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Abstract - An energy principle is constructed for an axisymmetric system such that its variation with respect to an artificial time parameter t is demonstrated to yield the components of the magnetohydrodynamic (MHD) force $\vec{F}=\vec{j}\times\vec{B}-\vec{\nabla}p-\rho_M(\vec{V}\cdot\vec{\nabla})\vec{V}$ when $\vec{V}$ is only toroidal. An accelerated steepest descent method is applied to the Fourier moments of these MHD forces in an inverse flux coordinate representation to reach a minimum energy state that corresponds to an MHD equilibrium with isothermal toroidal mass flow. Applications to a JET Tokamak configuration show the outward displacement of the pressure surfaces away from the flux surfaces induced by the plasma rotation. For fixed global and peak values of the total plasma energy content (thermal pressure plus directed flow), however, the position of the magnetic axis does not shift with the rotation.

1. INTRODUCTION

The application of unbalanced high power neutral beam injection in Tokamaks for the purposes of heating the plasma induces also a bulk mass flow. Toroidal plasma rotation velocities of up to 2×10^7cm/s have been measured in the ISX-B and PDX Tokamaks (ISLER et al., 1983; BRAU et al., 1983). Poloidal rotation velocities have also been detected in the PDX device with on-axis and off-axis neutral beam injection. The magnetic pumping effect of a plasma moving across a spatially varying magnetic field, however, rapidly damps these poloidal flows.

The effects of mass flows on the magnetohydrodynamic (MHD) plasma equilibria in fusion confinement devices have not been extensively investigated. Tokamak MHD equilibria with toroidal plasma rotation have been calculated analytically by MASCHKE and PERRIN (1980) and numerically using finite element methods (KERNER and JANDL, 1984; SEMENZATO et al., 1984; ELSAESSER and HEIMSOTH, 1986). SEMENZATO et al. (1984) have also generated MHD equilibria with combined toroidal and poloidal mass flows.

In this article, we shall develop an energy principle and demonstrate that its variation reproduces the cylindrical MHD force components in covariant representation for an axisymmetric plasma with isothermal toroidal mass flow. The positive definite nature of this energy principle guarantees that the minimum energy state corresponds to an MHD equilibrium. An accelerated steepest descent procedure is applied to advance the Fourier amplitudes of the inverse coordinates $R(\rho,\theta)$ and $Z(\rho,\theta)$, and of a poloidal angle renormalisation parameter $\lambda(\rho,\theta)$ until a minimum in the energy is reached (HIRSHMAN and WHITSON, 1983; HIRSHMAN and LEE, 1986). Also, an improved finite differencing scheme in the radial coordinate ρ is employed that is particularly useful for treating and eliminating convergence difficulties near the magnetic axis (HIRSHMAN et al., 1986).

It should be mentioned that LAO (1984) has formulated a variational moments method to solve this problem that differs significantly from our approach in two important aspects. First, the variational principle he employs is not positive definite and second, he uses a direct Jacobian inversion technique to solve the problem.

The representation of the magnetic field and the coordinate system is discussed in Section 2. In Section 3, we present the force balance relation in isothermal toroidally rotating axisymmetric plasmas and the profiles that have to be prescribed to obtain MHD equilibria. The definitions of beta (β) are provided in Section 4. In Section 5, we construct an energy principle and derive the force components that result from its variation with respect to an artificial time parameter. We also demonstrate how two of these components correspond to cylindrical MHD forces in covariant representation. The MHD force balance components in magnetic flux coordinates are investigated in Section 6. An application to the Joint European Torus (JET) is presented in Section 7. Finally, in Section 8, the summary and the conclusions are discussed.

2. THE MAGNETIC FIELD

In a magnetic flux coordinate system (ρ,θ,ϕ), it is assumed that a coordinate $0<\rho<1$ exists such that the magnetic field satisfies the condition $\vec{B}\cdot\vec{\nabla}\rho=0$. This, together with the Maxwell equation $\vec{\nabla}\cdot\vec{B}=0$ and axisymmetry implies that the magnetic field in contravariant representation can be written as

$$\vec{B} = \vec{\nabla}\phi \times \vec{\nabla}\psi + \sqrt{g}\,(\vec{B}\cdot\vec{\nabla}\phi)\,\vec{\nabla}\rho \times \vec{\nabla}\theta\,, \tag{1}$$

where it is convenient to identify the coordinate $0<\phi<2\pi$ with the geometric toroidal angle, so that the Jacobian $\sqrt{g}$ of the transformation from the cylindrical to the flux coordinates acquires the simple form

$$\sqrt{g} = R\left(\frac{\partial R}{\partial \theta}\frac{\partial Z}{\partial \rho} - \frac{\partial R}{\partial \rho}\frac{\partial Z}{\partial \theta}\right)\,. \tag{2}$$

The coordinate $0<\theta<2\pi$ represents the poloidal angle. In a magnetic flux coordinate system (ρ,θ_*,ϕ) in which the magnetic field lines are straight,

$$\vec{B} = \vec{\nabla}\phi \times \vec{\nabla}\psi + \vec{\nabla}\Phi \times \vec{\nabla}\theta_*\,, \tag{3}$$

where $2\pi\psi$ and $2\pi\Phi$ correspond to the poloidal and toroidal magnetic fluxes, respectively. The particular angle θ_* that makes the field lines straight is related to any arbitrary poloidal angle θ by the relation

$$\theta_* = \theta + \lambda(\rho,\theta) , \tag{4}$$

where λ is a periodic renormalisation parameter (HIRSHMAN and WHITSON, 1983).

