

# Quasi-isodynamic Configuration With $N = 12$ and High $\beta$

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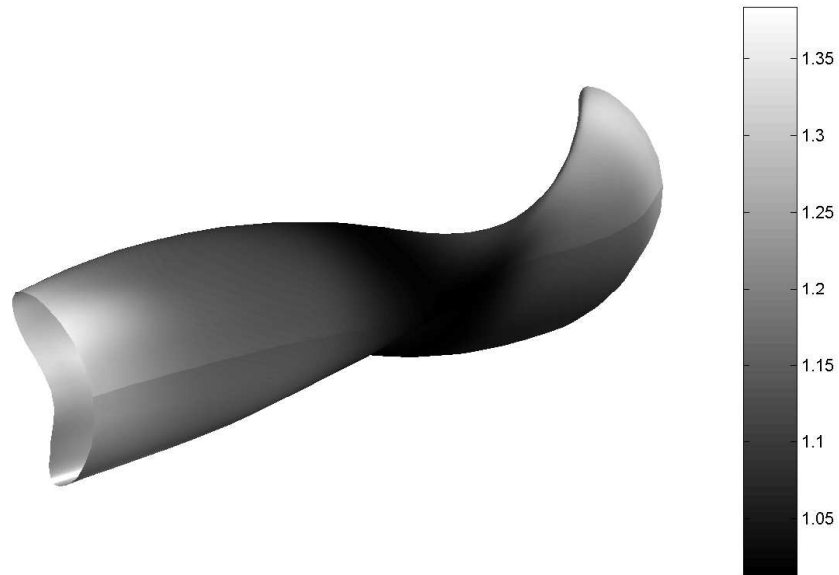
**Abstract.** Results of an optimization toward quasi-isodynamicity for a stellarator with a comparatively large number of periods,  $N = 12$ , are presented. The following set of physics properties to be achieved was used: 1) good long-time collisionless confinement of  $\alpha$ -particles; 2) small neoclassical transport in the  $1/\nu$  regime; 3) small bootstrap current; 4) high stability- $\beta$  limit. As a result, the boundary magnetic surface of a configuration is found that satisfies the above requirements for  $\langle\beta\rangle \approx 20\%$ .

**Keywords:** stellarator, computational optimization, quasi-isodynamicity.

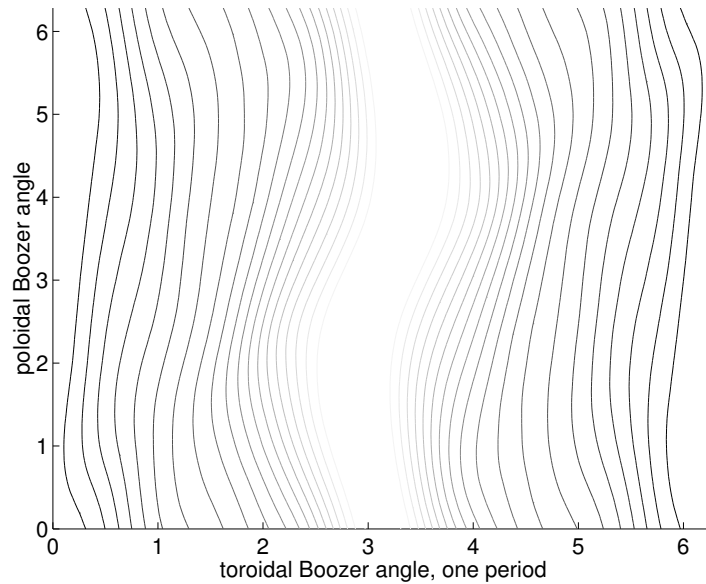
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## INTRODUCTION

