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ABSTRACT

A finite beta effect on the tearing modes is studied by using the incompressible helical equations in a cylindrical geometry. In a linear analysis, the dependence of the growth rate on the resistivity is obtained. As the beta value increases, the dependence becomes weaker to approach the ideal MHD approximation. In the nonlinear analysis, the width of the saturated magnetic island is evaluated by using the harmonics of a single helicity.

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1. INTRODUCTION

The major disruption phenomena in a tokamak are characterized by the negative spike in the loop voltage with the rapid drop in the soft X-ray signal (the electron temperature) followed by the decay of the plasma current. This indicates the rearrangement of the magnetic field lines or of the plasma current density and the thermal interaction with limiters and a wall. Prior to the negative voltage spike, the fluctuation of the magnetic field usually with the $m = 2/n = 1$ component is observed. Therefore, tearing modes driven by the plasma current are considered to be the trigger of the disruption in a low beta plasma. Two different mechanisms have been proposed. One is due to the nonlinear coupling of the $m = 2/n = 1$ and the $m = 3/n = 2$ modes [1] and the other is due to the $m = 2/n = 1$ mode enhanced by the $m = 1/n = 1$ resistive mode and the radiation near the plasma boundary [2,3]. The linear growth rate has the dependency, $\gamma \sim \eta^{3/5}$, where η is the inverse magnetic Reynolds number [4]. In the nonlinear evolution, the reconnection of the magnetic field lines forms a magnetic island. The width of the magnetic island grows linearly in time to saturate [5].

As the electron temperatures increases, the inverse magnetic Reynolds number decreases and the linear growth rate is expected to be smaller. However, the pressure driven mode may appear for a finite poloidal beta. In this report, we study the finite beta effect on the tearing modes by using the incompressible helical equation in a cylindrical geometry. The dependence of the linear growth rate on η is investigated for the $m = 2/n = 1$ and $m = 3/n = 2$ modes. The nonlinear evolution of the modes is also shown for a finite beta.

2. BASIC EQUATIONS AND PROFILES

In this study, we assume the incompressibility,

$$\nabla \cdot v = 0 \tag{1}$$

instead of the perpendicular incompressibility,

$$\nabla \cdot v = 0 \quad \text{and} \quad v_{\parallel} = 0 \tag{2}$$

This approximation includes the finite beta effect through the parallel velocity, $v_\parallel$, and the perturbation of the toroidal magnetic field. In the ideal MHD limit, $\eta = 0$, the slow wave and the Alfvén spectra are included, and the fast wave is suppressed. The equations of motion and the inductive equations are described as follows:

$$\frac{\partial U}{\partial t} + \vec{v}\cdot\nabla\left(U + \frac{2k}{m\alpha^2}v_\zeta\right) - \frac{k^2}{m^2\alpha^2}\frac{\partial v_\zeta^2}{\partial\theta} = \vec{B}\cdot\nabla J - \frac{k^2}{m^2\alpha^2}\frac{\partial B_\zeta^2}{\partial\theta} \tag{3}$$

$$\frac{\partial v_\zeta}{\partial t} + \vec{v}\cdot\nabla v_\zeta = \vec{B}\cdot\nabla B_\zeta \tag{4}$$

$$\frac{\partial\psi}{\partial t} = \vec{B}\cdot\nabla\phi + \alpha\eta J + E_w(t) \tag{5}$$

$$\frac{\partial B_\zeta}{\partial t} = -\alpha\vec{v}\cdot\nabla\left(\frac{B_\zeta}{\alpha}\right) + \alpha\vec{B}\cdot\nabla\left(\frac{v_\zeta}{\alpha}\right) - \frac{2k}{m\alpha}\vec{B}\cdot\nabla\phi + \alpha\eta R \tag{6}$$

where

$$\zeta = m\theta - kz \tag{7}$$

$$\vec{v} = \vec{e}_\zeta\times\nabla\phi + \vec{e}_\zeta v_\zeta \tag{8}$$

$$\vec{B} = \vec{e}_\zeta\times\nabla\psi + \vec{e}_\zeta B_\zeta \tag{9}$$

$$\alpha = 1 + (kr/m)^2 \tag{10}$$

$$J = \Delta^*\psi + \frac{2k}{m\alpha^2}B_\zeta \tag{11}$$

$$U = \Delta^* \Phi \tag{12}$$

and

$$R = \Delta^* B_\zeta - \frac{2k}{ma} J \tag{13}$$

The operator, Δ^*, is the Laplacian in the helical coordinate (r, Θ, ζ). The boundary conditions for ϕ and Ψ are

$$\phi(r = 1) = 0 \tag{14}$$

and

$$(\partial\Psi/\partial r)\ (r = 1) = \text{const.}\ (0 \leq r \leq 1). \tag{15}$$

