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ABSTRACT

A limited numerical optimisation of the current and pressure profiles in Tokamaks of various shapes and aspect ratios has led to a simple scaling law for the maximum volume averaged β which can be reached before the onset of ideal MHD instabilities: $\beta < 2.0\text{-}2.5 \mu_0 I/Ba$, where I is the total current, B the toroidal field at the centre of the plasma and a the minor plasma radius in the equatorial plane [1]. The limit is set by the appearance of a n = 1 free boundary instability, neglecting the possible stabilizing effect of a shell. The highest βs attained in tokamaks equipped with strong auxiliary heating are shown to be very near the limit given by this scaling law in spite of their very different conditions of operation. Possible implications are discussed.

One of the main problems which may affect the future of the Tokamak configuration is the possible existence of a maximum plasma pressure which can be confined without immediate loss of confinement. Ideal MHD instabilities, which grow typically on the microsecond Alfvén time scale, are prime candidates for such a limit. In the last 10 years many attempts have been made to find numerically this MHD limit, [2-7], without knowing if it is relevant to experiments. Tokamaks do operate in a restricted range of current and density [8], limited by the onset of disruptions, but the orthodox interpretation of disruption is that it is due to resistive MHD instabilities followed by overlap of islands and bears no relation to any β limit. With the availability of large neutral beam power many attempts have been and are still being made to heat the plasma to high β values and test the existence of a pressure limit. The saturation of β due to a degradation of confinement seen in the L-regime [9] is generally considered as evidence of an approaching soft β limit due to the onset of internal instabilities, maybe ballooning instabilities or internals kinks, but ASDEX sees no saturation in the H-regime up to the highest power level available. The fact that free boundary instabilities are expected to appear at lower β than the optimized ballooning limit is usually discarded on the basis of two arguments. Firstly, the comparison between the β reached in some Tokamaks and theoretical predictions obtained with numerical codes, using what was believed to be reasonable current and pressure profiles, have seemed to indicate that the experimental value was much above the calculated β limit [10,11]. Secondly, the free boundary calculation assumes a sharp plasma boundary and neglects the tenuous plasma which surrounds the plasma column in the shadow of limiters or in the scrape-off layer. The shell and the limiter are conductors which may stabilize on the short time scale the fast growing kinks which are then stabilized on the long time scale by some other phenomena.

In this letter we show that, except for a couple of exceptions which will be discussed in some detail, the highest values of β obtained in present devices correspond, both in magnitude and in its dependence on the plasma parameters, to the n=1 free boundary stability limit found by numerical optimisation of the pressure profile in a number of different configurations. It means that the n=1 ideal MHD

β limit has not yet been convincingly overcome. It also suggests that this limit could be a hard one to cross. It is certainly strange that such a variety of experiments could all reach values of β close to the n=1 limit and that this would only be due to a lack of heating power. Here are the details.

In the last 3 years we have made a number of numerical studies of β limits in Tokamaks of various shapes and elongations, optimizing with a few parameters the pressure and current profiles to reach the highest value of β stable to all ideal MHD modes while keeping the current and the plasma shape fixed [1,12,13]. The qualitative and quantitative results are not fundamentally different from the earlier results. But when comparing these results some general trend becomes evident. First, without any shell stabilization, the n=1 free boundary stability limit gives the lowest β value. In all the equilibria tested where the ballooning criterion became violated at a lower β than the n=1 free boundary limit, it was possible by a small adjustment of the pressure profile to push the ballooning limit at or above the n=1 limit, at least as long as q_0 remains a little above 1. Secondly, the ratio

$$R = \beta aB/\mu_0 I,$$

where β is the maximum stable value, a the horizontal half-width of the plasma, B the toroidal field in the middle and I the total current, was found to always lie between 2.0 and 2.5, up to a maximum value of the current corresponding to the true safety factor at the plasma surface slightly larger than 2 [1]. Beyond that point we have not found any simple general result, except that β drops strongly. The residual variation of R is due in part to the numerical inaccuracy associated with the discretisation, to the optimisation procedure but also to some residual dependence on the other parameters, in particular on q_s and on the geometry, which we have not fully identified. Any optimisation is limited by the number of free parameters and the class of functions used to describe the profiles. We have tried to improve the β by extending the choice of expressions and parameters, but we have not changed so far the result significantly. This does not mean it is not possible to do better by using very different expressions.

