

Single particle orbits in anisotropic fully shaped plasmas

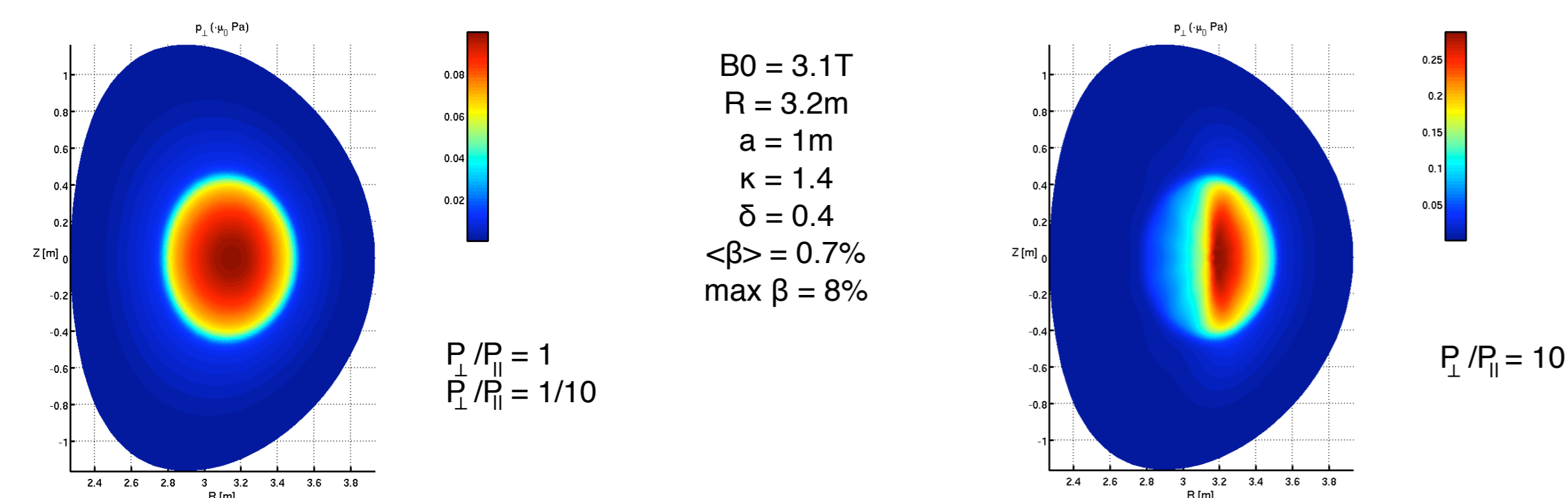
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Overview

