JULY 1981 LRP 184/81

EFFECTS OF MAGNETIC FIELD CURVATURE ON ALFVEN WAVE HEATING OF TOKAMAK PLASMAS

K. Appert, B. Balet, and J. Vaclavik

EFFECTS OF MAGNETIC FIELD CURVATURE ON ALFVEN WAVE HEATING OF TOKAMAK PLASMAS

K. Appert, B. Balet, and J. Vaclavik

Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne
CH - 1007 Lausanne / Switzerland

ABSTRACT

It is shown that the curvature of magnetic field lines may enhance the rate of energy absorption of surface quasi-modes by a few orders of magnitude at the Alfvén resonance surfaces located well into the plasma interior.

PACS numbers: 52.50.Gj, 52.35.Bj

It has been argued recently[1] that the absorption of surface quasi-modes (kink-like modes) at the Alfvén resonance surface cannot be efficiently used to heat the interior of tokamak plasmas. The reason given is that the loading resistance of the antenna rapidly decreases as the position of the resonance surface is shifted towards the plasma axis.[2] These theories, however, are based on simplifying assumptions. Although the problem is investigated in a cylindrical geometry the equations used are, to a great extent, equivalent to those derived for the case of a slab geometry with straight magnetic field lines.[3] On the other hand, numerical calculations[4] performed using a complete set of ideal MHD equations in a cylindrical geometry show that an efficient energy absorption can take place at the innermost plasma surfaces. The aim of the present letter is to demonstrate that this discrepancy is due to the curvature of the magnetic field lines, which was neglected in the previous treatments.

We consider low-frequency small-amplitude oscillations in a cold current-carrying plasma. The linearized ideal MHD equations are then appropriate to describe the plasma motion. They can be written in the form

$$\mu_o \rho \frac{\partial \vec{v}}{\partial t} = \text{curl } \vec{B} \times \vec{b} + \text{curl } \vec{b} \times \vec{B} , \tag{1}$$

$$\frac{\partial \vec{b}}{\partial t} = - \text{curl } \vec{E}, \tag{2}$$

$$\vec{E} = - \vec{v} \times \vec{B} , \tag{3}$$

where $\vec{v}$ is the plasma velocity, $\vec{b}$ and $\vec{B}$ are the oscillating and equilibrium magnetic fields respectively, $\vec{E}$ is the electric field and ρ is the equilibrium mass density. On introducing the displacement vector $\vec{\xi}$, defined by $\vec{v} = \partial\vec{\xi}/\partial t$, Eqs. (1)-(3) can be reduced to

$$\mu_o \rho \frac{\partial^2}{\partial t^2} \vec{\xi} = -\mu_o \nabla \tilde{p} + [\vec{B} \cdot \nabla \text{curl}(\vec{\xi} \times \vec{B})] + [\text{curl}(\vec{\xi} \times \vec{B}) \cdot \nabla \vec{B}], \tag{4}$$

where

$$\tilde{p} \equiv \frac{\vec{B} \cdot \vec{b}}{\mu_o} = \frac{1}{\mu_o} \vec{B} \cdot \mathrm{curl}(\vec{\xi} \times \vec{B}) \tag{5}$$

is the perturbed magnetic pressure. We now adopt a cylindrical geometry and assume that the equilibrium quantities are functions of radius r only. We then have $\vec{B} = B_z\hat{z} + B_\theta\hat{\theta}$. Moreover, we may take the time and space dependence of $\vec{\xi}$ and $\tilde{p}$ in Eqs. (4) and (5) as $\exp\{i[kz + m\theta - (\omega + i\nu)t]\}$, where $\nu \to 0_+$. To proceed further we introduce a local coordinate system with $\hat{r}$, $\hat{e} = \hat{e} \times \hat{r}$, $\hat{e} = \vec{B}/B$ and assume $|B_\theta/B_z| << 1$. On projecting Eq. (4) on $\hat{r}$ and expanding its right-hand side up to the first order in B_θ/B_z, we find

$$\rho A\xi_r = \frac{d\tilde{p}}{dr} + i\frac{2B_\theta}{\mu_o r} k_\parallel B_z \xi_\perp . \tag{6}$$

