}) + Z((\nabla I_1)^T \mathbf{t}_s - I_0(\mathbf{y}))| \mathrm{d}\Omega$$

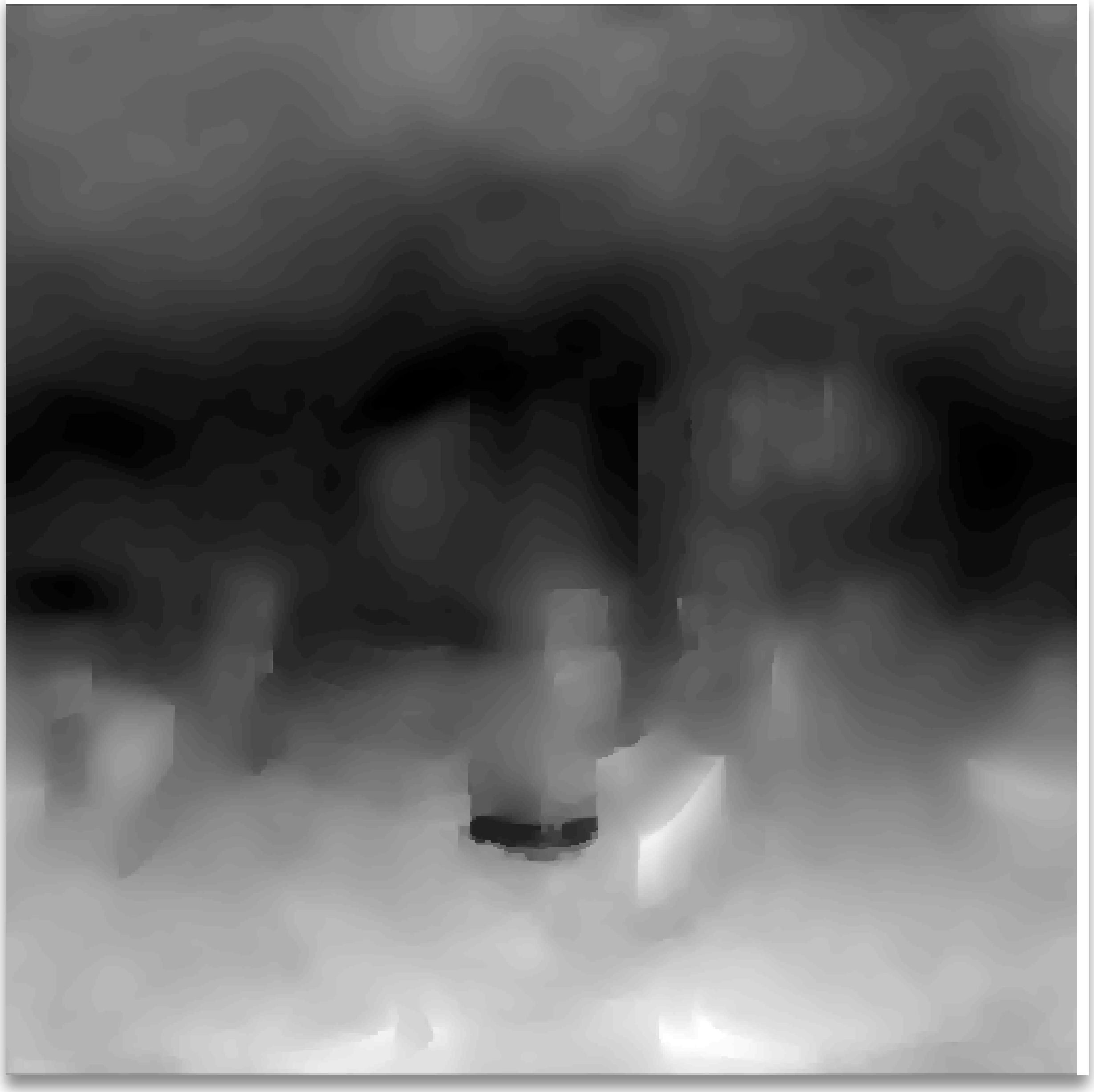
K is an auxiliary variable close to **D**