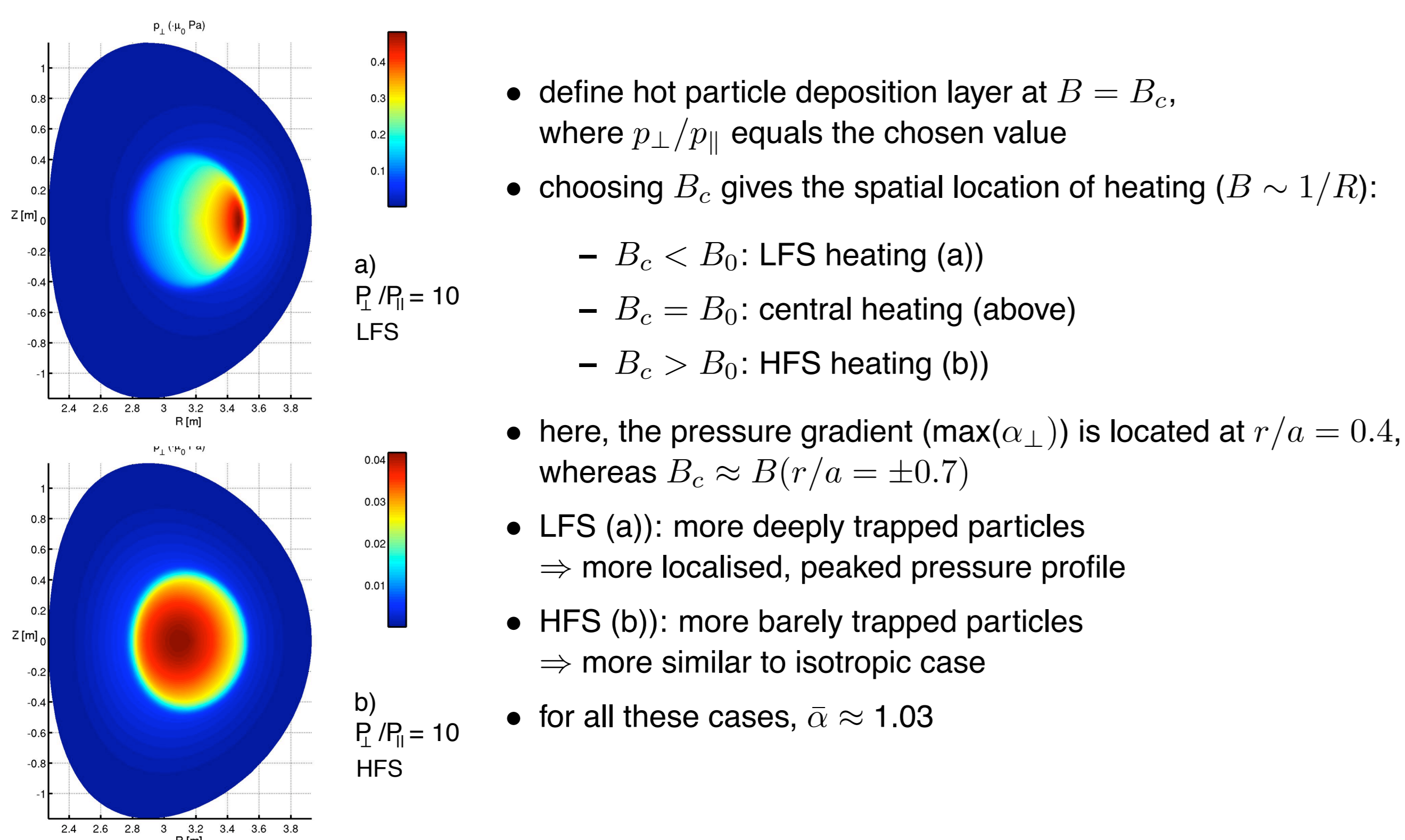
Anisotropic equilibria which are important in the context of e.g. ICRF heating are studied and its implications shown by the examples of the average toroidal magnetic drift frequency and the identification of tear drop orbits in the poloidal well of the magnetic field. Shown are the consequences of on- and off-axis heating, with either parallel or perpendicular pressure anisotropy, using newly derived exact canonical equations of motion. In general, due to the diamagnetic effect on the toroidal field, it is found that the perpendicular component of the pressure tensor, and its poloidal variation, determines the role of anisotropic pressure on single particle orbits.

$$p_{\perp}/p_{\parallel} \gg 1 \Rightarrow p = p(r, \theta)$$



- Large pressure gradient at $r/a = 0.4$, for enhanced diamagnetic effect on the toroidal field^{3,4}
- $\bar{\alpha} \equiv \langle -R_0 q^2 / 2B_0^2 (p') \rangle_{\theta} \approx 1.03$
- Introduced Bi-Maxwellian distribution function into the equilibrium code VMEC^{5,6}
 \Rightarrow parallel and perpendicular hot pressures can have different values everywhere
 \Rightarrow pressure is no longer a flux surface quantity but has poloidal dependence

