Analysis of Iterative Phase Retrieval Approach to Optimize Amplitude Measurement Parameters

Sudheer K. Jawla, J.-P. Hogge and S. Alberti

***Abstract*—This paper describes a thorough analysis made with the Iterative Error-Reduction Approach (IERA) for the determination of phase profiles of the output microwave beam of a gyrotron from known amplitudes. Based on theoretical field profiles of 170GHz/2MW coaxial gyrotron, a numerical/statistical approach is used to study the accuracy of Iterative phase retrieval algorithm. Several measurement parameters are optimized to get more accuracy during the infrared measurements of intensity thermograms of the microwave beam.**

***Index Terms*— Millimeter wave, Gyrotron, Phase Retrieval, Infrared Imaging.**

I. Introduction

The motivation of this study is to develop an appropriate methodology to calculate the accurate phase profile of a microwave beam from measured intensity thermograms. For this purpose IERA [1] is studied in more detail emphasizing on some basic parameters, which has never been considered before, issues like scan separation, plane dimension, mesh size and measurement accuracies which become more of an issue at higher frequencies.

Though other methods are also being studied for phase reconstruction, the error reduction approach seems to be most accurate in general in case of gyrotrons. Other methods, like Transport of Intensity Equation (TIE) [2], solve the real part of wave equation considering it as an elliptic partial differential equation. The main difficulty with this equation is its boundary behavior because the value of phase at the boundary is difficult to measure. Therefore a number of people suggested alternative algorithms to solve this problem. Also, a proper phase solution must satisfy the other part of wave equation which to our knowledge has not been considered in the case of TIE approach. Furthermore, in TIE an accurate estimate of intensity derivative along z direction needs closely spaced intensity measurements that depend on the wavelength of the microwave beam (Gaussian) and it's spreading along the propagation direction.

The moment irradiance approach [3] works well only if the beam pattern contains a minimal fraction of higher order modes.

Here we present a detailed analysis of the iterative error reduction phase reconstruction approach made with the theoretical field profiles of 'EU 170GHz/2MW coaxial cavity gyrotron for ITER' [4].

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II. Iterative Phase Retrieval Approach

The IERA is outlined briefly as follows: the field of an electromagnetic wave at *i*-th plane can be written as, $u_i\left(x,y,z_i\right)=A_i\,(x,y)\exp[i\phi_i\,(x,y)]$. Where $A_i(x,y)$ and $\phi_i(x,y)$ are wave amplitude and phase at the *i*-th plane respectively. The field u_j at a later distance z_j is given as

$$u_j\left(x,y,z_j\right)=F^{-1}\left\{\exp[i(z_j-z_i)k_z]\times F[u_i(x,y,z_i)]\right\} \tag{1}$$

where F and F^{-1} denotes the Fourier transform and its inverse respectively. Iteration method is shown below in figure (1),

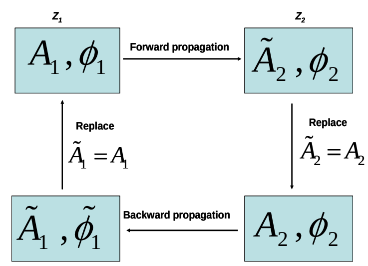

Fig. 1 Phase reconstruction in two cross sections.

where, A_1, A_2 and $\tilde{A}_1, \tilde{A}_2$ is measured and calculated field amplitudes in two planes z_1 and z_2 respectively. The criterion to stop the iteration is expressed as,

$$E_j^{(n)}=\int_{S_j}\left|A_j-\tilde{A}_j^{(n)}\right|^2 dS_j<\varepsilon_j \tag{2}$$

where the subscript means the value after the n^{th} iteration and ε_j is a set of given small values. The integral is performed on the surface of S_j. The phase reconstruction algorithm can be easily extended to any number of cross section and in this work three cross sections are considered.

The main issues which affect the reconstruction algorithm and needs to be incorporated during the measurements and measured data processing are discussed as follows.

1). The propagation method used in the above algorithm is the angular spectrum propagation [5] as it is better suited for near field propagation because of sampling reasons. Analysis is made depending on sampling frequency in Fourier space and distance of propagation. Zero padding is used on the measured data in accordance with the distance of propagation.

2). The behavior of the algorithm is also strongly related to the choice of the initial phase choice; if it resembles a close approximation to the true phase then convergence to a good solution will be achieved with small effort. In general the initial choice should be either a flat phase or a quadratic phase function which normally resemble the phase of a Gaussian beam. The initial phase choice is the only parameter, which can be altered during the retrieval process with measured data because when measurements are done, the scan separation and plane dimensions are fixed. A certain amount of randomness needs to be associated with these choices in order to fully assess the 'phase problem' from a statistical point of view.

