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SPECTRA OF LOWER-HYBRID WAVES REQUIRED TO SUSTAIN SIGNIFICANT CURRENTS

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Abstract

A simple quasilinear model is used to compute the current which can be sustained by a power source with the spectrum of a given shape. It is shown, firstly, that the current observed in PETULA B can be explained by theoretically determined broad spectra. Secondly, it is shown that narrower spectra (with which it is impossible to sustain a significant current) can be made very efficient by minute modifications. A high-$N_\parallel$ wing, containing only a few percent of the total power, is sufficient to enhance the current generated by a few orders of magnitude. This current is of the same order as that obtained with a broad spectrum.

1. INTRODUCTION

Experiments on lower-hybrid current drive have recently been very successful in sustaining toroidal currents in tokamaks [1]. It has repeatedly been stated [1, 2] that the "classical" theory of Fisch [3], which is a simple consequence of the quasilinear theory [4], can satisfactorily account only for the observed efficiencies (in terms of current per incident rf power) and not for the observed currents. For instance, on PLT [5], the injected waves have seemingly phase velocities that are too high to interact with a significant number of electrons. The currents obtained from the theory are several orders of magnitude smaller than the measured ones.

If it were not for this fundamental discrepancy one could regard the quasilinear theory as relevant to the experiments. Indeed, additional discrepancies as mentioned in Ref. 2 have lately found possible explanations based on quasilinear theory: the density limit observed in experiments is strongly correlated with the onset of wave-ion interactions [6] which constitute a energy-loss channel for current generation, and first indications of relaxation oscillations, observed in experiments [7], have now been found theoretically using a quasilinear model [8].

At present, the situation is absolutely open and speculations are permitted. In this paper, we speculate that the quasilinear theory holds and that the spectra of lower-hybrid waves inside the plasma are not necessarily the same as those obtained from grill [9, 10] and propagation theory [11]. To explain such a discrepancy one might invoke spectral broadening due to the scattering of the LH-waves on low-frequency density fluctuations [12] or the fact that the geometry of a real grill is more complex than that treated by theory.

Once these assumptions are made we ask for the shape of the spectra required to sustain significant currents. We show that the spectra obtained from the grill theory do not need modification, or at most mild modification, in order to be efficient for current drive. We also show that small uncertainties in the knowledge of the spectrum are compatible with discrepancies of orders of mangitude in the sustained currents. We shall conclude, therefore, that it is premature to exclude quasilinear theory as a valuable theory for LH current drive.

2. QUASILINEAR MODEL

In order to show an order-of-magnitude effect, as we intend to do, it is legitimate to adopt a very simple model. For this reason, we do not address the spatial problem of current drive which must include at least a ray-tracing calculation [11], but restrict ourselves to a quasilinear theory in a homogeneous plasma. We argue that in the lowest approximation, the power density absorbed by a homogeneous plasma has the same spectral distribution (in $N_\parallel$, the longitudinal index of refraction) as the total power P entering the plasma.

In our model the electron distribution function is governed by

$$\frac{\partial f}{\partial t} = \frac{\nu}{2}\{(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp^2 + \frac{\partial}{\partial v_\parallel} v_\parallel)\frac{1}{v^3}[2+(\frac{2}{v^2} - 1 - Z)(v_\perp \frac{\partial}{\partial v_\perp} + v_\parallel \frac{\partial}{\partial v_\parallel})]$$

$$+(1+Z)(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\frac{v_\perp}{v}\frac{\partial}{\partial v} + \frac{\partial}{\partial v_\parallel}\frac{1}{v}\frac{\partial}{\partial v_\parallel})\} f + \frac{\partial}{\partial v_\parallel} D \frac{\partial f}{\partial v_\parallel}, \qquad (1)$$

where the term involving $\nu = \omega_{pe}^3 \ln\Lambda/(4\pi n v_{te}^3)$ represents the electron-electron and electron-ion collisional effects and Z denotes the ion charge. The quasilinear diffusion coefficient resulting from the resonant Čerenkov interaction between the electrons and the lower-hybrid waves is given by

$$D = \frac{1}{2\pi v_\parallel^3}\int_0^{\pi/2} d(\cos\Theta)\, \cos\Theta\, W(k = v_\parallel^{-1}, \Theta). \qquad (2)$$