3. FORCE BALANCE

The MHD force balance relation for an isothermal toroidally rotating axisymmetric plasma reduces to

$$\vec{F} = -\left.\frac{\partial p}{\partial \rho}\right|_R \vec{\nabla}\rho + \vec{J} \times \vec{B} , \tag{5}$$

after we have set its component along the magnetic field lines to 0 and we have applied the Ohm's Law $\vec{\nabla} \times (\vec{V} \times \vec{B}) = 0$. The subscript indicates that the derivative of p with respect to ρ is to be evaluated at fixed R and the plasma pressure is given by the expression

$$p(\rho,R) = M(\rho)\left[\frac{\Phi'(\rho)}{H(\rho)}\right]^{\Gamma} h(\rho,R) , \tag{6}$$

where $M(\rho)$ is the mass function, the prime indicates the derivative of a flux surface quantity with respect to ρ, Γ is the adiabatic index (or ratio of specific heats) which we take to be 5/3,

$$h(\rho,R) = \exp\left[U(\rho)R^2\right] , \tag{7}$$

and

$$H(\rho) \equiv \langle h(\rho, R) \rangle \equiv \iint d\theta \, d\phi \sqrt{g} \exp\left[U(\rho) R^2\right] , \tag{8}$$

where $U(\rho) = 0.25 M_i \Omega^2(\rho)/T(\rho)$ represents the plasma flow function, M_i is the ion species atomic mass, T is the plasma temperature, and $\Omega \equiv \vec{V} \cdot \vec{\nabla}\phi$ is the toroidal angular rotation frequency.

Note that in the static limit $[U(\rho)=0]$, $H(\rho)=V'(\rho)$ is the differential volume profile, namely

$$V'(\rho) = 2\pi \int_0^{2\pi} d\theta \sqrt{g} . \tag{9}$$

The profiles that must be prescribed to generate flux-conserving isothermal toroidally rotating Tokamak MHD equilibria are $M(\rho)$, $\Phi(\rho)$, the rotational transform $\iota(\rho) \equiv \Psi'/\Phi'$, and $U(\rho)$. These functions also have the property that they are conserved in the energy minimisation scheme that is described in Section 5. The adiabatic conservation of mass between neighbouring flux surfaces is what makes the prescription of $M(\rho)$ as the input profile more advantageous than that of the function $P(\rho) \equiv M(\rho) \; [\Phi'(\rho)/H(\rho)]^{\Gamma}$ that caracterise the previous formulations of this problem.

4. BETA DEFINITIONS

The plasma beta associated with the thermal pressure is defined to be

$$\beta_p \equiv \frac{\iint d\rho \, d\theta \sqrt{g} \; p(\rho, R)}{\iint d\rho \, d\theta \sqrt{g} \; B^2/(2\mu_0)} , \tag{10}$$

and the plasma beta associated with the rotational energy density is defined to be

$$\beta_R \equiv \frac{\iiint d^3x \, \frac{1}{2} \rho_M \Omega^2 R^2}{\iiint d^3x \, B^2/(2\mu_0)} = \frac{\iint d\rho \, d\theta \sqrt{g} \, U(\rho) R^2 p(\rho, R)}{\iint d\rho \, d\theta \sqrt{g} \, B^2/(2\mu_0)} , \tag{11}$$

where ρ_M is the mass density. Thus the total beta is

$$\beta = \beta_P + \beta_R . \tag{12}$$

The peak values of beta that are calculated are approximately given by

$$\beta_P(0) = \frac{P(0) \exp\{U(0)[R_0(0)]^2\}}{[B_0(0)]^2/(2\mu_0)} , \tag{13}$$

$$\beta_R(0) = U(0) [R_0(0)]^2 \beta_P(0) , \tag{14}$$

where $R_0(0)$ and $B_0(0)$ are the m=0 Fourier amplitudes of R and B at the magnetic axis, respectively, and

$$\beta(0) \equiv \beta_P(0) + \beta_R(0) . \tag{15}$$

5. ENERGY PRINCIPLE VARIATION

We construct the energy W as

$$W = \iiint d^3x \left[\frac{B^2}{2\mu_0} + \frac{p(\rho,R)}{(\Gamma-1)}\right] . \tag{16}$$