The simplicity of this result has led to a search through the published experimental data to compare both the parametric dependence and the absolute value of the β limit. The data assembled is listed in Table 1 and compared in Figure 1 with the scaling law. Only near record values of β ($R > 1.8$) have been compiled since we are only interested in verifying the magnitude and the parametric dependence of the highest β achieved in each installation in a wide range of conditions. We only use published data in which there is a minimum of information to compute a credible value of R. We have nonetheless encountered a number of difficulties.

The first difficulty is the lack of information on the plasma halfwidth a, defined as the half-width of the last closed magnetic surface which does not intersect a limiter. When the distance between inside and outside limiters is not quoted we define a as half this distance. This leads to a systematic overestimate of the ratio R which can become important at high β, whenever the positioning system which centers the plasma does not fix a given flux surface, as in ASDEX, but only keeps the position of the center of gravity of the current.

Another problem is in the different definitions of β. In the scaling law derived numerically, $\beta = \int p dV / \int B^2/2\mu_0 dV$. Experimentalists use preferentially, while still calling it volume average, the definition $\beta = \int p dV/(B^2/2\mu_0 V)$, where B_0 is the vacuum field at the geometric center, or, when derived from a β_p measurement, the surface average value $\beta = \int p dS/(B^2/2\mu_0 S)$. From all these definitions the first one used in the scaling gives always the lowest value, but the difference is only a few percents except for strongly elongated plasmas or at high β_p. We do not make any adjustements as it is within the accuracy of the numerical results, although the fact that it is systematic should be kept in mind when doing the comparison between experiment and theory.

The most serious difficulty is to assess the accuracy of the measurements of β. The integration of the pressure profile obtained from local and instantaneous measurements of n_e, T_e and T_i is in principle a foolproof method to calculate β and guarantee the absence of anomalies in the profiles associated with the substantial nonthermal contribution or a possible rotation due to the strong neutral injection heating. This can be done for n_e and T_e with Thomson scattering but only PDX can perform a full instantaneous measurement of these profiles. Discharges with near record β values are usually not sufficiently reproducible to allow a shot by shot measurement of these profiles so that global measurements and some inversion or fitting must be used instead. The PDX results obtained with Thomson scattering at high power reveal an extreme radial asymmetry of the density while the electron temperature remains symmetric. This is a warning that the equilibrium does not always have an isotropic pressure or is nonstatic, and that straightforward inversion of global measurements, specially if the scan covers only part of the discharges, should be viewed as one element of information to be carefully cross-checked. Another point not mentionned in any of the papers is the fact that the ion density may be less than the electron density, due to impurities. This is already important at reasonably low Z_{eff} and worsens rapidely if there are any high Z impurities. Finally the beam contribution to β is calculated, which implies additional assumptions on the profiles. Thus this determination of β should be taken with caution.

Magnetic measurements are more direct. Measurement of the poloidal flux and the vertical field around the plasma allows a reconstruction of the equilibrium, if assumed static and isotropic, and gives $\beta_p+l_i/2$. An even better, remarkably accurate measurement of β_p, can be made with a diamagnetic loop. Apart from the well known technical difficulty of measuring such a small change of toroidal flux, the accuracy of this measurement depends on the aspect ratio, elongation and β_p but we have found in numerical simulation that the exact value lies always within a few percents of the value calculated from the flux through the diamagnetic loop. But in many of the experiments reported here the two values of β_p obtained through the poloidal flux and the diamagnetic loop are very different in the high β regime. This is usually interpreted by an anisotropic ion distribution due to the beam or a rotation but no reconstruction of the equilibrium has been made to verify this interpretation because of lack of a suitable equilibrium code which incorporates these new elements. The extreme asymmetry of the radial density profile in the equatorial plane of PDX supports this interpretation. The end result is an uncertainty on the final β value which cannot be easily quantified. This difficulty is now recognized and has led to a reappraisal of some published results in subsequent papers and explain that groups keep reanalysing their data and give apparently contradictory values of β.

