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Abstract

The wave induced fluxes of energetic trapped ions during ICRF heating of tokamak plasmas are calculated using the quasilinear equations. A simple single particle model of this transport mechanism is also given. Both a convective flux proportional to $k_\phi |E_+|^2$ and a diffusive flux proportional to $k_\phi^2 |E_+|^2$ are found. Here, k_ϕ is the toroidal wavenumber and E_+ is the left-hand polarized wave field. The convective flux may become significant for large k_ϕ if the wave spectrum is asymmetric in k_ϕ. But for the conditions of most previous experiments, these calculations indicate that radial transport driven directly by the ICRF wave is unimportant.

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1 Introduction

Recent experimental successes[1,2,3] in heating tokamak plasmas with waves in the Ion Cyclotron Range of Frequencies (ICRF) have demonstrated that this is one of the most promising methods of plasma heating to reactor relevant temperatures. At the same time, however, the experimental data indicates that the application of high levels of radiofrequency (RF) power may lead to changes in confinement properties of heated plasmas. A question arises as to whether these changes may be associated, at least partly, with interaction between the ICRF waves and ions. It is therefore of great importance to ascertain how the ICRF waves can directly affect ion radial transport.

Some aspects of this problem have recently been examined from the single particle (test particle) perspective[4] and the quasilinear perspective[5,6]. The purpose of the present paper is to provide a more complete analysis of the problem, to show the relationship between the single particle and quasilinear approaches, and to assess the importance of ICRF-driven transport for RF-created energetic ions as well as α-particles in ignited devices.

2 Basic Equations

Consider a collisionless plasma immersed in a magnetostatic field $\vec{B}$. In the presence of an oscillating electromagnetic field $\tilde{\vec{E}}$, $\tilde{\vec{B}}$, the distribution function f of species with charge q and mass M obeys the Vlasov equation

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{q}{M}\left[\tilde{\vec{E}} + \frac{1}{c}\vec{v}\times\left(\vec{B}+\tilde{\vec{B}}\right)\right]\cdot\frac{\partial f}{\partial \vec{v}} = 0. \tag{1}$$

We split f into an average distribution function F and an oscillating distribution function $\tilde{F}$. Moreover, we transform Eq. (1) into the guiding-centre phase space. On averaging Eq. (1) over a statistical ensemble and over the particle gyrophase α we then obtain the quasilinear equation for F in the form

$$\left(\frac{\partial}{\partial t} + v_{\parallel}\frac{\partial}{\partial \ell} + \vec{v_d}\cdot\nabla\right)F + \left\langle\frac{\partial}{\partial \vec{v}}\cdot\vec{\Gamma}_v\right\rangle_{\alpha} + \nabla\cdot\left\langle\frac{1}{\Omega}\vec{\Gamma}_v\times\vec{e_{\parallel}}\right\rangle_{\alpha} = 0, \tag{2}$$

where

$$\vec{\Gamma}_v = \frac{q}{M}\left\langle\left(\tilde{\vec{E}} + \frac{1}{c}\vec{v}\times\tilde{\vec{B}}\right)\tilde{F}\right\rangle \tag{3}$$

is the ensemble-averaged velocity space flux induced by the electromagnetic field, $\vec{e_{\parallel}} = \vec{B}/B$, ℓ is the path length along $\vec{e_{\parallel}}$, Ω is the particle cyclotron frequency and $\vec{v_d}$ is the guiding-centre drift due to an inhomogeneity of the magnetostatic field given by

$$\vec{v_d} = -v_{\parallel}\vec{e_{\parallel}}\times\nabla\frac{v_{\parallel}}{\Omega}. \tag{4}$$

We now integrate Eq. (2) over the velocity space and average it over a magnetic surface. On defining the surface-averaged density as

$$\bar{N} = \left(\oint \frac{d\ell}{B}\right)^{-1} \oint \frac{d\ell}{B} \int d\vec{v} F = \left(\oint \frac{d\ell}{B}\right)^{-1} 2\pi \sum_{\sigma} \int d\mu d\varepsilon \oint \frac{d\ell}{|v_{\parallel}|} F, \tag{5}$$

where $\sigma = \mathrm{sign}(v_{\parallel})$, $\varepsilon = v^2/2$, and $\mu = v_{\perp}^2/(2B)$, this yields the transport equation for $\bar{N}$:

$$\frac{\partial \bar{N}}{\partial t} + \left(\oint \frac{d\ell}{B}\right)^{-1} 2\pi \sum_{\sigma} \int d\mu d\varepsilon \oint \frac{d\ell}{|v_{\parallel}|} \left(\vec{v_d} \cdot \nabla F + \nabla \cdot \left\langle \frac{1}{\Omega} \vec{\Gamma}_v \times \vec{e_{\parallel}} \right\rangle_{\alpha}\right) = 0. \tag{6}$$

For axisymmetric tokamaks we have

$$\vec{B} = \nabla \phi \times \nabla \Psi + B_T R \nabla \phi \equiv \nabla \beta \times \nabla \Psi, \tag{7}$$

where ϕ is the toroidal angle, Ψ is the poloidal flux function, B_T is the toroidal field and R is the distance from the symmetry axis. Equation (7) implies

$$\left(\frac{\partial F}{\partial \beta}\right)_{\ell} = \left(\frac{\partial \left\langle \vec{\Gamma}_v \right\rangle_{\alpha}}{\partial \beta}\right)_{\ell} = 0.$$

