THE CORRELATION DIMENSION OF BROADBAND FLUCTUATIONS IN A TOKAMAK

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ABSTRACT

The correlation integral has been studied in detail for broadband magnetic and density fluctuations observed in the TCA tokamak [Proceedings of the 11th Symposium on Fusion Technology (CEC, Luxembourg, 1981), Vol.I, p.601]. These studies, which provide information on the correlation dimension ν, have shown no evidence for a low dimensional attractor. It has therefore been concluded that the broadband fluctuations are associated either with a high dimensional system ($\nu > 10$), or with a non-stationary state for which the concept of the correlation dimension has no meaning.

I INTRODUCTION

Tokamak plasmas exhibit a diverse variety of fluctuations. At low frequencies, coherent phenomena ($\Delta\omega \ll \omega$) are observed, such as sawteeth and Mirnov oscillations. These phenomena, which have been studied extensively, are associated with macroscopic plasma behavior. There also exists, extending to much higher frequencies, a broadband spectrum of fluctuations resulting from the turbulent nature of the plasma. These broadband fluctuations are of strong interest since they are believed to be the cause of the observed anomalous transport in tokamak plasmas. Although various models have been proposed to describe the broadband fluctuations,[1] their origin is still unclear.

Since a tokamak plasma is a system possessing a very large number of degrees of freedom, it is pertinent to ask whether it is possible to distinguish these broadband fluctuations from random noise. This is, in principle, possible by calculating from experimental data the correlation dimension ν using the algorithm proposed by Grassberger and Procaccia.[2] A finite value of ν implies that the system tends asymptotically with time (after the decay of transients) to some subset of the total phase space, called the attractor. A "strange" attractor[3] is associated with chaotic motion that, when Fourier analyzed, reveals generally a broadband spectrum. Dimensional analysis has been recently applied to many chaotic systems in widely varying fields of research. It readily provides a measure for chaotic systems that would be difficult to characterize by standard techniques. Since there exists no theory to predict the dimension of turbulence in a tokamak plasma, experimental determination is necessary. The application of dimensional analysis to tokamak data has been mainly concerned with low frequency coherent oscillations;[4-6] this has yielded, not surprisingly, low values of correlation dimension, $\nu \simeq 2-3$. In this paper, we study the dimension of broadband fluctuations [7-10] measured during ohmic discharges of the TCA tokamak.[11]

II THE CORRELATION INTEGRAL

The correlation dimension is defined, for a system yielding a time series $\{\vec{X}_i\}_{i=1,N}$ of points on the attractor, through the correlation integral

$$C(r) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i \neq j=1}^{N} H\left(r - \left|\vec{X}_i - \vec{X}_j\right|\right) , \qquad (1)$$

where H is the Heaviside function.[2] In practice, C(r) may be calculated from a time series of a single physical quantity, x(t), by reconstructing an m-dimensional vector using delay coordinates, $\vec{X}_i(t) = [x(t_i), x(t_i+p), \ldots, x(t_i+(m-1)p]$. For $m \geq \nu$, the correlation integral has the property that $C(r) \sim r^{\nu}$ for small r.[2,3]

The application of this algorithm requires appropriate choices for the total number of data points N, and the time delay p used in the construction of the m-dimensional vectors. In practice, the N data points are acquired at a sampling interval of Δt. Clearly, N and Δt must be chosen so as to yield sufficient resolution over the desired frequency range of the Fourier spectrum of x(t). In addition, N must be sufficiently large so that the limit in Eq. (1) is well approximated. This may, for the investigation of experimental data, involve some compromise since the parameters of the system should remain constant (to avoid transients) during the total time $\sim N\Delta t$ for which data are recorded. For proper reconstruction of the attractor, the time delay p must be chosen not too small (otherwise, $x(t_i) \simeq x(t_i+p) \simeq \ldots\ldots \simeq x(t_i+(m-1)p)$, leading to a strong distorsion in the reconstructed attractor) or not too large (otherwise, distant values in the same vector $\vec{X}_i$ are uncorrelated).[2] Denoting τ as the autocorrelation time for the time series x(t), a value of p should therefore be chosen such that $p \simeq \tau$. Finally, the acquired signal must be digitized with sufficient resolution so that $C(r) \sim r^{\nu}$ over a measurably large range of r.