Here, $A = (\omega + i\nu)^2 - k_\parallel^2 c_A{}^2$, $k_\parallel B = kB_z + m/rB_\theta$ and c_A is the Alfvén speed. Likewise, the projection of Eq. (4) on $\hat{e}$ and the subsequent expansion in B_θ/B_z yields

$$\rho A\xi_\perp = ik_\perp\tilde{p} - i\frac{2B_\theta}{\mu_o r} k_\parallel B_z \xi_r , \tag{7}$$

where $k_\perp B = m/rB_z - kB_\theta$. Next, we expand the right-hand side of Eq.(5) to obtain

$$\frac{1}{r}\frac{d}{dr}(r\xi_r) = -ik_\perp\xi_\perp - \frac{\mu_o\tilde{p}}{B^2} . \tag{8}$$

Finally, substituting $\xi_\perp$ from Eq. (7) into Eqs. (6) and (8), we arrive at two coupled first-order equations for the quantities ξ_r and $\tilde{p}$

$$A\frac{1}{r}\frac{d}{dr}(r\xi_r) = -\frac{2B_\theta}{\mu_o r}\frac{B_z}{\rho}k_\parallel k_\perp \xi_r + (k_\perp^2 c_A^2 - A)\frac{\mu_o}{B^2}\tilde{p}, \tag{9}$$

$$A\frac{d\tilde{p}}{dr} = \left[\rho A^2 - \left(\frac{2B_\theta}{\mu_o r}\right)^2 \frac{k_\parallel^2 B_z^2}{\rho}\right]\xi_r + \frac{2B_\theta}{\mu_o r}\frac{B_z}{\rho}k_\parallel k_\perp \tilde{p} . \tag{10}$$

It is worth mentioning that Eqs. (9) and (10) are, in fact, a particular approximation (cold plasma, small B_θ) to the equations derived earlier[5] for the case of a general cylindrically-symmetric equilibrium. For our purpose, however, the former are sufficiently accurate. If we now dispense with the terms which contain B_θ explicitly, Eqs. (9) and (10) are reduced to

$$A\frac{1}{r}\frac{d}{dr}(r\xi_r) = (k_\perp^2 c_A^2 - A)\frac{\mu_o}{B^2}\tilde{p}, \tag{11}$$

$$\frac{d\tilde{p}}{dr} = \rho A \xi_r . \tag{12}$$

These equations are equivalent to those used in Ref. 2 and to the MHD limit of those used in Ref. 1. They can be obtained directly from the equations pertinent to a slab geometry,[3] and take into account a shear of magnetic field lines but not their curvature. On the contrary, Eqs. (9) and (10) contain both. The curvature has an important effect on the behavior of $\tilde{p}$ near the singularity $A(r_o) = 0$. It may easily be shown from Eqs. (9) and (10) that $\tilde{p} \sim \xi_r \sim \ln(r-r_o)$ whereas

Eqs. (11) and (12) imply $\xi_r \sim \ln(r-r_o)$ and $p \sim (r-r_o)^2\ln(r-r_o)$. Thus, the curvature makes the equations of plasma motion somewhat "more singular". We shall see shortly that this feature strengthens the coupling of the plasma global motion (surface quasi-modes) to the Alfvén continuum and increases, therefore, the rate of energy absorption.

We now assume that the plasma oscillations are excited by an idealized antenna, consisting of a sheet current of given frequency and single helicity, which is located at a radius r_A in the vacuum region between the plasma column and a perfectly conducting wall of radius r_W:

$$\vec{J}_A = J_o(k\hat{\theta} - \frac{m}{r_A}\hat{z})\delta(r-r_A)\cos(\omega t)\cos(m\theta+kz).$$