Thus

$$\oint \frac{d\ell}{|v_{\parallel}|} \left(\vec{v_d} \cdot \nabla F + \nabla \cdot \left\langle \frac{1}{\Omega} \vec{\Gamma}_v \times \vec{e_{\parallel}} \right\rangle_{\alpha}\right)$$

$$= \oint \frac{d\ell}{|v_{\parallel}|} \left(v_{d\Psi} |\nabla \Psi| \frac{\partial F}{\partial \Psi} + \nabla \Psi \cdot \frac{\partial}{\partial \Psi} \left\langle \frac{1}{\Omega} \vec{\Gamma}_v \times \vec{e_{\parallel}} \right\rangle_{\alpha}\right). \tag{8}$$

From Eq. (4) we find

$$v_{d\Psi} |\nabla \Psi| = v_{\parallel} R B_T \frac{\partial}{\partial \ell} \frac{v_{\parallel}}{\Omega} \tag{9}$$

Hence

$$\oint \frac{d\ell}{|v_{\parallel}|} v_{d\Psi} |\nabla \Psi| \frac{\partial F}{\partial \Psi} = -R B_T \frac{\partial}{\partial \Psi} \oint d\ell \frac{|v_{\parallel}|}{\Omega} \frac{\partial F}{\partial \ell}. \tag{10}$$

On using Eqs. (8) and (10) we can write Eq. (6) as

$$\frac{\partial \bar{N}}{\partial t} + \frac{\partial}{\partial \Psi} \left(\Gamma_{\Psi}^{\mathrm{clas}} + \Gamma_{\Psi}^{\mathrm{neo}}\right) = 0, \tag{11}$$

where

$$\Gamma_{\Psi}^{\mathrm{clas}} = \left(\oint \frac{d\ell}{B}\right)^{-1} 2\pi \sum_{\sigma} \int d\mu d\varepsilon \oint \frac{d\ell}{|v_{\parallel}|} \nabla \Psi \cdot \left\langle \frac{1}{\Omega} \vec{\Gamma}_v \times \vec{e_{\parallel}} \right\rangle_{\alpha}, \tag{12}$$

$$\Gamma_{\Psi}^{\rm neo} = -\left(\oint \frac{d\ell}{B}\right)^{-1} RB_T 2\pi \sum_{\sigma} \int d\mu d\varepsilon \oint d\ell \frac{|v_{\|}|}{\Omega}\frac{\partial F}{\partial \ell}. \tag{13}$$

To proceed further, we need to know $\partial F/\partial \ell$. We shall now determine this quantity by solving Eq. (2) in powers of $\lambda \equiv \rho/R \ll 1$, where ρ is a typical gyroradius of the particle. We adopt the following ordering scheme

$$v_{\|}\frac{\partial}{\partial \ell} \sim \mathcal{O}(\lambda), \qquad \frac{\partial}{\partial t} \sim \vec{v}_d \cdot \nabla \sim \left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha} \sim \mathcal{O}\left(\lambda^2\right),$$

$$\nabla \cdot \left\langle \frac{1}{\Omega} \vec{\Gamma}_v \times \vec{e}_{\|} \right\rangle_{\alpha} \sim \mathcal{O}\left(\lambda^3\right), \qquad F = F_0 + F_1 + \dots . \tag{14}$$

In the first order, Eq. (2) becomes $\partial F/\partial \ell = 0$, which implies $F_0 = F_0(\Psi)$. In the second order we have

$$\frac{\partial F_0}{\partial t} + v_{\|}\frac{\partial F_1}{\partial \ell} + \vec{v}_d \cdot \nabla F_0 + \left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha} = 0. \tag{15}$$

If we set

$$F_1 = -RB_T \frac{v_{\|}}{\Omega}\frac{\partial F_0}{\partial \Psi} + F_1', \tag{16}$$

Eq. (15) reduces to

$$\frac{\partial F_0}{\partial t} + v_{\|}\frac{\partial F_1'}{\partial \ell} + \left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha} = 0. \tag{17}$$

Dividing Eq. (17) by $v_{\|}$ and integrating over ℓ we obtain

$$-\frac{\partial F_0}{\partial t} = \left(\oint \frac{d\ell}{v_{\|}}\right)^{-1} \oint \frac{d\ell}{v_{\|}} \left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha} \equiv \overline{\left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha}}. \tag{18}$$

Combining Eqs. (17) and (18) then yields

$$v_{\|}\frac{\partial F_1'}{\partial \ell} = \overline{\left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha}} - \left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha}. \tag{19}$$

This is the desired result. Substituting expressions (16) and (19) into Eq. (13) we get

$$\Gamma_{\Psi}^{\rm neo} = RB_T 2\pi \sum_{\sigma} \int d\mu d\varepsilon \oint d\ell \frac{\sigma}{\Omega} \left(\left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha} - \overline{\left\langle \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v \right\rangle_{\alpha}} \right). \tag{20}$$

Noting that

$$2\pi \sum_{\sigma} \int d\mu d\varepsilon \sigma \langle \dots \rangle_{\alpha} = \sum_{\sigma} \int \frac{B d\mu d\varepsilon d\alpha}{B\left|v_{\|}\right|} v_{\|} = \frac{1}{B}\int d\vec{v}\, v_{\|}\,, \tag{21}$$

Eq. (20) can be rewritten as

$$\Gamma_{\Psi}^{\text{neo}} = \left(\oint \frac{d\ell}{B}\right)^{-1} RB_T \oint \frac{d\ell}{\Omega B} \int d\vec{v}\, v_{\|} \left(\frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v - \overline{\frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma}_v}\right). \tag{22}$$

As one can see from Eq. (18), the last term in Eq. (22) vanishes for trapped particles.