Calculation of the correlation integral as defined in Eq. (1) may not lead to a reliable determination of the correlation dimension if the number of data points N is limited. For certain values of r, the value of C(r) may be over-estimated, especially for large values of embedding dimension m. The result is that there may not exist a sufficiently large range of r for which $C(r) \sim r^{\nu}$, or alternatively, a false value of the correlation dimension may be deduced. The origin of this phenomenon may be shown to be the over-weighting of correlations arising when vectors $\vec{X}_i$ and $\vec{X}_j$ are considered, even though the time difference $|t_i-t_j|$ is not large compared to the autocorrelation time. To avoid counting these pairs of vectors that are close in space only because their components are close in time, a modification of Eq. (1) has been proposed:[12]

$$C_k(r) = \lim_{N \to \infty} \frac{2}{N^2} \sum_{i=1}^{N-k} \sum_{j=i+k}^{N} H\left(r-|\vec{X}_i-\vec{X}_j|\right) . \qquad (2)$$

Note that setting k = 1 recovers the standard formulation of Eq. (1). In general, k should be chosen so that $k\Delta t > \tau$, the values of $C_k(r)$ saturating for k sufficiently large. This requirement is not overly restrictive, since it should still be possible to satisfy $k \ll N$. Thus for correctly sampled and treated time series data, the above modification to eliminate the "spurious" correlations is minor in terms of the total number of pairs of vectors considered. A more detailed treatment of the consequences of this modification may be found elsewhere.[10,12]

We have tested our implementation of the above-described algorithm by analyzing time series from mathematical systems with known attractors, and have obtained the already tabulated values for the correlation dimension. In addition, the algorithm has been applied to mathematical and experimental data from a simple model system.[7,10]

III APPLICATION TO TOKAMAK DATA

Three different diagnostic techniques are employed to measure fluctuations in the TCA tokamak:[8,13-15] (i) magnetic probes placed at various poloidal and toroidal locations in the shadow of the limiters, (ii) a triple Langmuir probe for scrape-off plasma measurements, and (iii) a CO_2 laser phase contrast diagnostic[16] that measures chord-averaged density fluctuations. All three diagnostics yield broadband fluctuation spectra in the kHz - MHz range of frequency. For discharges with significant mode activity, the magnetic fluctuation spectra are dominated at low frequencies by the coherent

Mirnov oscillation peak ($f_{Mirnov} \simeq 10$ kHz). Typical power spectra for the poloidal magnetic field (differentiated) $\dot{\tilde{b}}_\theta$ and ion saturation current $\tilde{i}_s$ in the plasma scrape-off layer, and the density fluctuations (filtered below 20 kHz) $\int \tilde{n}_e dl$ averaged along a central vertical chord, are shown in Fig. 1 (a) - (c). These spectra were obtained for the following tokamak operating parameters: $B_\phi = 15$ kG, $q(a) = 3.2$, $n_e = 3 \times 10^{13}$ cm^{-3}, and $T_e(0) = 800$ eV.

A detailed study of the modified correlation integral $C_k(r)$ has been undertaken by analyzing many different tokamak discharges. Since the three diagnostics described above measure fundamentally different physical quantities that are not simply related, dimensional analysis has been conducted using signals from each diagnostic. The signals were digitized into records of 8192 samples with 8 bit resolution, using a sampling frequency of 2 MHz. As the flat-top of TCA discharges lasts for up to 150 ms, the fluctuation data could be acquired over a time much shorter than the characteristic time for changes in the plasma equilibrium. This is important if transient behavior is to be avoided. The autocorrelation time of the fluctuation signals is, in the absence of significant Mirnov oscillations, rather short, $\tau \simeq 2 - 5$ µs. For the construction of the required m-dimensional vectors, a time delay of $p = 2$ µs was generally chosen for signals obtained from magnetic probes and the phase contrast diagnostic. A larger value, for example $p = 5$ µs, appeared more suitable for Langmuir probe data due to the larger low-frequency component in these signals. To eliminate the spurious correlations mentioned above, it was appropriate to consider $C_k(r)$ with $k \geq 5$.

Figures 2 and 3 show, for the same data as used in Fig. 1, logarithmic plots of $C_1(r)$ and $C_{10}(r)$ versus r for embedding dimension $2 \leq m \leq 12$. Comparing these figures, it can be seen that there is a marked effect of the spurious correlations for the Langmuir probe data. (Similar results were obtained with a probe measuring the plasma floating potential in the scrape-off layer.) The effect is much less for the magnetic probe and phase contrast data, however it can become clearly evident for discharges with a significant level of Mirnov oscillations.[10] The observed differences between the behavior of $C_1(r)$ for the various signals are associated with the differences in the level of the low-frequency component.

