LRP 338/87 December 1987

Singular Domains of the Low-Frequency Cold-Plasma Wave Equation

K. Appert, T. Hellsten, J. Vaclavik, and L. Villard

Singular Domains of the Low-Frequency Cold-Plasma Wave Equation

K. Appert, T. Hellsten*, J. Vaclavik, and L. Villard

Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne
21, Av. des Bains, CH-1007 Lausanne

*JET Joint Undertaking, Abingdon, Great Britain

Abstract - The system of two partial differential equations describing, on the basis of a cold plasma model, the propagation of low-frequency waves in an axisymmetric, current-carrying torus is shown to be singular in a two-dimensional domain of the independent variables. Regularized by means of a causal damping the singular domain becomes a resonantly absorbing domain.

A few years ago it has been suggested by HELLSTEN and TENNFORS (1984) that the partial differential equations describing the propagation of ICRF (ion cyclotron range of frequency) waves in cold, current-carrying, axisymmetric plasma possess singularities which coincide with magnetic surfaces. To determine whether a given magnetic surface is a singular surface one has to look for periodic solutions of a specific ordinary differential equation in a poloidal coordinate (HELLSTEN and TENNFORS, 1984). In this equation the variable labelling the magnetic surface appears as a parameter. If the equation possesses a non-trivial solution the corresponding magnetic surface is a singular surface of the system of partial differential equations. The singularities can physically be interpreted as the loci where resonance absorption (BUDDEN, 1986) takes place. They can also be interpreted as the places where the fast magnetosonic wave is mode converted into an ion-cyclotron wave. Tacitly it has been assumed (HELLSTEN and TENNFORS, 1984) that there is a finite number of discrete singular surfaces. A little later it was verified numerically (APPERT et al., 1986) that the singularities are indeed aligned with magnetic surfaces. Due to the fact that the physically relevant quantities like the total power absorption, the power deposition profiles and simply-structured wave fields converged in the numerical sense it was not remarked at that time that in some cases the number of resonance surfaces depends on the number of mesh points. With the numerical resolution presently available we are capable of showing that the singular magnetic surfaces which encounter an ion-cyclotron resonance $\omega = \omega_{ci}$ (wave frequency equals an ion-cyclotron frequency) form a two-dimensional continuum. The partial differential wave equation is therefore singular in a two-dimensional domain.

The cold-plasma wave equation and the method used to solve it are described in detail by VILLARD et al. (1986). In the present context we stress, however, the fact that a finite element discretization is used in both spatial directions $s = \sqrt{\psi}$ (ψ: flux coordinate) and χ (a measure for the poloidal angle). The most striking results are obtained for the so-called mode conversion scenario in a deuterium/hydrogen mixture in a small tokamak, Fig. 1. The physical parameters used for this figure are typical of the TCA tokamak (DE CHAMBRIER et al., 1982) : major radius R_0=0.61m, minor radius a=0.18m, B_0=1.5T, $n_D=10^{19}m^{-3}$, n_H/n_D=10%, frequency f=22MHz, toroidal wavenumber n=4, high-field-side antenna. The causality is ensured by adding an imaginary part δ to the dielectric tensor as described by SAUTER et al. (1987). With the introduction of a small but finite δ the equation is, strictly speaking, no longer singular but it can now be analyzed numerically. The power absorption density in Fig. 1a has been obtained on a discrete mesh with N_s=160 radial and N_χ=80 poloidal mesh points. One remarks that the field possesses high amplitude regions which are aligned with specific magnetic surfaces. If one of these high amplitude regions is picked out one finds that the ψ-component of the electric wave-field behaves radially like $1/(s - s_r)$, a clear indication for a singularity at s_r.

If now the number of poloidal mesh points is doubled, Fig. 1b, the number of resonance surfaces doubles. The same result can be obtained by using N_χ=80 poloidal mesh points only, but redistributing them in such a way that the poloidal mesh density doubles in the neighborhood of the ion-cyclotron resonance. From this fact one concludes that in the torus the ion-cyclotron resonance is a singular line of the cold-plasma wave equation. This singularity is, however, much weaker than the radial one.