It can be shown that in collisional transport theory[7] the radial fluxes scale as

$$\Gamma_{\text{col}}^{\text{clas}} \sim \frac{(F_{\text{col}})_p}{B_T}, \; \Gamma_{\text{col}}^{\text{neo}} \sim \frac{(F_{\text{col}})_{\|}}{B_p}, \tag{23}$$

where $\vec{F}_{\text{col}}$ is the collisional friction force density, and the subscript p indicates the component along the poloidal magnetic field B_p. Comparing Eqs. (12) and (22) we see that the RF-induced fluxes satisfy the same scaling as in Eq. (23) but with $\vec{F}_{\text{col}}$ replaced by $\vec{F}_W$, the latter being the force density due to the resonant interaction between the RF waves and particles, i.e. the wave momentum destruction rate. This finding is in agreement with a recent work.[8]

3 Ion Radial Fluxes

We shall now evaluate expressions (12) and (22) considering trapped ions that interact with ICRF waves. For doing this it is necessary to determine the velocity space flux (3) which, it turn, requires the knowledge of the oscillating distribution function $\tilde{F}$. Let us assume the wave field to be of the form

$$\tilde{\vec{E}} = \sum_{\vec{k}} \vec{E}_{\vec{k}} \exp\left[i\left(\int \vec{k} \cdot d\vec{x} - \omega_{\vec{k}} t\right)\right]. \tag{24}$$

The same is true for $\tilde{\vec{B}}$ and $\tilde{F}$. The function $\tilde{F}$ can then be easily found by solving Eq. (1) linearized around F_0. We obtain, noting that F_0 is to be evaluated at the banana centers,

$$\tilde{F}_{\vec{k}} = \frac{iq}{M} \frac{\exp\left[i\left(\xi \sin\varphi - n\varphi\right)\right]}{n\Omega - \omega_{\vec{k}}} \left[E_+ J_{n-1}(\xi) + E_- J_{n+1}(\xi)\right] \frac{v_\perp}{n\Omega}$$
$$\times \left[n\Omega\left(\frac{\partial}{\partial \varepsilon} + \frac{1}{B}\frac{\partial}{\partial \mu}\right) + \frac{1}{\Omega}\left(\vec{k} \times \vec{e}_{\|} \cdot \nabla\Psi - k_{\|} R B_T\right)\frac{\partial}{\partial \Psi}\right] F_0, \tag{25}$$

where

$$\xi = \frac{k_\perp v_\perp}{\Omega}, \qquad \tan\varphi = \frac{\vec{k}_\perp \times \vec{v}_\perp \cdot \vec{e}_{\|}}{\vec{k}_\perp \cdot \vec{v}_\perp}, \tag{26}$$

$$E_\pm = \frac{1}{2}(E_1 \pm iE_2)$$

$$E_1 = \frac{1}{k_\perp}\vec{E}_{\vec{k}} \cdot \vec{k}_\perp, \qquad E_2 = \frac{1}{k_\perp}\vec{k}_\perp \times \vec{E}_{\vec{k}} \cdot \vec{e}_\|, \tag{27}$$

and J_n is the Bessel function. In writing Eq. (25) we have retained only the term corresponding to the nth harmonic of Ω since $\omega_{\vec{k}} \sim n\Omega$ is assumed. Moreover, we have neglected terms proportional to $\vec{k}\cdot\vec{v}_d/\Omega$, $k_\| v_\|/\Omega$ and $\tilde{E}_\|$, which is a good approximation for ICRF waves.

We now eliminate $\tilde{\vec{B}}$ via Faraday's law and combine Eqs. (3), (24), and (25). Substituting the resulting expression into Eqs. (12) and (22), performing the averaging over α and the integration over ℓ then yields

$$\Gamma_\Psi^{\rm clas} = -\left(\frac{2\pi q}{M}\right)^2 \left(\oint \frac{d\ell}{B}\right)^{-1} \int_{t.p.} d\mu d\varepsilon \sum_{\vec{k}} \frac{\vec{k}\times\vec{e}_\| \cdot \nabla\Psi}{\left|v_\| n\Omega'\right|\Omega} \left|E_+ J_{n-1} + E_- J_{n+1}\right|^2 \frac{v_\perp^2}{n^2\Omega^2}$$
$$\times \left[n\Omega\left(\frac{\partial}{\partial\varepsilon} + \frac{1}{B}\frac{\partial}{\partial\mu}\right) - \frac{1}{\Omega}k_\phi BR\frac{\partial}{\partial\Psi}\right] F_0\Bigg|_{\ell=\ell_{\rm res}}, \tag{28}$$