The resulting boundary-value problem can then be solved using the Maxwell equations in the vacuum, the familiar boundary conditions at the plasma-vacuum interface,[6] and Eqs. (9) and (10) or (11) and (12) in the plasma. For the equilibrium quantities we choose profiles which are typical for tokamak plasmas: $\rho = \rho_o(1-.9x^2)$, $B_z=B_o$ = const. and $B_\theta = B_o\alpha x(3-3x^2+x^4)$, where α is a small parameter, $x = r/a$ and a is the plasma radius. As for the antenna current we take $J_o = aB_o/(2\mu_o)$. The equations are integrated numerically by means of the Runge-Kutta method with an appropriate choice of $\nu/\omega \ll 1$. Once the solution is found we calculate the power delivered by the antenna per unit length of the plasma according to

$$P = -\mathrm{Re}\int \vec{J}_A \cdot \vec{E}\, dV_A/2.$$

A typical result is shown in Fig. 1 where the power is plotted versus the position of the resonance surface. The parameters used were $m = 1$, $k = .6/a$, $\alpha = .1$, $r_A = 1.2a$, $r_W = 1.5a$, and the frequency was varied such that the resonance surface moved along the plasma radius. The line A was obtained using Eqs. (9) and (10), i.e. with the curvature terms included, whereas the line B was computed by means of Eqs. (11) and (12), which do not take into account the curvature. It is easily seen that the curvature has a dramatic effect on the absorbed power. For the inner resonance surfaces the power is enhanced by a few orders of magnitude in the presence of the curvature. This can be understood if we invoke the previous argument about the coupling of the plasma global motion to the Alfvén continuum. As shown in Fig. 2 the perturbed magnetic pressure, which is associated with the global motion, is considerably increased all along the plasma radius in the case where the curvature terms are included in the calculation. In this figure the resonance surface is at $r = .4a$, corresponding to the frequency $\omega = .9\ c_{Ao}/a$, $\nu/\omega = 10^{-4}$ and the other parameters used are the same as for Fig. 1. It has been demonstrated recently[4] that for a given equilibrium and fixed value of m the absorbed power has a maximum at a definite value of $k(\omega)$ for each resonance surface. The dependence of such a maximum upon the position of the resonance surface is shown in Fig. 3 for the case where $m = 1$, $\alpha = .1$. Here again the lines A and B correspond to the calculations with and without the curvature, respectively. We notice that the optimal powers are comparable. However, the position of the optimal surface is appreciably shifted towards the plasma axis when the curvature is taken into account.

In conclusion, we have demonstrated that the curvature of the magnetic field lines plays a very important role in determining:
1) the rate of energy absorption due to the coupling of surface quasi-modes to shear Alfvén waves, 2) the position of the optimal resonance surface. The former is enhanced, in general, while the latter is pushed towards the plasma interior. Both features are favorable for the Alfvén wave heating scheme of tokamak plasmas.

The authors wish to acknowledge the useful discussions with Dr. R. Gruber, Prof. A. Hasegawa, and Prof. F. Troyon. They also acknowledge Dr. J.B. Lister for reading the manuscript.

This work has been supported by the Swiss National Science Foundation, the Ecole Polytechnique Fédérale de Lausanne and by Euratom.

REFERENCES

[1] D.W. Ross, G.L. Chen, and S.M. Mahajan, presented at the Fourth Topical Conference on RF Heating in Plasma, Austin, Texas, February 9-11, 1981.

[2] R. Keller et al., presented at the Fourth Topical Conference on RF Heating in Plasma, Austin, Texas, February 9-11, 1981.

[3] L. Chen and A. Hasegawa, Phys. Fluids 17, 1399 (1974).

[4] K. Appert, B. Balet, R. Gruber, F. Troyon, and J. Vaclavik, presented at the Eighth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Brussels, July 1-10, 1980, IAEA-CN-38/D-1-1.

[5] K. Appert, R. Gruber, and J. Vaclavik, Phys. Fluids 17, 1471 (1974).

[6] B.B. Kadomtsev, in Review of Plasma Physics, edited by M.A. Leontovich (Consultants Bureau, New York, 1966), Vol. 2, p. 153.

FIGURE CAPTIONS

Fig. 1 Absorbed power versus the position of the resonance surface for $m = 1$, $k = .6/a$, $\alpha = .1$, $r_A = 1.2a$, $r_W = 1.5a$.
Line A: with the curvature. Line B: without the curvature.