Depth Estimation - synthetic data

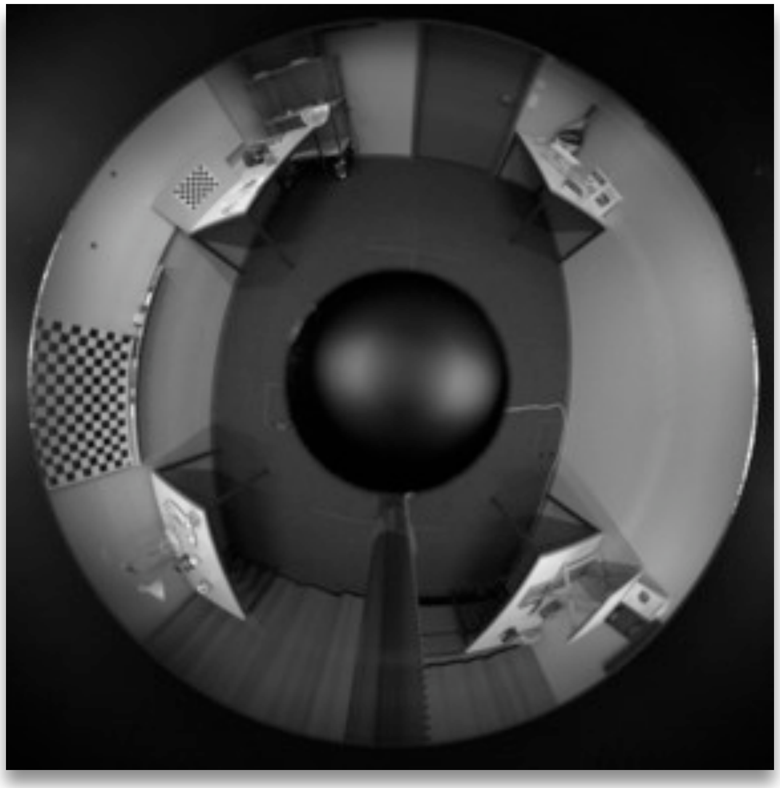
Ground Truth



Estimated depth map



Real 3D Reconstruction



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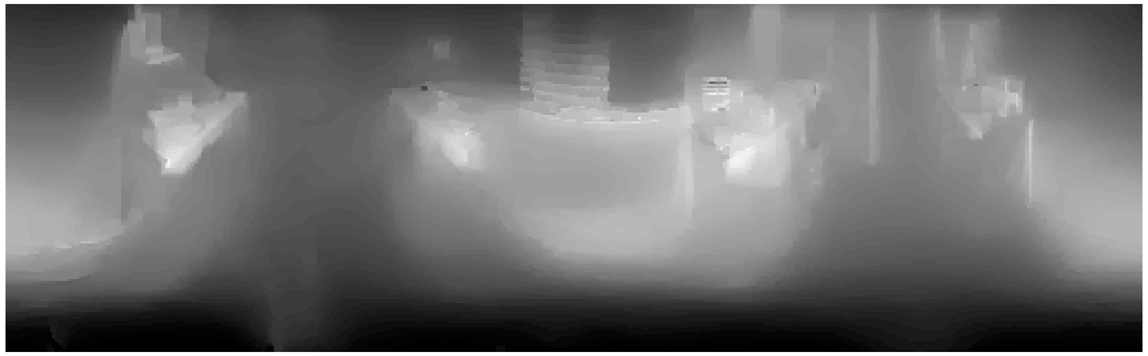
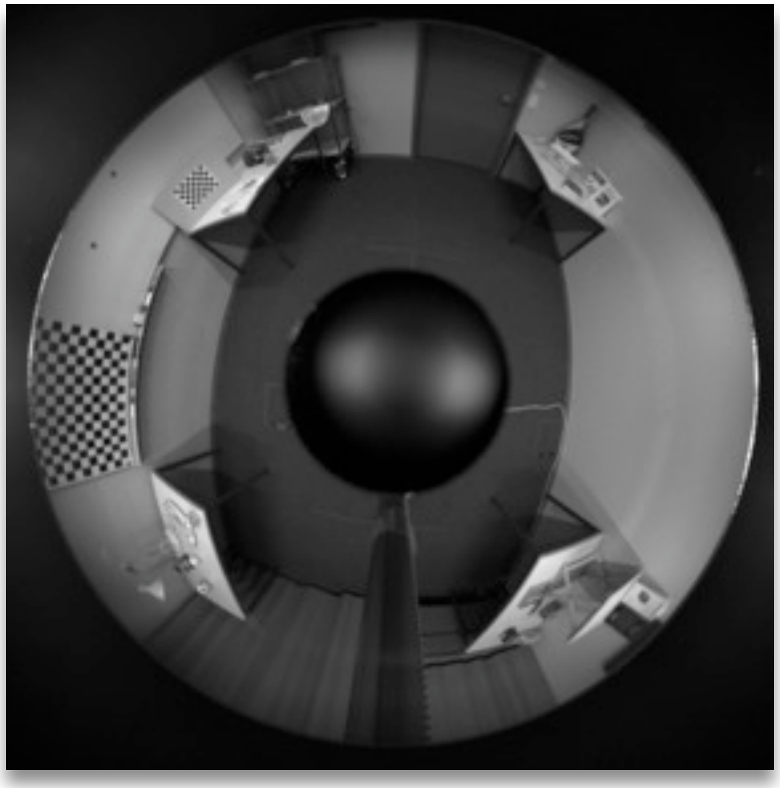
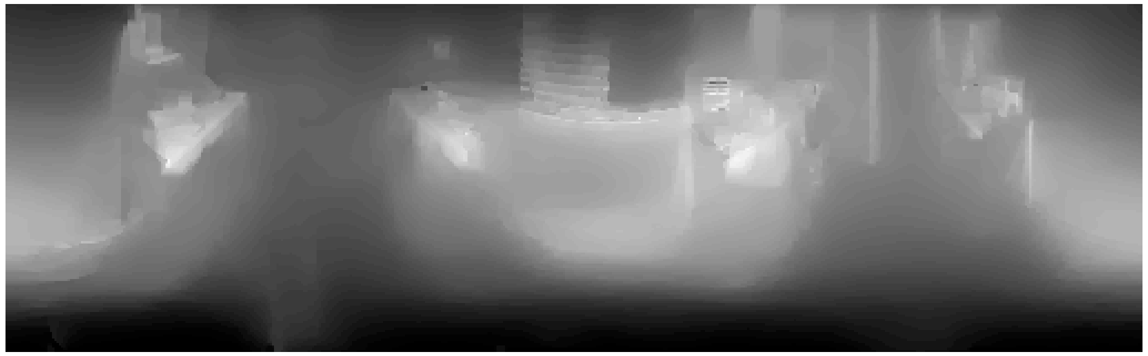