$$\Gamma_\Psi^{\rm neo} = \left(\frac{2\pi q}{M}\right)^2 RB_T \left(\oint \frac{d\ell}{B}\right)^{-1} \int_{t.p.} d\mu d\varepsilon \sum_{\vec{k}} \frac{k_\|}{\left|v_\| n\Omega'\right|\Omega} \left|E_+ J_{n-1} + E_- J_{n+1}\right|^2 \frac{v_\perp^2}{n^2\Omega^2}$$
$$\times \left[n\Omega\left(\frac{\partial}{\partial\varepsilon} + \frac{1}{B}\frac{\partial}{\partial\mu}\right) - \frac{1}{\Omega}k_\phi BR\frac{\partial}{\partial\Psi}\right] F_0\Bigg|_{\ell=\ell_{\rm res}}, \tag{29}$$

where $\Omega' = \partial\Omega/\partial\ell$, $\ell_{\rm res}$ is defined by $\omega_{\vec{k}} = n\Omega(\ell_{\rm res})$, and we have noted $k_\| B_T R - \vec{k}\times\vec{e}_\| \cdot \vec{\nabla}\psi = k_\phi RB$. The notation $\int_{t.p.} d\mu d\varepsilon$ means that we are only integrating over the region of velocity space representing trapped particles. The contribution of passing particles to transport is smaller (and will be ignored here) because their orbit shifts are smaller than the banana orbit widths of trapped particles (see Sec. 4 for more discussion of this). Equations (28) and (29) can be combined to yield

$$\Gamma_\psi = \left(\frac{2\pi q}{M}\right)^2 RB \left(\oint d\ell/B\right)^{-1} \int_{t.p.} d\mu d\varepsilon \sum_{\vec{k}} \left\{ \frac{|E_+ J_{n-1} + E_- J_{n+1}|^2}{|v_\| n\Omega'|\Omega} \frac{v_\perp^2}{n^2\Omega^2} \right.$$
$$\left. \times \left[k_\phi n\Omega\left(\frac{\partial}{\partial\varepsilon} + \frac{1}{B}\frac{\partial}{\partial\mu}\right) - k_\phi^2 \frac{RB}{\Omega}\frac{\partial}{\partial\psi}\right] F_0 \right\}_{\ell=\ell_{\rm res}} \equiv \Gamma_\psi^A + \Gamma_\psi^S \tag{30}$$

As can be seen from Eq. (30), the RF-induced fluxes depend on the symmetry of the RF-spectra. Whereas Γ_ψ^S exists for any shape of the spectrum, Γ_ψ^A vanishes if the spectrum is symmetric in k_ϕ (the toroidal wavenumber). This, however, does not imply that Γ_Ψ^A

vanishes if the spectrum emitted by an antenna is symmetric. What is important is the symmetry of the local spectrum at the resonance points on a given magnetic surface. This local spectrum may have some degree of asymmetry because different spectral components have different propagation characteristics if the rotational transform is non-zero[9].

Estimating $\partial F_0/\partial r \sim F_0/a$, where a is the minor radius, and defining $\rho_\theta = \rho B/B_\theta$ we find that, for a single k_ϕ component, the symmetric flux is smaller than the asymmetric flux by the ratio

$$\left|\frac{\Gamma_\psi^S}{\Gamma_\psi^A}\right| \sim \frac{k_\phi \rho}{n}\frac{\rho_\theta}{a}. \tag{31}$$

For typical ICRF parameters with $k_\phi \rho \ll 1$, the asymmetric flux will dominate unless the k_ϕ asymmetry is very small.

The results derived above are sufficiently general and valid for any axisymmetric tokamak with arbitrary cross-section of the magnetic surfaces. The expressions for the fluxes can be considerably simplified if we consider large-aspect-ratio circular tokamaks. To this end, let (r, θ, ϕ) be the usual (right-handed) toroidal coordinates, where r is the distance from the magnetic axis and θ is the poloidal angle. In the limit $r/R_0 \ll 1$, the flux surfaces become concentric circles and we can use the following conventional model for the field:

$$\vec{B} = (0, \pm\frac{r}{q_s R_0}, 1)\frac{B_0}{1 + r\cos(\theta)/R_0}, \tag{32}$$

where R_0 is the major radius of the magnetic axis. Note that $B_\theta > 0$ for a plasma current flowing in the positive ϕ direction. Some useful relationships in this simplified geometry are

$$\int \frac{d\ell}{B} = \frac{2\pi q_s R_0}{B_0}, \quad \Omega' = \frac{\Omega_0 r \sin\theta}{q_s R^2}, \quad \Gamma_\Psi = R B_\theta \Gamma_r. \tag{33}$$

We now choose $n = 1$ and assume $k_\perp \rho \ll 1$. The Bessel functions in Eq. (30) can then be approximated as $J_0 = 1.0$ and $J_2 = 0.0$ and the integration over ε and μ becomes trivial. Thus, upon substituting expressions (33) into Eq. (30), we finally obtain

$$\Gamma_r = \Gamma_r^A + \Gamma_r^S, \tag{34}$$

$$\Gamma_r^A = -\frac{B}{B_\theta}\left[\frac{R^2}{R_0 r|\sin\theta|}\sum_{\vec{k}}\left|\frac{cE_+}{B}\right|^2 \frac{k_\phi}{\omega} N_{\text{t.p.}}\right]_{\text{res}}, \tag{35}$$

and

$$\Gamma_r^S = -\left(\frac{B}{B_\theta}\right)^2 \left[\frac{R^2}{R_0 r|\sin\theta|}\sum_{\vec{k}}\left|\frac{cE_+}{B}\right|^2 \frac{k_\phi^2}{\omega^2\Omega}\frac{\partial}{\partial r}\left(N_{\text{t.p.}}\overline{v_{\text{t}\perp}^2}\right)\right]_{\text{res}}, \tag{36}$$

where $N_{\rm t.p.} = \int_{\rm t.p.} d\vec{v}F_0$ is the density of trapped particles in the resonance layer, and $\overline{v_{\rm t\perp}^2} = \int_{\rm t.p.} d\vec{v}F_0 v_\perp^2/(2N_{\rm t.p.})$. Equations (30) and (34)–(36) represent the main results of this paper.

