THEORY OF ELECTRON TAIL·FORMATION IN
LOWER-HYBRID CURRENT-DRIVE EXPERIMENTS

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ABSTRACT

The combined effects of an electric field and obliquely propagating plasma waves on the tail of an electron distribution function are studied analytically. The runaway production rate is enhanced by many orders of magnitude if the source, even though weak, drives waves with parallel phase velocities around the critical velocity. A practical formula to quantify this phenomenon is derived. Depending on the source strength two different regimes may occur : one is stable and linear with the rf power, whilst the other, which saturates, may be unstable. Connections with experiments are discussed.

1. INTRODUCTION

Runaway electrons and related phenomena have been a recurrent topic since the earliest investigations on controlled fusion research. The present day experiments on lower hybrid heating (HARVEY et al., 1981) or on rf-driven current for steady-state tokamak operation (YAMAMOTO T. et al., 1980; OHKUBO K. et al., 1981; MAEKAWA T. et al., 1981) might well revive interest in the subject. Although in principle there should be no dc electric field in the steady-state operation - and de facto no runaways produced - present experiments always involve an electric field. Thus, there exist incentives for studying the role of imposed waves in the runaway kinetic theory. It has even been proposed to use the runaways deliberately as current carriers in a steady-state tokamak (BHADRA D.K., 1980).

In an earlier report (LIU C.S. et al., 1980) we showed that the quasilinear diffusion of the electrons by the waves of the lower hybrid branch could enhance the runaway production rate by many orders of magnitude. More recently, observations on lower hybrid experiments in the USA (HARVEY R.W. et al., 1981) and in Japan (OHKUBO K. et al., 1981; MAEKAWA T. et al., 1981) have been interpreted in terms of rf-induced runaway electrons, thus corroborating the first observations made in ATC (BOYD D.A. et al., 1976). In view of this, it seems desirable to have available a theory for this interesting effect. Our previous work was limited by an assumption about the spectrum; the latter was not self-consistent with the distribution function but given a priori, so that a direct estimation of the strength of the rf source involved was impossible. The different

approach used here allows us to remove this limitation and to provide a more comprehensive study of the phenomenon. Also the aim of this paper is to give a simple expression for the runaway production rate in terms of the source strength.

The paper is organised as follows. In section 2 we formulate the model and present some qualitative considerations. The stationary distribution and flux of runaway electrons are calculated in section 3. In section 4 we study the role of the anomalous Doppler interaction and conclude our work in section 5.

2. MODEL

2.1 Qualitative considerations

Let us consider a magnetised plasma, $\omega_{ce} > \omega_{pe}$ (where ω_{ce} and ω_{pe} are the electron cyclotron and plasma frequency, respectively), in the presence of a weak dc electric field $E \ll E_c$ which is parallel to the magnetic field. Here E_c is the critical electric field $E_c = 4\pi e^3 n \ln\Lambda \ (m v_e^2)^{-1}$, where $\ln\Lambda$ is the Coulomb logarithm and e,m,n and v_e are the electron charge, mass, density and thermal velocity, respectively. Under these circumstances the collisional drag due to Coulomb collisions dominates over the electric field acceleration for most of electrons, and only a small fraction of particles, varying exponentially, will run away. In the velocity space an important cross-over point v_c may be defined by balancing the electric field force against the frictional drag due to Coulomb collisions, i.e.

$$e E = m \, \nu_0 \, v_e^3 / v_c^2 \quad , \tag{1}$$

with the collision frequency $\nu_0 = 4\pi e^4 n \ln\Lambda \; (m^2 v_e^3)^{-1}$. Electrons with velocities $v > v_c$ will be gradually accelerated by the electric field towards the velocity of light. On the other hand, electrons with $v \ll v_c$ will probably remain in the Maxwellian distribution. Around $v = v_c$, the shape of the electron distribution function is very crucial in determining the fraction of electrons which pass over the critical velocity v_c and so in determining the runaway production rate. Now, this shape may be affected by the presence of waves with the resonant velocity in this domain; if the quasilinear diffusion due to the waves enhances the flow of electrons accross v_c an increased runaway production rate may be expected.

Let us now turn to the branches of electrostatic waves in the magnetised plasma which may resonate with electrons having velocities around v_c. Since we assume $E \ll E_c$ we have obviously $(v_c/v_e) = (E_c/E)^{1/2} \gg 1$ (typically in the range 4-10). Naturally, it is electron waves that we consider. Their dispersion relation in the cold plasma approximation reads (MIKHAILOVSKII A.B., 1974)

$$1 - \frac{\omega_{pe}^2}{\omega_{ce}^2} \cos^2\theta - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \sin^2\theta = 0 \quad . \tag{2}$$

Here θ is the angle between the magnetic field and the wave vector $\vec{k}$. From the two branches involved, the upper hybrid and the lower hybrid

ones, we will consider the latter whose dispersion relation in a strongly magnetised plasma reduces to

$$\omega = \omega_{pe} \cos \theta \quad . \tag{3}$$

Besides the consideration of simplicity, the choice of the lower hybrid branch is dictated by the existence of data from lower hybrid experiments (YAMAMOTO T. et al., 1980; HARVEY R.W. et al., 1981; OHKUBO K. et al., 1981; MAEKAWA T. et al., 1981). In fact, the adequate dispersion relation is (MIKHAILOWSKII A.B., 1974).

$$\omega^2 = \omega_{pe}^2 \left(\cos^2\theta + \frac{m_e}{m_i} \right) ,$$

where m_i is the mass of the ions. For waves which are not too perpendicular to the magnetic field ($\cos^2\theta \gg m_e/m_i$) this dispersion relation matches to equation (3).

The wave-particle resonances occur at $\omega = k_{//} v_{//}$ (Cerenkov resonance) and at $\omega - n\omega_{ce} = k_{//} v_{//}$, where n is an integer either positive (normal Doppler resonance) or negative (anomalous Doppler resonance). For $\omega_{ce} \gg \omega$ only the first two harmonics are to be retained, $n = \pm 1$.

In the experiments the launched waves may have two opposite directions with respect to the electric field. For waves propagating in the same direction as runaway electrons, $k_{//} > 0$, it is sufficient to consider the Cerenkov resonance at the velocity $v_r^c = \omega/k_{//}$ and the anomalous Doppler resonance at the velocity $v_r^d = (\omega + \omega_{ce})/k_{//} \approx \omega_{ce}/k_{//}$. In the case of a "standard" runaway tail the derivative of

the distribution function is negative and the Cerenkov interaction leads to a damping of the waves; resonant electrons are then accelerated and may pass over the critical point v_c. In contrast, the anomalous Doppler interaction leads to an emission of waves and a pitch-angle scattering of the electrons (PARAIL V.V. and POGUTSE O.P., 1976).