The profile of the safety factor and the plasma pressure are chosen as

$$q = q_0(1 + (r/0.6)^8)^{1/4} \tag{16}$$

and

$$p = p_0(1 - r^2) \quad (0 \leq r \leq 1). \tag{17}$$

3. NUMERICAL RESULTS

Assuming that $\gamma \sim \eta^\alpha$, we study the dependence of α on β_p, s and ε, where

$$\beta_p = 4\int_0^1 r\,dr\,p / B_\theta^2(1) \tag{18}$$

$$s = r_s q'/q, \tag{19}$$

$$\varepsilon = k/n \tag{20}$$

and r_s is the radius of the rational surface. Figures 1(a), (b) and (c) show the dependence of α on β_p, s and ε, respectively. For the profiles (16) and (17), the m = 3/n = 2 ideal MHD mode becomes unstable for $\beta_p > 1$ and $q_0 = 1.44$. The m = 2/n = 1 ideal MHD mode is stable up to $\beta_p \sim 3$ due to the strong shear near the q = 2 surface. When the ideal mode becomes unstable, the growth rate approaches the ideal one and the dependence on η vanishes.

From these figures, we have

$$\alpha = 3/5 - C\,\varepsilon\,\beta_p/s \quad \text{for } \gamma \gg \gamma_I, \qquad (21)$$

where C takes 1.25 in this equilibrium and γ_I denote the growth rate of the ideal MHD mode. Figures 2(a) and (b) show the perturbation of Ψ in the linear analysis for the m = 2/n = 1 and the m = 3/n = 2 modes. The solid line and the dashed line denote the cases of $\beta_p = 0$ and $\beta_p = 3$, respectively. The ideal MHD mode is stable for the m = 2/n = 1 mode. As the growth rate approaches the ideal one, $\Psi(r_s)$ decreases, i.e. the mode structure becomes close to the ideal one in the linearly growing phase. However, $\Psi(r_s)$ is still finite and the reconnection occurs in the nonlinear evolution.

The nonlinear evolution of a magnetic island is shown in Figs. 3(a) and (b) for the m = 2/n = 1 and the m = 3/n = 2 modes, respectively. We use $\eta = 10^{-4}$, $N_r = 51$ and N = 10 for the m = 2/n = 1 mode and $\eta = 10^{-5}$, $N_r = 101$ and N = 10 for the m = 3/n = 2 mode, where N_r and N are the radial mesh points and the number of the harmonics. The time is normalized by the poloidal Alfvén time in a plasma radius. As the m = 2/n = 1 ideal MHD mode is stable, the width of the saturated magnetic island coinsides with that obtained by the Δ' - analysis [5].

When the ideal mode becomes unstable, the width of the saturated magnetic island increases as β_p for $\beta_p < 2$ and it decreases for $\beta_p > 2$. For $\beta_p > 2$, $\Psi(r_s)$ becomes small and it may reduce the island width. Figure 4 shows the time evolution of the nonlinear growth rate defined by $\gamma_K = (dE_K/dt)/E_K$, where E_K is the kinetic energy. For smaller β_p ($\beta_p < 1$), there appears a linearly growing phase called the Rutherford regime. However, the unstable mode grows exponentially in time and staturates.

4. CONCLUSIONS

The scaling of the growth rate with η is given by $\gamma \sim \eta^{\alpha}$ and $\alpha = 0.6 - C\,\varepsilon\,\beta_p/s$ (C = 1.25) when the ideal MHD mode is stable and γ tends to that of the ideal MHD instability. In the nonlinear evolution, the island width and the kinetic energy increase by the ideal MHD growth rate when γ becomes close to γ_I. This indicates that the time scale of the tearing mode becomes insensitive to the magnetic Reynolds numbers and the characteristic time is the order of the poloidal Alfvén frequency.