4 A Single Particle Model

RF driven transport can also be analyzed from the single particle viewpoint. As a particle passes through a resonance where $\omega - k_\parallel v_\parallel = \Omega(R)$, it will absorb or lose some energy from the RF. For small $k_\parallel v_\parallel/\omega$, cyclotron damping primarily affects $v_\perp$, which changes the width of a banana orbit but not the radial location of the banana tip. But for $k_\parallel \neq 0$, there are small changes in $v_\parallel$ which cause the banana tip to move radially, thus causing net transport.

As an example of the use of the single particle model, we will briefly consider standard neoclassical transport driven by collisions. Banana orbits in an axisymmetric tokamak can be examined using conservation of energy, magnetic moment, and toroidal angular momentum P_ϕ[10]:

$$P_\phi = RMv_\phi + \frac{q}{c}RA_\phi = RMv_\phi - \frac{q}{c}\Psi(r)$$

where $RA_\phi = -\Psi(r)$ is the poloidal flux function. The radial position $r_{\rm tip}$ of the banana tip of a trapped particle is given by $P_\phi = -\frac{q}{c}\Psi(r_{\rm tip})$, since $v_\phi \approx v_\parallel = 0$ there. Using $\partial\Psi/\partial r = RB_\theta$, we find that if collisions induce a change $\delta v_\parallel$ at some point along the particle's orbit, then the banana tip will be displaced radially by the distance

$$\delta r = -\frac{\delta v_\phi}{\Omega_\theta}. \tag{37}$$

($\Omega_\theta = qB_\theta/Mc$ is positive for plasma current in the positive ϕ direction.) This leads to a diffusion coefficient for trapped particles given by

$$D_{\rm t.p.} = \frac{\langle(\delta r)^2\rangle}{2\langle\delta t\rangle} = \frac{\nu_{ii}v^2}{2\Omega_\theta^2} \tag{38}$$

assuming that $v_\phi \ll v$ so that the main cause of δv_ϕ is pitch angle scattering. In the $v \gg v_{th,i}$ limit, ν_{ii} is given by

$$\nu_{ii} = \frac{9.009 \times 10^{-8} Z^2 Z_{eff} n_e \log\Lambda}{A^{1/2}W^{3/2}}{\rm sec}^{-1}$$

where A and Z are the particle's atomic mass and charge, n_e is the electron density per cm^3, and W is the particle's energy in eV. The radial transport of passing particles will be ignored because their orbit shifts are typically much smaller than the banana widths of trapped particles. The fraction of particles which are trapped is $\approx .9\sqrt{r/R}$. The thermal conductivity is found from the appropriate average of $D_{\text{t.p.}}$ over all trapped particles, $\chi_i \approx .9\sqrt{r/R}(\int dE \exp(-E/T_i)ED_{\text{t.p.}})/(\int dE \exp(-E/T_i)E)$. This expression is within 10% of the rigorous derivation of χ_i in the banana regime[7]:

$$\chi_i = \frac{2}{3}\sqrt{\frac{r}{R}}\frac{\nu_{ii}(2T_i/M_i)}{\Omega_\theta^2}$$

where ν_{ii} is evaluated at $W = \frac{3}{2}T_i$.

We now return our attention to the case of RF driven transport. The rate of change of P_ϕ due to asymmetric fields is

$$\frac{dP_\phi}{dt} = -\frac{\partial H}{\partial \phi} = \frac{q}{c}\vec{v}\cdot\frac{\partial \vec{A}}{\partial \phi} - q\frac{\partial U}{\partial \phi}.$$

Using $\vec{E} = \text{Real}\left(\vec{E}_0 \exp(i\int \vec{k}\cdot d\vec{x} - i\omega t)\right) = -\nabla U - (1/c)\partial\vec{A}/\partial t$ and the Coulomb gauge $\nabla\cdot A = 0$, this can be written as

$$\frac{dP_\phi}{dt} = \frac{qRk_\phi}{\omega}\left[\vec{v}\cdot\vec{E} - \frac{\vec{k}\cdot\vec{E}}{k^2}(\vec{k}\cdot\vec{v} - \omega)\right] \tag{39}$$

The electric field is highly oscillatory so that averaging over time leads to zero net change in P_ϕ, except when a resonance occurs between $\vec{E}$ and the gyrofrequency oscillations in $\vec{v}$. The total displacement of a particle's banana tip after it has passed through a resonance can be related to its change in energy by integrating Eq. (39):

$$\delta r = -\frac{1}{MR\Omega_\theta}\int dt\frac{dP_\phi}{dt} = -\frac{k_\phi \delta W}{M\omega\Omega_\theta} \tag{40}$$

where we have noted that the second term in the brackets of Eq. (39) is a perfect time derivative and has no resonant contribution. We have also assumed that the leading R of Eq. (39) can be treated as a constant during the integral over a resonance. Including the small gyroradius oscillations in R could lead to additional transport, but it would be smaller by a factor of $\sim (kR)^{-1}$. A result similar to Eq. (40) can be found by starting with the constraint on quasilinear heating found by Kennel and Engelmann[12], $v_\perp^2 + (v_\parallel - \omega/k_\parallel)^2 = \text{constant}$, expanding to find $\delta v_\parallel = k_\parallel \delta v^2/(2\omega)$, and inserting into Eq. (37) to get $\delta r \approx -k_\parallel \delta W/(M\omega\Omega_\theta)$.