From depth map to 3D model

Real 3D Reconstruction



Estimated depth map



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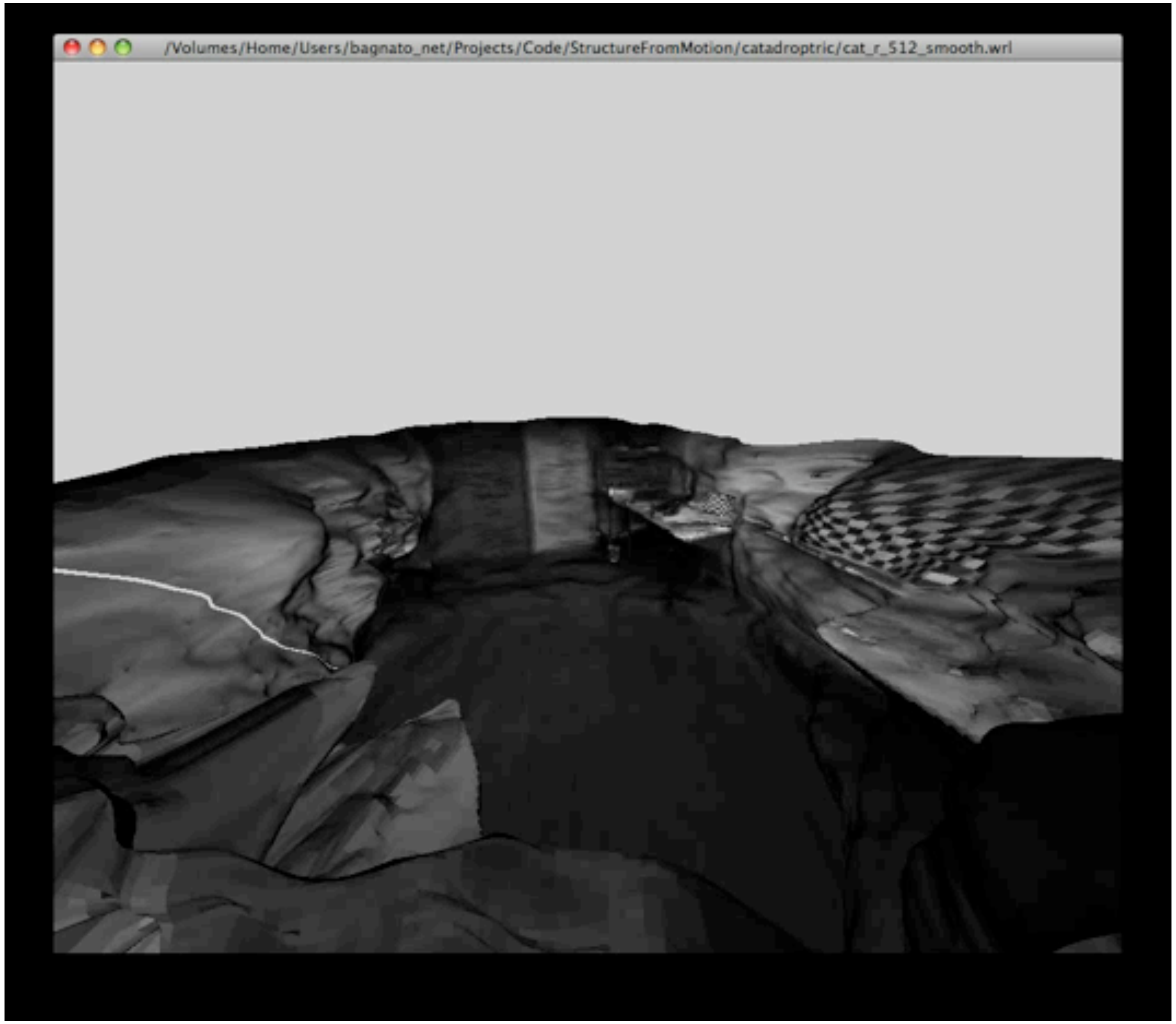

Conclusions

- Adapted TVL1 algorithm for omnidirectional optical flow
  - Valid for all single effective viewpoint devices
  - Graceful handling of irregular sampling grid
  - Numerically stable
- Novel algorithm for dense depth map estimation
  - no correspondences to solve
  - test on real sequences are convincing
- Real time implementation (work in progress)!