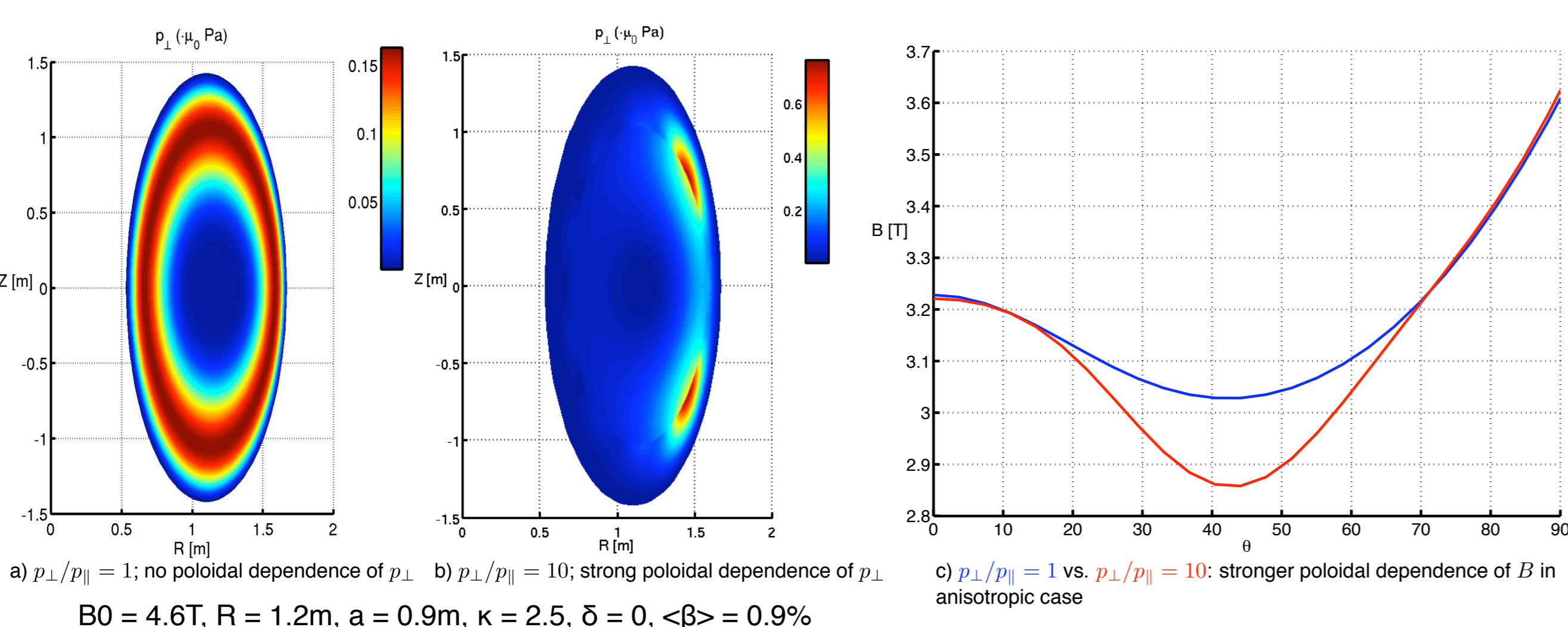
Off-axis heating



- define hot particle deposition layer at $B = B_c$, where p_{\perp}/p_{\parallel} equals the chosen value
- choosing B_c gives the spatial location of heating ($B \sim 1/R$):
 - $B_c < B_0$: LFS heating (a)
 - $B_c = B_0$: central heating (above)
 - $B_c > B_0$: HFS heating (b)
- here, the pressure gradient ($\max(\alpha_{\perp})$) is located at $r/a = 0.4$, whereas $B_c \approx B(r/a = \pm 0.7)$
- LFS (a): more deeply trapped particles \Rightarrow more localised, peaked pressure profile
- HFS (b): more barely trapped particles \Rightarrow more similar to isotropic case
- for all these cases, $\bar{\alpha} \approx 1.03$

$$p = p(r, \theta) \Rightarrow B = B(r, \theta)$$

- poloidal pressure dependence opens the way for new equilibria.
 \Rightarrow magnetic wells in poloidal direction can be generated or at least deepened



a) $p_{\perp}/p_{\parallel} = 1$; no poloidal dependence of p_{\perp} b) $p_{\perp}/p_{\parallel} = 10$; strong poloidal dependence of p_{\perp} c) $p_{\perp}/p_{\parallel} = 1$ vs. $p_{\perp}/p_{\parallel} = 10$: stronger poloidal dependence of B in anisotropic case
 $B_0 = 4.6T$, $R = 1.2m$, $a = 0.9m$, $\kappa = 2.5$, $\delta = 0$, $\langle \beta \rangle = 0.9\%$