Fig. 2 Radial profile of the perturbed magnetic pressure (plasma global motion) for the case where the resonante surface is at $r = .4a$. $\omega = c_{Ao}/a$, $\nu/\omega = 10^{-4}$, and the other parameters used are the same as in Fig. 1. Line A: with the curvature. Line B: without the curvature.

Fig. 3 Maximal absorbed power versus the position of the resonance surface for $m = 1$, $\alpha = .1$, $r_A = 1.2a$, $r_W = 1.5a$.
Line A: with the curvature. Line B: without the curvature.

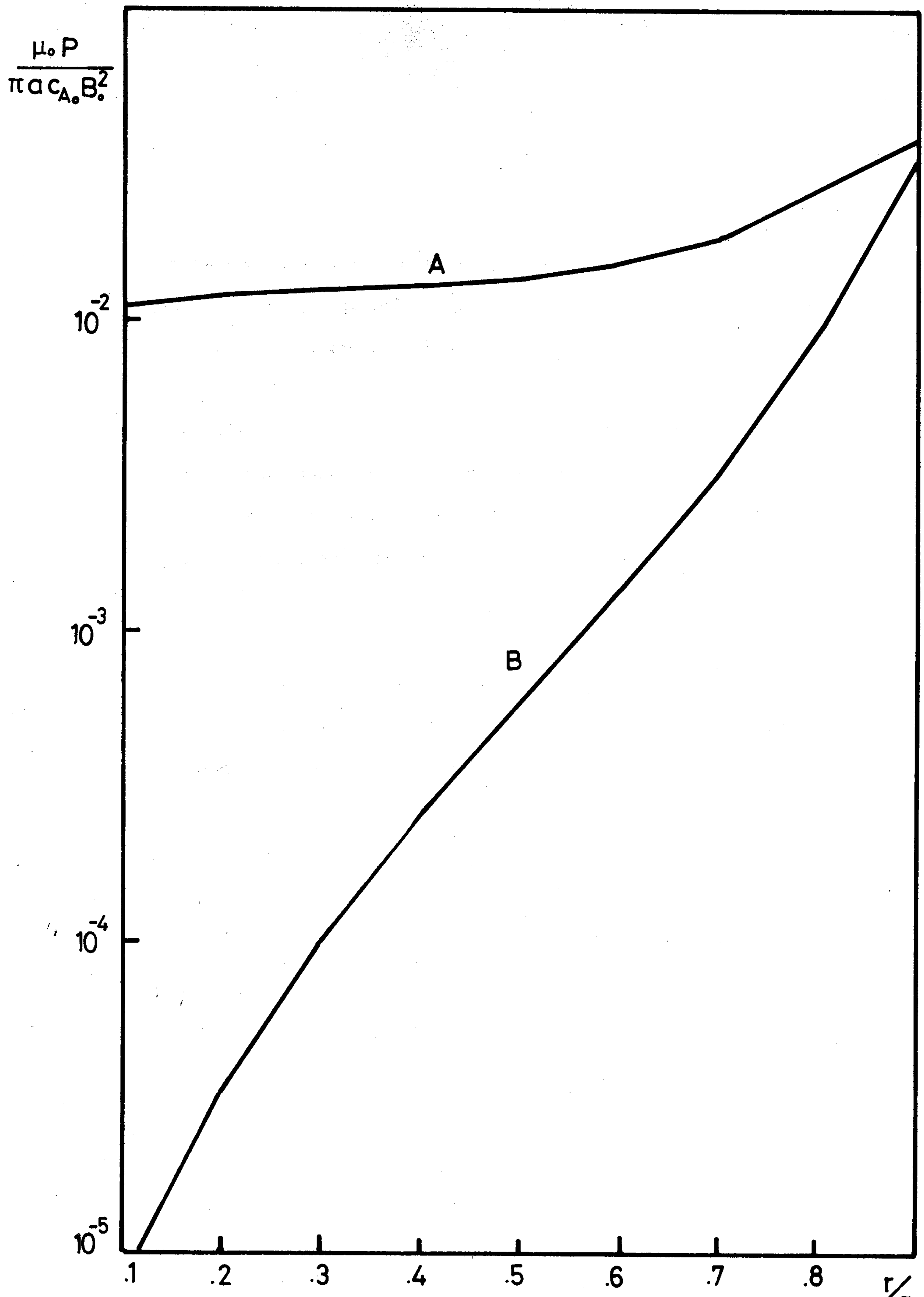

Fig. 1

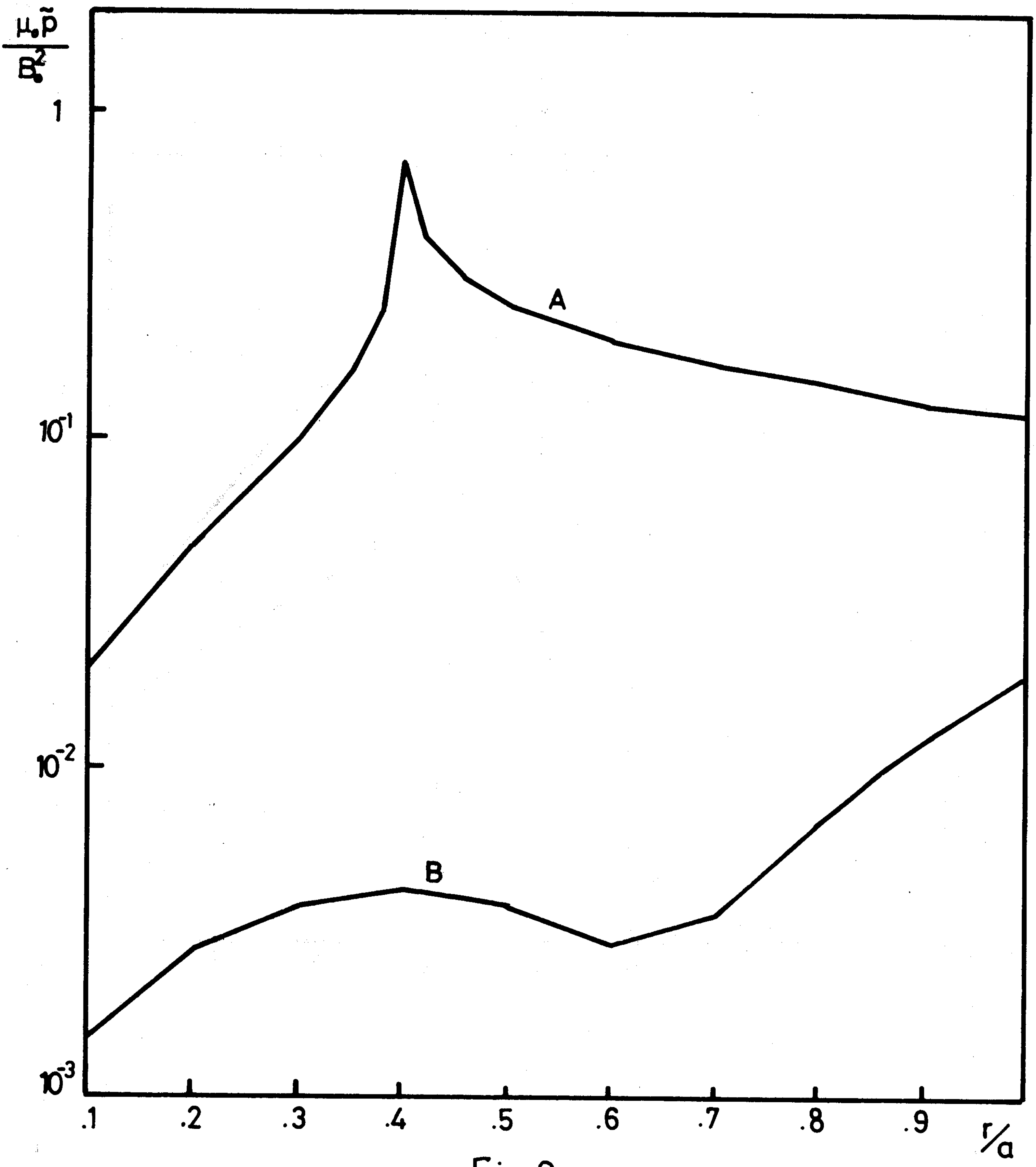

Fig. 2

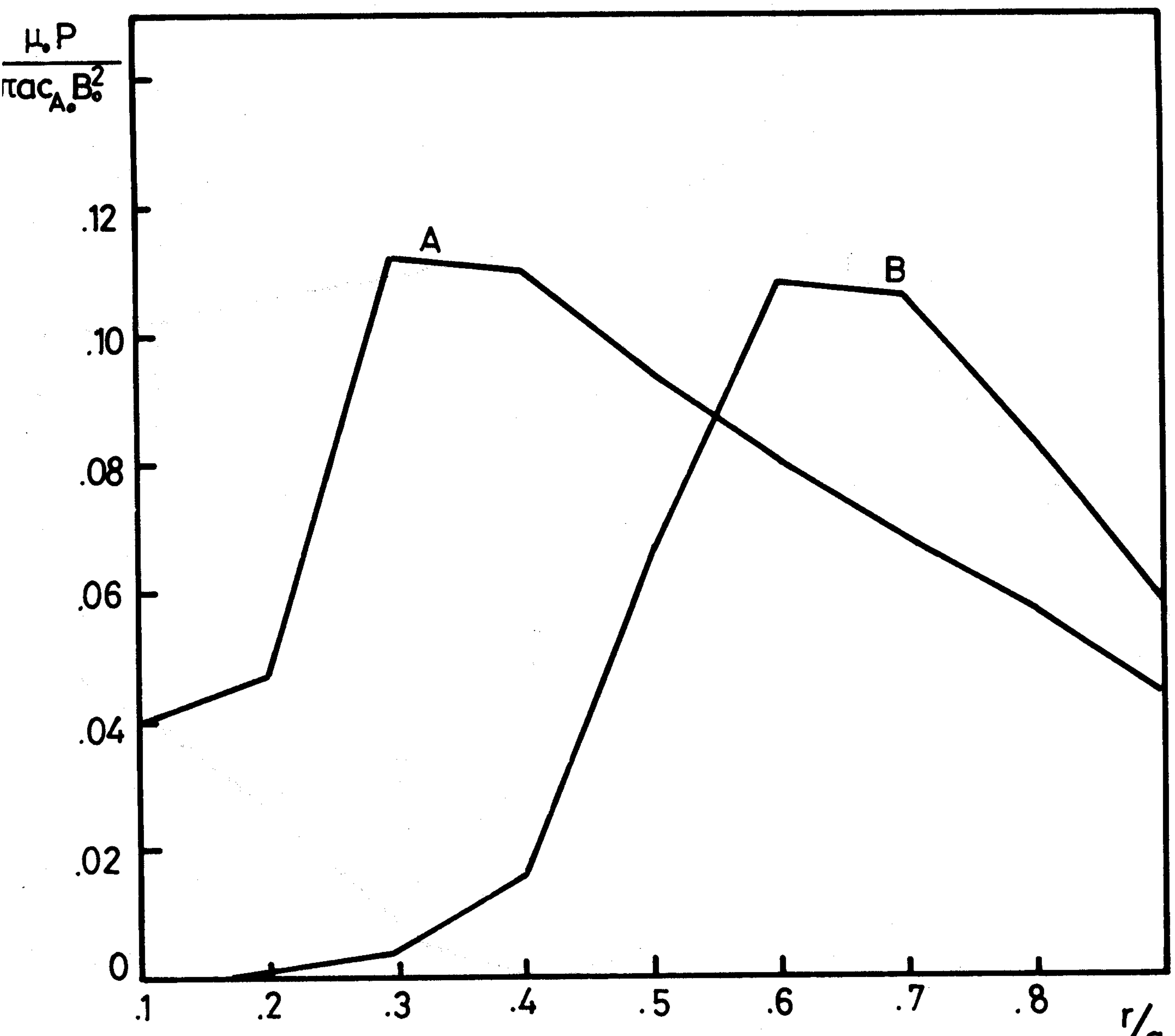

Fig. 3