In Fig. 4 is plotted a graph of the slope of log $C_{10}(r)$ versus log r, as a function of m, for the magnetic probe data. (Due to the available finite value of N, for $m > 10$ there does not exist a significant range of r for which the slope is constant.) There is clearly no saturation in slope for increasing values of m. Indeed, the slope is equal to the embedding dimension up to the largest value of m considered, this being a characteristic of random noise. Similar behavior is obtained for the Langmuir probe and phase contrast data. In fact, for none of the large sample of different tokamak discharges with a variety of operating conditions was any clear indication of saturation observed. (Technically, it should be noted that the apparent saturation in slope which may be seen for small values of r in Fig. 2(b) can be ascribed to the influence of spurious correlations.)

IV DISCUSSION AND CONCLUSIONS

From the present study of broadband fluctuations in the TCA tokamak, we therefore conclude that there is no evidence for an attractor of low dimension. A possible interpretation of the results of this study is that the broadband fluctuations are associated with a system whose dimension is high, $\nu > 10$ for the available resolution. This suggests that the fluctuations result from a nonlinear system, having a large number of coupled degrees of freedom, as is traditionally assumed by plasma turbulence theories.[1] This interpretation is consistent with the fact that the measured fluctuation spectra are extremely broad (see Fig. 1), showing no evidence of coherent modes at frequencies above the frequency of the Mirnov oscillations and their low order harmonics.

An alternative interpretation may be that the fluctuations result from a transient system which, in the context of the present study, can never be considered as reaching an equilibrium state. For such a non-stationary state, the concept of a dimension has no meaning : in particular, the algorithm for calculation of the correlation dimension assumes that transients have decayed before the data are acquired. This alternative interpretation is also consistent with an extremely broad spectrum of fluctuations. In addition, however, it is interesting to note that data recorded at different times during a tokamak discharge give rise to correlation integrals having the same behavior. A data record taken during the current ramp-up phase of the discharge, during which strong transients are undoubtedly present, yields a correlation integral having the same behavior as for a data record taken at the end of the flat-top of the discharge. Under these conditions it is reasonable to question if the application of the Grassberger-Procaccia algorithm to calculate the correlation dimension of the system is relevant.

It may be considered that a tokamak is a rather inflexible device on which to undertake a preliminary study of the application of dimensional analysis to plasma turbulence. Independent of operational parameters, the fluctuation spectra observed in tokamaks are always extremely broad, indicating a well-developed turbulent state. This, however, is not the case with all plasma devices; in fact, the transition to a turbulent state has been experimentally studied in a Q-device.[17] The application of dimensional analysis to these, presumably lower dimensional, plasma systems would be of likely interest.

In conclusion, a detailed study of the correlation integral has been undertaken for broadband magnetic and density fluctuations in a tokamak. We have determined that, for a wide range of operating conditions, the broadband fluctuations do not originate from a system of measurably low dimension.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

FIG. 1. Power spectra for fluctuations in (a) poloidal magnetic field $\dot{\tilde{b}}_\theta$, (b) ion saturation current $\tilde{i}_s$, and (c) chord-averaged density $\int \tilde{n}_e \, dl$.

FIG. 2. Logarithmic (base 10) plot of $C_1(r)$ versus r for (a) $\dot{\tilde{b}}_\theta$, (b) $\tilde{i}_s$, and (c) $\int \tilde{n}_e \, dl$.

FIG. 3. Logarithmic (base 10) plot of $C_{10}(r)$ versus r for (a) $\dot{\tilde{b}}_\theta$, (b) $\tilde{i}_s$, and (c) $\int \tilde{n}_e \, dl$.

FIG. 4. Slope of log $C_{10}(r)$ versus log r as a function of the embedding dimension m, for $\dot{\tilde{b}}_\theta$ as shown in Fig. 3(a).