Analytic expansions [1, 2] and numerical optimizations [1, 2, 3] have previously demonstrated that approaching quasi-isodynamicity (qi) conditions in configurations with poloidally closed contours of  $B$  leads to a significant improvement of fast-particle collisionless confinement (by definition), diminished neoclassical transport and small bootstrap current. It has also been shown that the secondary currents close on themselves within each period of the system in the quasi-isodynamic configurations under consideration [3]. There is no clear analytical indication that the approximate fulfillment of qi conditions is at odds with a high stability- $\beta$  limit. Computational investigations of such type of stellarators with different period numbers indicate that the stability limit increases with the number of periods. In the present paper, results will be presented for the optimization of an  $N = 12$  configuration with high stability  $\beta$ -limit. In the optimized qi configuration identified with this procedure, Mercier, resistive-interchange and stellarator-symmetric local ballooning modes are found to be stable for  $\langle\beta\rangle = 20\%$ . At this value of  $\beta$ , there are no collisionless losses of  $\alpha$ -particles born at  $1/2$  of the minor plasma radius for typical power plant parameters ( $B = 5T, V = 1000m^3$ ) up to a time of flight of 1 sec. The neoclassical transport is similar to that of W7-X [4] and the value of the bootstrap current is small. The aspect ratio of the configuration found is about 36. Preliminary results of global-mode stability will be presented.



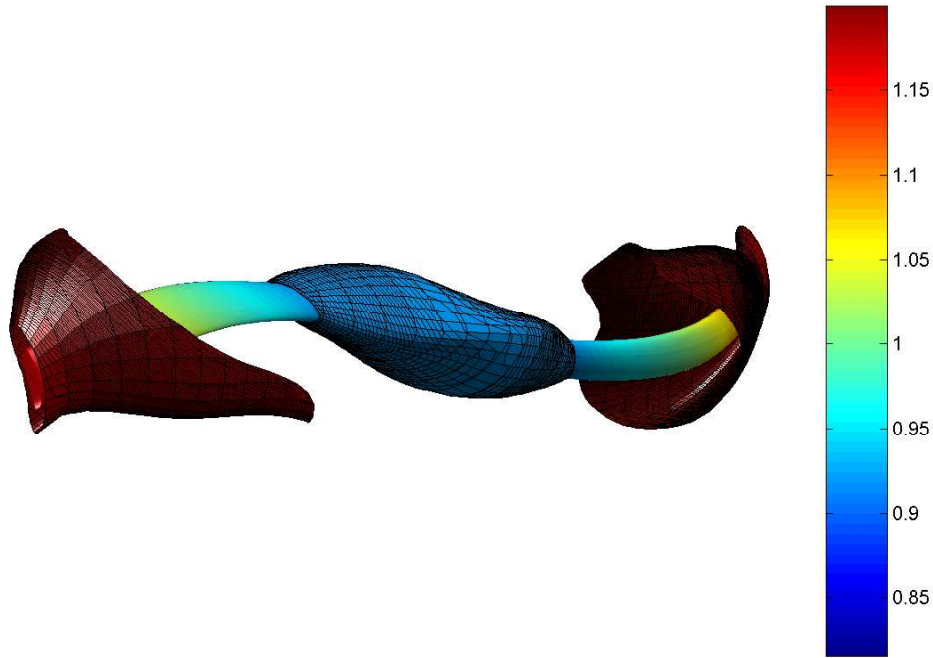
**FIGURE 1.** One period of boundary magnetic surface of an optimized  $N = 12$  stellarator. The shading defines the magnetic field strength.



**FIGURE 2.** Contours of  $B$  on a magnetic surface corresponding to  $1/2$  of plasma minor radius.

## INITIAL CHOICE AND GOALS OF OPTIMIZATION

In the optimization the VMEC code [5] was used to calculate equilibria for fixed boundaries; the transition to magnetic coordinates was performed by the JMC code [6]; a set of codes was used to calculate the contours of  $B$  on magnetic surfaces, the contours of the second adiabatic invariant, Mercier and ballooning-mode stability as well as values