The wave spectrum, W, evolves according to

$$\frac{\partial W}{\partial t} = 2(\gamma - Z\nu/4)W + S(k,\Theta), \qquad (3)$$

with the quasilinear damping rate

$$\gamma = \frac{\pi}{2}\frac{\cos\Theta}{k^2}\int d^2v \frac{\partial}{\partial v_\parallel} f(v_\parallel = k^{-1}). \qquad (4)$$

Here, the dispersion relation of the waves is assumed to be $\omega = \cos\Theta$, Θ being the angle between the magnetic field and the wave vector $\vec{k}$. The term S, in Eq. (3), represents the spectrum of an external power source. For implicity, its form is assumed to be

$$S(k,\Theta) = S_0 s(k)\, \delta(\cos\Theta - \cos\Theta_0), \qquad (5)$$

where S_o is a constant and the profile function s(k) satisfies

$$\int s(k)\, k^2\, dk = 1. \qquad (6)$$

All the quantities in Eqs. (1)-(6) are normalized according to $k \to k/\lambda_D$, $v \to v v_{te}$, $t \to t/\omega_{pe}$, $f \to f n/v_{te}^3$, and $W \to W 4\pi n T_e \lambda_D^3$.

The function $k^2 s(k)$ can be identified with the shape of the spectral distribution $p(N_\parallel)$ of the total power P:

$$k^2\, s(k)\, dk = p(N_\parallel)\, dN_\parallel,\; N_\parallel = k\, c/v_{te} \qquad (7)$$

The equations (1) and (3) were solved numerically using the finite-element method with the additional ansatz: $f = F(v_\parallel)\exp[-v_\perp^2/2T(v_\parallel)]/2\pi T(v_\parallel)$. The details of the procedure can be found elsewhere [13]. The distribution function (initially Maxwellian) and the wave spectrum (initially the thermal noise) were advanced in time until the system reached a steady-state. The electron current density was then calculated according to $j = \int v_\parallel F(v_\parallel) dv_\parallel$. In convenient units:

$$I[kA] = 6.7 \times 10^4\; n[10^{19}\ m^{-3}]\, T_e^{1/2}[keV]\, a^2[m]\, j, \qquad (8)$$

where a is the radius of the current column. The power required to sustain the current is given by:

$$P[kW] = 2.9 \times 10^{11}\; R[m]\, a^2[m]\, n^{3/2}[10^{19}\ m^{-3}]\; T_e[keV]\; S_o \qquad (9)$$

Note that the uncertainty concerning the size of the radius a (in a simple model like ours) does not translate into an uncertainty concerning the dependence of I on P as long as j depends linearly on S_o. From preliminary calculations and for values of a and P compatible with experimental data, we know that j indeed depends more or less linearly on S_o.

3. RESULTS

The results presented below were obtained for typical PETULA parameters: $n = 1.5 \times 10^{19}\ m^{-3}$, $T_e = 1$ keV, R = 0.72 m and a = 0.12 m. For simplicity all the calculations have been done with Z = 1 and an accessibility limit at $N_\parallel = 1$. Correct modelling of these two quantities would, in general, lower the current j by as much as a factor of 2. Also, the value of $\cos\theta_o$ was fixed at 0.7 for convenience.

Figure 1 shows a calculated power spectrum $p(N_\parallel)$ [10] for PETULA (phase diffence between guides is 90 °) together with the electron distribution function $F(v_\parallel)$ which results from the injection of a total power P = 190 kW ($S_o = 3.5 \times 10^{-8}$). The domains of positive and negative $N_\parallel$ have been treated separately using the identification Eq. (7), but normalising $p(N_\parallel)$ in the domain $-\infty < N_\parallel < +\infty$. The currents carried by the non-Maxwellian tails are j = +0.075 and j = -0.005 and a first conclusion from FIG. 1 is that the produced counter-current is unimportant. The main result from FIG. 1, however, is that there is no striking discrepancy between theory and experiment in the case of PETULA: The total computed current, j = 0.07, corresponds to ~ 100 kA as compared to the value of 130 kA observed in the experiment [7]. A further result from FIG. 1 and accompanying computations is that the wings at high $N_\parallel$ ($N_\parallel$ > 6-8) have no influence on the current at all. The power in these wings is Landau damped without an appreciable effect on the distribution function. It therefore appears, for the power levels considered, that the discussion of the main lobe of the spectrum is sufficient.

