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BETA LIMIT OF A COMPLETELY BOOTSTRAPPED TOKAMAK

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Beta Limit of a Completely Bootstrapped Tokamak

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A beta limit is given for a completely bootstrapped tokamak. The beta limit is sensitive to the achievable Troyon factor and depends directly upon the strength of the tokamak bootstrap effect.

Key Words: tokamak, bootstrap current, Troyon limit

Recent theoretical work[1,2] suggests that it may be possible to drive the tokamak current completely via the bootstrap effect.[3,4] The motivation behind this idea is the goal of obtaining a steady-state tokamak reactor, without relying on an inductive loop voltage or external current drive. That is, since the amount of volt-seconds provided by the loop voltage is always finite, inductive tokamaks can only operate in a pulsed mode. There are also severe limitations[5] on external current drive, due to the large recirculating power involved. The most efficient current drive schemes, which employ lower hybrid waves,[6] also have the problem that the waves cannot penetrate to the magnetic axis.[7] The completely bootstrapped tokamak avoids all these difficulties, but there are naturally other issues to consider. For example, a steady-state, completely bootstrapped tokamak relies on the tokamak plasma being sufficiently resilient to tearing modes, so that the finite level of magnetohydrodynamic (MHD) turbulence required to completely bootstrap the tokamak does not cause the plasma to disrupt. The finite level of turbulence is required, because there would otherwise be a violation of Cowling's theorem.[8] That is, the bootstrap effect, considered by itself, cannot give a true tokamak steady-state, since the effect does not create poloidal magnetic flux.[3,4] Instead, bootstrapped tokamaks push poloidal magnetic flux out of regions of high plasma pressure, thus hollowing the tokamak current profile. The tearing modes associated with the hollow bootstrap current profile, however, might actually be beneficial, since they generate additional poloidal magnetic flux. The combination of the bootstrap effect and tearing modes might therefore allow for true tokamak steady-state operation.[1] This was the issue discussed in a previous work.[2] In this Comment, we address another issue, that of the stability of completely bootstrapped tokamaks against pressure driven modes. Our result is a beta limit for completely bootstrapped tokamaks operating in the first region of pressure stability. The beta limit we determine is sensitive to the achievable Troyon factor and depends directly upon the strength of the tokamak bootstrap effect.

We begin with a set of definitions for the toroidal and poloidal betas,[9]

$$\beta_t \equiv 2\mu_0 \langle p \rangle / B_0{}^2 \qquad (1)$$

and

$$\beta_p \equiv 2\mu_0\langle p\rangle/B_p{}^2. \tag{2}$$

In definitions (1) and (2), $\langle p\rangle$ is the volume averaged plasma pressure,

$$\langle p\rangle \equiv (1/V_0) \int_0^{V_0} p(V)\, dV, \tag{3}$$

B_0 is the applied toroidal magnetic field, and B_p is the poloidal field,

$$B_p \equiv \mu_0 I_0/(2\pi a\kappa). \tag{4}$$

In relation (4), I_0 is the total toroidal current, a is the plasma minor radius, and κ is the plasma elongation,

$$\kappa \equiv A/(\pi a^2), \tag{5}$$

with A the area of the plasma cross section. We also need the definition of the kink safety factor,

$$q_* \equiv aB_0/(R_0B_p), \tag{6}$$

where R_0 is the plasma major radius.

Let us consider a tokamak operating in the usual first region of pressure stability. Then a fundamental limit imposed by pressure stability against ballooning-kink modes is the Troyon limit,[10]

$$\beta_{tmax} = g\,(I_0/aB_0). \tag{7}$$

In equation (7), the current I_0 is expressed in megamps and β_{tmax} is given in per cent. The number g is known as the Troyon factor. Alternative forms for the Troyon limit can be obtained using the definitions (1) - (6). That is, alternative forms for equation (7) are

$$\beta_{tmax} = \gamma_{tr}\,(a/R_0)\,(\kappa/q_*) \tag{8}$$

and

$$\beta_{pmax} = \gamma_{tr} (R_0/a) (\kappa q_*), \tag{9}$$

where we designate

$$\gamma_{tr} = g/20 \tag{10}$$

as the Troyon coefficient. For example, a Troyon factor $g = 3.5$ corresponds to a Troyon coefficient $\gamma_{tr} = 0.175$.

