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MICROINSTABILITIES IN TOKAMAKS

C.S. Liu*

*on leave from University of Maryland
Department of Physics and Astronomy

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Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne
CH-1007 Lausanne / Switzerland

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Department of Physics and Astronomy

ABSTRACT

The recent progress in the microinstability theory of Tokamaks is reviewed.

I INTRODUCTION

One of the most important problems of the controlled fusion research is the observed anomalous transport across the magnetic field in magnetically confined plasmas. The basic idea of magnetic confinement of plasma is that the magnetic field restricts the charged particle motion to a gyroradius around the magnetic field line so that, upon colliding with other particles, they random walk across the magnetic field with a small step, of the order of their gyroradius. The classical diffusion coefficient is therefore $D_c \simeq \nu_{ei} \rho_e^2 \propto T^{-1/2} B^{-2}$, where ν_{ei} is the electron - ion collision frequency and ρ_e is the mean electron gyroradius. The confinement therefore is expected to improve with increasing magnetic field and temperature. In Tokamak geometry, there are geometric factors that enhance the classical diffusion coefficient (neoclassical diffusion), which however is still small compared with the observed particle and energy transport. To account for the anomalous transport is therefore a central problem for Tokamak research and is yet poorly understood. The major energy loss in Tokamaks is through the enhanced crossfield electron heat conductivity which is inversely proportional to plasma density (Alcator scaling) and has a magnitude of the order

$$\chi_e = 10^4 \, n_{14}^{-1} \quad cm^2/sec \tag{1}$$

which is about one to two orders of magnitude higher than neoclassical prediction. χ_e may also depend inversely on T_e and M_i .

To explain this anomalous crossfield transport has been the primarily motivation for studying the microinstabilities in Tokamaks. For a confined plasma is always in a state of non thermal equilibrium with excess free energy due to the deviation from a homogeneous, Maxwellian distribution. This excess frequency can feed into many natural oscillations in plasmas leading to plasma instabilities and anomalous transport. To confine a plasma pressure by magnetic field requires a current perpendicular to both the magnetic field and the pressure gradient

$$\vec{j} \times \vec{B} = c \nabla p \tag{2}$$

The drift velocity associated with this current

$$v_d = \frac{j_\perp}{ne} = \frac{cT}{eB} \frac{d \ln n}{dx} = c_s \frac{\rho_i}{L_n} \tag{3}$$

is much below the ion sound speed $c_s = (T_e/M)^{1/2}$, as $\rho_i \ll L_n = (d\ln n/dx)^{-1}$ where we have assumed the temperature to be uniform. This relative motion between electrons and ions across the magnetic field generates a new set of modes with phase velocity comparable to the v_d and frequency $\omega \approx \vec{k} \cdot \vec{v}_d$ which is usually much below the ion cyclotron frequency Ω_i for $k\rho_i^2/L_n < 1$, where ρ_i is the mean ion gyroradius. Because they are generically of the same origin, we will call all these low-frequency modes drift modes. In Tokamaks the toroidal geometry introduces rich varieties of drift modes, particularly as a result of the so-called trapped particles, i.e. particles trapped by the inhomogeneous magnetic field. Those instabilities which own their existence to the trapped particle are usually called trapped particle instabilities. In Section II we will review the theory of drift waves in the slab geometry with sheared magnetic field, which has seen much progress in the past few years both in linear stability analysis and in nonlinear development.

II ELECTROSTATIC DRIFT WAVES IN SLAB GEOMETRY - LINEAR THEORY

a) Local Analysis

Consider a potential perturbation $\varphi(x) \exp i(k_y y + k_z z - \omega t)$ in a slab plasma with density $n(x) = n_0 (1 + x/L_n)$ immersed in a static magnetic field $B\hat{z}$. Assuming $\omega \ll \Omega_i = eB/mc$, $k\rho_i \ll 1$ and $T_i = 0$, we may calculate the ion density response using the guiding center approximation in which the ion experiences a $E \times B$ drift and a polarization drift

$$\vec{v}_E = \frac{c\,\vec{E} \times \vec{B}}{B^2} \quad , \quad \vec{v}_p = -\frac{c}{\Omega B}\left(\frac{\partial \vec{E}_\perp}{\partial t} + \vec{v}\cdot\nabla \vec{E}_\perp\right) \tag{4}$$

Using the continuity equation for ions

$$\frac{\partial n_i}{\partial t} + \vec{v}_E \cdot \nabla n_i + n_i \nabla \cdot \vec{v}_p = 0 \tag{5}$$

we then obtain the perturbed ion density as

$$\frac{\delta n_i}{n_0} = \frac{e\varphi}{T_e}\left[\frac{\omega_*}{\omega} + \frac{\omega}{\Omega_i} k^2 \rho_s^2\right] \tag{6}$$

where $\omega_* = \frac{k_y c T}{eB} \frac{d \ln n}{dx}$ and $\rho_s^2 = T_e / M\Omega_i$.