We then proceed to vary W, which is positive-definite for $\Gamma>1$, with respect to an artificial time parameter t to obtain

$$\frac{dW}{dt} = \iiint d\rho\, d\theta\, d\phi\, \frac{\partial}{\partial t}\left(\frac{\sqrt{g}\,B^2}{2\mu_0}\right) + \int_0^1 d\rho\, \frac{M(\rho)\left[\Phi'(\rho)\right]^{\Gamma}}{(\Gamma-1)} \frac{\partial}{\partial t}\left[H(\rho)\right]^{1-\Gamma}$$

$$= \iiint d\rho\, d\theta\, d\phi \left\{\frac{\partial}{\partial t}\left(\frac{\sqrt{g}\,B^2}{2\mu_0}\right) - p(\rho,R)\frac{\partial\sqrt{g}}{\partial t} - \sqrt{g}\,M(\rho)\left[\frac{\Phi'(\rho)}{H(\rho)}\right]^{\Gamma} \left.\frac{\partial h}{\partial R}\right|_{\rho} \dot{R}\right\}, \tag{17}$$

where we have used $\partial h/\partial t=(\partial h/\partial R)\dot{R}$ and we have defined $\dot{R}=\partial R/\partial t$. Because the derivative of h with respect to R in equation (17) is evaluated at fixed ρ, we have that

$$M(\rho)\left[\frac{\Phi'(\rho)}{H(\rho)}\right]^{\Gamma} \left.\frac{\partial h}{\partial R}\right|_{\rho} = \frac{\partial}{\partial R}\left\{M(\rho)\left[\frac{\Phi'(\rho)}{H(\rho)}\right]^{\Gamma} h(\rho,R)\right\}\Bigg|_{\rho} = \left.\frac{\partial p}{\partial R}\right|_{\rho} . \tag{18}$$

In addition, noting that we can express

$$\frac{\partial\sqrt{g}}{\partial t} = R\left(\frac{\partial\dot{R}}{\partial\theta}\frac{\partial Z}{\partial\rho} + \frac{\partial R}{\partial\theta}\frac{\partial\dot{Z}}{\partial\rho} - \frac{\partial\dot{R}}{\partial\rho}\frac{\partial Z}{\partial\theta} - \frac{\partial R}{\partial\rho}\frac{\partial\dot{Z}}{\partial\theta}\right) + \frac{\sqrt{g}}{R}\dot{R} , \tag{19}$$

and

$$\frac{1}{2\sqrt{g}}\frac{\partial}{\partial t}\left[(\sqrt{g})^2 B^2\right] = \frac{(\Phi')^2}{\sqrt{g}}\left[\iota^2\left(\frac{\partial R}{\partial\theta}\frac{\partial \dot{R}}{\partial\theta}+\frac{\partial Z}{\partial\theta}\frac{\partial \dot{Z}}{\partial\theta}\right)+R\dot{R}\left(1+\frac{\partial\lambda}{\partial\theta}\right)^2\right]$$
$$+\frac{(\Phi')^2}{\sqrt{g}}R^2\left(1+\frac{\partial\lambda}{\partial\theta}\right)\frac{\partial\dot{\lambda}}{\partial\theta}\quad, \tag{20}$$

we can obtain, after an integration by parts, that

$$\frac{dW}{dt} = -\iiint d\rho\, d\theta\, d\phi\, F_R \dot{R} - \iiint d\rho\, d\theta\, d\phi\, F_Z \dot{Z} - \iiint d\rho\, d\theta\, d\phi\, F_\lambda \dot{\lambda}$$
$$+\iint_{\rho=1} d\theta\, d\phi\, R\left(p+\frac{B^2}{2\mu_0}\right)\left(\frac{\partial Z}{\partial\theta}\dot{R}-\frac{\partial R}{\partial\theta}\dot{Z}\right)\quad, \tag{21}$$

where the residual forces are given by

$$F_R = \frac{\partial}{\partial\rho}\left[R\frac{\partial Z}{\partial\theta}\left(p+\frac{B^2}{2\mu_0}\right)\right]-\frac{\partial}{\partial\theta}\left[R\frac{\partial Z}{\partial\rho}\left(p+\frac{B^2}{2\mu_0}\right)-\frac{(\iota\Phi')^2}{\sqrt{g}}\frac{\partial R}{\partial\theta}\right]$$
$$+\frac{\sqrt{g}}{R}\left[\left(p+\frac{B^2}{2\mu_0}\right)-\frac{R^2(B^\phi)^2}{\mu_0}\right]+\sqrt{g}\left.\frac{\partial p}{\partial R}\right|_\rho\quad, \tag{22}$$

$$F_Z = \frac{\partial}{\partial\theta}\left[R\frac{\partial R}{\partial\rho}\left(p+\frac{B^2}{2\mu_0}\right)+\frac{(\iota\Phi')^2}{\sqrt{g}}\frac{\partial Z}{\partial\theta}\right]-\frac{\partial}{\partial\rho}\left[R\frac{\partial R}{\partial\theta}\left(p+\frac{B^2}{2\mu_0}\right)\right], \tag{23}$$

and

$$F_\lambda = \frac{\partial}{\partial\theta}\left[\frac{R^2(\Phi')^2}{\sqrt{g}}\left(1+\frac{\partial\lambda}{\partial\theta}\right)\right]\quad, \tag{24}$$

where $B^\phi \equiv \vec{B}\cdot\vec{\nabla}\phi$. It is straightforward to demonstrate that equations (22) and (23) correspond to the $\sqrt{g}R\vec{\nabla}\phi\times\vec{\nabla}Z$ and the $\sqrt{g}R\vec{\nabla}R\times\vec{\nabla}\phi$ components of the force balance relation (5), respectively.