- For more details: L.Bagnato,P.Vandergheynst,P.Frossard. A Variational Framework for Structure from Motion in Omnidirectional Video Sequences. Submitted to IEEE Transactions in Image Processing

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Thank you!

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References

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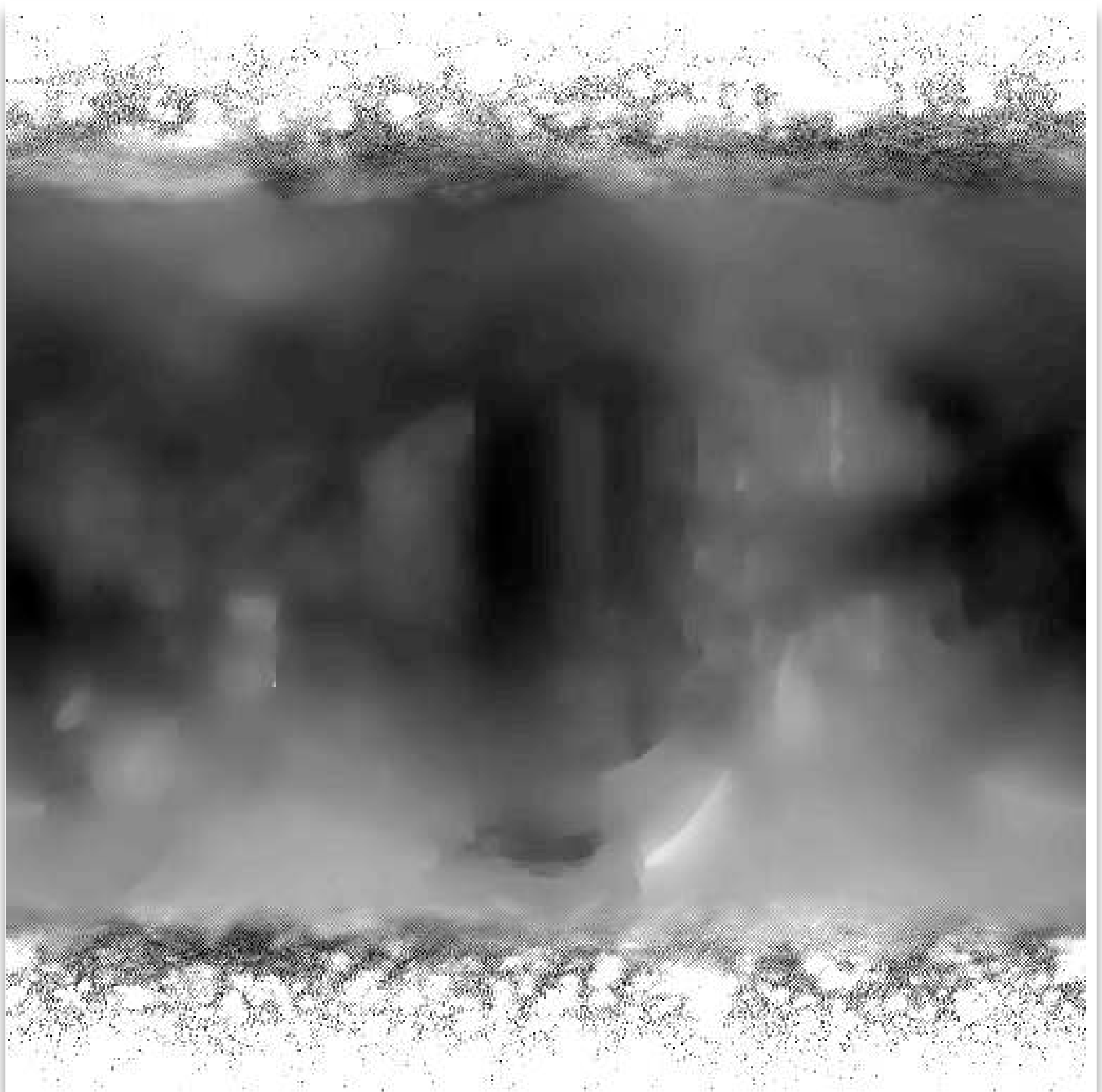
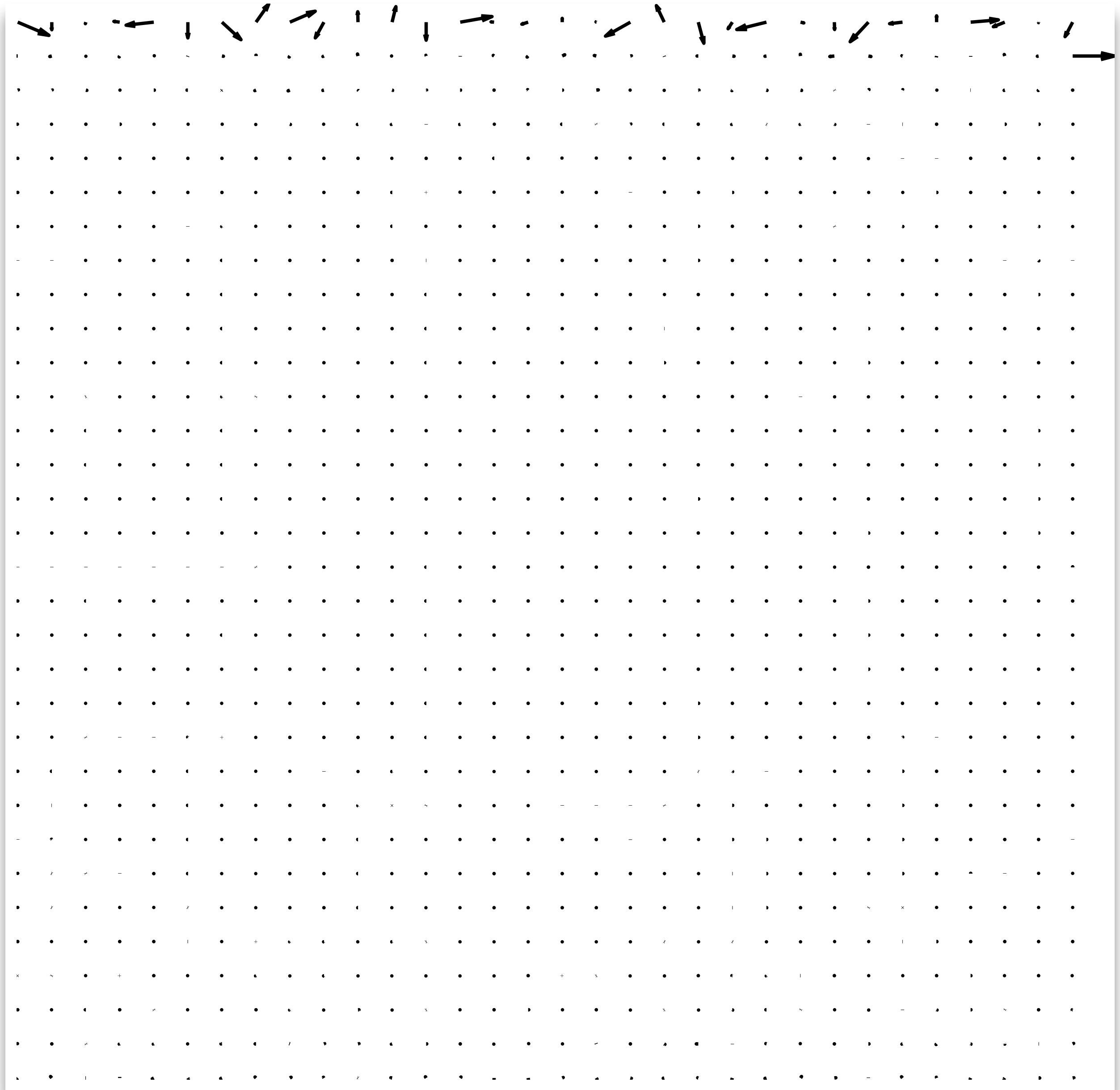
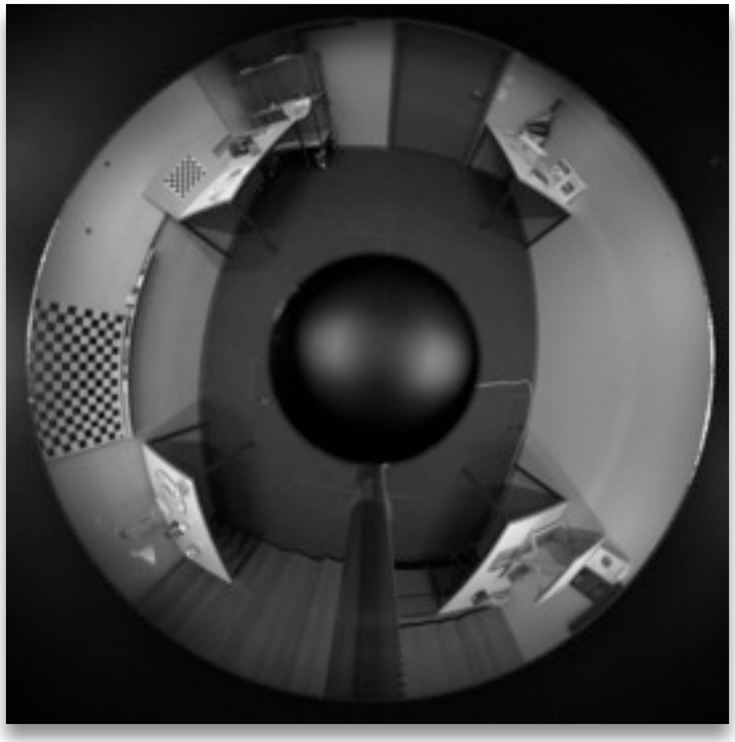
Naive Discretization