3). One of the important tasks in the algorithm is to choose mesh spacing and mesh dimension. To obtain correct results, it is crucial that the mesh spacing should be sufficiently small and mesh dimensions sufficiently large. Since, during the measurements the camera pixels are fixed, therefore one needs to interpolate the measured data. Also, for the angular spectrum propagation, we can choose equal object and target pixels. Some of the results are shown below for an optimization of scan separation and mesh dimensions starting for a theoretically calculated RF beam. In figure (2), reconstruction efficiency is plotted for three planes with scan separations d1 and d2 between first to second and first to third planes respectively. In figure (3), the reconstructed and theoretical phases are plotted in x and y cross section for plane number 2 for case A of figure (2).

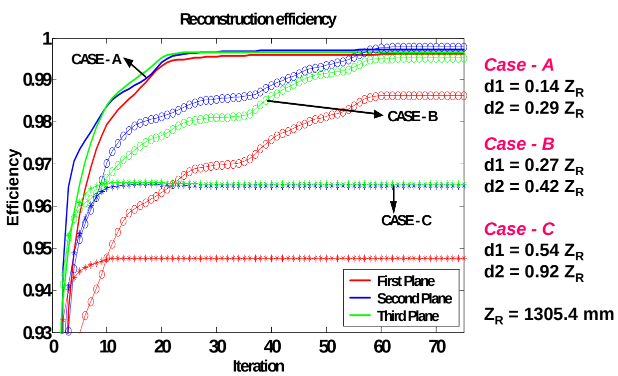

Fig.-2: The reconstruction efficiency is plotted with respect to the number of iterations using different scan separations as cases A, B and C for an initial random phase guess, d1 is the distance between plane 1 and 2, d2 is the distance between plane 1 and 3, Z_R represents the Rayleigh length.

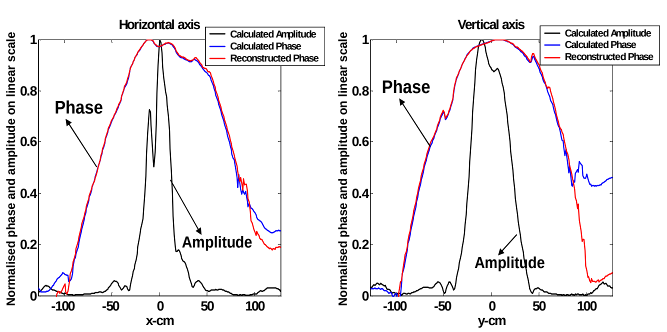

Fig-3: Calculated and reconstructed phase and amplitude profiles in plane 2 on a linear scale. The plane dimension is 256mm with a mesh size of 256. Plane 1 is the window plane and 2 and 3 are at 180mm and 380mm respectively. No. of iterations used are 75.

4), For experimentally measured intensity patterns, one should strictly aligned the field amplitude profiles within the accuracy of a fraction of a wavelength in order to avoid errors in the phase retrieval method. For this purpose, the expression in eq. (1) must be corrected for the shift in the beam centre of gravity at each plane location for experimentally measured profiles. For an arbitrary shaped beam propagating in a uniform media, the beam center of gravity x-coordinate varies linearly with z and is calculated as,

$$\left\langle I_{z_j}, x\right\rangle = \left\langle I_{z_i}, x\right\rangle + \frac{(z_j - z_i)}{2\pi}\frac{\iint \frac{k_x'}{k_z'}\left|\tilde{u}_{z_i}(k_x', k_y')\right|^2 dk_x' dk_y'}{\iint \left|\tilde{u}_{z_i}(k_x', k_y')\right|^2 dk_x' dk_y'}$$

A similar expression holds for y-coordinate accordingly. The correction term based on the above expression is introduced in eq. (1) as,

$$u_j(x,y,z_j) = F^{-1}\left\{\begin{array}{l} F\left[u_i(x,y,z_i)\right]\times\exp\left[i(z_j - z_i)k_z\right]\times \\ \underbrace{\exp\left[i\left(k_x\left\langle I_{z_i}, x\right\rangle + k_y\left\langle I_{z_i}, y\right\rangle\right)\right]}_{correction\ factor}\end{array}\right\}$$

The above discussed analysis and an appropriate data filtering technique/defective pixel correction method will be applied to the measured data to calculate the phase of the output microwave beam of 118GHz and 170GHz gyrotron. For the IR measurements, the target material to be used in the experiment is also optimized for the thickness at the two frequencies using free space method. From the measured complex permittivity, material thickness is optimized to minimize the back reflections from the target. For the materials like macor/robax, a wrong selection of the thickness can give rise to large back reflection inside the gyrotron that can seriously effect the whole operation of the gyrotron.

CONCLUSION

This analysis concludes that if the measurements are made close to the beam waist position (gyrotron window plane) with small scan separations and an optimized choice of other parameters, one can determine the phase to a good accuracy.

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