The last term in equation (21) corresponds to the contribution from the plasma-vacuum interface. For the fixed boundary calculations we are interested in developing in this work, this term vanishes. Thus, by expansions in Fourier series, we can express equation (21) as

$$\frac{dW}{dt} = -2\pi^2 \sum_{m\geq 0} \int dV \left[F_{Rm}(\rho)\dot{R}_m(\rho) + F_{Zm}(\rho)\dot{Z}_m(\rho) + F_{\lambda m}(\rho)\dot{\lambda}_m(\rho) \right]$$
$$-2\pi^2 \int dV\, F_{R0}(\rho)\dot{R}_0(\rho) \quad , \tag{25}$$

where $dV = V'(\rho)d\rho$,

$$F_{Rm}(\rho) = \frac{2}{2\pi V'}\int_0^{2\pi} d\theta\, F_R(\rho,\theta)\cos(m\theta) - \frac{\delta_{m,0}}{2\pi V'}\int_0^{2\pi} d\theta\, F_R(\rho,\theta) \quad , \tag{26}$$

$$F_{Zm}(\rho) = \frac{2}{2\pi V'}\int_0^{2\pi} d\theta\, F_Z(\rho,\theta)\sin(m\theta) \quad , \tag{27}$$

$$F_{\lambda m}(\rho) = \frac{2}{2\pi V'}\int_0^{2\pi} d\theta\, F_\lambda(\rho,\theta)\sin(m\theta) \quad , \tag{28}$$

and $R_m(\rho)$, $Z_m(\rho)$, and $\lambda_m(\rho)$ are the Fourier amplitudes of $R(\rho,\theta)$, $Z(\rho,\theta)$ and $\lambda(\rho,\theta)$, respectively.

The path of steepest descent corresponds to

$$\dot{R}_m(\rho) = F_{Rm}(\rho) \quad , \tag{29}$$

$$\dot{Z}_m(\rho) = F_{Zm}(\rho) \quad , \tag{30}$$

and

$$\dot{\lambda}_m(\rho) = F_{\lambda m}(\rho) \quad . \tag{31}$$

However, the convergence of these equations can be accelerated with a second-order Richardson scheme consisting of a set of non-linear coupled ordinary differential equations with the addition of constraints to minimise the spectral width. Further details about the numerical procedure can be found in HIRSHMAN and WHITSON (1983), HIRSHMAN and LEE (1986), and HIRSHMAN et al. (1986).

6. FORCE BALANCE IN FLUX COORDINATES

The radial component of force balance is obtained by taking the dot product of $\sqrt{g}\vec{\nabla}\theta\times\vec{\nabla}\phi$ with equation (5),

$$F_\rho = -\left.\frac{\partial p}{\partial \rho}\right|_R - \sqrt{g}\,(\vec{j}\cdot\vec{\nabla}\phi)(\vec{B}\cdot\vec{\nabla}\theta) + \sqrt{g}\,(\vec{j}\cdot\vec{\nabla}\theta)(\vec{B}\cdot\vec{\nabla}\phi) , \quad (32)$$

where using Ampère's Law, the toroidal current density is

$$\vec{j}\cdot\vec{\nabla}\phi = \frac{1}{\mu_0\sqrt{g}}\left(\frac{\partial B_\theta}{\partial \rho} - \frac{\partial B_\rho}{\partial \theta}\right) , \quad (33)$$

and the poloidal current density is

$$\vec{j}\cdot\vec{\nabla}\theta = -\frac{1}{\mu_0\sqrt{g}}\;\frac{\partial B_\phi}{\partial \rho} , \quad (34)$$

from which we derive the relation

$$F_\rho = -\left.\frac{\partial p}{\partial \rho}\right|_R - \frac{\Phi'}{\mu_0\sqrt{g}}\left[\iota\frac{\partial B_\theta}{\partial \rho} + \left(1+\frac{\partial \lambda}{\partial \theta}\right)\frac{\partial B_\phi}{\partial \rho}\right] + \frac{\iota\Phi'}{\mu_0\sqrt{g}}\,\frac{\partial B_\rho}{\partial \theta} , \quad (35)$$

where B_ρ, B_θ, and B_ϕ constitute the components of the magnetic field in the covariant representation.