For waves propagating in a direction opposite to the runaway electrons, $k_{\shortparallel} < 0$, the important resonances are that of Cerenkov again and the normal Doppler resonance at the velocity $v_r = (\omega - \omega_{ce})/k_{\shortparallel} \approx \omega_{ce}/|k_{\shortparallel}|$. The Cerenkov interaction accelerates the resonant electrons to more negative velocities but the electric field brings them back towards the bulk of the distribution function, so that no change in the runaway rate may be expected. The usual Doppler interaction acts, however, on the side of the distribution with positive velocities; the waves are damped and the runaway tail tends to become isotropic, which leads to the formation of a positive slope. The absorption of a plasmon via the normal Doppler effect is characterized by an increase in the gyration energy of an electron $\Delta E = \hbar\omega_{ce}$, and by a decrease in its parallel energy $\Delta E = \hbar(\omega_{ce}-\omega)$. For obliquely propagating waves in a magnetised plasma, one has $\omega_{ce} \gg \omega$ and the plasmon energy may be neglected in comparison with the kinetic energy transfer. Thus, the electrons with velocities v_r are pitch-angle scattered by the externally driven waves along the lines of nearly-constant energy. Therefore, the runaway electrons with velocities $v_{\shortparallel} < v_r$ whilst accelerated by the electric field will pile up around v_r; as a result a positive slope will appear on the

runaway distribution preventing the electrons from running away. This situation, which is analogous to the runaway kinetic instability described by many authors (PARAIL V.V. and POGUTSE O.P., 1976; LIU C.S. et al., 1977; LIU C.S. and MOK Y., 1977; PAPADOPOULOS K. et al., 1977; HUI B.H. and WINSOR N., 1978; PARAIL V.V. and POGUTSE O.P., 1978; MUSCHIETTI L. et al., 1981), is complicated because of the unstable spectrum. A quantitative investigation would require the assistance of a numerical code and a separate study which is out of the scope of this paper. Let us say simply that the cyclotron emission is expected to be enhanced and the runaway production rate to be cut down or even cut off in the case of a strong source.

Henceforth, we consider the case of waves propagating in the same direction as the runaway electrons $(k_{\parallel} > 0)$, for which analytic calculations may be performed.

2.2 Basic equations

The kinetic equation for the electron distribution is then

$$
\frac{\partial}{\partial t} f(v_{\parallel}, v_{\perp}, t) = \frac{\partial}{\partial v_{\parallel}} D_0 \frac{\partial}{\partial v_{\parallel}} f - E \frac{\partial}{\partial v_{\parallel}} f
$$

$$
+ \left(\frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \right) D_1 \left(\frac{\partial}{\partial v_{\parallel}} f - \frac{v_{\parallel}}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} f \right) \quad (4)
$$

$$
+ \frac{\partial}{\partial v_{\parallel}} \nu(v_n) \left(v_{\parallel} f + \frac{\partial}{\partial v_n} f \right)
$$

with

$$D_o \left(v_{\shortparallel}, t \right) = 2\pi \int\limits_{k_{\shortparallel} > 0} \frac{d^3 k}{(2\pi)^3} \; \mathcal{E}\left(\vec{k}, t \right) \delta\left(\frac{k_{\shortparallel}}{k} - k_{\shortparallel} v_{\shortparallel} \right) \tag{5}$$

$$D_1 \left(v_{\shortparallel}, v_{\perp}, t \right) = \frac{\pi}{2} \int\limits_{k_{\shortparallel} > 0} \frac{d^3 k}{(2\pi)^3} \left(\frac{k_{\shortparallel}}{k} \right)^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}} \right)^2 \mathcal{E}\left(\vec{k}, t \right) \delta\left(\omega_{ce} - k_{\shortparallel} v_{\shortparallel} \right). \tag{6}$$

The spectrum of the waves obeys the quasi-linear equation

$$\frac{\partial}{\partial t} \mathcal{E}\left(\vec{k}, t \right) = 2 \left(\gamma_o + \gamma_1 - \frac{\nu_o}{2} \right) \mathcal{E}\left(\vec{k}, t \right) + S\left(\vec{k} \right) \tag{7}$$

where γ_0 corresponds to the Cerenkov interaction

$$\gamma_o = \frac{\pi}{2} \frac{k_{\shortparallel}}{k^3} \int d^3 v \; k_{\shortparallel} \frac{\partial}{\partial v_{\shortparallel}} f \; \delta\left(\frac{k_{\shortparallel}}{k} - k_{\shortparallel} v_{\shortparallel} \right) \tag{8}$$

and γ_1 to the anomalous Doppler interaction

$$\gamma_1 = \frac{\pi}{8} \frac{k_{\shortparallel}}{k^3} \int d^3 v \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}} \right)^2 k_{\shortparallel} \left(\frac{\partial}{\partial v_{\shortparallel}} f - \frac{v_{\shortparallel}}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} f \right) \delta\left(\omega_{ce} - k_{\shortparallel} v_{\shortparallel} \right). \tag{9}$$

The quantities are normalized according to $k \rightarrow k/\lambda_D$, $v \rightarrow v \, v_e$, $t \rightarrow t/\omega_{pe}$, $f \rightarrow fn/v_e^3$, $\varepsilon_k \rightarrow \varepsilon_k 4\pi nT \lambda_D^3$ and $E \rightarrow E(4\pi nT)^{1/2}$. Here λ_D is the Debye length and T the electron temperature. In equation (4) we have modelled the collisions by a Vedenov (VEDENOV A.A., 1968) term with $\nu(v) = \nu_0 \, v^{-3}$. This term is linear and simulates situation in which the dissipation energy is removed into a thermal reservoir (KULSRUD R.M. et al., 1973). On the other hand, this model is convenient for our purposes since it describes well the competition between

the externally driven waves and the relaxation of the distribution function towards a Maxwellian, which leads to the creation of a substantial current. It is worth mentionning that in our units the ratio $(v_0/E)^{1/2}$ is just the critical velocity v_c.

The term $S(\vec{k})$ in equation (7) represents the external source that drives the waves. For simplicity we will assume that it is unidirectional and constant within a certain range of wavenumbers determined by the parallel refractive index and the electron temperature:

$$S(\vec{k}) = \begin{cases} S \ \delta(\cos\theta - \cos\theta_o) & \text{for} \ k_2 < k < k_1 \\ \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Two points are still worth mentioning. Firstly, since the local shape of the distribution is important with respect to the parallel velocity only, the Maxwellian ansatz discussed by MUSCHIETTI L. et al. (1981) may be used

$$f(v_{//}, v_{\perp}, t) = F(v_{//}, t) \ \frac{1}{2\pi T_{\perp}(v_{//}, t)} \ \exp\left[-\frac{v_{\perp}^2}{2 T_{\perp}(v_{//}, t)}\right]. \tag{11}$$

Secondly, the anomalous Doppler interaction and the Cerenkov interaction have neither the same "efficiency" in diffusing particles nor the same resonant velocity. We have $v_r^d = \omega_{ce}/(k\cos\theta) = v_r^c \ \omega_{ce}/\cos\theta$. Since $\omega_{ce} > 1$ and $v_r^c \gg 1$ the resonant velocity v_r^d may lie above the velocity of light for very obliquely propagating waves as in lower hybrid experiments. In this case the only resonance to be

considered is the Cerenkov one. If the anomalous Doppler interaction has to be considered its "efficiency" is reduced by the factor $(k\ v\ /2\omega_{ce})^2$ as compared to the Cerenkov one so that we may treat it by means of a perturbation method (cf. section 4). The next section will be devoted to the Cerenkov interaction only.