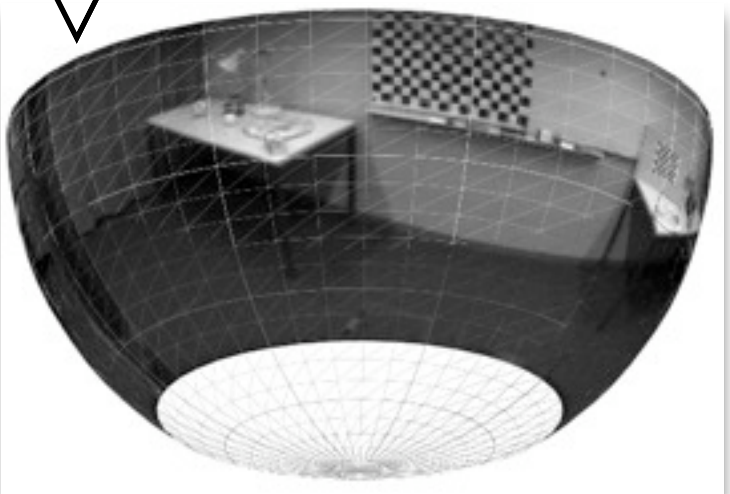


Structure from Motion

Omnidirectional image



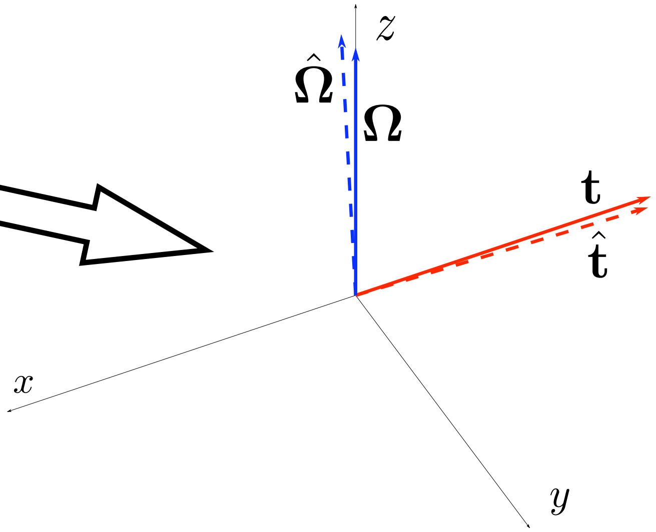
Projection on 2-Sphere

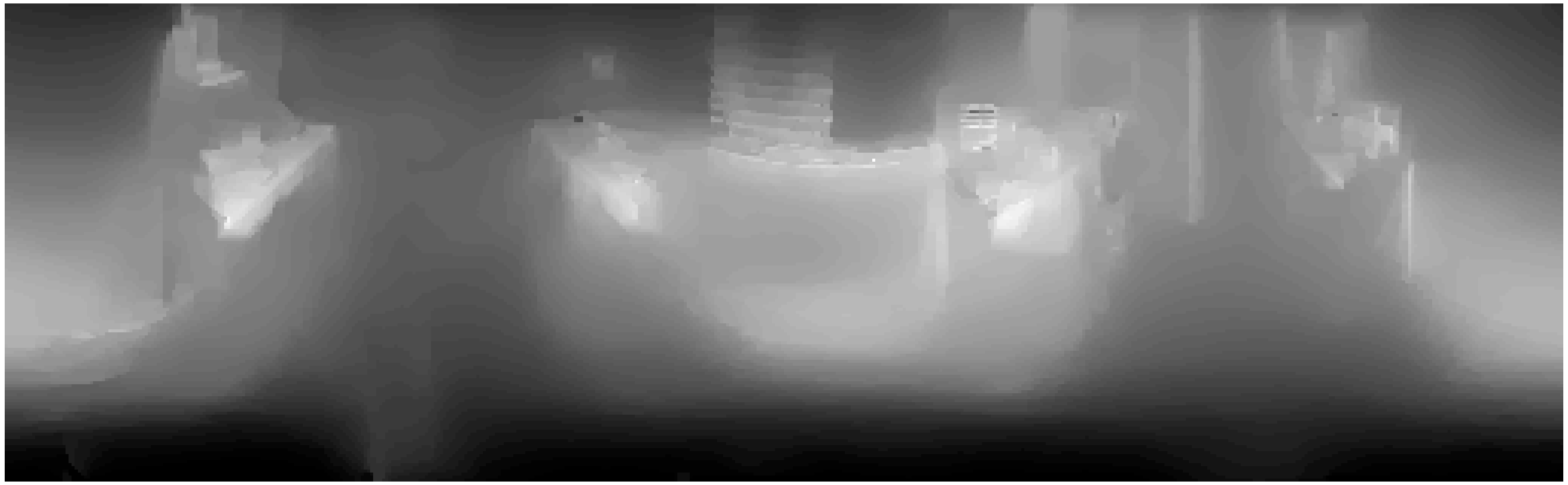


Input: 2 omnidirectional images from a video sequence

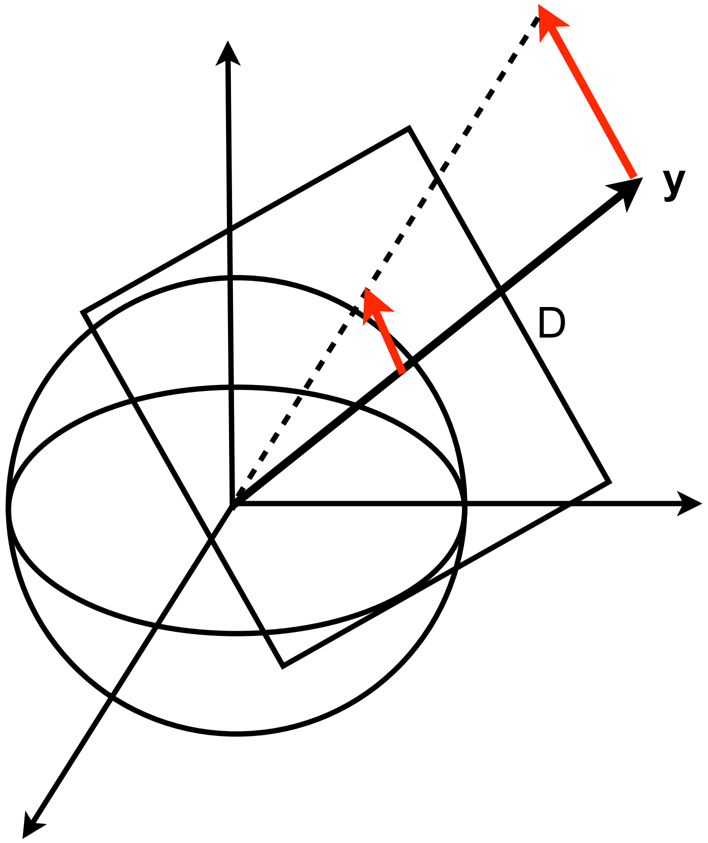
Algo: Graph based TVL1 variational approach