The power contained in the main lobe is P = 100 kW and the spectral distribution is modelled by

$$p(N_\parallel) \sim \begin{cases} 0 \quad , & N_\parallel < 1 \\ \exp\left[-\left(\frac{N_\parallel - N_o}{\Delta N}\right)^2 \ln 2\right] - \exp\left[-\left(\frac{N_m - N_o}{\Delta N}\right)^2 \ln 2\right], & 1 < N_\parallel < N_m \\ 0 \quad , & N_\parallel > N_m \end{cases} \quad (10)$$

The quantities N_o, ΔN and N_m are free parameters.

Figure 2 shows the dependence of the results obtained for PETULA on eventual uncertainties concerning the width of the spectrum ΔN. The spectrum drawn as a dotted line is the best fit to the main lobe in FIG. 1 and has been obtained with N_o = 2.5, N_m = 6 and ΔN = 1.4. The resulting distribution function carries a current of j = 0.065, which is slightly lower than that obtained in FIG. 1. From FIG. 2 we remark that only a narrowing of the PETULA spectrum influences the current remarkably: the current drops by almost an order of magnitude when ΔN decreases by a factor 2 (curve b). We conclude that significant currents can be sustained with N_o = 2.5, N_m = 6 and ΔN > 1.4. The dependence of j on ΔN is weak in this range.

The strongest (and well-known) effect is demonstrated in FIG. 3. Here the spectra corresponding to PETULA (curve a) and to PLT [5] (curve b) are shown together with the produced distribution functions. As the PLT spectrum extends only to $N_\parallel$ = 4, corresponding to the resonant velocity $v_\parallel$ = 5.7, very few resonant particles are available to carry a current (curve b). Its value is j = 9 x 10^{-5} which is almost three orders smaller than the PETULA value, j = 0.065 (curve a). Our important and most spectacular result is shown with the dashed line. A minute wing which contains 4% of the total power is added to the PLT spectrum from $N_\parallel$ = 4 to $N_\parallel$ = 6. This wing is sufficient to "bridge the gap": enough particles can diffuse from $v_\parallel$ = 3.8 to $v_\parallel$ = 5.7 where they are accelerated and maintained by the main lobe of the spectrum. The present wing contains much less power than that found previously [14].

The inacceptable discrepancy between theory and experiment has therefore been reduced to an easily acceptable uncertainty on whether or not LH power spectra exhibit small wings containing a few percent of the total power. Therefore, it cannot be excluded that LH current drive works in just the way it had originally been imagined.

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FIGURE CAPTIONS

FIG: 1 : Shape of power spectrum, p, for PETULA B versus parallel index of refraction, $N_{\parallel}$, and the parallel electron velocity distribution $F(v_{\parallel})$ resulting from an injection of 190 kW.

FIG: 2 : Gaussian power spectra, $p(N_{\parallel})$, and parallel electron velocity distribution, $F(v_{\parallel})$, resulting from an injection of 100 kW.
The dotted line corresponds to PETULA B ($\Delta N = 1.4$) whereas the lines a and b have $\Delta N = 2$ and $\Delta N = 0.8$ respectively.

FIG: 3 : Spectra, $p(N_{\parallel})$, and parallel electron velocity distribution, $F(v_{\parallel})$, resulting from an injection of 100 kW. The spectra can be attributed to PETULA B (curve a, $N_o = 2.5$, $N_m = 6$, $\Delta N = 1.4$) and to PLT (curve b, $N_o = 2.2$, $N_m = 4$, $\Delta N = 0.8$). The dotted lines represent a sligthly modified PLT spectrum and the corresponding distribution function.