3. DISTRIBUTION AND FLUX OF RUNAWAY ELECTRONS IN PRESENCE OF PLASMA WAVES

Let us neglect the terms due to the anomalous Doppler interaction in the quasilinear equations (4) and (7), and introduce the ansatz given by equation (11) for the electron distribution function and use spherical coordinates for the spectrum. One obtains the following equations where v signifies $v_{\parallel}$:

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial v}\left[D_o(v,t)\frac{\partial F}{\partial v} - E F + \frac{\nu_o}{v^3}\left(v F + \frac{\partial F}{\partial v}\right)\right] , \qquad (12)$$

with

$$D_o(v,t) = \frac{1}{2\pi\,v^3}\int_0^{\pi/2} d(\cos\theta)\ \cos\theta\ \mathcal{E}\left(k=v^{-1}, \theta, t\right) , \qquad (13)$$

and

$$\frac{\partial}{\partial t}\mathcal{E}(k,\theta,t) = 2\left(\gamma_o - \frac{\nu_o}{2}\right)\mathcal{E} + S(k,\theta) , \qquad (14)$$

with

$$\gamma_o = \frac{\pi}{2}\ \frac{\cos\theta}{k^2}\ \frac{\partial F}{\partial v}\bigg|_{v=k^{-1}} \qquad . \qquad (15)$$

Under the influence of the electric field and the quasilinear damping of the waves the distribution F grows a tail towards v = + ∞. A stationary self-consistent state is eventually established in the system.

3.1 Stable stationary solution and runaway production rate

After dispensing with the time derivative in equation (12) a first trivial integration yields

$$\frac{\gamma_0}{v^3}\left(v F + \frac{\partial F}{\partial v}\right) + D_0 \frac{\partial F}{\partial v} - E F + A = 0 \quad , \qquad (16)$$

where the constant of integration A is identified as the flux of runaway electrons in the limit v → ∞: A = E F(v → ∞). Of course, a stationary distribution function involving a loss of particles at +∞ implies an equivalent source at v = 0. A simple way to cope with this problem is to ensure a constant number of particles by keeping F at the Maxwellian value at v = 0. This assumption provides us with the boundary condition which is necessary for integrating equation (16). γ_0 is expected to be negative (an assumption that will be checked a posteriori in section 3.2) and the spectrum is sustained by the source S only (equation (10)). From equations (14) (15) we find

$$\mathcal{E}(k,\theta) = \frac{S\,\delta(\cos\theta - \cos\theta_0)\,[\,H(k-k_2) - H(k-k_1)\,]}{\gamma_0 - \pi \cos\theta\, k^{-2}\, \frac{\partial F}{\partial v}\Big|_{v=k^{-1}}} \quad , \qquad (17)$$

where H is the Heaviside function. Now the one-to-one correspondence introduced by the Cerenkov resonance condition between the k-space and

v-space allows us to use the v variable only. Thus on combining equations (13) and (17) we have

$$D_0(v) = \frac{S \cos \theta_0}{2 \pi v^3} \left[H(v - v_1) - H(v - v_2) \right] \left(V_0 - \pi \cos \theta_0 \, v^2 \frac{\partial F}{\partial v} \right)^{-1} \quad (18)$$

which may be simply substituted in equation (16). The range of resonant velocities is limited by $v_1 = k_1^{-1}$ and $v_2 = k_2^{-1}$. At this point, two opposite cases must be distinguished: either the level of spectrum is controlled by the Landau damping or the collisions prevent it from being infinite (cf. Fig. 1).

Let us consider first the case $\left| 2\gamma_0 \right| \gg \nu_0$, an inequality which will be interpreted in section 3.2 in terms of the source strength. We may then write for F the simple linear differential equation

$$\dot{F} = \left(\tilde{E} v^3 - v \right) F - \tilde{A} v^3 + \frac{\tilde{S}}{v^2} \Delta \left(v_1, v_2, v \right) \quad , \quad (19)$$

where the dot signifies the derivative with respect to v and the notations $\tilde{E} = E/\nu_0$, $\tilde{A} = A/\nu_0$, $\tilde{S} = S/(2\pi\nu_0)$, and $\Delta(v_1, v_2, v) = H(v - v_1)$ - $H(v - v_2)$ have been introduced. The general solution reads

$$F = \left\{ c + \int_v^\infty dz \left[\tilde{A} z^3 - \frac{\tilde{S}}{z^2} \Delta(v_1, v_2, v) \right] exp \left(\frac{z^2}{2} - \tilde{E} \frac{z^4}{4} \right) \right\}$$

$$\times \, exp \left(\tilde{E} \frac{v^4}{4} - \frac{v^2}{2} \right) \quad . \quad (20)$$

We set the constant of integration $c = 0$ in order to have a bounded solution for $v \to +\infty$ and determine $\tilde{A}$ by matching the distribution to a Maxwellian value $(2\pi)^{-1/2}$ at $v = 0$:

$$F(0) = \tilde{A} I_1 - \tilde{S} I_2(v_1, v_2)$$

with

$$I_1 = \int_0^\infty dz \, z \, \exp\left(\frac{z^2}{2} - \tilde{E} \frac{z^4}{4}\right)$$

$$I_2(v_1, v_2) = \int_{v_1}^{v_2} \frac{dz}{z} \, \exp\left(\frac{z^2}{2} - E \frac{z^4}{4}\right) \quad .$$

I_1 is easy to calculate via the change of variables $y = \tilde{E}^{1/2} Z^2/2 - (2\tilde{E})^{-1/2}$

Since $v_c \gg 1$ implies $(\tilde{E})^{-1/4} \gg 1$ one obtains simply

$$I_1 = \frac{\sqrt{\pi}}{\tilde{E}^{3/4}} \, \exp\left(\frac{1}{4\tilde{E}}\right) \quad .$$

Thus we recover the "classical" runaway rate (PARAIL V.V. and POGUTSE O.P., 1978).

$$\tilde{A}_0 = \frac{1}{\sqrt{2\pi} \, I_1} = \frac{\tilde{E}^{3/4}}{\sqrt{2} \, \pi} \, \exp\left(- \frac{1}{4\tilde{E}}\right) \quad . \tag{21}$$

The second integral I_2 (v_1, v_2) may be evaluated by the saddle point method. We write

$$I_2(v_1, v_2) = \int_{v_1}^{v_2} dz \, \exp\left(-\tilde{E} \frac{z^4}{4} + \frac{z^2}{2} - z \ln z\right) \quad .$$

The argument of the exponential $G(z) = -\tilde{E} z^4/4 + z^2/2 - z \ln z$

has a distinct maximum for $z_0 = (2\tilde{E})^{-1/2} \left[1 + (1 - 8\tilde{E})^{1/2}\right]^{1/2}$

and becomes negative above $z_m \approx (\tilde{E})^{-1/2} \left[1 + (1 + 4\tilde{E} \ln \tilde{E})^{1/2}\right]^{1/2}$.