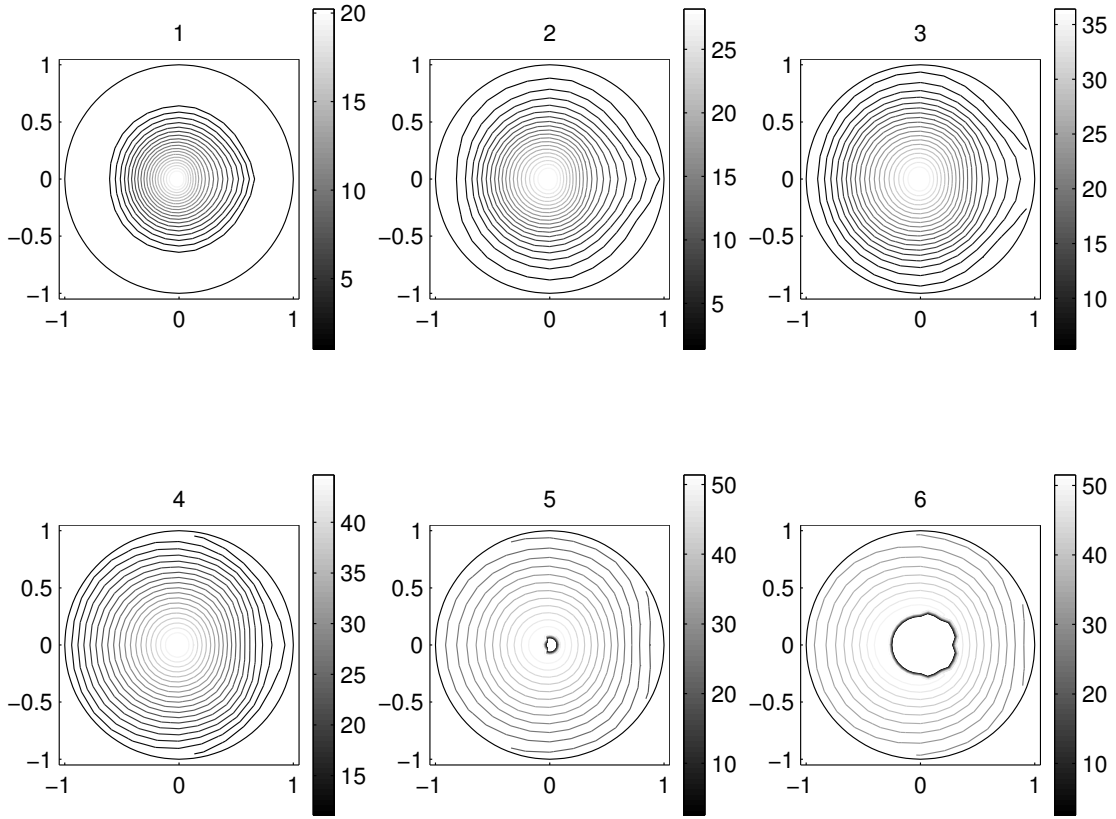


**FIGURE 3.** Near-axis magnetic surface and three surfaces of constant strength of the magnetic field.

of the effective ripple [4] and the structural factor of bootstrap current in the  $1/\nu$  regime (see, e.g. [7]). The initial geometry of the boundary magnetic surface cross-sections was chosen similar to that of W7-X, namely, bean-shaped cross-section in the region of the maximal magnetic field on the magnetic axis and triangular cross-section in the region of minimal  $B$  on the magnetic axis. It is worth to mention that in W7-X the regions of extrema of the magnetic field strength on the magnetic axis coincide with the regions with almost zero (minimal  $B$  on axis) and maximal (maximal  $B$  on axis) curvature of the magnetic axis. In the configurations considered here this is different: the curvature of the magnetic axis is almost zero in both – due to the stellarator symmetry – up-down symmetric cross-sections. The optimizations were performed for fixed  $\beta$  values. A few sets of calculations were made to find the maximal stability  $\beta$  limit with acceptable behavior of other physics properties. Here, results are presented for  $\langle\beta\rangle = 0.22$ .

## RESULTS OF OPTIMIZATION

In Fig. 1 a 3D view is shown of this optimized  $N = 12$  stellarator (one period is shown). The shading defines the magnetic field strength on the boundary magnetic surface. The geometry of the cross-sections is similar to that of W7-X. The main difference from W7-X is that the curvature of the magnetic axis is small not only in the region of minimal  $B$ , but in the region of maximal  $B$ , too. Correspondingly, almost all contours of  $B$  are poloidally closed (see Fig. 2). In Fig. 3 a near-axis magnetic surface is shown together with surfaces  $B = \text{const}$ . It is seen that there is an absolute minimum of  $B$  near the magnetic axis in the region of minimal  $B$  on the magnetic axis, so that a closed surface