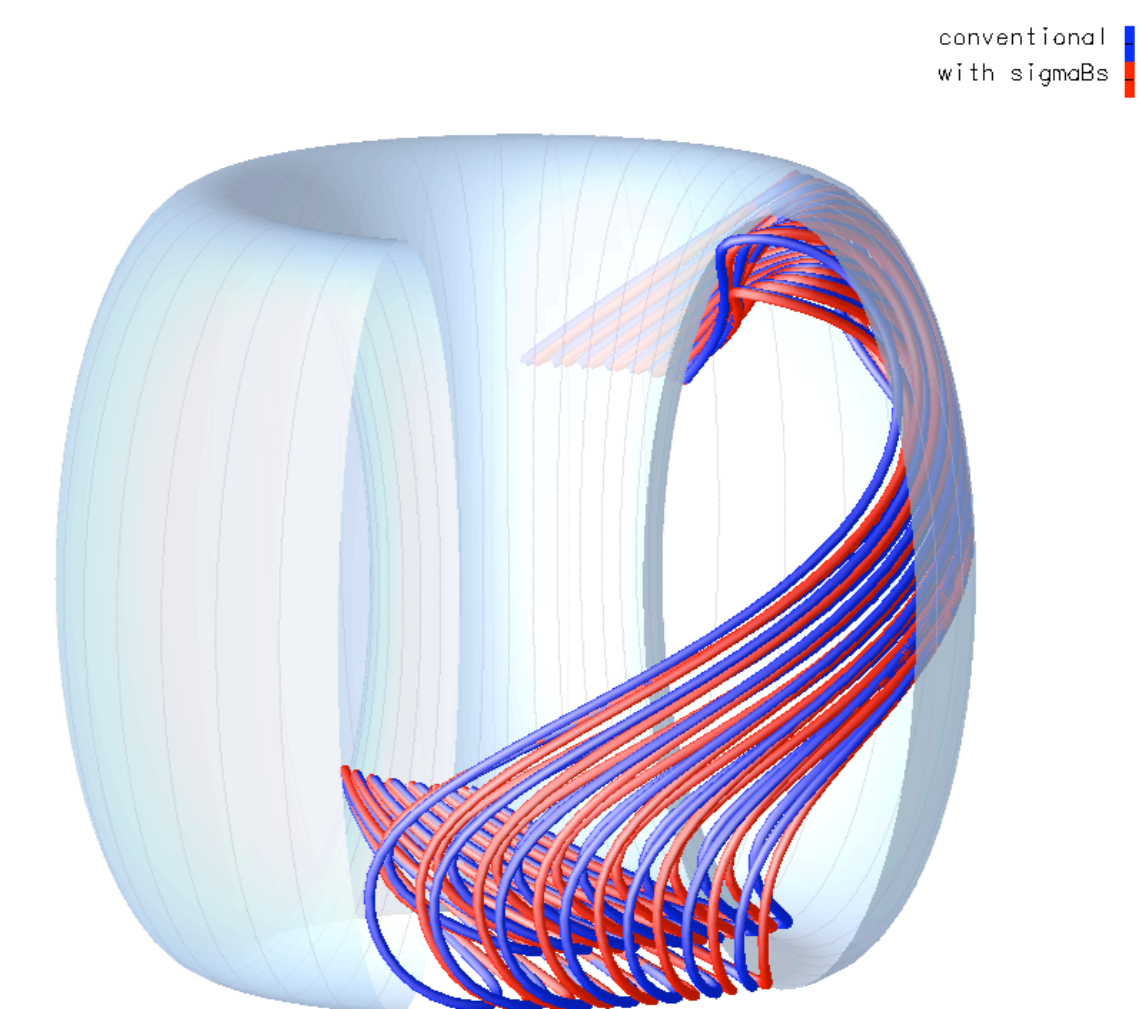
Exact canonical formulation

- Implemented new, exact canonical formulation in the single particle orbit code VENUS, including higher order radial magnetic field terms¹
- The orbits can now exactly be identified with the drift velocity equation satisfying Liouville's theorem²

$$\mathbf{v}_d = \frac{e\rho\sigma[\mathbf{B} + \nabla \times (\rho_{\parallel}\sigma\mathbf{B})]}{\gamma m_0(1 + \rho_{\parallel}\mu_0\mathbf{K} \cdot \mathbf{B}/B^2)}$$