Recalling $\tilde{E} = v_c^{-2}$ one has more insight into the behaviour of $G(z)$:

the maximum is situated around $(1-\tilde{E})V_c$ and the cutoff around $\sqrt{2}(1-\tilde{E} \ln v_c)$ v_c. Consequently, the waves play a significant role in the rate of runaways if the interval $[v_1, v_2]$ contains v_c, confirming the qualitative discussion of section 2.1. On the other hand, waves with high resonant velocities, above $\sqrt{2}v_c$, do not play any role.

Let us assume now that the critical velocity lies well within the interval $[v_1, v_2]$ $(v_c - v_1 > 2, v_2 - v_c > 2)$. One has

$$I_2 (v_1, v_2) \approx \int_{v_1}^{v_c} dz\, exp \left[G(z_o) - (z - z_o)^2 \left(1 - 4\tilde{E} - 2\tilde{E}^2 \right) \right]$$

$$= exp\left[G(z_o) \right] \int_{v_1 - z_o}^{v_2 - z_o} dy\, exp \left[-y^2 \left(1 - 4\tilde{E} - 2\tilde{E}^2 \right) \right]$$

$$\approx \sqrt{\pi}\, exp \left[\frac{v_c^2}{4} - \ell_n \left(v_c^2 - 2 \right) \right] \quad . \tag{22}$$

Hence

$$\tilde{A} = \left(I_1 \right)^{-1} \left[(2\pi)^{-\frac{1}{2}} + \tilde{S}\, I_2 (v_1, v_2) \right]$$

$$= \frac{\tilde{E}^{3/2}}{\sqrt{\pi}}\, exp\left(\frac{-1}{4\tilde{E}} \right) \left[\frac{1}{\sqrt{2\pi}} + \tilde{S} \sqrt{\pi}\, \frac{\tilde{E}}{1 - 2\tilde{E}}\, exp\left(\frac{1}{4\tilde{E}} \right) \right]$$

$$= \tilde{A}_o + \tilde{S}\, \frac{\tilde{E}^{5/2}}{1 - 2\tilde{E}} \quad . \tag{23}$$

Obviously, for a small electric field ($\tilde{E}$ equals a few percent) the classical term $\tilde{A}_0$, which decreases exponentially, becomes negligible and one has the convenient formula

$$\tilde{A} = \tilde{S}\, \frac{\tilde{E}^{5/2}}{1 - 2\tilde{E}} \tag{24}$$

which shows that in this regime the runaway production rate is linearly proportional to the power of the source. Nevertheless, it must not be forgotten that this formula has been derived under the condition $|2\gamma_0| \gg \nu_0$. It is therefore clear that the case of a very strong source creating a plateau on the distribution function is not treated by this formula.

In next section we investigate the conditions imposed on the source by the above inequality whilst the opposite case $|2\gamma_0| \gg \nu_0$ is deferred to section 3.3. Anticipating the results we may emphasize that there is a surprisingly broad range of $\tilde{S}$ to enhance the runaway rate many orders above the classical rate before a plateau appears on the distribution function and formula (24) breaks down.

3.2 Validity of the solution and source strength

On considering equation (19) it is clear that the positive term of the rhs $\tilde{S}\Delta(v_1,v_2,v)/v^2$ makes the slope of F less negative, or possibly changes even its sign. We must check that the condition for the Cerenkov interaction to control the spectrum, $-2\gamma_0 \gg \nu_0$, holds. Consequetly, conditions for the values of $\tilde{S}$ will be found. For this investigation equation (19) has been integrated numerically and the behaviour of the solution has been studied. It turns out that it is sufficient to consider the conditions which may be obtained analytically at the special points v_1 and v_c.

From equations (15) and (19) the condition $-2\gamma_0 \gg \nu_0$ reads

$$\pi \cos\theta_0 \left[\left(v^3 - \frac{v^5}{v_c^2} \right) F(v) + \tilde{A} v^5 - \tilde{S} \right] \gg \gamma_0 \quad ,$$

or by using equation (24)

$$\pi \cos\theta_0 \left[\left(1 - \frac{v^2}{v_c^2}\right) v^3 F(v) + \tilde{S}\left(\frac{v^s}{v_c^s} + 2\tilde{E}\frac{v^s}{v_c^s} - 1\right)\right] \gg \nu_0 \quad .$$

For $v > v_c$ the first term is negative and the inequality holds for S sufficiently big. In fact, it is sufficient to satisfy the condition at v_c so that

$$\tilde{S} \gg \frac{\nu_0}{2\pi\cos\theta_0} v_c^2 \quad . \tag{25}$$

However, the source term S must not be so big that it violates the condition at $v_1 < v_c$:

$$\tilde{S}\left(1 - \frac{v_1^s}{v_c^s} - 2\tilde{E}\frac{v_1^s}{v_c^s}\right) \ll v_1^3 F(v_1)\left(1 - \frac{v_1^2}{v_c^2}\right) - \frac{\nu_0}{\pi\cos\theta_0} \quad .$$

If $v_1/v_c \ll 1$ it reduces to

$$S \ll v_1^3 F(v_1) - \frac{\nu_0}{\pi\cos\theta_0} \equiv \tilde{S}_c \tag{26}$$

where $F(v_1)$ is the quasi-Maxwellian value (cf. Appendix). It is worth noting that the second term, proportional to ν_0, may be neglected for typical tokamak parameters unless nearly perpendicular waves are in question. As we shall see in the next section the critical value for the source $\tilde{S}_c$ is important because it determines the frontier between the linear regime and the regime of a strong source.