We follow the procedure outlined by Stix[11] for calculating the change in energy as a particle passes through a resonance. Our result will be valid for fundamental absorption at $\Omega = \omega$ in the $kv/\omega \ll 1$ limit, where $\delta W \approx \delta W_\perp$ and finite gyroradius effects involved in the usual Bessel functions or the Doppler shift of the resonance layer can be ignored. The perpendicular components of the equation of motion are

$$\frac{\partial v_x}{\partial t} - \Omega(t)v_y = \frac{q}{M}\text{Real}\left(E_x e^{-i\omega t}\right)$$

$$\frac{\partial v_y}{\partial t} + \Omega(t)v_x = \frac{q}{M}\text{Real}\left(E_y e^{-i\omega t}\right),$$

where $\Omega(t)$ is the local gyrofrequency seen by the particle as it moves along the field line. Defining $u = v_x + iv_y$ and $E_\pm = \frac{1}{2}(E_x \pm iE_y)$, and ignoring the non-resonant E_- term, the solution can be written as

$$u(t_1) = e^{-i\int_{t_0}^{t_1}\Omega dt}[u(t_0) + \frac{q}{m}E_+\int_{t_0}^{t_1} e^{-i\int_{t_0}^{t}(\omega-\Omega)dt'}dt].$$

The integral is approximated by the method of stationary phase, expanding $\Omega(t) \approx \omega + \dot{\Omega}t$ near the resonance, where $\dot{\Omega} = v_\parallel \partial\Omega/\partial\ell$. The change in energy as the particle passes through a resonance is given by

$$\delta W_\perp = \frac{M}{2}\left[|u(t_1)|^2 - |u(t_0)|^2\right] = \frac{q^2|E_+|^2\pi}{M|\dot{\Omega}|} + \text{Real}\left(u^*(t_0)qE_+e^{-i\dot{\Omega}t_0^2/2+i\pi/4}\sqrt{\frac{2\pi}{|\dot{\Omega}|}}\right).$$

Particles may either absorb or lose energy, depending on whether they are in or out of phase with E_+. Assuming that the wave-particle phase is random, the average change in energy as a particle passes through the resonance layer is

$$\langle\delta W_\perp\rangle = \frac{\pi q^2|E_+|^2}{M|v_\parallel\partial\Omega/\partial\ell|}, \tag{41}$$

where all quantities are evaluated at resonance. It is also useful to know that $\left\langle(\delta W_\perp)^2\right\rangle - \langle\delta W_\perp\rangle^2 = 2W_\perp\langle\delta W_\perp\rangle$. Defining the time required for a particle to travel from one banana tip to the other as $\tau_B = \int d\ell/|v_\parallel|$, and noting that a particle passes through the resonance layer twice in this time, we find that the RF causes particle banana tips to move radially at the velocity:

$$V_{r,rf} = \frac{\langle\delta r\rangle}{\tau_B/2} = -\frac{2k_\phi\langle\delta W_\perp\rangle}{M\omega\Omega_\theta\tau_B}. \tag{42}$$

Equation 41 says that particles which pass through the resonance layer slower (have smaller $\left(v_\parallel\right)_{\text{res}}$) will absorb more energy. The singularity at $v_\parallel = 0$ can be eliminated by including finite $k_\parallel$ or higher order time derivatives in the expansion of $\Omega(t) \approx \omega - k_\parallel v_\parallel(t) + \dot{\Omega}t + \ddot{\Omega}t^2/2$ near the resonance. In many cases these corrections can be ignored since the $v_{\|}$ singularity in Eq. (41) is integrable. As Stix found, integrating Eq. (41) over all particles leads to a finite expression for the absorbed RF power. Likewise, integrating Eq. (42) over all particles leads to a finite total particle flux. The number of particles per second which pass through the resonance layer is given by $2 * 2\pi R|B_\theta/B| \int d^3v f|v_{\|}|$. Each of these particles' banana tip will move radially by a distance $\langle \delta r \rangle \propto 1/|v_{\|}|$. Ignoring the small contribution of passing particles to the transport, we calculate the total radial convective flux to be

$$\Gamma_r^A = \frac{4\pi R|B_\theta/B| \int_{t.p.} d^3v f|v_{\|}|\langle \delta r \rangle}{2\pi r 2\pi R_0} = -\frac{B}{B_\theta}\left(\frac{R^2}{R_0 r|\sin\theta|}\frac{k_\phi}{\omega}\left|\frac{cE_+}{B}\right|^2 N_{\text{t.p.}}\right)_{\text{res}},$$

where $N_{\text{t.p.}}$ is the density of trapped particles in the resonance layer. Integrating over the wave k_ϕ spectrum produces Eq. (35).