**FIGURE 4.** Contours of the second adiabatic invariant in polar representation  $s, \theta_B$  for a set of  $B_{reflect}$ , starting from nearly minimal value of  $B_{reflect}$  (top left) and finishing at almost maximal value of  $B_{reflect}$  (bottom right).

is seen. This minimum is created by the diamagnetic effect. Such a structure of surfaces  $B = \text{const}$  leads to creation of a maximum of the second adiabatic invariant near the magnetic axis (see Fig. 4), in contrast to a vacuum-field case [1]. The high value of  $\beta$  makes this maximum pronounced. As the contours of the second adiabatic invariant are closely linked with the trajectories of the banana centers of the reflected particles and as these contours are well closed inside the plasma volume, particles that satisfy the condition of adiabaticity should be confined for a long time in the collisionless regime. The direct calculation of drift motion of 1000  $\alpha$ -particles started at  $\frac{1}{2}$  of the minor plasma radius verifies this. For power plant parameters all reflected particles are confined during 1 sec. Fig. 5 shows the radial profile of effective ripple, as defined in Ref. [4]. It is worth mentioning that the diffusion coefficient is proportional to  $\varepsilon^{3/2}$  and inversely proportional to the square of aspect ratio for fixed small plasma radius. Thus, in the configuration found the diffusion coefficient for fixed small plasma radius is similar to W7-X. The bootstrap current in the configuration considered here is small and changes its sign along the flux coordinate. Mercier and resistive-interchange modes are stable except at a few medium or higher-order resonant surfaces with weakly unstable behavior. In addition, the condition of stability of stellarator-symmetric local ballooning

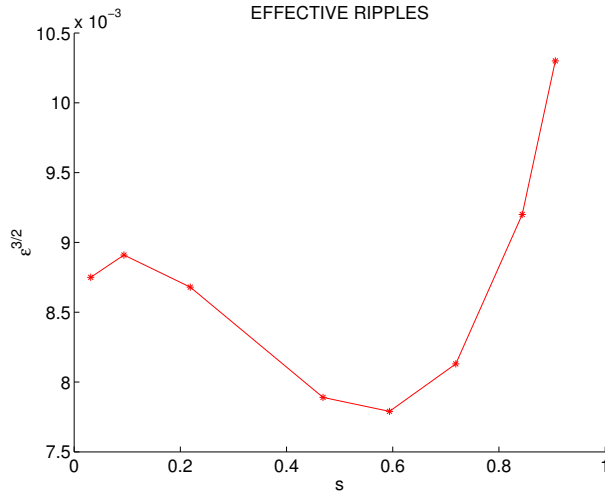


FIGURE 5. The radial profile of effective ripple for  $N = 12$  configuration.