3.3 Limit of a strong source

We now turn to the opposite case $|2\gamma_0| \ll \nu_0$. The diffusion coefficient D_0 (equation (18)) reads simply

$$D_0(v) = \frac{b \; \nu_0}{v^3} \; \Delta(v_1, v_2, v) \tag{27}$$

with

$$b = \frac{S \cos\theta_0}{2\pi \, \nu_0^2} = \frac{\pi \cos\theta_0 \, \tilde{S}}{\nu_0} \qquad .$$

On substituting equation (27) in equation (16) one obtains

$$\dot{F}\left(1 + b\,\Delta(v_1, v_2, v)\right) = \left(E \, v^3 - v\right) F - A \, v^3 \quad . \tag{28}$$

It is clear that $\dot{F}$ is always negative. If it were not the case some $v > v_c$ with $(\tilde{E} \, F - \tilde{A})v^3 - vF > 0$ would exist. With increasing v this inequality would be more and more satisfied. However, since $\tilde{A} = \tilde{E}F(v \to \infty)$ (cf. equation (16)) it breaks down certainly at $+ \infty$.

Now the negative slope of the distribution function is lessened in the interval $[v_1, v_2]$ by the presence of waves. In order to satisfy the inequality $|2\gamma_0| \ll \nu_0$ large values of b (b $\gg$ 1) are necessary. This imposes

$$\frac{\pi \cos\theta_0 \, v^2}{b} \left[\left(v - \tilde{E} \, v^3\right) F + \tilde{A} \, v^3\right] \ll \nu_0 \qquad .$$

On expressing b in terms of S we find

$$\tilde{S} \gg \left(v^3 - E \, v^5\right) F + \tilde{A} \, v^5 \quad , \quad v \in \left[v_1, v_2\right] \quad . \tag{29}$$

The rhs of this inequality is decreasing with v so that it is sufficient to evaluate it in v_1. We have seen in section 3.1 that the interesting case is that with the ordering $v_1 < v_c < v_2$. One then has

$$\tilde{S} \gg v_1^3 F(v_1)$$

(30)

which implies again the critical power density $S_c = v_1{}^3 F$ (v_1) (cf. equation (26)) estimated with the quasi-Maxwellian value $F(v_1)$ given in Appendix.

The solution to equation (28) has been considered in a previous report (LIU C.S. et al., 1980). The runaway production rate was found to be

$$\tilde{A} = \frac{1}{\sqrt{2\pi}} \tilde{E}^{3/2} \exp\left[-\frac{v_1^2}{2}\left(1 - \frac{v_1^2}{v_c^2}\right)\right]$$

$$\times \left[\frac{v_c(v_2^2 - v_1^2)}{(v_2^2 - v_c^2)(v_c^2 - v_1^2)} + \left(\frac{\pi}{4b}\right)^{1/2}\left(erf(x_2) + erf(-x_1)\right)\right]^{-1}$$

with the conditions $\left|v_i{}^2 - v_c{}^2\right| > 4\,v_c$ and the definitions

$$X_i = \left(v_i^2 - v_c^2\right)\left(\tilde{E}/4b\right)^{1/2} \quad , \quad i = 1, 2 \ .$$

For large values of b the rate is saturating at

$$\tilde{A} = \frac{1}{\sqrt{2\pi}} \tilde{E} \exp\left[-\frac{v_1^2}{2}\left(1 - \frac{v_1^2}{2v_c^2}\right)\right]\left(v_2^2 - v_1^2\right)^{-1}$$

$$\times \left[\frac{1}{\left(\frac{v_2^2}{v_c^2} - 1\right)\left(1 - \frac{v_1^2}{v_c^2}\right)} + \frac{1}{2b}\right]^{-1} \ .$$

(31)

Alternately, the saturation of the rate with an increasing value of $\tilde{S}$

may be deduced from the inequality (29) evaluated in v_c,

$$\tilde{S} \gg \tilde{A}\, v_c^5 \qquad \text{or} \qquad \tilde{S}\, \tilde{E}^{5/2} \gg \tilde{A} \quad .$$

This clearly imposes that the rate $\tilde{A}$ ceases to increase linearly with the source, as $\tilde{S}\, \tilde{E}^{5/2}$, yet saturates and lets $\tilde{S}$ to grow alone. The characteristic value of b where the rate begins to saturate is given by

$$b_A = \frac{1}{2}\left(\frac{v_2^2}{v_c^2} - 1 \right)\left(1 - \frac{v_1^2}{v_c^2} \right)$$

which corresponds to a source of value

$$\tilde{S}_A = \frac{\nu_o}{2\pi \cos\theta_o}\left(\frac{v_2^2}{v_c^2} - 1 \right)\left(1 - \frac{v_1^2}{v_c^2} \right) \quad . \tag{32}$$

If v_1 is not very high one has $\tilde{S}_c \gg \tilde{S}_s$ so that the regime of a strong source ($\tilde{S} \gg \tilde{S}_c$) is the saturated regime.

4. ROLE OF ANOMALOUS DOPPLER INTERACTION

The anomalous Doppler interaction may falsify the results of section 3 in two manners. On the one hand, the distribution could become unstable due to the increased number of particules in the tail. Oblique plasma waves would then be excited and the resonant runaway electrons pitch-angle scattered (PARAIL V.V. and POGUTSE O.P., 1976). This instability would result in a quasi-steady state similar to that described by MUSCHIETTI L. et al. (1981). On the other hand, the driven waves themselves could interact with the runaway electrons via the anomalous Doppler resonance. The associated loss of parallel momentum of the resonant electrons would then result in a reduction of the runaway production rate.

4.1 Stability with respect to the anomalous Doppler effect

In order to test the stability of the stationary runaway distribution (equation (20)), we introduce in the growth rate γ_1 (equation (9)) the Maxwellian ansatz (equation (11))

$$\gamma_1 = \frac{\pi}{4} \frac{\sin^2\theta \cos\theta}{\omega_{ce}^2} \left[\frac{\partial}{\partial v} \left(T_\perp F \right) + \frac{\omega_{ce}}{k \cos\theta} F \right] \quad . \tag{33}$$

The term $\omega_{ce}/(k \cos\theta)$ is the resonant velocity v_r^d which is related to the Cerenkov resonant velocity by $v_r^d = v_r^c \, \omega_{ce}/\cos\theta$.
It ranges typically from 10 upwards and renders the destabilizing term in equation (33) more important than the term with the derivative. If we look for a sufficient criterion for stability we may simply balance

$$\frac{\pi}{4} \frac{\sin^2\theta \cos\theta}{\omega_{ce}^2} v_r^d F\left(v_r^d \right) < -\gamma_0 = -\frac{\pi}{2} \cos\theta \, v^2 \frac{\partial F}{\partial v}\bigg|_v \quad , $$

where the variable v is chosen to be the Cerenkov resonant velocity. Therefore, $v_r^d = \omega_{ce} \, v/\cos\theta$ and the criterion reads

$$\frac{\sin^2\theta}{\omega_{ce}} v \, F\left(v_r^d \right) < -2\cos\theta \, v \frac{\partial F}{\partial v}\bigg|_v \quad . $$

Let us replace $F(v_r^d)$ by its approximation for high velocities (cf. Appendix), $\partial F/\partial v|_v$ by means of the differential equation (19) and $\tilde{A}$ by its expression in terms of the source (equation (23)).