Output: **Depth Map** and **Ego-motion**





Spherical Optical Flow Field



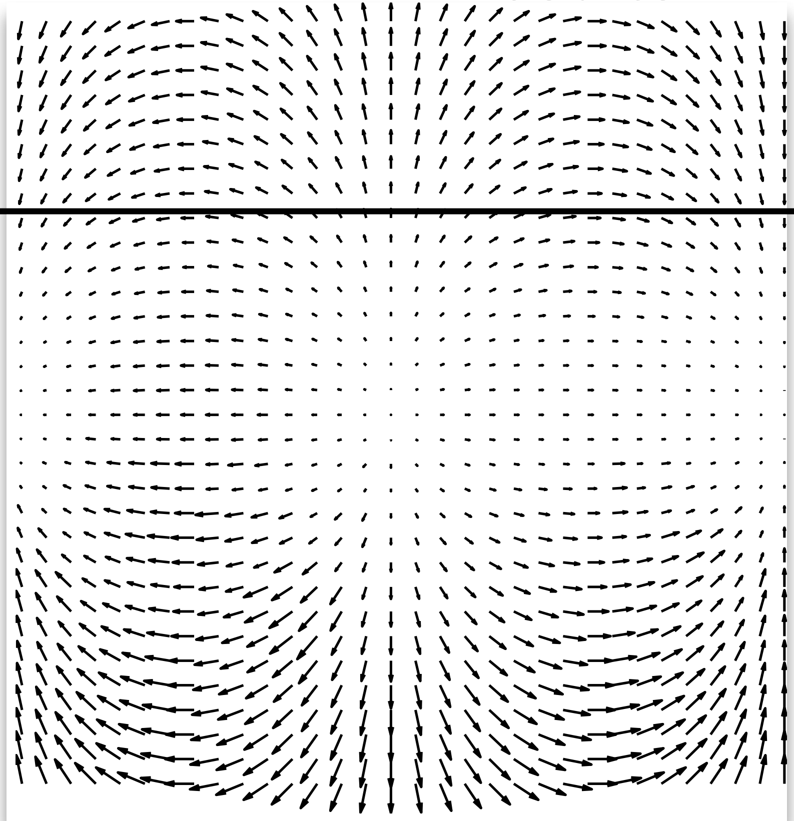
Brightness consistency

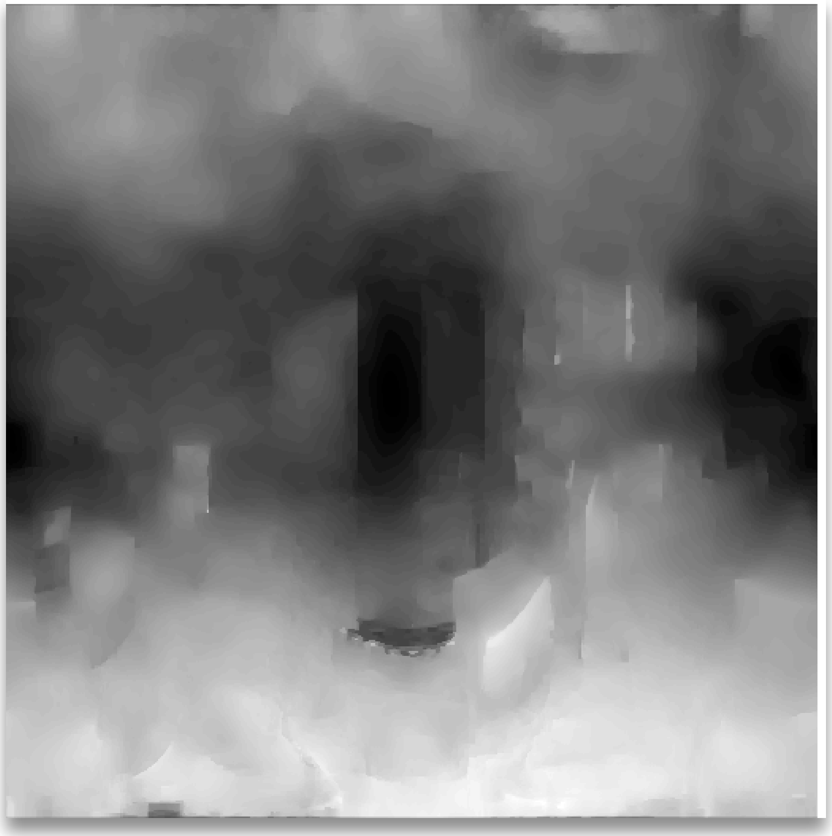
$$I_0(\mathbf{y}) - I_1(\mathbf{y}+\mathbf{u}) = 0$$