As the thermal velocity of electrons v_e is much greater than the parallel phase velocity $\omega/k_\parallel$, the bulk electrons can readily adjust their parallel pressure to balance the electric force

$$n m \frac{dv_\parallel}{dt} = - T_e \frac{\partial n}{\partial z} + ne \frac{\partial \phi}{\partial z} = 0 \tag{7}$$

so as to achieve a Boltzmann distribution

$$n_e = n_0 \, e^{e\phi/T_e} \tag{8}$$

Because the low frequency and long wavelength of the potential, the quasi neutrality condition must hold

$$n_e = n_i \tag{9}$$

Using (9) and substituting Eq. (8) into Eq. (4) while neglecting the last term of (4) due to the polarization drift, we find the following linear equation

$$\frac{\partial \phi}{\partial t} - v_d \frac{\partial \phi}{\partial y} = 0 \tag{10}$$

Note that we have used the electron Boltzmann distribution Eq. (8) without linearization. Thus Boltzmann electrons do not contribute to nonlinear mode coupling. Thus by neglecting the ion polarization drift, one finds that any form of $\varphi(y - V_d t)$ is a solution to Eq. (10).

We have so far neglected the "resonant electrons" whose parallel velocity is close the parallel phase velocity $v_{\parallel} = \omega_*/k_{\parallel}$ and thus in Cerenkov resonance with the wave, causing induced emission and absorption. To find their response to the potential perturbation, we use the drift kinetic equation which is simply the reduced Vlasov equation for the guiding center distribution $f(\vec{X}, v_{\parallel}, t)$ where $\vec{X}, v_{\parallel}$ are guiding center position and parallel velocity :

$$\frac{\partial f}{\partial t} + v_{\parallel}\frac{\partial f}{\partial z} + c\frac{\vec{E}\times\vec{B}}{B^2}\cdot\nabla_{\perp} f + \frac{eE_{\parallel}}{m}\frac{\partial f}{\partial v_{\parallel}} = 0 \tag{11}$$

Assume that $f = f_0(X, v_{\parallel}) + f_1(x)\exp i(k_y y + k_z z - \omega t)$ and $f_1 \ll f_0$, we may linearize Eq. (11) to find

$$f_1 = \frac{e\varphi}{m}\left(\frac{k_y}{\Omega_e}\frac{\partial f_0}{\partial X} - k_z\frac{\partial f_0}{\partial v_{\parallel}}\right)\Big/(\omega - k_z v_{\parallel}) \tag{12}$$

Integrating over $v_{\parallel}$ to obtain the density perturbation which for a Maxwellian equilibrium distribution :

$$f_0(v_{\parallel}, x) = \frac{n_0(x)}{\pi^{1/2} v_e} e^{-v_{\parallel}^2/v_e^2}$$

becomes

$$\frac{\delta n_e}{n_0} = \frac{e\varphi}{T_e}\left[1 + \frac{\omega - \omega_*}{k_{\parallel} v_e} Z\left(\frac{\omega}{k_{\parallel} v_e}\right)\right] \tag{13}$$

where

$$Z(\zeta) = \frac{1}{\pi}\int d\eta\, e^{-\eta^2}/(\eta - \zeta) \tag{14}$$

the plasma dispersion function with the contour defined in the Landau sense (i.e. ζ has a small positive imaginary part). For $\omega \ll k_\parallel v_e$,

$$\frac{\delta n_e}{n} = \frac{e\varphi}{T_e}\left[1 + i\sqrt{\pi}\,\frac{\omega - \omega_*}{|k_\parallel| v_e} \right] \tag{15}$$

Note that for $\omega = \omega_*$, the resonant electrons make no contribution. Substituting Eq. (15) and Eq. (5) into quasi-nautrality condition Eq. (19), we find the dispersion relation for drift wave

$$\omega_k = \omega_*(1 - k_y^2\rho_s^2), \qquad \gamma_k = \frac{\omega_k^2\, k^2\rho_s^2}{|k_\parallel|\, v_e} > 0 \tag{16}$$