One obtains

$$\frac{\sin^2\theta}{\omega_{ce}} \, \tilde{S} \, \tilde{E} \left(1 + \frac{v_c^2 \cos^2\theta}{v^2 \omega_{ce}^2} \right) < 2 \cos\theta \, v \left[\left(1 - 2\tilde{E} \right) \left(v - \tilde{E} \, v^3 \right) F(v) \right.$$

$$\left. + \tilde{S} \, \tilde{E}^{5/2} v^3 - \left(1 - 2\tilde{E} \right) \frac{\tilde{S}}{v^2} \, \Delta \left(v_1, v_2, v \right) \right] .$$

This complicated expression is approximated at $v = v_1$ by

$$\tilde{S} < v_1^3 F(v_1) \left(1 + \frac{\sin^2\theta \; v_1}{\cos\theta \; \omega_{ce} \; v_c^3} \right)^{-1} , \qquad (34)$$

reduces at $v = v_c$ to

$$\sin\theta \, tg\,\theta < 4 \, \omega_{ce}$$

and is estimated at $v > v_2$ by

$$\sin\theta \, tg\,\theta < 2 \, \omega_{ce} v_c^2 .$$

Thus, unless we consider very oblique waves, for which the resonant velocity $v_r{}^d$ lies anyway above the velocity of light, both latter inequalities are obviously satisfied and a sufficient criterion for stability is obtained from equation (34)

$$\tilde{S} < v_1^3 F(v_1) .$$

Therefore, in so far as one disregards the case of a strong source, the distribution function found in section 3 may be declared as stable.

Let us now turn to the case of a strong source. The distribution function is nearly flat from v_1 upwards and the growth rate γ_1 has to

overcome the collisional damping only for an instability to occur. The threshold is given by (cf. equation (33))

$$\frac{\pi}{2} \frac{\sin^2\theta \cos\theta}{\omega_{ce}^2} \left[\frac{\partial}{\partial v} \left(T_{\perp} F \right) + v F \right] > \nu_o \quad ,$$

where the variable v is chosen to be the anomalous Doppler resonant velocity. The optimum angle corresponds to $(\cos \theta)^2 = 1/3$. The high velocity portion of the distribution plays a role so that F may be evaluated by (cf. Appendix)

$$F = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{v_{\parallel}^2}{2}\left(1 - \frac{v_{\parallel}^2}{2v_c^2}\right) \right] \left(\frac{v_{\perp}^2}{v_c^2} - 1\right)\left(1 - \frac{v_{\parallel}^2}{v_c^2}\right)\left(\frac{1 + \frac{v_c^2}{v^2}}{v_2^2 - v_1^2} \right) \quad ,$$

where equation (32) with $b > b_s$ has been used. Hence, a necessary condition for instability is derived

$$v\left(1 + \frac{v_c^2}{v^2}\right) \tilde{E}^2 \frac{\left(v_2^2 - v_c^2\right)\left(v_c^2 - v_1^2\right)}{v_2^2 - v_1^2} \exp\left[-\frac{v}{2}\left(1 - \frac{v_1^2}{2v_c^2}\right) \right]$$
$$> 2 \omega_{ce}^2 \nu_o \quad . \tag{35}$$

As an example, for $\tilde{E} = 2\%$, $v_1 = 5$, $v_2 = 15$, $\omega_{ce} = 2$, $\nu_0 = 2 \cdot 10^{-6}$ runaway electrons around 20 v_e might already drive the instability.

4.2 Stimulated diffusion by the anomalous Doppler interaction

As the runaway electrons are gradually accelerated by the electric field towards the velocity of light they may encounter the anomalous Doppler resonance of the imposed waves $v_r^d = \omega_{ce}/(k \cos \theta_0)$. Of course, this event implies that v_r^d lies below the velocity of light and that θ_0 is not too close to $\pi/2$. In lower hybrid experiments, for example, this could happen in the periphery of the

plasma. The interaction pitch-angle scatters the electrons that diffuse in velocity space. Nevertheless, since there is no instability but simply a stimulated diffusion due to the externally imposed waves, the strength of this process depends on the source. We shall see that the action of a weak source (in the sense $S \ll S_c$) does not stop the runaways but simply diminishes temporarily their parallel momentum. This effect results in a correction to the runaway production rate that will be calculated by means of a perturbation method.

Each of the terms in equation (16) represents the different particle fluxes involved. Now the term due to the anomalous Doppler interaction in the kinetic equation (4) may also be integrated once after having used the Maxwellian ansatz (equation (11)). Thus, we obtain the particle flux term originating from the anomalous Doppler interaction

$$\Phi_{\parallel}(v) = -\overline{D}_{\parallel} \left[\frac{\partial}{\partial v} (T_{\perp} F) + v F \right]$$

with

$$\overline{D}_{\parallel} = \frac{1}{4\pi} \int_0^{\pi/2} d\theta \int_0^{\infty} dk \, k^4 \sin^3\theta \cos^2\theta \, \frac{\mathcal{E}_k}{\omega_{ce}^2} \, \delta\left(\omega_{ce} - k \, v \cos\theta \right) \quad (36)$$

The minus sign indicates that a backward flux, directed towards the bulk of the distribution, is in question. However, this term is assumed to be small in comparison to A and will be evaluated by means of the solution of equation (19), which is considered as the zero order solution.

The pitch-angle diffusion due to the anomalous Doppler interaction is elastic (PARAIL V.V. and POGUTSE O.P., 1976) so that

the loss of parallel energy must balance the gain in the perpendicular energy. Therefore

$$\frac{\partial}{\partial t}\left(\mp T_{\perp}\right)\bigg|_{Doppler} = -\Phi_{\parallel}\, v \qquad ,$$

However, in a stationary state this increase in perpendicular energy must be compensated by the net convection of the electrons along the electric field which is given by the runaway production rate A. Thus we have

$$\frac{dT_{\perp}}{dv} = -\frac{1}{A}\,\Phi_{\parallel}\, v \quad . \tag{37}$$

Insofar as we have $\Phi_{\parallel} \ll A$ we may expect that $\partial(T_{\perp}F)/\partial v \ll v\,F$. Thus, on introducing the asymptotic expression of F for high velocities given in Appendix, one obtains

$$\Phi_{\parallel}(v) = -\overline{D}_{\perp}\,\widetilde{A}\,v_c^{2}\left(v + \frac{v_c^{2}}{v}\right) \quad . \tag{38}$$