Although we will not go through the details here, a complete Fokker-Planck equation for the evolution of f can be derived from this single particle model. There will be terms proportional to $\langle \delta W_\perp \rangle$ and $\left\langle (\delta W_\perp)^2 \right\rangle$ which describe quasilinear heating, a term proportional to $\langle \delta r \rangle$ which results in Γ_r^A found above when integrated over all particles, and other transport terms proportional to $\langle (\delta r)^2 \rangle$ and $\langle \delta r \delta W_\perp \rangle$. In particular, the diffusive Γ_r^S flux of Eq. (36) is proportional to the radial diffusion coefficient

$$D_{r,rf} = \frac{\langle (\delta r)^2 \rangle - \langle \delta r \rangle^2}{2(\tau_B/2)} = \frac{2k_\phi^2 W_\perp}{M^2\omega^2\Omega_\theta^2}\frac{\langle \delta W_\perp \rangle}{\tau_B}. \tag{43}$$

5 Discussion

It is possible to assess the importance of ICRF-driven transport using Eqs. (42) and (43) without a detailed knowledge of $\langle \delta W_\perp \rangle$, instead using a simple energy balance to estimate its order of magnitude. During ICRF minority heating, the ICRF wave directly heats a small concentration of resonant minority ions, which then transfer their energy to the rest of the plasma via collisions. The resonant minority ions heat up until a balance occurs between the RF power per particle, which is of order $\langle \delta W_\perp \rangle / (\tau_B/2)$, and the collisional drag, which is of order $(W - \frac{3}{2}T)/\tau_W$, where τ_W^{-1} is the rate of energy equilibration between the minority ions and the thermal plasma which is at temperature T. (A similar result could be obtained by using Eq. (41) relating $\langle \delta W_\perp \rangle$ to $|E_+|^2$, and using Stix's quasilinear formulas[11] to relate $|E_+|^2$ to the average energy of the minority ions.) Assuming that the RF power is large enough so that the average energy of the minority ions, $\sim W$, satisfies $W \gg \frac{3}{2}T$, we can use the balance between RF heating and collisional drag,

$$\frac{\langle \delta W_\perp \rangle}{\tau_B/2} \sim \frac{W}{\tau_W}, \tag{44}$$

to write the order of magnitude of RF-driven convection and diffusion as

$$V_{r,rf} \sim \overline{k_\phi} \rho \frac{\rho_\theta}{\tau_W} \qquad D_{r,rf} \sim \overline{k_\phi^2} \rho^2 \frac{\rho_\theta^2}{\tau_W},$$

where $\overline{k_\phi}$ is an average of k_ϕ over the local wave spectrum. We emphasize that the E_+ dependence of $V_{r,rf}$ and $D_{r,rf}$ has been hidden in the velocity dependence of these expressions. This form allows easy comparison with neoclassical transport using Eq. (38),

$$\frac{\Gamma_r^A}{\Gamma_r^{\rm neo}} \sim \frac{V_{r,rf}}{D_{\rm t.p.}/a} \sim \overline{k_\phi} R \frac{a^2}{R^2 q} \frac{1}{\tau_W \nu_{ii}}.$$

The quantity $\tau_W \nu_{ii}$ measures the amount of pitch angle scattering that occurs in a slowing down time, and can be written as

$$\tau_W \nu_{ii} = \frac{Z_{eff}}{2A \langle Z_i^2/A_i \rangle \left(1 + W^{3/2}/W_c^{3/2}\right)},$$

where $\langle Z_i^2/A_i \rangle = \sum n_i Z_i^2/A_i$, and $W_c = 14.8\, A T_e \langle Z_i^2/A_i \rangle^{2/3}$ is the 'critical' energy above which drag due to electrons exceeds drag due to ions. At moderate energies below W_c, $\tau_W \nu_{ii} \sim 1$, while at higher energies pitch angle scattering becomes negligible. If there is a very strong asymmetry in the wave spectrum so that $\overline{k_\phi} R \sim 10 - 20$, then RF driven convection could be comparable to neoclassical diffusion if the minority ion energy is below W_c, while RF driven convection could dominate if the RF power is sufficiently large to make the average minority ion energy exceed W_c. The RF drives particles inward if the wave is travelling in the same direction as the plasma current, or outwards for counter-going waves. It is conceivable that antennas which preferentially launch waves in one direction may be able to improve the confinement of the minority ions, or may be able to remove unwanted helium ash. In the usual case however, the wave spectrum will be nearly symmetric and RF driven convection will be small compared to neoclassical diffusion. From Eq. (31) we see that RF-driven diffusion, in contrast to convection, will almost always be negligible.

Strictly speaking, neoclassical *particle* transport proceeds at a very slow electron transport rate which is $\sqrt{M_e/M_i}$ slower than neoclassical ion heat transport because of momentum conservation constraints. However, it is experimentally observed that particle transport is much faster than neoclassical predictions, and there is apparently a strong, anomalous inward convective term as well. Therefore, we have been comparing RF driven transport with a neoclassical diffusion coefficient which includes ion-ion collisions, Eq. (38). Although Eq. (38) is theoretically only appropriate for heat transport, it is also of the same order of magnitude as experimentally observed particle transport.

A term similar to Γ_r^S of Eq. (36) has been obtained in a recent work[4]. Among other differences, their answer is smaller than ours by a factor of $\sim (B_\theta/B)^2$ because of an error in their Eq. (39) which used $\partial\Psi/\partial r = RB$ instead of $\partial\Psi/\partial r = RB_\theta$. They do not mention the existence of a convective term which depends on the asymmetry in the k_ϕ spectrum. Furthermore, they compare RF-driven transport with neoclassical *electron particle* transport, which is negligible compared to experimental observations. Although the results of S.C. Chiu[6] are not in a form which can be easily compared with ours, he does point out the existence of symmetric and asymmetric ICRF transport terms and also concludes that they are usually small compared to neoclassical transport. Our results are in disagreement with another recent paper[5] which found an expression for RF-driven transport that was independent of k_ϕ.