The perpendicular component of force balance results from the dot product of $\sqrt{g}(\vec{B}\times\vec{\nabla}\rho)/B^2$ with equation (5),

$$F_{\perp} = -\sqrt{g}\,\vec{J}\cdot\vec{\nabla}\rho = -\frac{1}{\mu_0}\frac{\partial B_\phi}{\partial \theta} \quad , \tag{36}$$

which demonstrates that $F_\lambda = -\Phi' F_\perp$. An equilibrium that is achieved by minimizing the cylindrical MHD forces can be diagnosed by analysing whether it satisfies the criteria $F_\rho \approx 0$ and $F_\perp \approx 0$.

7. NUMERICAL RESULTS

We have developed the computer code ATRIME (Axisymmetric Toroidally Rotating Isothermal Moments Equilibrium) to descend the energy of a rotating Tokamak plasma to its minimum state. It consists basically of an axisymmetric version of the VMEC code (HIRSHMAN et al., 1986) with the cylindrical MHD force components appropriately modified to account for plasma mass flow as expressed in equations (22) and (23).

The effects of isothermal toroidal plasma rotation on MHD equilibria are investigated for a standard JET Tokamak configuration. The plasma boundary is parameterised by

$$\begin{aligned} R &= R_a + a\cos(\theta + \delta\sin\theta) , \\ Z &= E a \sin\theta , \end{aligned} \tag{37}$$

where R_a=2.96m, a=1.25m, E=1.68 and δ=0.3 (TROYON et al., 1984; SEMENZATO et al., 1984). A Fourier series is found for this boundary curve using a scheme that minimises the spectral width and thus the harmonic content of the series but yet accurately and economically represents the curve (HIRSHMAN and MEIER, 1985). The Fourier amplitudes R_m and Z_m that result are

$$\begin{aligned}
&R_0(1)=2.7675, && Z_0(1)=0,\\
&R_1(1)=1.254, && Z_1(1)=2.0582,\\
&R_2(1)=1.943\times10^{-1}, && Z_2(1)=3.683\times10^{-2},\\
&R_3(1)=-3.39\times10^{-3}, && Z_3(1)=-4.375\times10^{-2},\\
&R_4(1)=3.0\times10^{-5}, && Z_4(1)=8.36\times10^{-3},\\
&R_5(1)=-8.9\times10^{-4}, && Z_5(1)=-1.3\times10^{-3}.
\end{aligned} \tag{38}$$

The dimensions of these amplitudes are expressed in metres. The sequences of JET equilibria we generate in this study all have 41 flux surfaces and 8 poloidal modes. The mass function profile is given by

$$M(\rho) = M_0 (1-\rho)^2 , \tag{39}$$

the rotational transform profile is expressed as

$$\iota(\rho) = 1 - \frac{2}{3}\rho , \tag{40}$$

and the toroidal flux function is written as

$$\Phi(\rho) = \frac{1}{2}\rho . \tag{41}$$

Typically an equilibrium calculation takes 15s on a CRAY-1 computer.

First, we generate a static JET equilibrium with $\beta=\beta_P=5.5\%$ and $\beta(0)=\beta_P(0)=15.8\%$ by choosing $M_0=0.0685$. The flux surfaces and the pressure surfaces for this equilibrium are shown in Fig. 1. Next, we generate a toroidally rotating JET equilibrium by choosing the plasma rotation profile as

$$U(\rho) = U_0 = 0.023, \tag{42}$$

and $M_0=0.065$ to obtain a case with the same total $\beta=\beta_P+\beta_R$ and $\beta(0)=\beta_P(0)+\beta_R(0)$ as in the static example. The specific values are $\beta_P=4.5\%$, $\beta_R=1\%$, $\beta_P(0)=12.9\%$, and $\beta_R(0)=2.9\%$. This choice for $U(\rho)$ constitutes a case in which the profile of the square of the rotational angular frequency coincides with the temperature profile. The Mach number at the magnetic axis (defined as $2R_0(0)\sqrt{U_0}$) is 0.95. The flux surfaces and the pressure surfaces for this equilibrium are shown in Fig. 2. The pressure surfaces in this case are displaced outwards away from the axis of symmetry compared with the flux surfaces as is expected. The position of the magnetic axis, however, does not significantly change when we compare the static example with the rotating example. We find that $R_0(0)\approx 3.122$m, and calculations with $0<U_0<0.023$ give approximately the same answer. Consequently, we conclude that the position of the magnetic axis is roughly invariant with plasma rotation provided that the total plasma energy content, as reflected by the values of β and $\beta(0)$, does not change. Previous analytic and numerical studies (MASCHKE and PERRIN, 1980; KERNER and JANDL, 1984; SEMENZATO et al., 1984) have reported an outward displacement of the magnetic axis with plasma rotation. Those calculations were carried out by prescribing and fixing the $P(\rho)$ profile (rather than the mass function) as the value of U_0 is varied.