From Eq. (17) the spectrum driven by the source reads

$$\mathcal{E}_k = S\,\delta(\cos\theta - \cos\theta_0)\,\frac{k^2}{\pi\cos\theta_0\,\dot{F}(1/k)} \quad , \quad k_2 < k < k_1 \quad . \tag{39}$$

The exact dependence of $\dot{F}$ on v is given by the differential equation (19) and is very complicated between v_1 and v_2 (cf. Fig. 1). We shall model it roughly here with a v^{-4} dependence. Since we have $F(v_c) = -2\widetilde{S}\widetilde{E}^2$ from equations (19) and (24) we may use this value for

normalization and write

$$\dot{F}(v) = -2 \frac{\tilde{S}}{v^4} \quad , \quad v_1 < v < v_2 \; .$$

Performing the integrations in equation (36) where the spectrum is given by equation (39) one obtains simply

$$\overline{D}_1 = \frac{\sin \theta_0 \; \dot{\gamma}_0}{4 \cos^2 \theta_0 \; v^3}$$

which is introduced in equation (38) to evaluate the backward flux

$$\dot{\phi}_{\shortparallel}(v) = - \frac{\sin \theta_0}{4 \cos^2 \theta_0} A \frac{v_c^2}{v^2} \left(1 + \frac{v_c^2}{v^2} \right) \; . \tag{40}$$

Combining the latter with equation (37) yields

$$\frac{d T_{\perp}}{d v} = \frac{\sin \theta_0}{4 \cos^2 \theta_0} \frac{v_c^2}{v} \left(1 + \frac{v_c^2}{v^2} \right)$$

which implies approximately a logarithmic growth of $T_{\perp}$ with the velocity. It is worth noting that the pitch-angle scattering by the Coulomb collisions results also in a logarithmic growth of $T_{\perp}$ (LIU C.S. and MOK Y., 1977).

We now calculate the correction to the runaway production rate caused by the backward flux $\phi_{\shortparallel}$. Inspection of equation (16) shows that the correction to F satisfies

$$\dot{F}_\phi = \left(\tilde{E} \, v^3 - v \right) F_\phi + \dot{\phi}_{\shortparallel}(v) \frac{v^3}{\dot{\gamma}_0} \Delta \left(\frac{v_1 \, \omega_{ce}}{\cos \theta_0} , \frac{v_2 \, \omega_{ce}}{\cos \theta_0} , v \right) . \tag{41}$$

We look for a particular solution which vanishes at $v_2 \omega_{ce}/\cos \theta_0$.

It reads

$$F_{\Phi}(v) = -\frac{1}{\gamma_0} \int\limits_{v}^{v_2 \omega_{ce}/\cos\theta_0} dz \, \Phi_{\shortparallel}(z) \, z^3 \exp\left(\frac{z^2}{2} - \tilde{E}\frac{z^4}{4}\right) H\left(z - \frac{v_1 \omega_{ce}}{\cos\theta_0}\right)$$

$$\times \exp\left(\tilde{E}\frac{v^4}{4} - \frac{v^2}{2}\right) \quad .$$

Very likely one has $v_1 \omega_{ce}/\cos\theta_0 > v_c$ so that equation (40) may be used for $\Phi_{\shortparallel}$. Then

$$F_{\Phi}(v) = -\int\limits_{v}^{v_2 \omega_{ce}/\cos\theta_0} dz \, \tilde{\Phi} \, z \exp\left(\frac{z^2}{2} - \tilde{E}\frac{z^4}{4}\right) H\left(z - \frac{v_1 \omega_{ce}}{\cos\theta_0}\right)$$

$$\times \exp\left(\tilde{E}\frac{v^4}{4} - \frac{v^2}{2}\right) \quad . \qquad (42)$$

with

$$\tilde{\Phi} = \frac{-\sin\theta_0}{4\cos^2\theta_0} \, \tilde{A} \, v_c^2 \quad .$$

Following the same procedure as for equation (20) we have to evaluate equation (42) at $v = 0$:

$$F_{\Phi}(0) = -\tilde{\Phi} \, I_3$$

with

$$I_3\left(\frac{v_1 \omega_{ce}}{\cos\theta_0} , \frac{v_2 \omega_{ce}}{\cos\theta_0}\right) = \int\limits_{v_1 \omega_{ce}/\cos\theta_0}^{v_2 \omega_{ce}/\cos\theta_0} dz \, z \exp\left(\frac{z^2}{2} - \tilde{E}\frac{z^4}{4}\right) \quad .$$

Hence the corrected runaway production rate is

$$\tilde{A} = \left(\tilde{A}_0 + S \frac{\tilde{E}^{5/2}}{1-2\tilde{E}} \right) \left\{ 1 \right.$$

$$\left. - \frac{\sin\theta_0}{4\sqrt{\pi}} \frac{V_c}{V_1^2\omega_{ce}^2 - V_c^2\cos\theta_0} \exp\left[-\frac{\tilde{E}}{4\cos^4\theta_0}\left(V_1^2\omega_{ce}^2 - V_c^2\cos^2\theta \right)^2 \right] \right\} .$$

(43)

This correction may be important when compared to the "classical" runaway rate $\tilde{A}_0$ but is small when compared to the enhanced rate due to Cerenkov $\tilde{S}$ $\tilde{E}^{5/2}$. Because of $(\cos\theta_0)^{-4}$ in the exponent the correction decreases very fast as the angle increases and the stimulated diffusion takes place far above the critical velocity.

5. DISCUSSION

We have shown that the runaway production rate may be greatly enhanced due to plasma waves with parallel phase velocities around the critical velocity. The simple analytical theory, which has been developed, provides a quantitative estimate of this phenomenon. Also, a practical formula for the runaway production rate is derived:

$A = 2\sqrt{2\pi}/(\pi^3\ln\Lambda)$ $(E/E_c)^{5/2}$ (P/T) $(sec^{-1}cm^{-3})$ where $\ln\Lambda$ is the Coulomb logarithm and P the rf power density.

This simplicity has a counterpart: as in the classical runaway kinetic theories, our model is based not only on the usual assumption of an infinite homogeneous medium, but also on the hypothesis of a constant electric field. Because of this an application to lower-hybrid current-drive experiments is not straightforward. In fact, in

toroidal discharges the decay time of the current is too long to change the total current during the rf pulse. As a result, a drop in the loop voltage, and so in the dc electric field, is observed. In spite of this, parallels between our work and experiments (YAMAMOTO et al., 1980; OHKUBO et al, 1981; MAEKAWA et al., 1981) may be drawn:

1) While, consistently with the drop in the electric field, the production of MeV-runaways is cut down (hard X-ray emission suppressed) the observations strongly suggest, in agreement with our model, that there exists an important population of runaway electrons in the keV-range which carry the rf-driven current.