Let us look at the results of Table 1 for each experiment.

The results of PDX are drawn from a systematic study of the high β regime at very high beam power [14]. The first five values at B=1.2 T are the end points from power scans carried out at fixed currents (Figure 6 of the reference). Each value represents an average of 3 to 5 discharges with the same current, using all the neutral beam power available. The value at 0.7 T is calculated from β_p=1, average value of 5 discharges as seen in Fig. 2 of the reference. The value of R calculated from Fig. 7 in the same reference is 10 % higher but since the experimental points seem to be the same as in Fig. 2 we have chosen to compute R from β_p=1 which should be accurate but does not include any anisotropic effect. At 1 T we use Fig. 7, since β_p is not given anywhere, remembering that it may not be fully consistent with the result at 0.7 T. There is more information in this reference but it is not sufficient to compute R so we disregard it. Nevertheless all the data supports an empirical scaling law $\beta \sim I_N^{1.15}$ which is indistinguishable from the scaling proposed in this paper in the operating range of PDX. As mentionned above the radial profiles of T_e and n_e measured in a single shot, single time, during injection are incompatible with a scalar pressure static equilibrium. This seems to be always the case in PDX, with quasi-perpendicular injection, and it is not explained. The fact that the highest βs achieved are, within the accuracy of the measurements and of the calculation of β from the data, below or on the limit predicted, is thus rather surprising.

The results of Doublet III are drawn from 2 papers [15 and 16] and a set of values provided by T.N. Todd. The two highest β values (beam included) have been obtained with elongated plasmas (1.46 and 1.6 respectively) while the lowest value is obtained with a circular plasma. The record value of 4,5 % has an estimated absolute error bar of ± 0.5 % but since it is an average between a diamagnetic measurement, an MHD equilibrium fit and a transport analysis the error quoted may be misleading. There is also a substantial pressure anisotropy and maybe some rotation.

ISX-B has been operating for a long time and a large amount of data has been collected. Both circular and elongated plasmas have been studied, up to an elongation of 1.45. In the early discharges [17] β was determined by poloidal field measurements cross-checked with partial profile informations, when available. The large anisotropic beam contribution is calculated and not measured. Full consistency was not achieved at that time between the magnetic measurements and the profile measurements. The first seven lines in Table 1 are from this early data bank and they include two record values for R, 2.77 and 2.96, the contribution from the beam being included. The next two results are extracted from a more recent paper [18] which contains studies of the parametric dependence of β versus power, current and magnetic field. There, β is computed exclusively from magnetic measurements and equilibrium reconstruction. The most recent results are extracted from a paper which describes ideal MHD stability tests made on these equilibria to show that they are marginally stable to the n=1 external mode [19]. A more detailed analysis of recent discharges has been reported [20]. In particular one of the discharges of [19] has been reanalysed and β has come down 10 %, which gives an idea of the error bar on β for this series of measurements. It is also stated that, while the values of β_p obtained from the diamagnetic loop and from equilibrium reconstruction agree very well at $\beta_p \ll 1$, they systematically differ at $\beta_p > 1$, the diamagnetic value being always smaller. This is the same behavior as in Doublet III, JFT-2 and T-11. The record values of R correspond to very high β_p (1.7-1.9) but the diamagnetic loop was apparently not yet available so that one does not have a measure of the error bar which should be attached to these values.