We now consider the tokamak bootstrap effect. Driving the tokamak current completely via the bootstrap effect requires the attainment of a certain poloidal beta. For a circular plasma cross section with elongation $\kappa=1$, the required poloidal beta is[3,4]

$$\langle\beta_{\theta bs}\rangle = (1/C_{bs}) (R_0/a)^{1/2}, \tag{11}$$

where C_{bs} is a number of order unity and we designate

$$\langle\beta_\theta\rangle = \beta_{pcir}, \tag{12}$$

which is just the poloidal beta (2) when the plasma cross section is circular. A tokamak with a strong bootstrap effect will have a large value for the coefficient C_{bs}. In a previous study[2] of completely bootstrapped tokamaks, which used the circular cylindrical approximation and peaked pressure profiles, values for C_{bs} were found in the neighborhood of $C_{bs} = 0.8$. If the plasma cross section is elongated, instead of being circular, then the relationship between β_p and $\langle\beta_\theta\rangle$ depends on the details of the plasma equilibrium, and usually requires numerical evaluation. For the general case, we can write

$$\beta_p = (\, \kappa^2 /\, f(\kappa)\,)\, \langle\beta_\theta\rangle, \tag{13}$$

where the function $f(\kappa)$ must obey $f(1) = 1$. Combining equations (11) and (13), the required poloidal beta for a completely bootstrapped tokamak with an elongated cross section is

$$\beta_{pbs} = (1/C_{bs})\, (\, \kappa^2/f(\kappa)\,)\, (R_0/a)^{1/2}. \tag{14}$$

An approximate form for the function $f(\kappa)$ can be obtained from the following argument. First, using the definitions (4) and (6), the total plasma current I_0 is

$$I_0 = (2\pi R_0/\mu_0 B_0)\ \kappa\ q_*\ B_p{}^2. \qquad (15)$$

The total bootstrap current I_{bs}, however, scales as[11]

$$I_{bs} \propto (2\pi R_0/B_0)\ (a/R_0)^{1/2}\ q\ \langle p \rangle, \qquad (16)$$

where q is the MHD safety factor, which is the inverse of the rotational transform. For a completely bootstrapped tokamak, we equate expressions (15) and (16), and find the required bootstrap poloidal beta

$$\beta_{pbs} = (1/C_{bs})\ \kappa\ (q_*/q)\ (R_0/a)^{1/2}. \qquad (17)$$

Of course, the elongation scaling between the kink safety factor q_* and MHD safety factor q depends on the details of the plasma equilibrium. For a flat q-profile, however, we have[12]

$$(q_*/q)_{app} = 2\kappa/(1+\kappa^2), \qquad (18)$$

which leads to the approximation

$$f_{app}(\kappa) = (1 + \kappa^2)/2. \tag{19}$$

For large κ, equations (13) and (19) indicate a gain of a factor of two in β_p over the circular value.

We now suppose that one is actually able to attain a disruption free, completely bootstrapped tokamak. The value of the poloidal beta β_{pbs}, however, cannot exceed the Troyon limit β_{pmax}. We must have

$$\beta_{pbs} < \beta_{pmax}. \tag{20}$$

In a completely bootstrapped tokamak, relations (9), (14), and (20) therefore limit the attainable kink safety factor to

$$q_* > (1/(C_{bs}\gamma_{tr})) (\kappa/f(\kappa)) (a/R_0)^{1/2}. \tag{21}$$

For the values C_{bs} = 0.8, γ_{tr} = 0.175, κ = 2.0, and (R_0/a) = 4.0, and using the function $f_{app}(\kappa)$ of equation (19), we have $q_* > 2.8$.