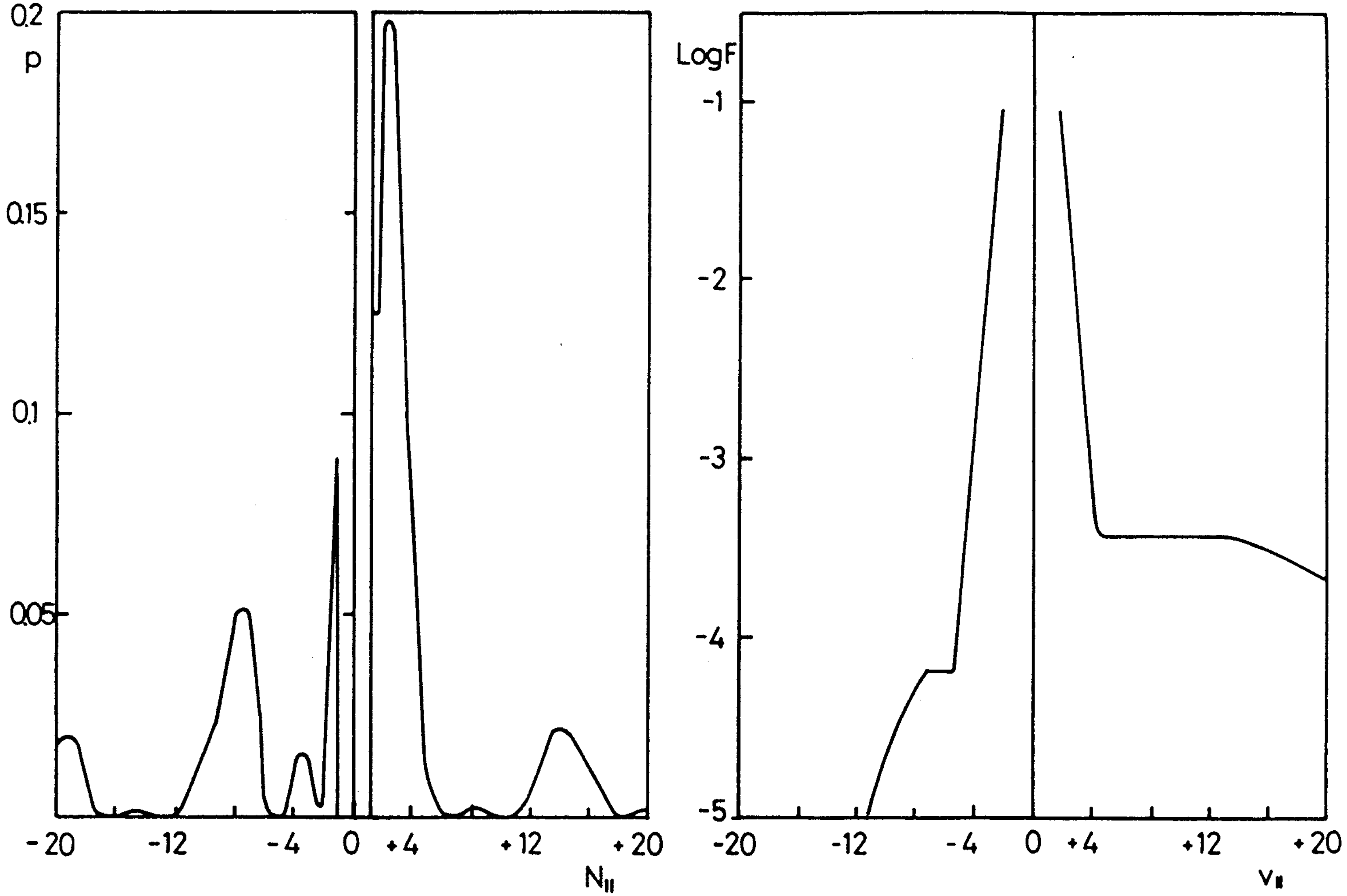
0.2
p
0.15
0.1
0.05
-20
-12
-4
0
+4
+12
+20
$N_{\parallel}$
LogF
-1
-2
-3
-4
-5
-20
-12
-4
0
+4
+12
+20
$v_{\parallel}$