2) In agreement with the observations of OHKUBO et al. (1981), we have found two different operating regimes depending on the source strength. For a weak source we expect a linear change in the run-runaways production rate and the experiments display a change in the loop voltage, soft X-ray and electron cyclotron emission which is linear in rf power. For a strong source, a saturation of these signals is observed, which agrees with the saturating runaway production rate calculated here. The critical power density (cf.equation(26)) that separates the two regimes may be written in physical units as $P_c = 2.3 \cdot 10^{14} \ (v_0/\omega_{pe}) \ (n_{13})^{3/2} \ T_{keV} \ \exp \ ((E/E_c) \ (v_1^4/4)$ $- v_1^2/2) \ (W/m^3)$, where n_{13} is normalized to $10^{13} \ cm^{-3}$, T_{keV} is normalized to 1 keV and v_1 is the minimum parallel phase velocity of the driven waves in units of the electron thermal velocity. The application of this formula for the parameters of the experiment gives a value of 5 kW/m³ which is an order below the experimental

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data of 40 kW/m^3. This discrepancy may be attributed to the inhomogenities which affect the density, temperature and parallel refractive index (BONOLI and OTT, 1981) in the experiment, and perhaps to the existence of a reflected power.

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REFERENCES

BHADRA D.K. (1980) Nucl. Fusion 20, 619.

BONOLI P.T. and OTT E. (1981) Phys. Rev. Lett. 46, 424.

BOYD D.A., STAUFFER F.J. and TRIVELPIECE A.W. (1976) Phys. Rev. Lett. 37, 98.

HARVEY R.W., RIORDAN J.C., LUXON J.L. and MARX K.D. (1981) 4th Topical Conference on RF Heating in Plasmas, Austin, Texas.

HUI B.H. and WINSOR N. (1978) Phys. Fluids 21, 940.

KULSRUD R.M., SUN Yung-Chiun, WINSOR N.K. and FALLON H.A. (1973) Phys. Rev. Lett. 31, 690.

LIU C.S., MOK Y.C., PAPADOPOULOS K., ENGELMANN F. and BORNATICI M. (1977) Phys. Rev. Lett. 39, 701.

LIU C.S. and MOK Y. (1977) Phys. Rev. Lett. 38, 162.

LIU C.S., MUSCHIETTI L., APPERT K., VACLAVIK J. and BOYD D.A. (1980) Lausanne Report LRP 175/80.

MAEKAWA T., SAITO T., NAKAMURA M., CHO T., KUBO S., SHIMOZAMA T., TERUMICHI Y., HAMADA Y. and TANAKA S. (1981) Phys. Lett. 85 A, 339.

MIKHAILOVSKII A.B., (1974) Theory of plasma instabilities, (Consultants Bureau, New-York - London), Vol. 1.

MUSCHIETTI L., APPERT K. and VACLAVIK J. (1981) Phys. Fluids 24, 151.

MUSCHIETTI L., APPERT K. and VACLAVIK J. (1981) Lausanne Report LRP 182/81.

OHKUBO K., TAKAMURA S., KAWAHATA K., TETSUKA T., MATSUURA K., NODA N., SAKURAI K., TANAHASHI S. and FUJITA J. (1981) Report IPPJ-524, Nagoya.

PAPADOPOULOS K, HUI B. and WINSOR N. (1977) Nucl. Fusion 17, 1087.

PARAIL V.V. and POGUTSE O.P. (1976) Fiz. Plazmy 2, 228 (Sov. J. Plasma Phys. 2, 126).

PARAIL V.V. and POGUTSE O.P., (1978) Nucl. Fusion 18, 303.

VEDENOV A.A. (1968) Theory of Turbulent Plasma, London Iliffe Book.

YAMAMOTO T., IMAI T., SHIMADA M., SUZUKI N., MAENO M., KONOSHIMA S., FUJII T., UEHARA K., NAGASHIMA T., FUNAHASHI A. and FUJISAWA N. (1980) Phys. Rev. lett. 45, 716.

APPENDIX: Asymptotic expressions of F

The distribution function F that is given by equation (20) reduces to simple explicit expressions in both limits of small and high velocities. We have

$$F(v) = \exp\left(\tilde{E}\frac{v^4}{4} - \frac{v^2}{2}\right) \left[\tilde{A} \int_v^\infty dz\, z^3 \exp\left(\frac{z^2}{2} - \tilde{E}\frac{z^4}{4}\right)\right.$$

(A1)

$$\left. - \tilde{S} \int_v^\infty dz\, \Delta(v_1, v_2, z)\, \frac{1}{z^2} \exp\left(\frac{z^2}{2} - \tilde{E}\frac{z^4}{4}\right)\right].$$

1. In the small velocity limit, $v < v_1 \ll v_c$, the two integrals of equation (A1) may be easily evaluated. The first one yields, after the change of variable $y = (v^2 - v_c^2)/(2v_c)$, $\pi v_c^3 \exp(1/(4E))$. The second one is identified as the integral $I_2 (v_1, v_2)$ calculated in equation (22). Therefore,

$$F(v) = \exp\left(\tilde{E}\frac{v^4}{4} - \frac{v^2}{2}\right) \left[\tilde{A}\sqrt{\pi}\, v_c^3 \exp\left(\frac{1}{4\tilde{E}}\right)\right.$$

$$\left. - \tilde{S}\sqrt{\pi}\, \exp\left(\frac{1}{4\tilde{E}}\right) \exp\left(-\ell n\left(v_c^2 - z\right)\right)\right]$$

or by using equations (21) and (23) for A

$$F(v) = \frac{1}{\sqrt{2\pi}} \exp\left(\tilde{E}\frac{v^4}{4} - \frac{v^2}{2}\right).$$

(A2)

2. In the high velocity limit, $v > v_2 > v_c$, the term which is proportional to S disappears and the remaining integral is evaluated by means of the change of variable as before. One obtains

$$F(v) = \frac{\tilde{A}}{\tilde{E}}\left(1 + \frac{v}{2} \frac{2v}{v^2 - v_c^2}\right) = \frac{\tilde{A}}{\tilde{E}}\left(1 + \frac{v_c^2}{v^2}\right) . \tag{A3}$$

FIGURE CAPTION

Shape of the driven spectrum in the linear regime. The strong source regime would be represented by a horizontal line. We display two cases of source strength $S = 5 \cdot 10^{-8}$ (dashed line) and $S = 1.5 \cdot 10^{-7}$ (dotted line), for the same electric field $\tilde{E} = 1$ %, angle $\theta_0 = 45^0$ and collision frequency $\nu_0 = 1.34 \cdot 10^{-6}$. Note that with increasing ν the spectrum becomes independent of S. The solid line indicates the approximation used to calculate the stimulated pitch-angle diffusion by the anomalous Doppler effect (cf. section 4.2).

$10^5 \times \mathcal{E} \, (k = v^{-1}, \theta_o)$