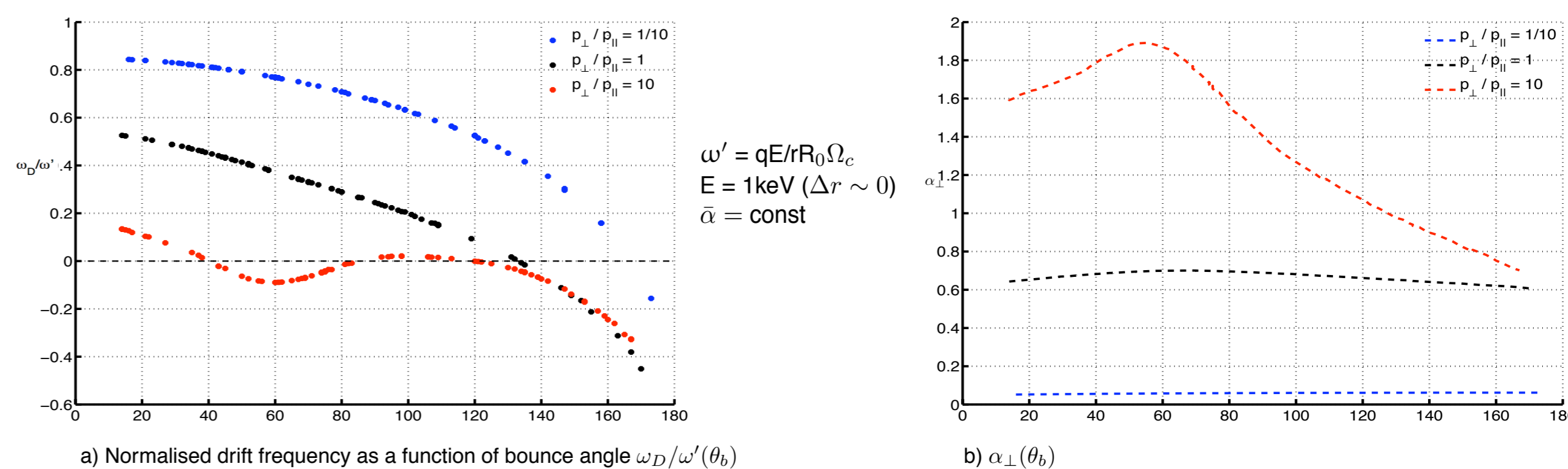
- These new terms give a contribution of the order of β .

The figure shows two orbits in a tight aspect ratio Tokamak ($B_0=5.6T, R_0=1.1m, a=0.9m, \kappa=2.5$) with $\beta=2\%$ and an energy of 500keV.