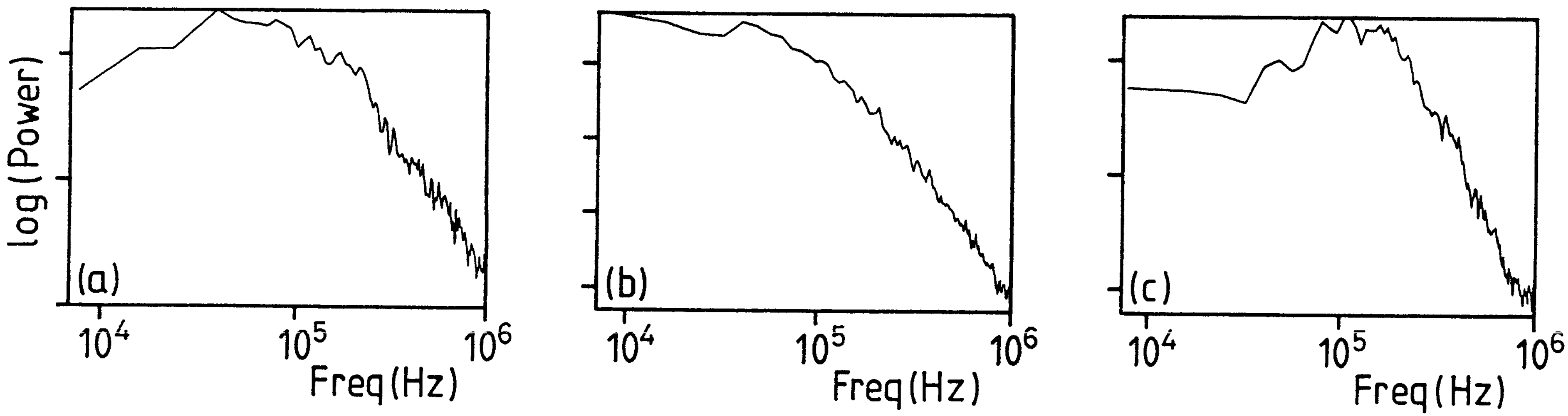

Figure 1

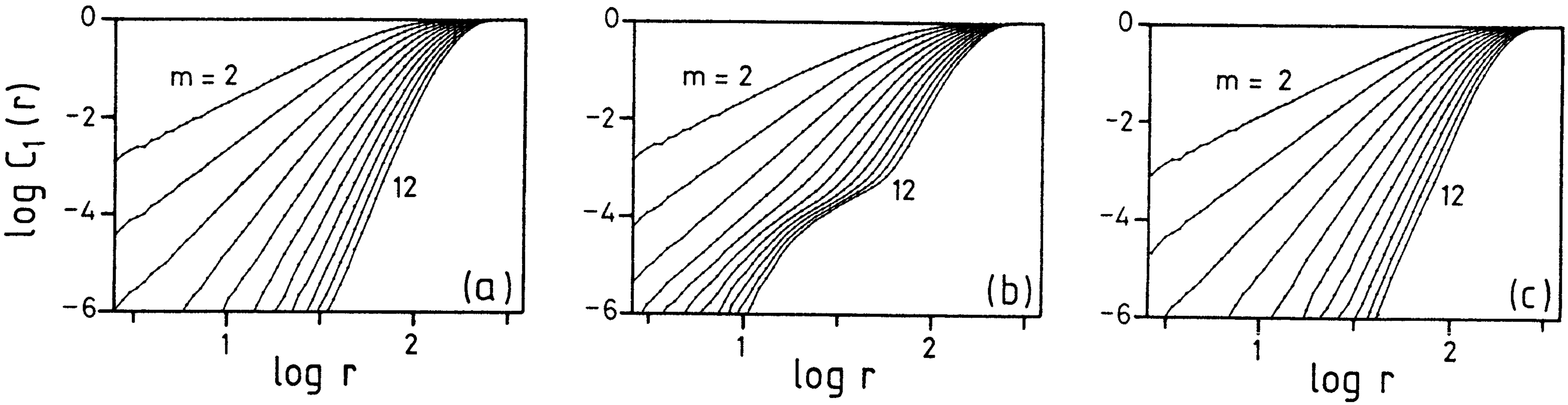

Figure 2

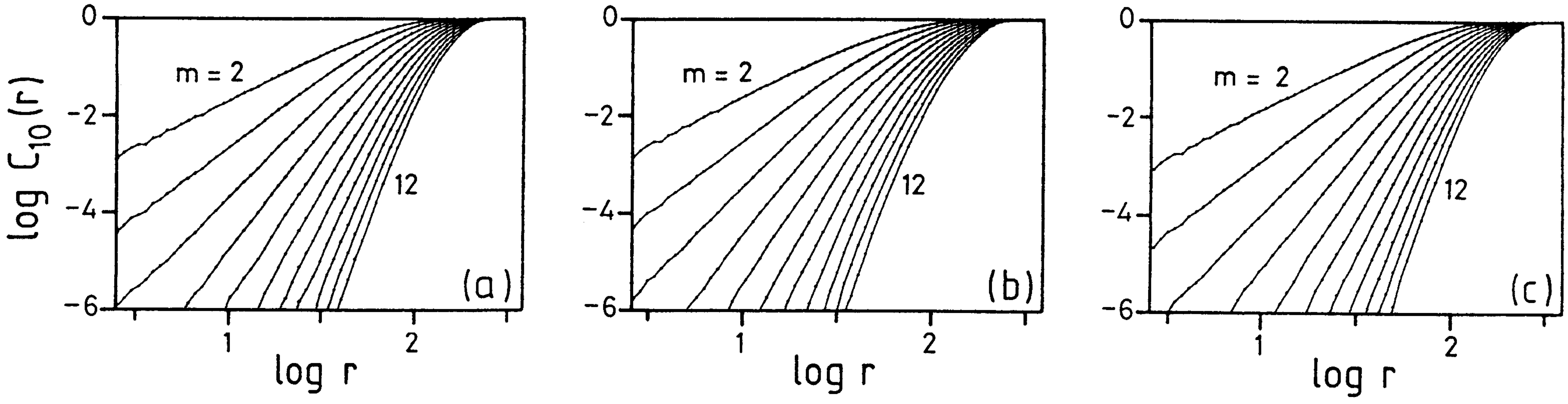

Figure 3

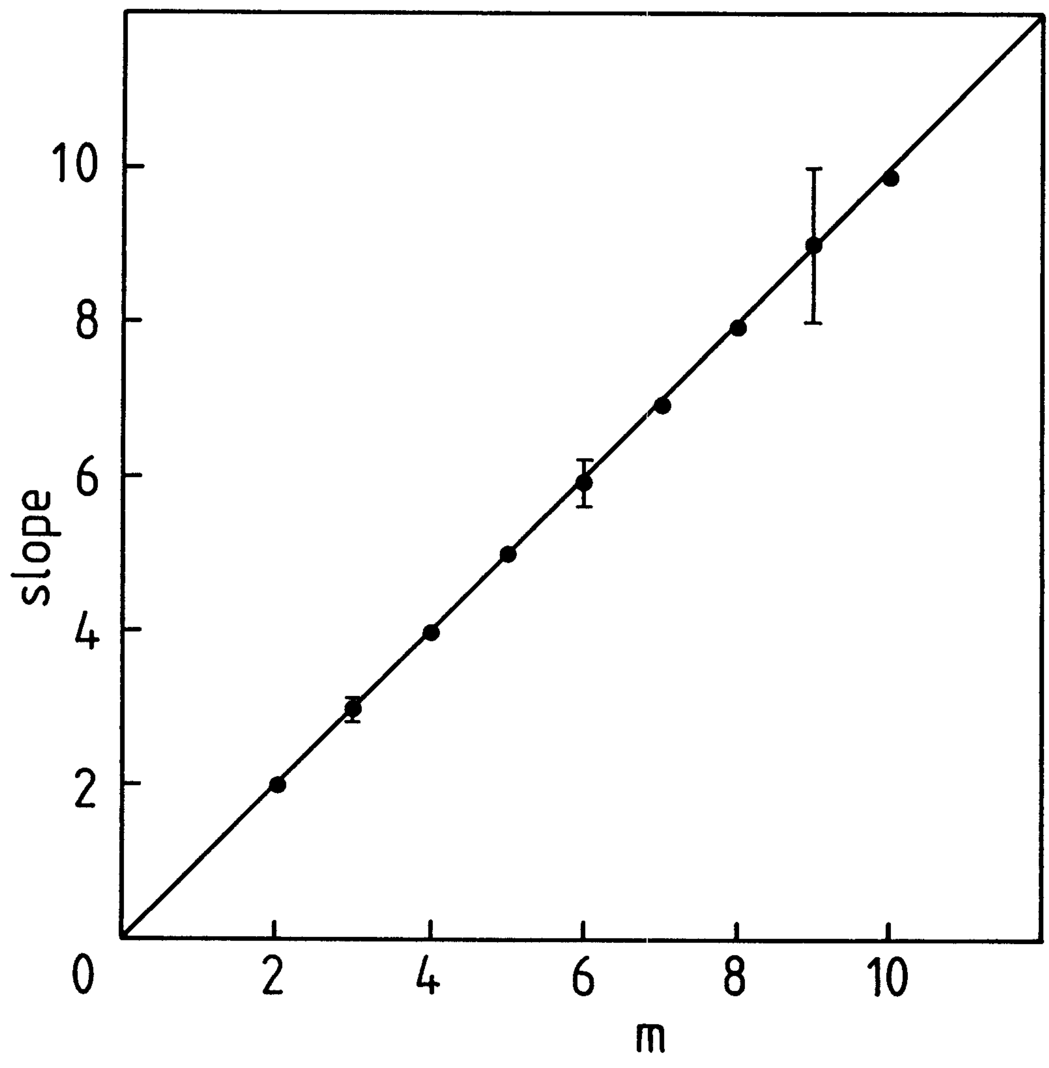

Figure 4

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ABSTRACT

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Since a tokamak plasma is a system possessing a very large number of degrees of freedom, it is pertinent to ask whether it is possible to distinguish these broadband fluctuations from random noise. This is, in principle, possible by calculating from experimental data the correlation dimension ν using the algorithm proposed by Grassberger and Procaccia.[2] A finite value of ν implies that the system tends asymptotically with time (after the decay of transients) to some subset of the total phase space, called the attractor. A "strange" attractor[3] is associated with chaotic motion that, when Fourier analyzed, reveals generally a broadband spectrum. Dimensional analysis has been recently applied to many chaotic systems in widely varying fields of research. It readily provides a measure for chaotic systems that would be difficult