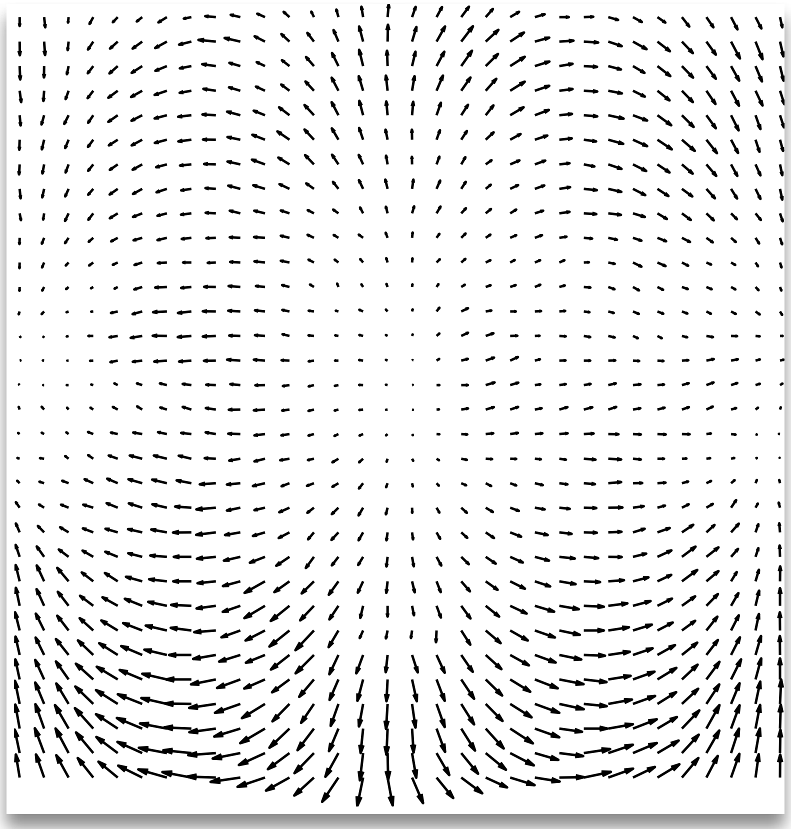
Linear approximation

$$I_1(\mathbf{y}+\mathbf{u}) - (\nabla I_1(\mathbf{y}+\mathbf{u_0}))^T(\mathbf{u}-\mathbf{u_0}) - I_0(\mathbf{y}) = 0$$

Result - our solution







Results - planar technique