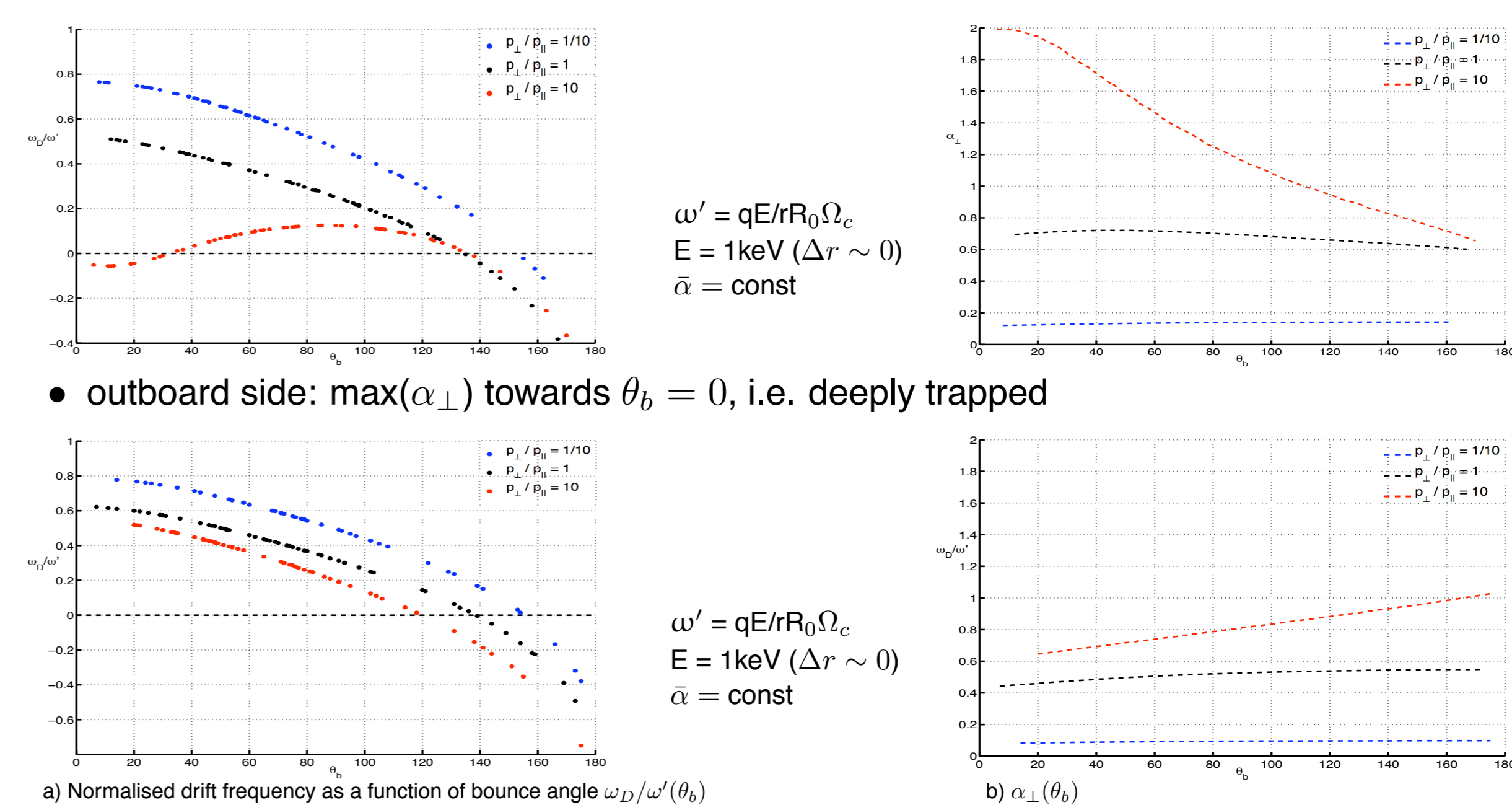
The newly introduced terms in the Hamiltonian formulation contribute to the orbit in the order of β . Blue: the orbit neglecting the additional terms, red: using exactly canonical equations of motion.

$$p = p(r, \theta) \Rightarrow \omega_D = \omega_D(\alpha_{\perp}(\theta))$$



- $p_{\perp} \neq p_{\parallel} \Rightarrow \alpha = -R_0 q^2 / B_0^2 (p'_{\perp} + p'_{\parallel}) / 2 \neq \alpha_{\perp} = -R_0 q^2 / B_0^2 (p'_{\perp})$
- Toroidal precession drift frequency depends on α_{\perp} , not α , i.e. on p'_{\perp}
- ω_D depends strongly on bounce angle due to poloidal dependence of α_{\perp}
- Keeping $\bar{\alpha} = \langle \alpha \rangle_{\theta}$ constant, ω_D is higher/lower for $p_{\perp}/p_{\parallel} < / > 1$