With that prescription, however, not only does β_R increment with increasing U_0, but so does β_p (and thus β). Consequently, the shift of the magnetic axis induced by the plasma rotation that was found is misleading.

Finally, we present a calculation of a JET equilibrium with a very peaked $U(\rho)$ profile. This corresponds to a case in which the $\Omega(\rho)$ profile is more peaked than the $T(\rho)$ profile. The form chosen is

$$U(\rho) = U_0(1-\rho)^4 = 0.023(1-\rho)^4 \,, \qquad (43)$$

with M_0=0.065. The Mach number on axis is 0.95 and the values of $\beta(0)$=15.8%, $\beta_p(0)$=12.9% and $\beta_R(0)$=2.9% are the same as in the constant $U(\rho)$ example previously presented. The global values are β_p=4.9%, β_R=0.4% and as a result β=5.3%. The flux surfaces and the pressure surfaces for this case are shown in Fig. 3. An outward shift of the pressure surfaces with respect to the flux surfaces, though not as significant as in the constant $U(\rho)$ example, is also evident in this figure. The position of the magnetic axis is $R_0(0)$=3.124m which indicates that its shift is actually more sensitive to the value of $\beta(0)$ than that of β. This is confirmed by undertaking a calculation that yields the same β=5.5% as the constant $U(\rho)$ case but has $\beta(0)$=16.3% to obtain $R_0(0)$=3.127m.

8. SUMMARY AND CONCLUSIONS

We have constructed the energy principle $W=\int\int\int d^3x[B^2/2\mu_0+p(\rho,R)/(\Gamma-1)]$ where the pressure function is given by

$p(\rho,R)=M(\rho)[\Phi'(\rho)]^{\Gamma}\exp[U(\rho)R^2]/\langle\exp[U(\rho)R^2]\rangle^{\Gamma}$ and we have demonstrated that its variation with respect to an artificial time parameter t yields the components of the MHD force $\vec{F}=\vec{j}\times\vec{B}-\vec{\nabla}p-\rho_M(\vec{V}\cdot\vec{\nabla})\vec{V}$ when $\vec{V}$ has only a toroidal component in the contravariant representation. The magnetohydrodynamic equilibrium state for an axisymmetric plasma with isothermal toroidal mass flow is achieved when these MHD forces simultaneously vanish. To reach the minimum energy state that corresponds to an equilibrium, a set of nonlinear coupled ordinary differential equations for the Fourier amplitudes of the inverse coordinates $R(\rho,\theta)$ and $Z(\rho,\theta)$, and the periodic poloidal angle renormalisation parameter $\lambda(\rho,\theta)$ in terms of the Fourier amplitudes of the MHD forces are solved using an accelerated steepest descent method. To obtain fixed boundary rotating equilibria, the surface functions that are conserved during the iteration procedure, $M(\rho)$, $U(\rho)$, $\iota(\rho)$ and $\Phi(\rho)$ and the Fourier amplitudes of R and Z at the plasma boundary must be prescribed.

Numerical toroidally rotating MHD equilibria for the JET Tokamak configuration are obtained with this method. Our investigations show that for fixed peak and global values of the total plasma beta, $\beta(0)$ and β, that the pressure surfaces shift away from the major axis with respect to the flux surfaces as a function of the plasma rotation, but that the position of the magnetic axis remains roughly invariant. Our calculations also show that the displacement of the magnetic axis is actually more sensitive to the peak value of the total β rather than its global value and that the shift of the pressure surfaces depends more strongly on β_R than $\beta_{R(0)}$.

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FIGURE CAPTIONS

Fig. 1.-The toroidal magnetic flux surfaces (solid contours) and the pressure surfaces (dashed contours) for a static JET MHD equilibrium with β_p=5.5% and $\beta_p(0)$=15.8%.

Fig. 2.-The toroidal magnetic flux surfaces (solid contours) and the pressure surfaces (dashed contours) for a toroidally rotating JET MHD equilibrium with β_p=4.5%, $\beta_p(0)$=12.9%, β_R=1%, $\beta_R(0)$=2.9%, and Mach number of 0.95 at the magnetic axis calculated with a flat $U(\rho)$ profile.

Fig. 3.-The toroidal magnetic flux surfaces (solid contours) and the pressure surfaces (dashed contours) for a toroidally rotating JET MHD equilibrium with β_p=4.9%, $\beta_p(0)$=12.9%, β_R=0.4%, $\beta_R(0)$=2.9%, and Mach number of 0.95 at the magnetic axis calculated with a peaked $U(\rho)$ profile.