A completely bootstrapped tokamak is therefore a low current device. If one contemplates operating the tokamak at the minimum attainable q_*, then relation (21) and equation (8) give a toroidal beta limit,

$$\beta_{tbs} = C_{bs}\, \gamma_{tr}^2\, (a/R_0)^{1/2}\, f(\kappa). \qquad (22)$$

Relation (22) suggests what parameters are required for a tokamak reactor with its current driven completely via the bootstrap effect. First, the reactor beta is very sensitive to the value of the safely achievable Troyon coefficient γ_{tr}. A large plasma elongation κ is very favorable. Beta also displays a dependence on the tightness of the aspect ratio (R_0/a). Further, we see that the beta value of a completely bootstrapped tokamak is limited by the value of the bootstrap coefficient C_{bs}. For the values $C_{bs} = 0.8$, $\gamma_{tr} = 0.175$, $\kappa = 2.0$, and $(R_0/a) = 4.0$, and again using the function $f_{app}(\kappa)$ of equation (19), we have $\beta_{tbs} = 3\%$. This number is, as mentioned, very sensitive to the assumed value for the achievable Troyon coefficient. But beta might be increased, if it is possible to maintain a tokamak plasma with a larger elongation or tighter aspect ratio.

Here, however, let us consider issues related to the bootstrap coefficient C_{bs}. Unless one makes several crude approximations, such as those made in Ref. 2, it is obviously difficult to theoretically determine a value for the coefficient C_{bs}. That is, since C_{bs} depends upon the strength of the bootstrap effect, this coefficient depends in detail upon the profiles of the plasma density n(r), the electron temperature $T_e(r)$, and the ion temperature $T_i(r)$. Of course, the form for the bootstrap current density j_{bs} is[13]

$$j_{bs} = - \delta^{1/2} (nT_e/B_\theta) \big(2.44 (1 + T_i/T_e) (n'/n) + 0.69 (T_e'/T_e) - 0.42 (T_i'/T_e) \big), \quad (23)$$

where $\delta = r/R_0$, B_θ is the poloidal magnetic field, and the primes denote radial derivatives. This equation indicates that the bootstrap current is driven primarily by the density gradient. But the density profile n(r) may be flat in a tokamak reactor. A flat density profile might lead to a small value of C_{bs} and, consequently, to a small achievable beta. In this context, it is useful to examine the higher order corrections to equation (23). Using the neoclassical transport theory,[14] we obtain the expression

$$j_{bs} = -\delta^{1/2} (nT_e/B_\theta) \Big((2.44 - 1.44\,\delta^{1/2}) (1 + T_i/T_e) (n'/n) + (0.69 + 0.31\,\delta^{1/2}) (T_e'/T_e) + (-0.42 + 0.25\,\delta^{1/2}) (T_i'/T_e) \Big). \qquad (24)$$

Equation (24) indicates that, if the reactor density profile n(r) is flat, then the higher order terms multiplying the temperature gradients are somewhat helpful for the tokamak bootstrap effect.

In conclusion, relation (22) gives a limit for the plasma beta in a completely bootstrapped tokamak. The beta limit is sensitive to the achievable Troyon factor and depends directly on the strength of the tokamak bootstrap effect. The strength of the bootstrap effect, however, depends in detail upon the form of the plasma profiles under reactor conditions. After completion of this work, we became aware of the high poloidal beta bootstrap current experiments on the JT-60 tokamak.[15] These experiments indicate a collapse of the plasma beta in a regime where the tokamak bootstrap current fraction is near 80%. The experimenters suggest that hollow plasma current profile formation may be occurring before the beta collapse. Further, the experimentally measured value of the Troyon factor before the collapse was slightly above g = 1.3, substantially below the Troyon limit. In this context, we mention that the Troyon limit (7) is obtained by a plasma profile optimization which requires, among other things, a central safety factor q_0 near unity. Considering the quadratic dependence on the Troyon factor in the beta limit (22), it is clearly important to know what the maximum achievable Troyon factor will be for the plasma profiles associated with a completely bootstrapped tokamak. In any case, the MHD features of the collapse reported in Ref. 15 appear to be characteristic of pressure driven instabilities. From the considerations presented here, it would clearly be of interest to know what the forms of the various plasma profiles are preceeding and during a beta collapse. Obviously, additional long pulse experiments with elongated, thermonuclear plasmas would be very useful for resolving issues related to the tokamak bootstrap effect.

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Notes

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