to characterize by standard techniques. Its application to tokamak data[4-6] has been mainly concerned with low frequency coherent oscillations; this has yielded, not surprisingly, low values of correlation dimension, $\nu \simeq 2$-3. In this letter, we study the dimension of broadband fluctuations measured during ohmic discharges of the TCA tokamak.

The correlation dimension is defined, for a system yielding a time series $\{\vec{X}_i\}_{i=1,N}$ of points on the attractor, through the correlation integral

$$C(r) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i \neq j=1}^{N} H\left(r-|\vec{X}_i-\vec{X}_j|\right) , \qquad (1)$$

where H is the Heaviside function.[2] In practice, C(r) may be calculated from a time series of a single physical quantity, x(t), by reconstructing an m-dimensional vector using delay coordinates, $\vec{X}_i(t) = [x(t_i),\ x(t_i+p), \ldots\ldots, x(t_i+(m-1)p]$. For $m \geqslant 2\nu+1$, the correlation integral has the property that $C(r) \sim r^\nu$ for small r.

The application of this algorithm requires appropriate choices for the total number of data points N, and the time delay p used in the construction of the m-dimensional vectors. In practice, the N data points are acquired at a sampling interval of Δt. Clearly, N and Δt must be chosen so as to yield sufficient resolution over the desired frequency range of the Fourier spectrum of x(t). In addition, N must be sufficiently large so that the limit in Eq. (1) is well approximated. This may, for the investigation of experimental data, involve some compromise since the parameters of the system should remain constant (to avoid transients) during the total time $\sim N\Delta t$ for which data are recorded. For proper reconstruction of the attractor, the time delay p must be chosen not too small (otherwise, $x(t_i) \simeq x(t_i+p) \simeq \ldots\ldots \simeq x(t_i+(m-1)p)$, leading to a strong distorsion in the reconstructed attractor) or not too large (otherwise, distant values in the same vector $\vec{X}_i$ are uncorrelated).[2] Denoting τ as the autocorrelation time for the time series x(t), a value of p should therefore be chosen such that $p \simeq \tau$. Finally, the acquired signal must be digitized with sufficient resolution so that $C(r) \sim r^{\nu}$ over a measurably large range of r.