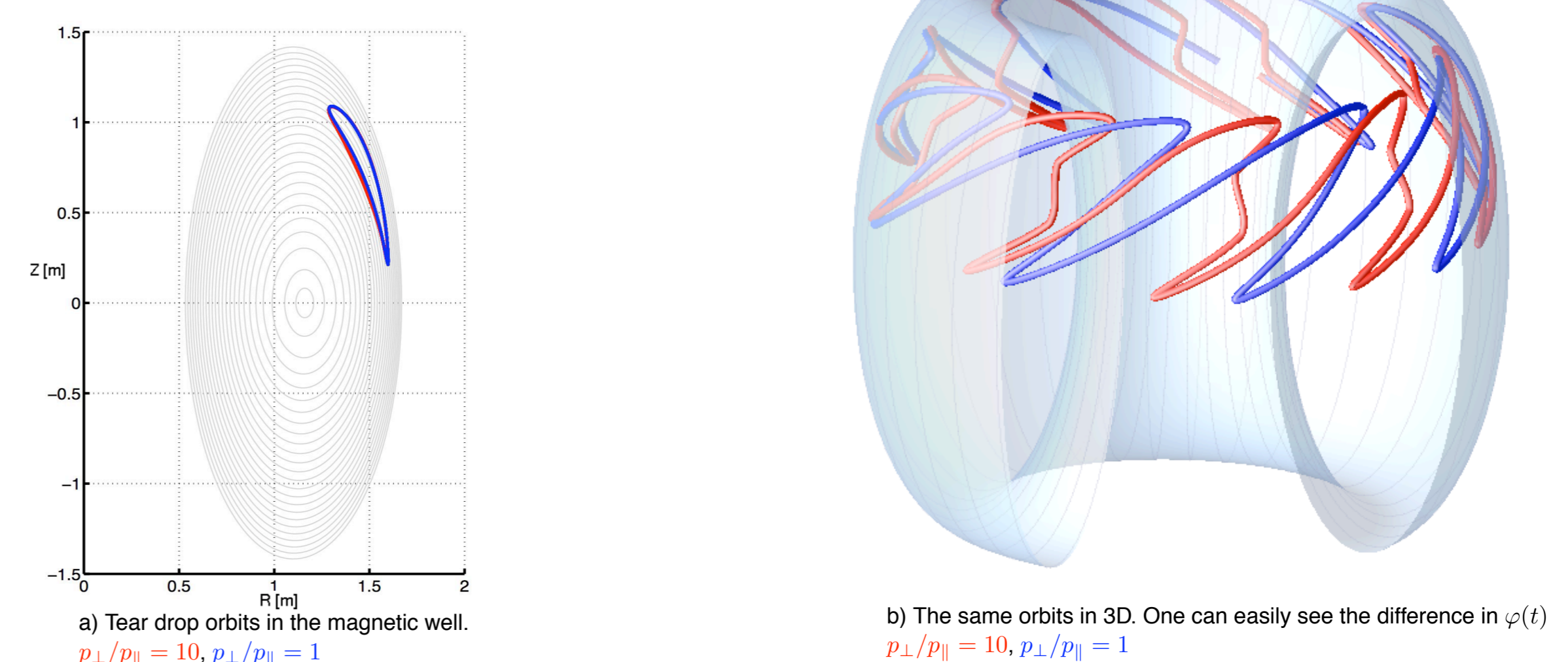
Heating location \Rightarrow location of maximum α_{\perp}



- outboard side: $\max(\alpha_{\perp})$ towards $\theta_b = 0$, i.e. deeply trapped
- inboard side: $\max(\alpha_{\perp})$ towards $\theta_b = \pi$, i.e. barely trapped

Non-standard orbits

- important toroidal orbit corrections due to anisotropy and off-axis heating
- here, $B \searrow \Rightarrow v_{\parallel} = \sqrt{1 - \lambda B} \nearrow \Rightarrow \phi(t) \nearrow$



a) Tear drop orbits in the magnetic well. $p_{\perp}/p_{\parallel} = 10, p_{\perp}/p_{\parallel} = 1$ b) The same orbits in 3D. One can easily see the difference in $\phi(t)$ $p_{\perp}/p_{\parallel} = 10, p_{\perp}/p_{\parallel} = 1$

References

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Acknowledgments

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