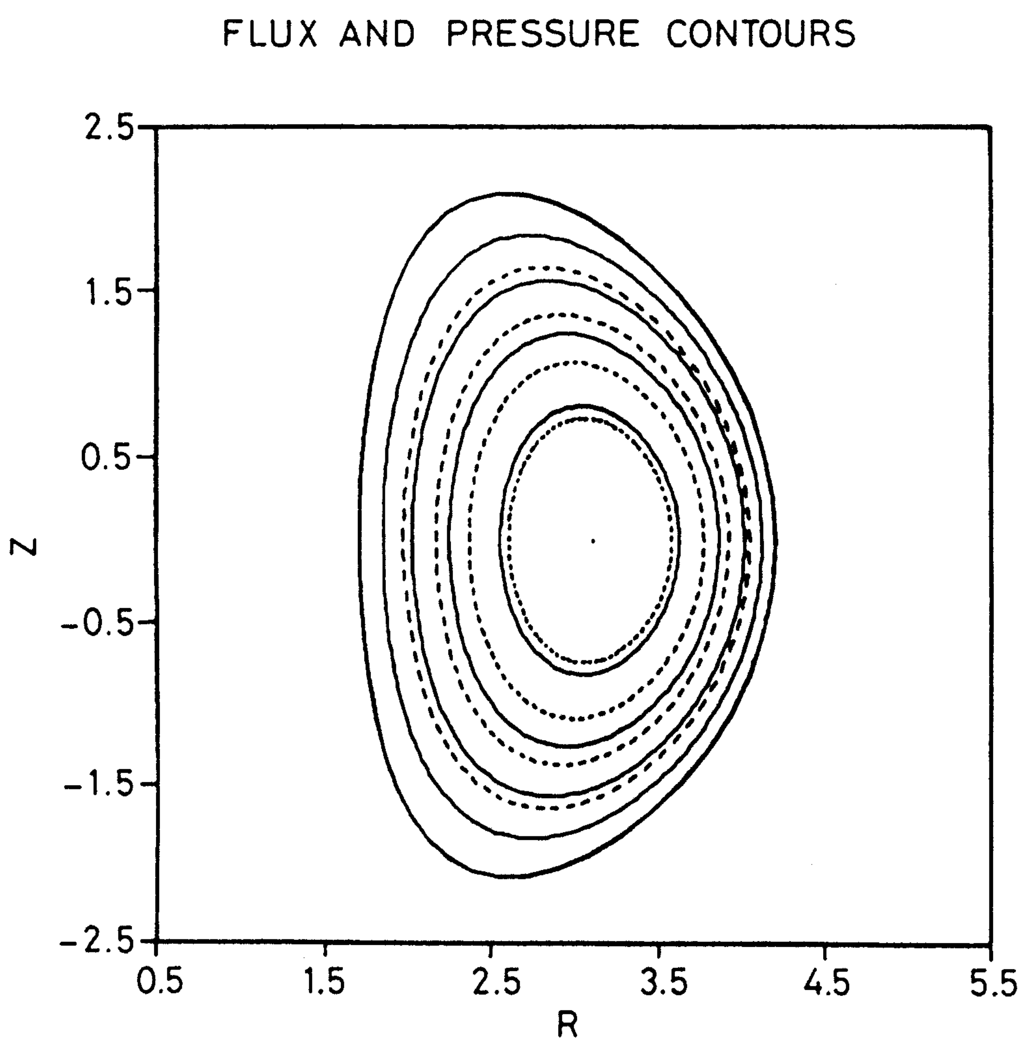

Fig. 1

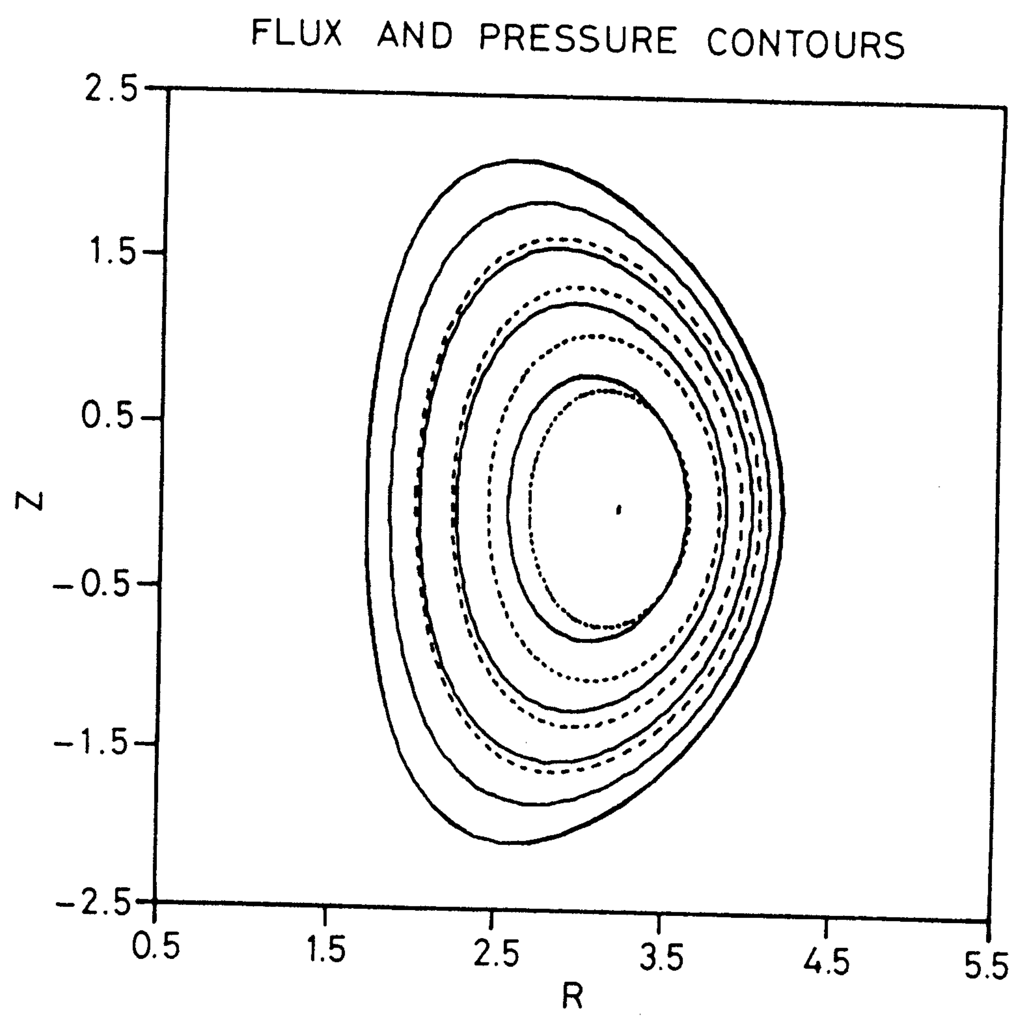