FIG. I

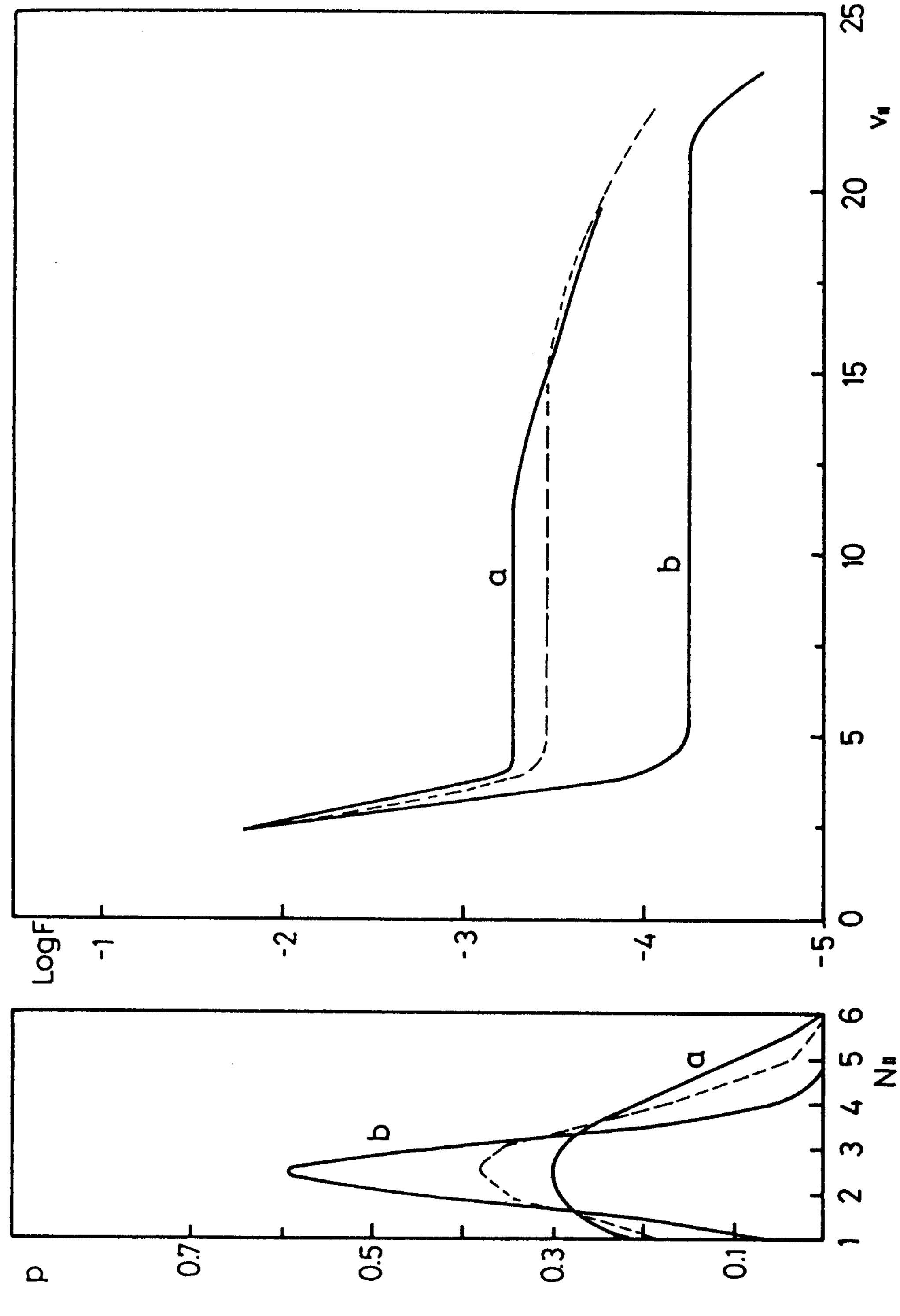

FIG. 2

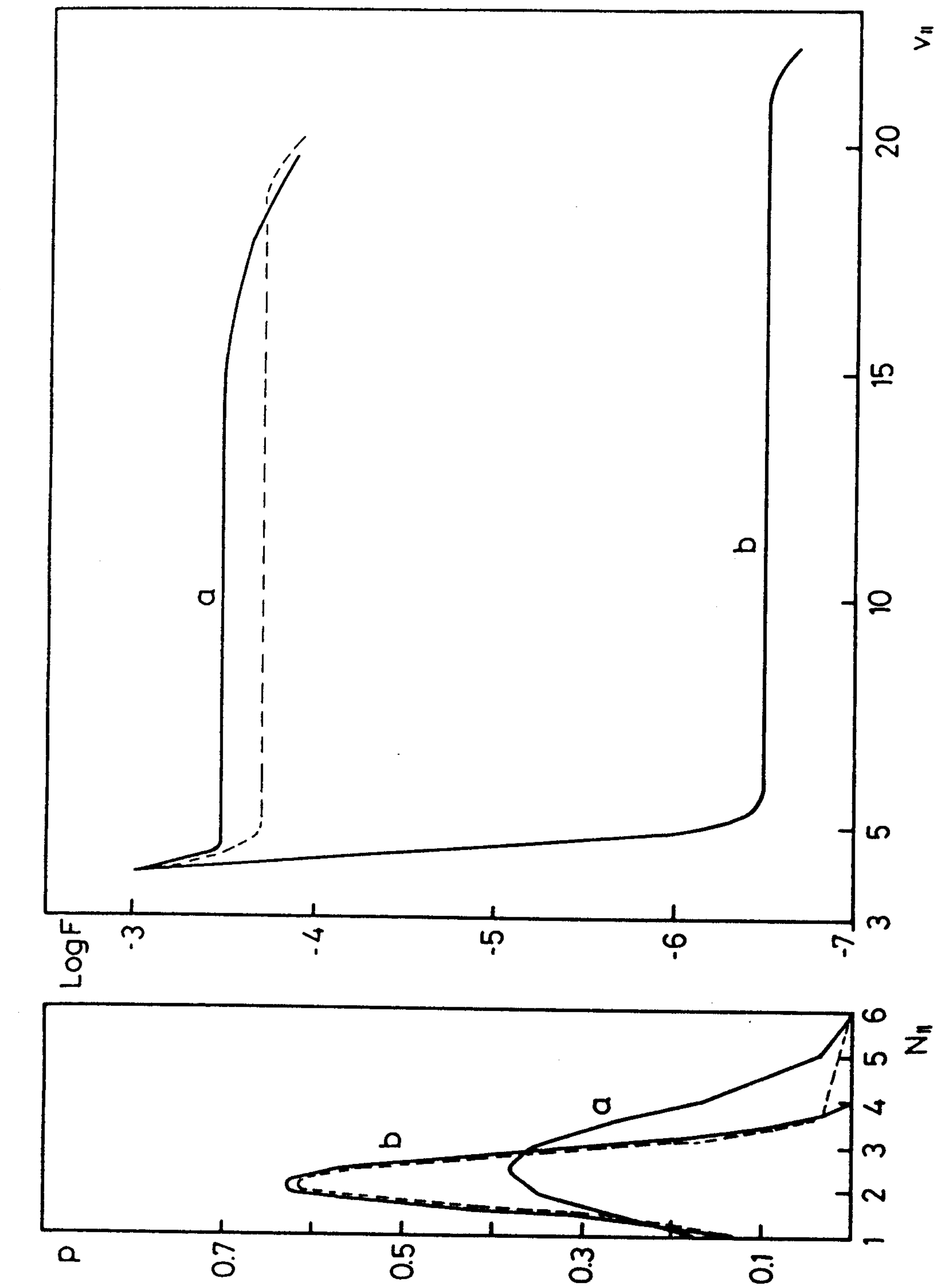
P
0.7
0.5
0.3
0.1
1
2
3
4
5
6
$N_{\parallel}$
a
b
LogF
-3
-4
-5
-6
-7
3
5
10
15
20
$v_{\parallel}$
a
b

FIG. 3