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FIGURE CAPTIONS

Fig. 1: Dependence of α on

(a) β_p for $\varepsilon = 1/3$ and $q_o = 1.44$,

(b) s at the rational surface for $\beta_p = 0.5$, $\varepsilon = 1/3$, and

(c) ε for $\beta_p = 0.5$ and $q_o = 1.44$.

Fig. 2: Perturbation of Ψ

(a) for the m = 2/n = 1 mode and

(b) for the m = 3/n = 2 mode.

The solid line and the dashed line denote the cases of $\beta_p = 0$ and $\beta_p = 3$, respectively.

Fig. 3: Time evolution of the magnetic island width

(a) for the m = 2/n = 1 mode and

(b) the m = 3/n = 2 mode.

For $\beta_p > 1$, the ideal MHD mode becomes unstable.

Fig. 4: Time evolution of the growth rate defined by $\gamma_K = (dE_K/dt)/E_K$. For $\gamma \sim \gamma_I$, the magnetic island saturates soon after the exponential grow.

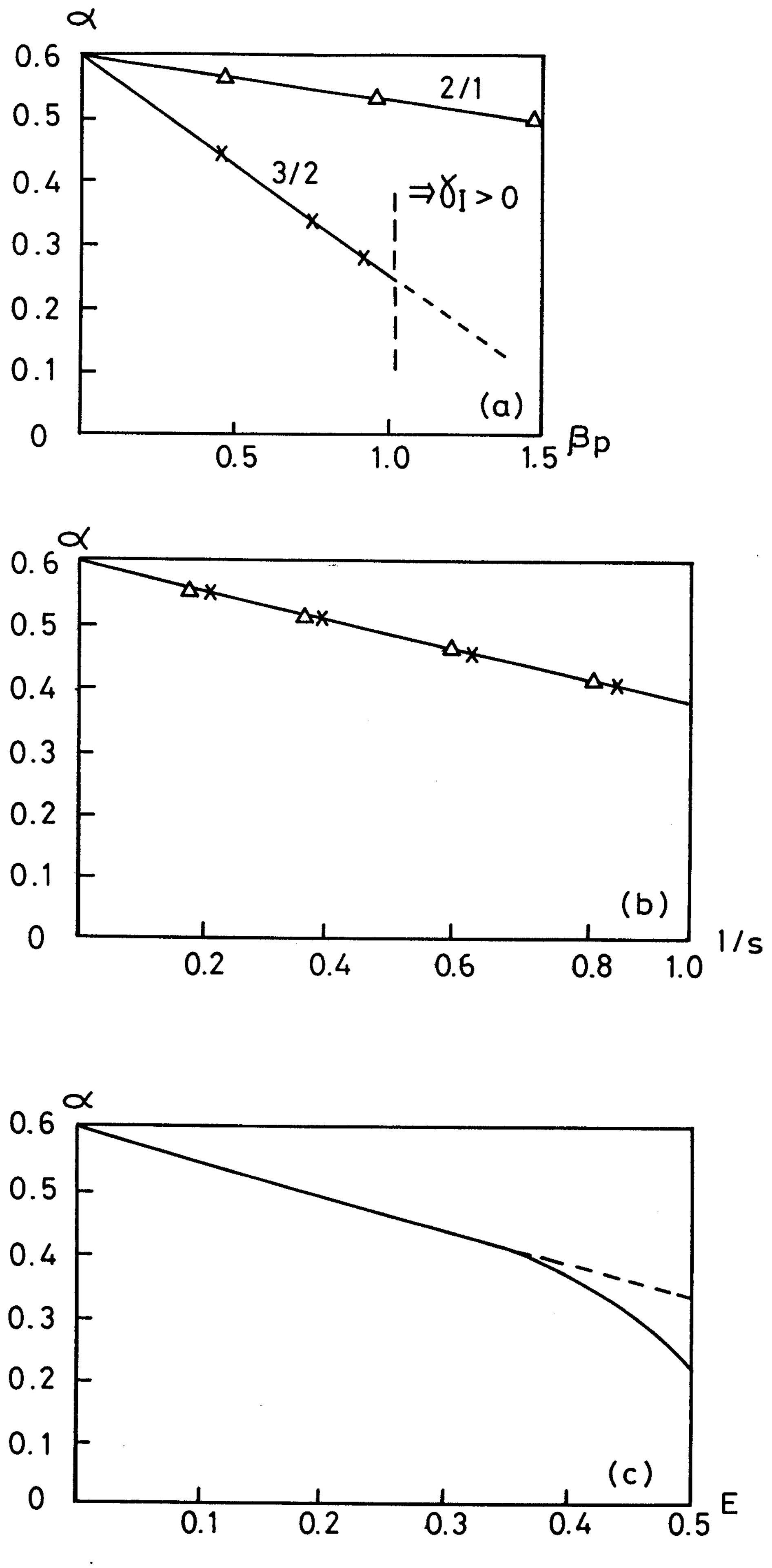

α
0.6
0.5
0.4
0.3
0.2
0.1
0
2/1
3/2
⇒ $\gamma_I > 0$
(a)
0.5
1.0
1.5
β_p
α
(b)
0.2
0.4
0.6
0.8
1.0
1/s
α
(c)
0.1
0.2
0.3
0.4
0.5
E