The ASDEX result [9] is remarkable as it has the highest poloidal β_p=2.65, determined from magnetic measurements and checked for consistency, and yet it still agrees with the scaling law. ASDEX has concentrated on the transport rather than the β limit so that there is little data usable for this study. Heating is also by neutral beam injection and the highest β_p is obtained only in the H-regime.

T-11 has a very distinctive feature. It has a shell with a time constant larger than the duration of the discharge. This allows easy operation at low q and one might expect a higher β limit. Operation at a β=3 % has apparently been achieved transiently but in such an unreproducible way that no diagnosing could be made [22]. The value quoted here is the largest β documented. The β=2.1 % is given in a first paper [21]. It is computed from a $\beta_p + l_i/2$ measurement. A diamagnetic loop gave a much smaller β_p, corresponding to a β of 1.3 % only, while the profile measurements gave an even lower β. The difference was ascribed to a very strongly anisotropic distribution due to the strong injection (0.8 MW), but such an extreme anisotropy is difficult to accept without questioning the whole procedure of determining β. In a more recent paper [22] a power scan in the same conditions gave a maximum value of β=1.7 %, shown in brackets in Table 1, but there is no mention of the anisotropic beam contribution.

The result of JFT-2, described in [23], gives a record R. It is so much above the scaling prediction that it needs a closer examination. The thermal component of β is determined by integration over the density and the temperature profiles. Magnetic measurements are said to be in agreement with this value within the experimental accuracy but this accuracy is not given. If true it would imply a strong pressure anisotropy. In a more recent paper [24] β_p is given but there is no comment on the beam contribution. There are some points of consistency which can be raised. The value of β on axis, calculated from the profiles shown, is β_0=6.6 %. The average value of 2.6 % quoted in the paper for the thermal component seems too high for this β_0. We have never seen such a small ratio of β_0/β in our numerical optimisation. Using a more acceptable ratio of 3 to 4 would bring the average thermal β in the 2 % range. The crucial elements are the three profiles, density, electron and ion temperatures, which look poorly determined (Figure 12 of ref. 24). In such a high density regime the electron temperature must have been determined by X-rays and it is known to be unreliable at temperatures of a few hundreds of eV. The density profile is fitted through one point. Also, the assumption $n_i=n_e$ must be made to obtain the 6.6 % quoted above from the profiles. It may be questioned since the radiated energy increases up to 20 % of the total input power of 1 MW and the loop voltage falls only by 40 %

while the electron temperature is claimed to double. This result does not seem to be sufficiently documented to consider the limit given by the scaling law as definitely surpassed.

In the experiments listed above, heating is done by NBI at extreme power levels and there is strong evidence in every experiment that the equilibrium is not a static isotropic equilibrium, but the data does not allow a quantification of the effect of a possible rotation or of a strongly anisotropic pressure on stability. It would be extremely interesting to have an equivalent study made with RF heating which would, independently of its usefulness, help to clarify the role of the non thermal beam component or of a rotation in the ultimate β that can be reached. The only evidence so far has been obtained in TOSCA with ECE [25] which showed that β scaled linearly with the current. TOSCA is a device in which the plasma continuously evolves. It makes the evaluation of the shape and size of the plasma very difficult so that it is difficult to calculate precisely R. Nevertheless the values of Table 1, selected as a typical subset of a dozen discharges provided by the TOSCA group [26], are at or below the predicted limit.