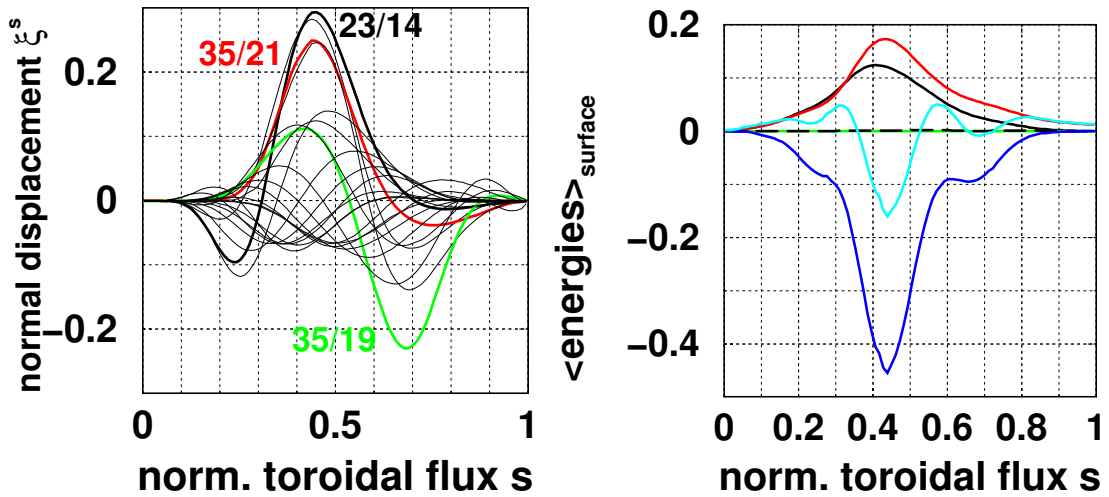
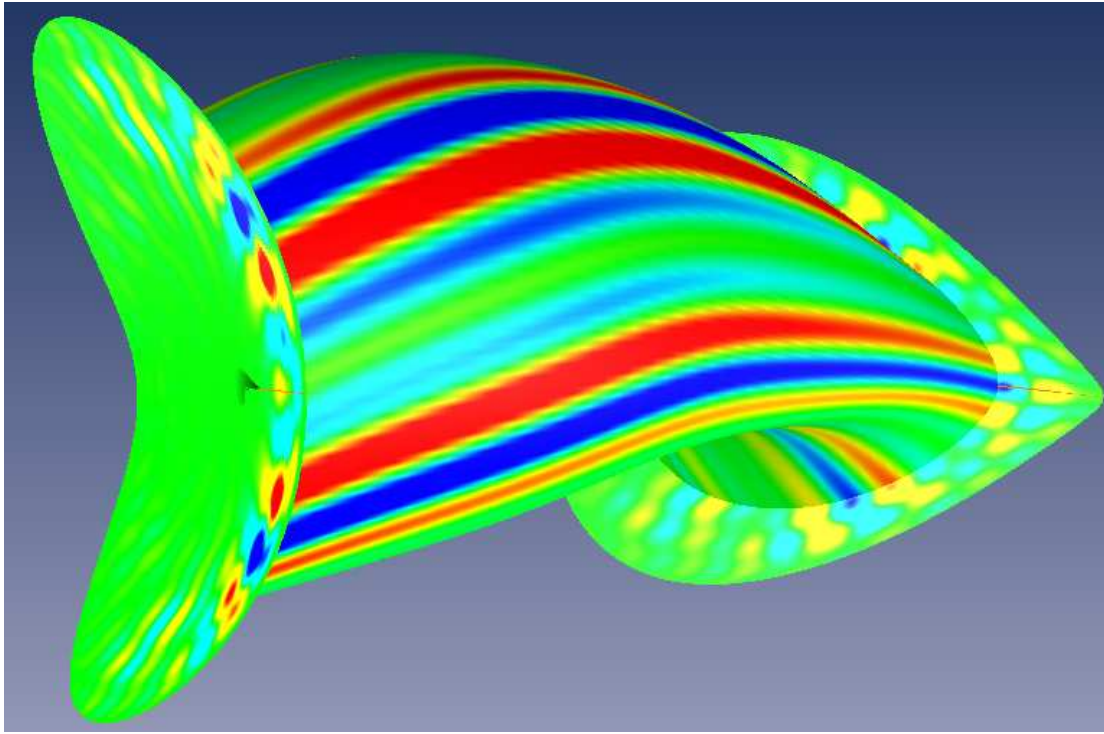


FIGURE 6. CAS3D normal displacement harmonics (left) and surface-averaged energy contributions (right) at  $\langle\beta\rangle = 0.22$ . Radial integration of the surface-averages of  $\delta^2W$  (curve oscillating around zero) shows it to be slightly positive.

modes was included into the optimization procedure. They are found to be stable for intervals in the covering space of flux surfaces of about ten periods. The global-mode fixed-boundary stability analysis was performed by the CAS-3D code [8]. It was found that  $\langle\beta\rangle=0.2$  is an approximately marginal point (see Fig. 6). A 3d view of the plasma perturbation (shown in Fig. 7) reveals its ballooning nature with strongest amplitudes on the outside of the torus.



**FIGURE 7.** Perturbation of Fig. 6: view of half a field period with normal displacement amplitude (dark for strong). An interior magnetic surface is shown located at  $s \approx 0.44$ , together with two meridional cuts: the triangular cross-section in the background and the bean-shaped cross-section in the foreground.

## CONCLUSIONS

It is shown by computational optimization that in quasi-isodynamic stellarators the considered conditions of improvement of plasma confinement are not in strict contradiction with each other and for  $N=12$  and  $A=36$  it is possible to fulfill all these conditions at high  $\beta$ ,  $\langle \beta \rangle = 20\%$  ( $\beta_0 = 50\%$ ).

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