Since the RF only heats the bulk plasma indirectly through the resonant minority ions, it is useful to ask how much broader the bulk plasma heating profile is than the initial RF absorption profile (which is theoretically predicted to be very peaked) due to radial transport of the intermediary resonant ions. The relevant criterion is the distance that a particle may be displaced in an energy transfer time τ_W. (This criterion comes from solving a model heat transport equation for the minority ions of density n_{m} and average energy $\frac{3}{2}T_{\mathrm{m}}$,

$$\frac{\partial}{\partial t}n_{\mathrm{m}}T_{\mathrm{m}} = 0 = \frac{2}{3}P_{rf} - \frac{n_{\mathrm{m}}(T_{\mathrm{m}} - T)}{\tau_W} + \nabla \cdot \left[-V_{r,rf}n_{\mathrm{m}}T_{\mathrm{m}} + \nabla\left(D_{r,rf}n_{\mathrm{m}}T_{\mathrm{m}}\right)\right]$$

in the limit of short τ_W and large P_{rf}.) Again using Eq. (44), we find RF-driven convection may lead to a displacement of order $\Delta r = V_{r,rf}\tau_W \sim \overline{k_\phi \rho}\rho_\theta$, RF-driven diffusion will lead to $\overline{\Delta r} = \sqrt{2D_{r,rf}\tau_W} \sim \sqrt{\left(\overline{k_\phi^2}\rho^2\rho_\theta^2/2\right)}$, and neoclassical diffusion will lead to $\left(\overline{\Delta r}\right)_{\mathrm{neo}} = \sqrt{2D_{\mathrm{t.p.}}\tau_W} \sim \rho_\theta\sqrt{\tau_W\nu_{ii}}$. Since the RF power absorbed by the minority ions is spread out by at least a distance of order $(r/R)^{1/2}\rho_\theta$ because of the banana width alone, we can see that broadening due to ICRF-driven transport is negligible if $k_\phi\rho \ll 1$, a condition which is satisfied in most experiments. Neoclassical transport may lead to some broadening of the heating profile for moderate minority energies $W < W_c$ where $\tau_W\nu_{ii} \sim 1$.

It is difficult for the RF-driven heat flux to be important because of the shortness of τ_W, but RF-driven particle transport can operate on a longer time scale because collisions conserve particles. In turn, this can affect the RF heating profile, since $P_{rf} \propto n_{\mathrm{m}}(r)\left|E_+(r)\right|^2/r$.[11] Most models in the past have assumed that the minority density profile $n_{\mathrm{m}}(r)$ is simply proportional to the electron density profile. The issue of particle transport in tokamaks is not very well understood, but it seems that ICRF may change $n_{\mathrm{m}}(r)$ either directly, if the k_ϕ spectrum is sufficiently asymmetric that Γ_r^A is large, or indirectly, if the minority ions become sufficiently energetic that they become decoupled from the assumed turbulence-driven inward pinch needed to explain the peaked density profiles observed in tokamaks.

Although our present calculations would need some generalization before detailed application to the case of RF-driven α transport (since our quasilinear calculations assumed $k_{\|}v_{\|}/\omega \ll 1$ and our single particle calculations of $\langle \delta W_\perp \rangle$ assumed $\vec{k}\cdot\vec{v}/\omega \ll 1$, while for CIT-type parameters, 3.5 MeV α-particles have a gyroradius $\rho \sim 3.0(v_\perp/v)cm$, $k_\phi \sim .1$ cm^{-1} and $k_\perp \sim 1.0$ cm^{-1}), it is still possible to draw some general conclusions based on Eq. (42). We can not use Eq. (44) as before since α's are energetic because they are produced by fusion reactions and not necessarily because they have absorbed much RF power. Instead noting that the RF power absorbed by the α's, $P_{rf\to\alpha}$, is of order $n_\alpha \langle \delta W_\perp \rangle_\alpha / (\tau_B/2)$, and that the α heating power of the main plasma, $P_{\alpha\to\mathrm{plasma}}$, is of order $n_\alpha W/\tau_W$ we find that the RF may displace an α-particle by the distance

$$\Delta r = V_{r,rf}\tau_W \sim \frac{P_{rf\to\alpha}}{P_{\alpha\to\mathrm{plasma}}}\overline{k_\phi}\rho\rho_\theta$$

during a slowing down time. We expect that as long as $P_{rf\to\alpha}$ is small compared to $P_{\alpha\to\mathrm{plasma}}$, RF-driven transport of α's can be ignored during their slowing down. Again, this differs from the conclusions of a recent paper[4] because they compare RF-driven transport with neoclassical transport, which is negligible for α's during their slowing down.

While transport driven directly by the RF is probably unimportant in most experiments, there are a number of other mechanisms which may be playing important roles. In some experiments, the RF may produce energetic ions which have such large banana widths that unconfined orbit losses are important. Additional transport may be caused by large banana width neoclassical effects, toroidal field ripple, sawtooth and fishbone instabilities, or other instabilities.

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