Calculation of the correlation integral as defined in Eq. (1) may not in all cases lead to a reliable determination of the correlation dimension. For certain values of r, the value of C(r) may be over-estimated, especially for large values of embedding dimension m. The result is that there may not exist a sufficiently large range of r for which $C(r) \sim r^{\nu}$, or alternatively, a false value of the correlation dimension may be deduced. The origin of this phenomenon may be shown to be the spurious correlations arising when vectors $\vec{X}_i$ and $\vec{X}_j$ are considered, even though the time difference $(t_i - t_j)$ is not large compared to the autocorrelation time. To avoid counting these pairs of vectors that are close in space only because their components are close in time, a modification of Eq. (1) has been proposed:[7]

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Figures 2 and 3 show, for the same data as used in Fig. 1, logarithmic plots of $C_1(r)$ and $C_{10}(r)$ versus r for embedding dimension $2 \leq m \leq 12$. Comparing these figures, it can be seen that there is a marked effect of the spurious correlations for the Langmuir probe data. (Similar results were obtained with a probe measuring the plasma floating potential in the scrape-off layer.) The effect is much less for the magnetic probe and phase contrast data, however it can become clearly evident for discharges with a significant level of Mirnov oscillations.

In Fig. 4 is plotted a graph of the slope of log $C_{10}(r)$ versus log r, as a function of m, for the magnetic probe data. There is clearly no saturation in slope for increasing values of m. Indeed, the slope is equal to the embedding dimension up to the largest value of m considered, this being a characteristic of random noise. Similar behavior is obtained for the Langmuir probe and phase contrast data. In fact, for none of the large sample of different tokamak discharges with a variety of operating conditions was any clear indication of saturation observed. (Technically, it should be noted that the apparent saturation in slope which may be seen for small values of r in Fig. 2(b) can be ascribed to the influence of spurious correlations.)

From the present study of broadband fluctuations in the TCA tokamak, we therefore conclude that there is no evidence for an attractor of low dimension. A possible interpretation of the results of this study is that the broadband fluctuations are associated with a system whose dimension is high, $\nu > 10$ for the available resolution. This suggests that the fluctuations result from a nonlinear system, having a large number of coupled degrees of freedom, as is traditionally assumed by plasma turbulence theories.[1] This interpretation is consistent with the fact that the measured fluctuation spectra are extremely broad (see Fig. 1), showing no evidence of coherent modes at frequencies above the frequency of the Mirnov oscillations and their low order harmonics.

An alternative interpretation may be that the fluctuations result from a transient system which, in the context of the present study, can never be considered as reaching an equilibrium state. For such a non-stationary state, the concept of a dimension has no meaning : in particular, the algorithm for calculation of the correlation dimension assumes that transients have decayed before the data are acquired. This alternative interpretation is also consistent with an extremely broad spectrum of fluctuations. In addition, however, it is interesting to note that data recorded at different times during a tokamak discharge give rise to correlation integrals having the same behavior. A data record taken during the current ramp-up phase of the discharge, during which strong transients are undoubtedly present, yields a correlation integral having the same behavior as for a data record taken at the end of the flat-top of the discharge. Under these conditions it is reasonable to question if the application of the Grassberger-Procaccia algorithm to calculate the correlation dimension of the system is relevant.

It may be considered that a tokamak is a rather inflexible device on which to undertake a preliminary study of the application of dimensional analysis to plasma turbulence. Independent of operational parameters, the fluctuation spectra observed in tokamaks are always extremely broad, indicating a well-developed turbulent state. This, however, is not the case with all plasma devices; in fact, the transition to a turbulent state has been experimentally studied in a Q-device.[15] The application of dimensional analysis to these, presumably lower dimensional, plasma systems would be of likely interest.

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FIGURE CAPTIONS

FIG. 1. Power spectra for fluctuations in (a) poloidal magnetic field $\dot{\tilde{b}}_\theta$, (b) ion saturation current $\tilde{i}_s$, and (c) chord-averaged density $\int \tilde{n}_e \, dl$.

FIG. 2. Logarithmic (base 10) plot of $C_1(r)$ versus r for (a) $\dot{\tilde{b}}_\theta$, (b) $\tilde{i}_s$, and (c) $\int \tilde{n}_e \, dl$.

FIG. 3. Logarithmic (base 10) plot of $C_{10}(r)$ versus r for (a) $\dot{\tilde{b}}_\theta$, (b) $\tilde{i}_s$, and (c) $\int \tilde{n}_e \, dl$.

FIG. 4. Slope of log $C_{10}(r)$ versus log r as a function of the embedding dimension m, for $\dot{\tilde{b}}_\theta$ as shown in Fig. 3(a).