An analysis of the wave equation along a specific magnetic surface yields a singularity of the type $(\chi - \chi_{ci}) \log|\chi - \chi_{ci}|$ (HELLSTEN and TENNFORS, 1984), where χ_{ci} is the angle at which the surface encounters ω_{ci}. In fact, it is this singularity which allows the differential equation defining a singular surface to possess non-trivial periodic solutions for all those magnetic surfaces which intersect the line $\omega=\omega_{ci}$.

It is tempting to interpret the formation of resonance surfaces as being due to a lack of resolution in the poloidal direction which, in principle, could lead to a reflection of the ion-cyclotron wave. We can show, however, that the resonance surfaces are not a numerical artefact, but their finite number is. First of all, it has been shown (VILLARD et al. 1986) that the numerical method used converges in a multitude of simple cases and for all global quantities. In the case of Fig. 1, the total power varies only by 10% when the number of mesh points is varied by a factor of 4. The radial power deposition profile varies only by 4%. In Fig. 2 we show the energy flux through the magnetic surfaces as a function of s, corresponding to Fig. 1a and 1b, respectively. Another argument in favour of the numerics is the fact that the solution has all the desirable physical properties of a resonantly absorbing system : the power absorbed is independent of the damping mechanism. For δ varied by a factor of 3, or with the introduction of either collisional damping $\nu/\omega=5\cdot10^{-3}$, or ion cyclotron damping with T_i=100 to 900eV, the total power and the power deposition profile remain the same (within the limits of numerical resolution stated above). We must therefore associate the present apparent lack of convergence in the wave field with some mathematical properties of the partial differential equation at hand. Secondly, the number of resonance surfaces is also proportional to the number of radial mesh points. Thirdly, we have analyzed the numerical wave field along the resonance magnetic surfaces and found that, in general, the real and imaginary parts of E_ψ have roughly the same amplitude with a phase of about $\pi/2$ between them. This indicates that the ion-cyclotron wave field does not contain a substantial part of standing wave; hence the resonance surfaces observed in Fig. 1 are not due to numerical reflection.

Moreover, we can show that the wave field inside the magnetic surfaces not crossing $\omega=\omega_{ci}$ is converged. This is demonstrated on Fig. 3 where the same geometrical parameters as in Fig. 1 were used, but here the frequency is f=10.5MHz and the plasma pure deuterium. The other parameters are $n_D=5\cdot10^{18}m^{-3}$, $B_0=1.5T$, toroidal wavenumber n=-8 and a helical antenna. Fig. 3a was obtained with $N_s=160$ radial and $N_\chi=80$ poloidal mesh points, Fig. 3b with the double number of poloidal mesh points. We clearly see that the two innermost resonance surfaces do not intersect ω_{ci} and are independent of the mesh. The third one in Fig. 3b is just tangent to ω_{ci}. The number of outer ones is proportional to the number of mesh points, as it was the case in Fig. 1. A careful examination shows that all detailed features of the wave field inside the surfaces not intersecting ω_{ci} are independent of the mesh. The wave field outside those surfaces strongly depends on the mesh as well as on the dampings. With increasing damping the fine structure ceases to exhibit surface aligned islands but again the total absorbed power and the energy flux versus s (analogous to Fig. 2) are roughly independent of the damping. This is, once again, what we expect for resonance absorption. In contrast to Fig. 1 this time the discrete numerical solution, Fig. 3, of our continuous mathematical problem exhibits a certain amount of standing wave.

In conclusion, we have discussed, by numerical means, the properties of a partial differential equation. We have shown that this equation is singular in a two-dimensional domain. As this equation should represent some physics where nothing ever is singular we have used the causality argument to regularize the solution and found that it describes resonance absorption.

Let us make an important final remark. It is impossible, on the basis of a cold-plasma wave equation, to separate the power absorbed by the ions at ω_{ci} from the power mode-converted into slow kinetic waves. An eventually *ad hoc* introduced ion-cyclotron damping acts as any other damping in this plasma model, i.e. it leads to resonance absorption. For reasonable plasma parameters, the ion-cyclotron damping is strong enough to hide the singular nature of the wave equation on surfaces which intersect the ion-cyclotron resonance.

Acknowledgements

We thank Drs. D. Batchelor, M. Brambilla, D. Faulconer, F. Jaeger, R. Koch, A. Samain, D. van Ester for animated discussions. This work was partly supported by the Fonds National Suisse de la Recherche Scientifique.