Fig. 2

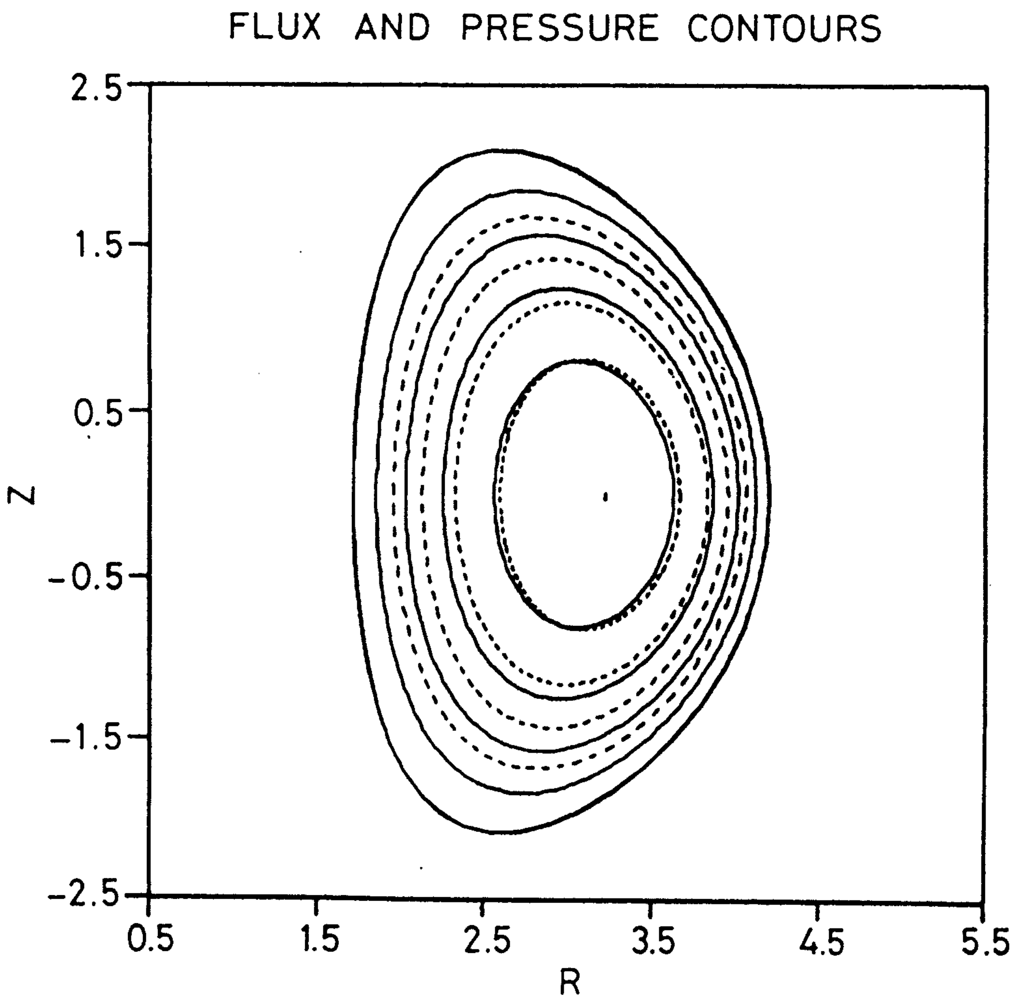

Fig. 3