In conclusion there is agreement between the proposed scaling law for the β limit and the highest β achieved in tokamaks. If it is not a coincidence this agreement can be considered as surprising since the limit has been calculated for a free boundary plasma with no shell and a partially optimized profile and there is a priori no reason that the limit be effectively reached. Coincidence cannot be excluded. For example the ballooning limit seems to obey a same scaling law with a larger coefficient [27] so that the degradation of the confinement may be due to these ballooning instabilities alone which will be overcome with much more power. But it may also be that the limit given here is a disruptive limit as the density limit independent of the soft ballooning β limit. The fact that the highest β in ASDEX is reached only in the H regime with no degradation but that there are no discharges yet above the scaling would be an indication of such a behaviour. More experiments are clearly needed to test this law, in particular in very different ranges of shapes, aspect ratios, magnetic fields and with different heating schemes. Since at high β the confinement time seems to be independent of the toroidal field and of density and increase linearly with the current, it should be possible, in every tokamak, to search for a minimum B limit at constant q, keeping the power constant, also the new experiment PBX [28] in which the ballooning limit should, in some circumstances, become infinite, while the scaling law proposed here, insensitive to the plasma shape predicts a rather low value, should resolve the question of which modes, low n kinks or high n balloonings, are responsible for the β limit observed until now.

TABLE 1.

Device	B (T)	I(MA)	a(cm)	β(%)	$I_N=\frac{\mu_0 I}{aB}$	Ref. number	$R=\frac{\beta}{I_N}$
PDX	1.2	0.25	42 [1]	1.25 [2]	0.61	14	1.96
	1.2	0.30	42 [1]	1.65 [2]	0.73	14	2.21
	1.2	0.35	42 [1]	2.10 [2]	0.85	14	2.41
	1.2	0.40	42 [1]	2.50 [2]	0.99	14	2.52
	0.7	0.23	42 [1]	2.50 [5]	0.98	14	2.55
	1.0	0.30	42 [1]	2.06	0.82	14	2.51
D-III	0.62	0.345	38	4.5	1.84	15	2.44
	0.80	0.39	39	3.5	1.57	16	2.23
	1.26	0.23	34	1.40	0.67	see text	2.09
ISX-B	1.44	0.097	26 [1]	0.65	0.33	17	1.97
	1.14	0.099	26 [1]	0.90	0.42	17	2.14
	1.18	0.172	26 [1]	1.49	0.70	17	2.13
	1.18	0.192	26 [1]	1.88	0.79	17	2.38
	1.18	0.187	26 [1]	2.13	0.77	17	2.77
	1.15	0.180	26 [1]	2.34	0.79	17	2.96
	0.8	0.175	26 [1]	2.30	1.06	17	2.17
	0.93	0.190	26 [1]	2.25	0.99	18	2.27
	1.22	0.185	26 [1]	1.5	0.73	18	2.06
	0.94	0.184	26 [1]	1.9	0.94	19	2.02
	0.88	0.199	26 [1]	2.2	1.09	19	2.02
	0.90	0.193	26 [1]	2.4	1.04	19	2.20 [4]
	0.92	0.181	26 [1]	2.6	0.95	19	2.73
ASDEX	2.2	0.20	40	0.55 [3]	0.29	9	1.90
JFT-2	1.1	0.145	21 [6]	2.6	0.79	23	3.30
T-11	0.87	0.100	19.5 [1]	2.1	0.76	21	2.77
				1.7		22	2.23
TOSCA	0.47	0.051	6	0.46	0.23	26	2.00
	0.475	0.043	6	0.35	0.18	26	1.94
	0.475	0.056	6	0.58	0.25	26	2.32
	0.475	0.056	6	0.51	0.25	26	2.07

1 Half-width given by limiters.

2 Average value of data.

3 Calculated from β_p=2.65 assuming a circular plasma. It is a surface averaged β.

4 This equilibrium has been reanalysed in [20] and β has come down to 2.2 %, which brings R to 2.0.

5 Calculated from β_p=1.

6 Half-width given by vertical limiters assuming a circular plasma.

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FIGURE CAPTION

Figure 1 : Record discharges in the R, I_N plane. $RI_N = \mu_0 I/aB$, $R = \beta/I_N$. The values linked by an arrow are for identical discharges which have been analysed twice. The 2 horizontal lines at R=2.0 and 2.5 delimit the range of maximum β found in the numerical computations [1].