References

APPERT K., COLLINS G.A., HELLSTEN T., VACLAVIK J., and VILLARD L. (1986) Plasma Phys. and Contr. Fusion 28,133

BUDDEN K.G. (1986) "Radio waves in the ionosphere", Cambridge University Press

DE CHAMBRIER A. et al. (1982) Plasma Phys. 24, 893

HELLSTEN T. and TENNFORS E. (1984) Physica Scripta 30, 341

SAUTER O., APPERT K., VILLARD L. and VACLAVIK J. (1987) Comp. Phys. Commun.46, 205

VILLARD L., APPERT K., GRUBER R. and VACLAVIK J. (1986) Comput. Phys. Rep. 4, 95

Figure Captions

Fig. 1a Equilines of the power absorption density obtained with a 160x80 mesh for a D/H ICRF mode conversion scenario. The dashed line indicates the position where $\omega=\omega_{ci}$.

Fig. 1b Equilines of the power absorption density obtained with a 160x160 mesh.

Fig. 2 Energy flux through magnetic surfaces labelled by s.

Fig. 3a Equilines of the power absorption density obtained with a 160x80 mesh for an Alfven Wave Heating scenario.

Fig. 3b Equilines of the power absorption density obtained with a 160x160 mesh.

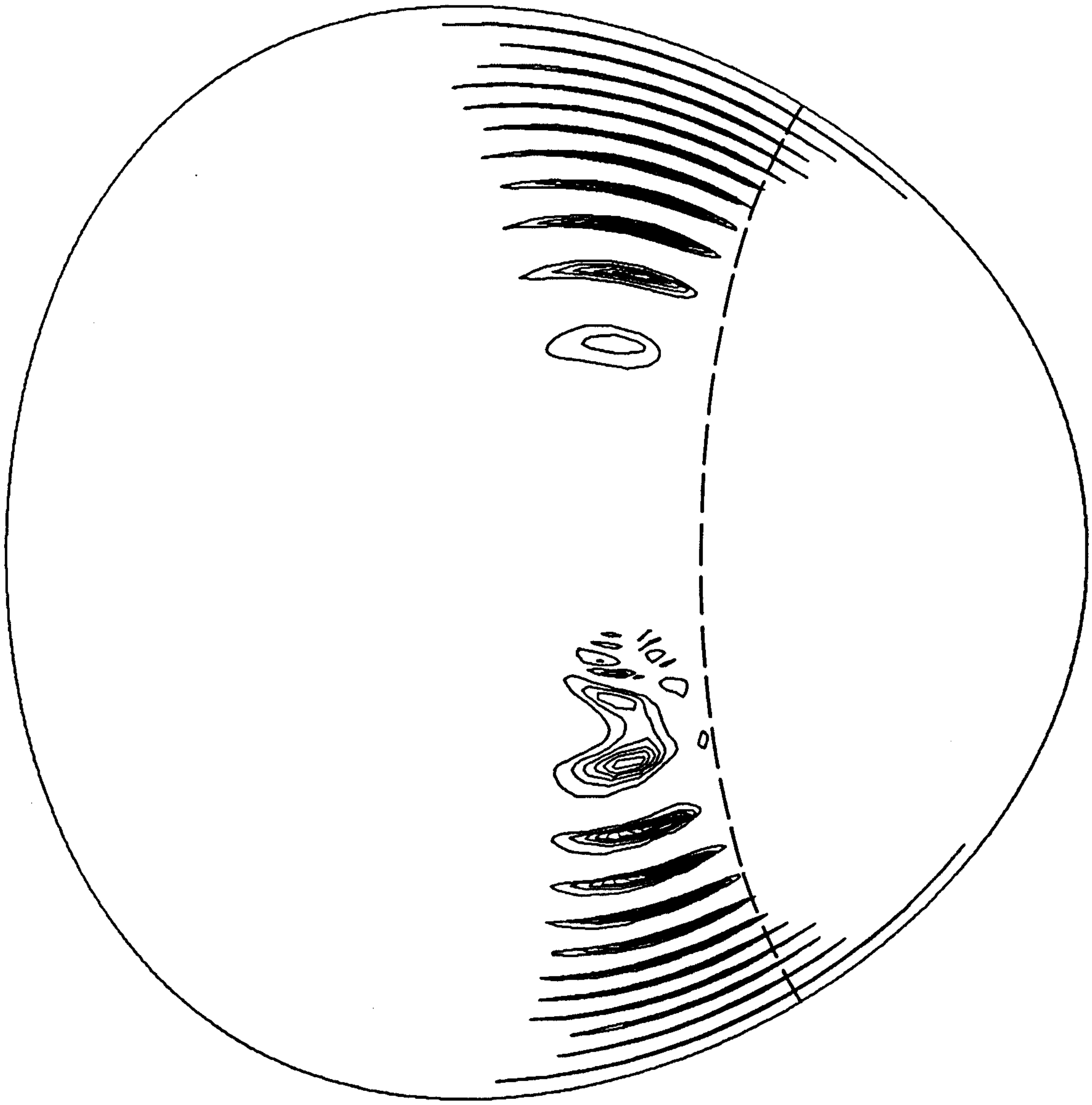

Fig. 1a

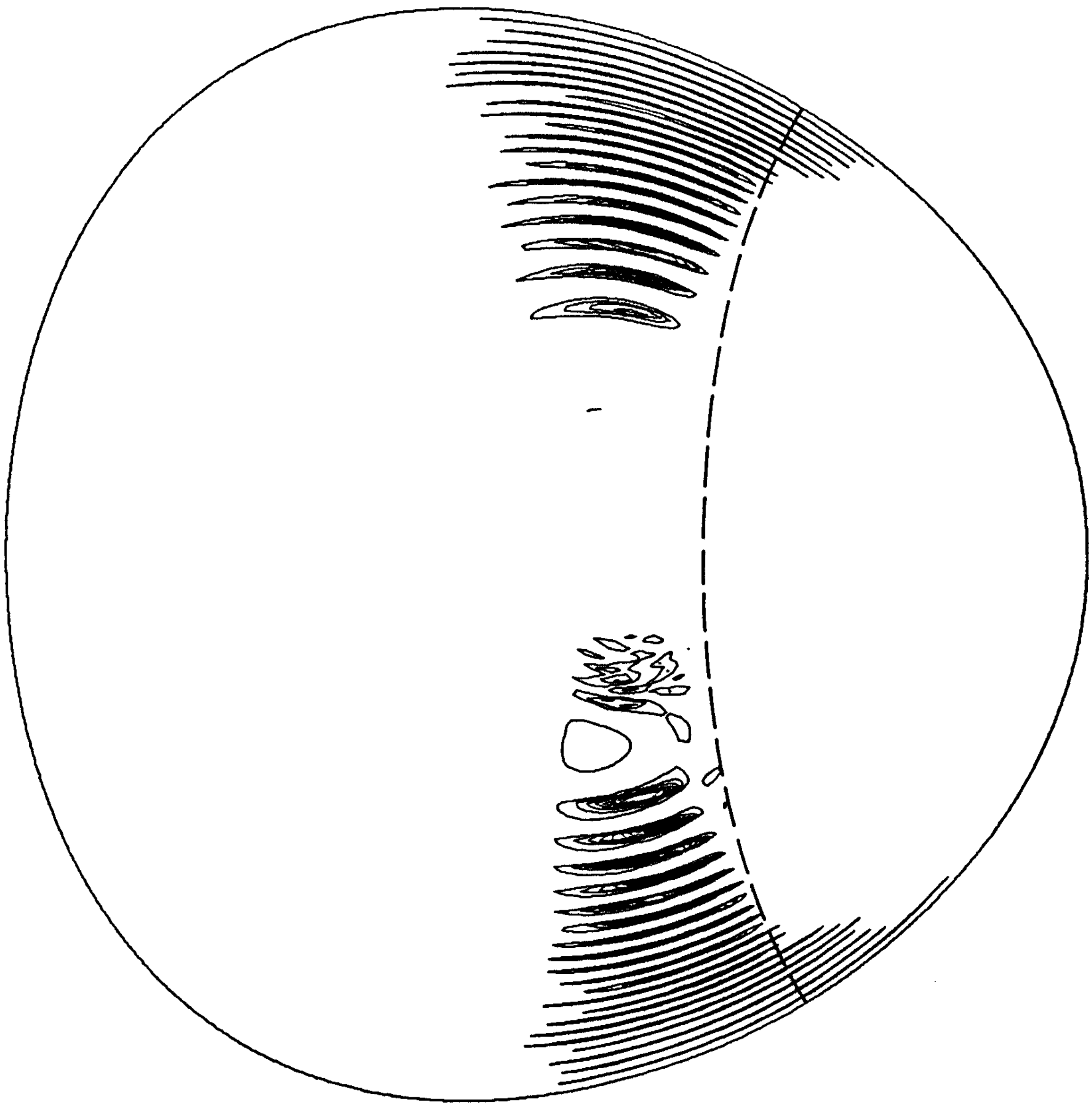

Fig. 1b

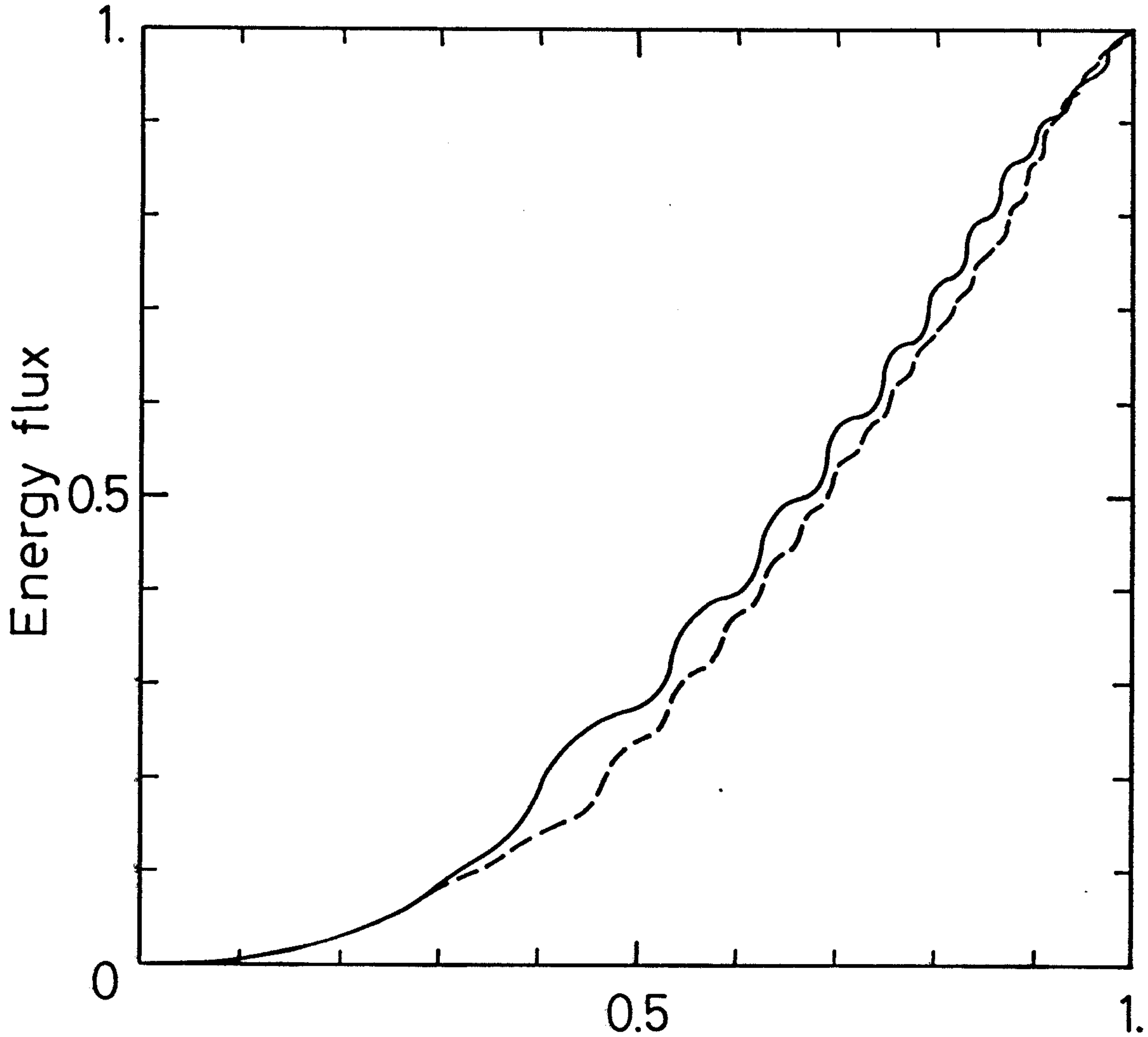

Fig. 2

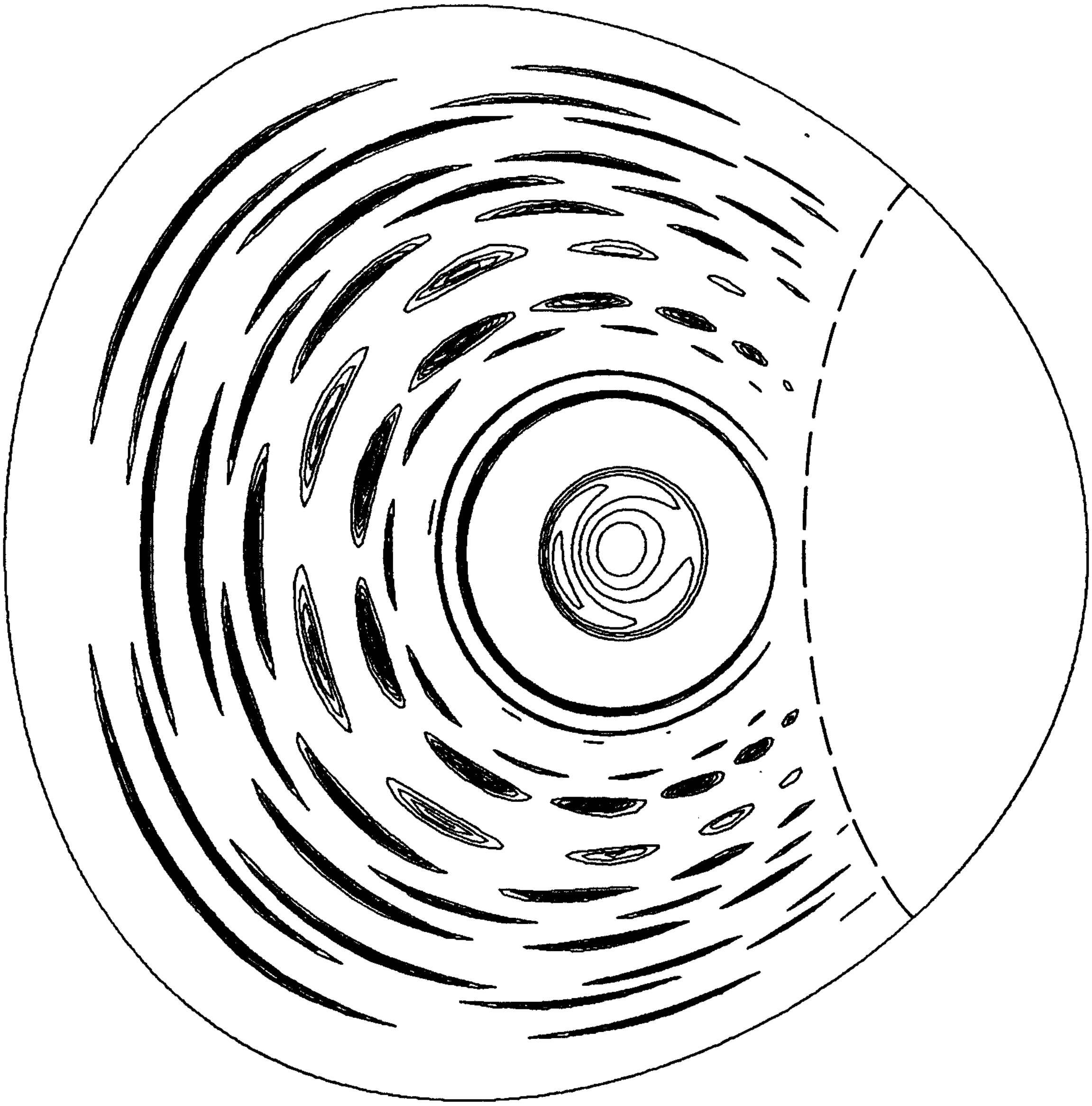

Fig. 3a

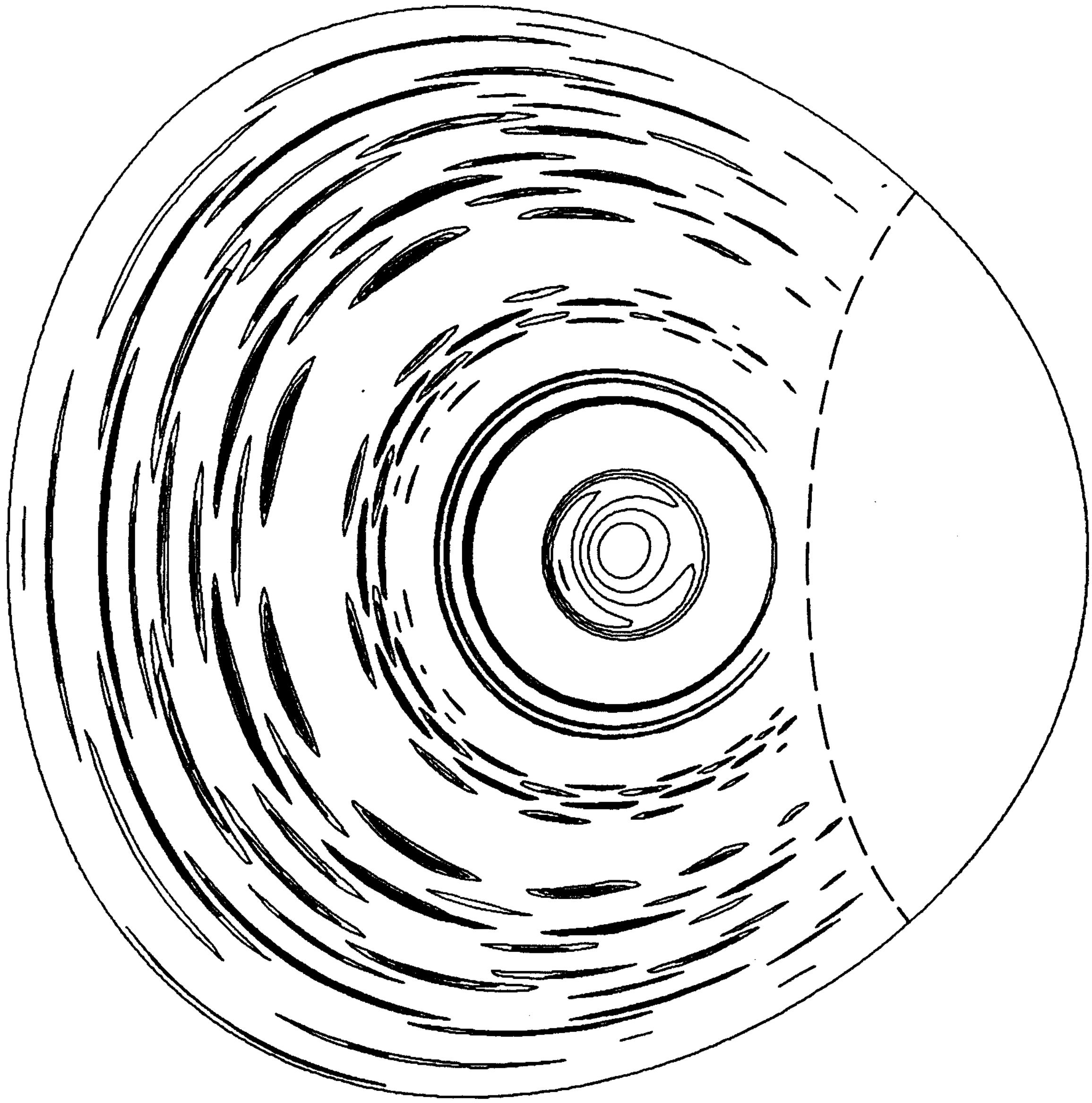

Fig. 3b