FIG. 1

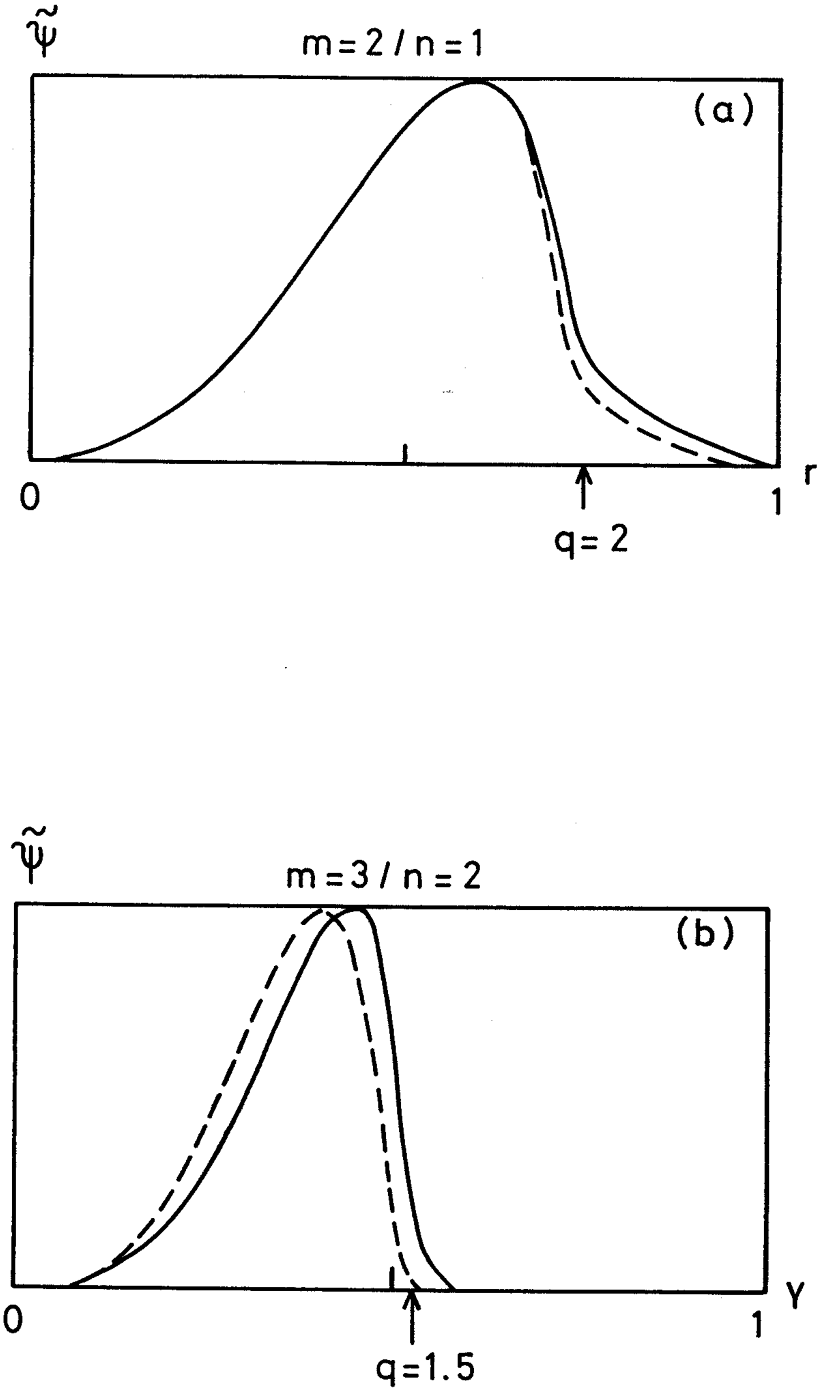


FIG. 2

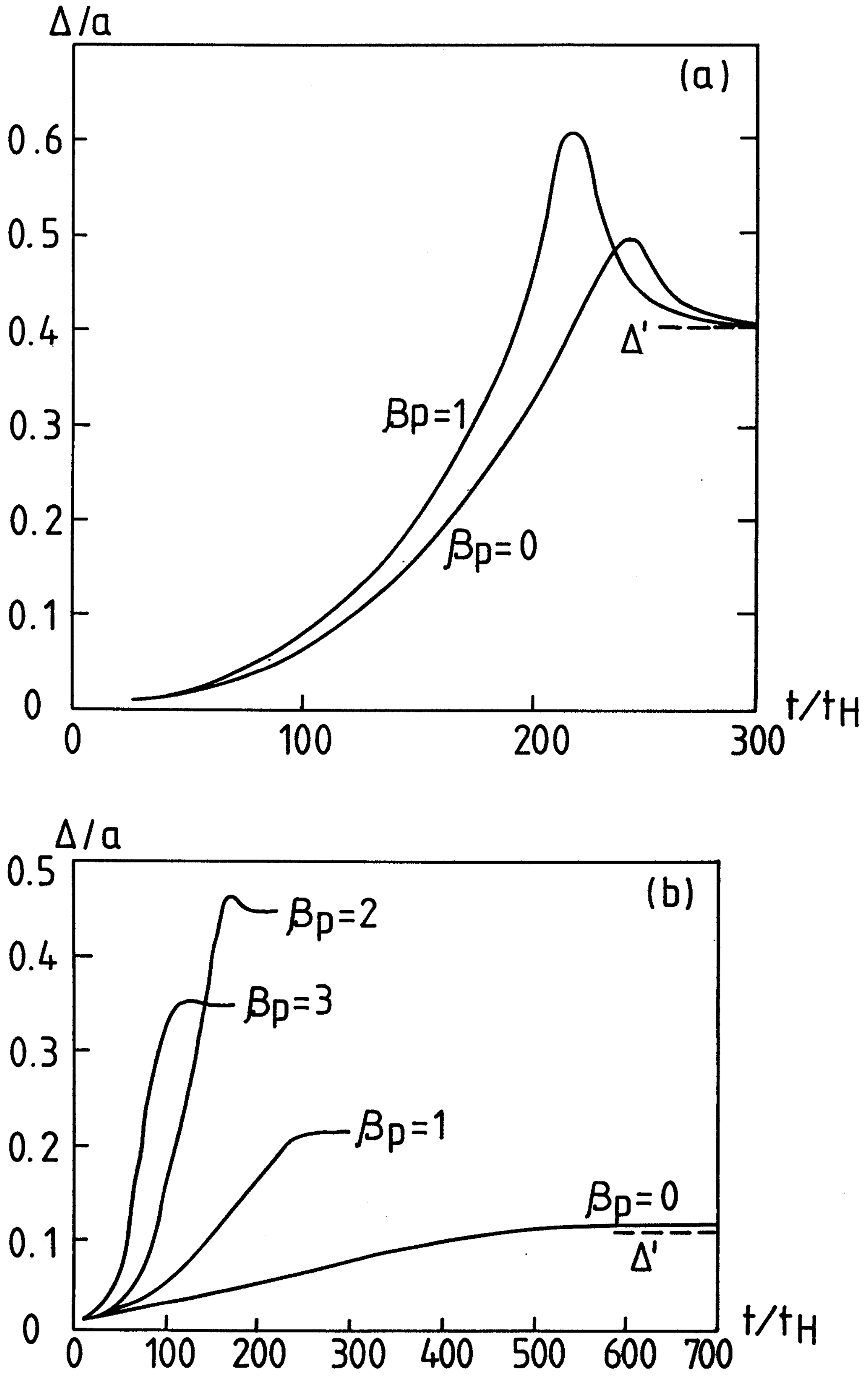
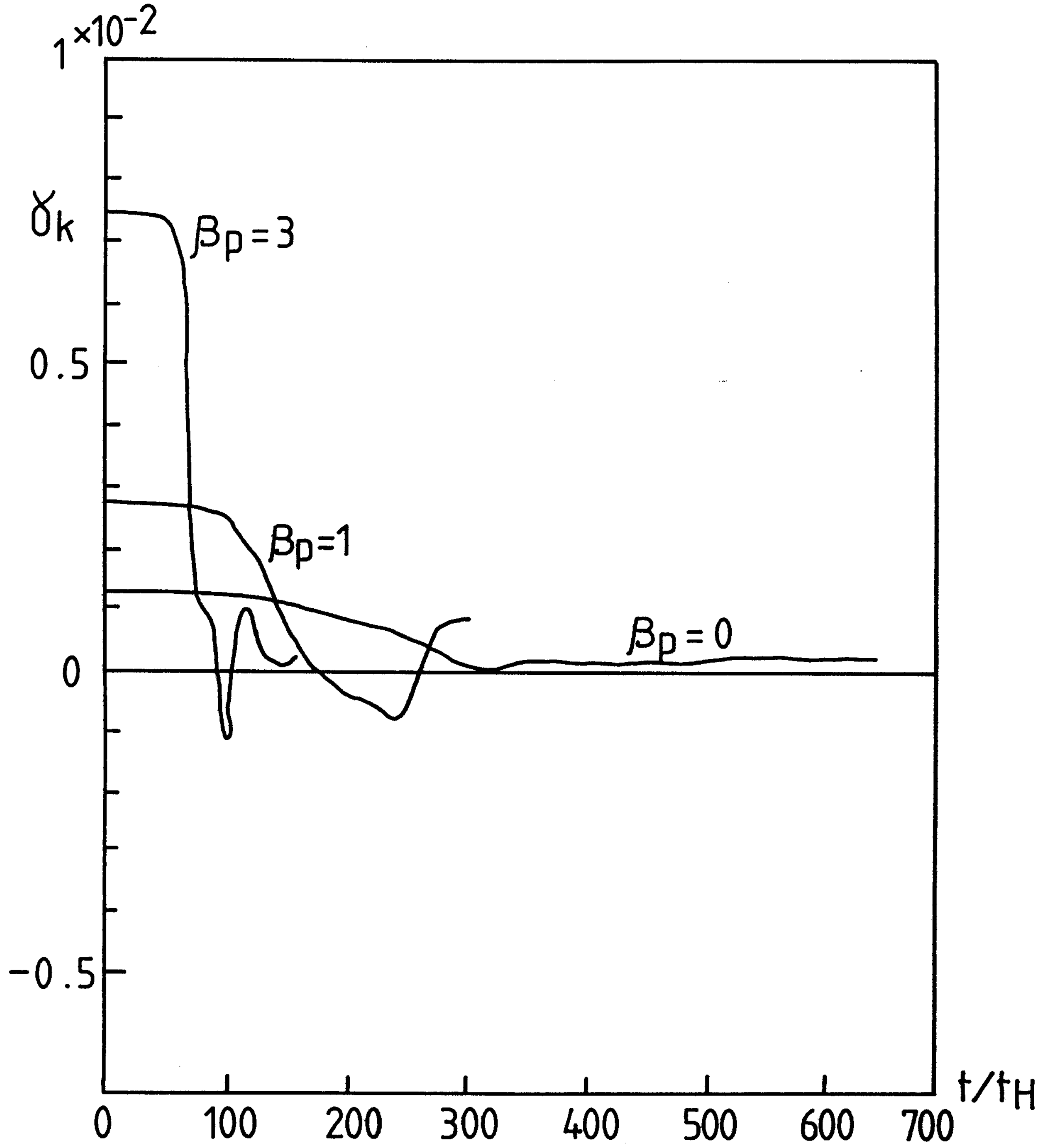

Δ/a
(a)
0.6
0.5
0.4
0.3
0.2
0.1
0
Δ'
βp=1
βp=0
0
100
200
300
t/tH
Δ/a
0.5
(b)
βp=2
0.4
βp=3
0.3
0.2
βp=1
βp=0
0.1
Δ'
0
0
100
200
300
400
500
600
700
t/tH

FIG. 3



1 ×10^-2
γ_k
$\beta_p = 3$
$\beta_p = 1$
$\beta_p = 0$
0.5
0
−0.5
0
100
200
300
400
500
600
700
t/t_H

FIG. 4