This is the <u>universal</u> instability, so named because it is thought to be prevalent in all magnetically confined plasmas. The ion polarization drift produces a downward frequency shift $\delta\omega_k = -\omega_*\, k_y^2\rho_s^2$ which makes the wave energy negative in the frame of electron diamagnetic drift in which the resonant electrons do not feel the effect of inhomogeneity

$$\mathcal{E} = \omega \left.\frac{\partial\, \mathrm{Re}\,\epsilon}{\partial\omega}\right|_{\omega=\omega_k} \frac{|E_k|^2}{8\pi} = \frac{\delta\omega_k}{k^2\lambda_D^2}\,\frac{\omega_*}{(\delta\omega_k)^2}\,\frac{|E_k|^2}{8\pi} < 0$$

The electron Landau damping thus makes the wave unstable.

In a collisional plasma with high electron - ion collision frequency, the density perturbation can be obtained from the electron fluid equations

$$\frac{\delta n_e}{n_0} = \frac{e\varphi}{T}\left[\frac{k_{\parallel}^2 D - i\omega_*\left(1 - i\frac{\omega}{\nu}\right)}{k_{\parallel}^2 D - i\omega\left(1 - i\frac{\omega}{\nu}\right)}\right] \tag{17}$$

$$\rightarrow \frac{e\varphi}{T}\left[1 - i\frac{(\omega_* - \omega)}{k_{\parallel}^2 D}\left(1 - i\frac{\omega}{\nu}\right)\right] \quad \text{for} \quad k_{\parallel}^2 D \gg \omega, \omega_*$$

where $D = T_e/m\nu$ in the parallel diffusion coefficient. From Eq. (6), (9) and (17) one obtains the growth rate for the dissipative drift mode for $\nu > \omega$

$$\gamma = \omega_*^2 k_y^2 \rho_s^2 / k_{\parallel}^2 D \tag{18}$$

b) Nonlocal Analysis

So far we obtained the linear dispersion relation neglecting the spatial variation of the potential along the density gradient $\varphi(x)$. Because of the plasma inhomogeneity, we should properly solve the eigenvalue problem, yielding not only the frequency and growth rate, but also the width of the region where the mode is localized (the eigenfunction). In the absence of sheared magnetic field, one expects the drift modes to be localized near the maximum density gradient. We may therefore expand the drift frequency around the maximum density gradient

$$\omega_* = \omega_{*0}\left(1 - x^2/L_n^2\right)$$

and $\varphi(x) = \varphi(x_0) + \varphi'(x-x_0) + \varphi''(x-x_0)^2/2$

to find the eigenvalue equation as Weber equation with a "potential well" due to the $\omega_{*0}\, x^2/L_n^2$ term :

$$\rho_s^2 \frac{d^2\varphi}{dx^2} + \frac{\omega_{*0}}{\omega}\left(1 - \frac{x^2}{L_n^2}\right)\varphi - \left(1 + \frac{\omega - \omega_*}{k_{\parallel} v_e}\, i\sqrt{\pi}\right)\varphi = 0 \qquad (19)$$

which has the fundamental-mode solution $\varphi \sim \exp -\alpha x^2$ with $\alpha^{-1/2} \simeq \sqrt{\rho_s L_n}$ as the width of localization. There is also a small frequency shift $\delta\omega = (\rho_s/L_n)\,\omega_*$. In the presence of a sheared magnetic field due to the radial variation of the rotational transform ι, which we model in the slab geometry by introducing a y-component magnetic field as $\vec{B}_y = \hat{y} B_0\, x/L_s$ so that the total magnetic field is $\vec{B} = B_0(\hat{z} + \frac{x}{L_s}\hat{y})$. This sheared magnetic field makes $k_{\parallel}$ a function of x: $k_{\parallel} = \vec{k}\cdot\vec{B}/B_0 = k_z + k_y x/L_s$.

In this case, the additional ion sound term arising from the perturbed parallel ion motion must be taken into account and Eq. (19) becomes

$$\rho_s^2 \frac{d^2\varphi}{dx^2} + \left(\frac{\omega_*}{\omega} - 1\right)\varphi - \left(\frac{\omega_*}{\omega}\frac{x^2}{L_n^2} - \frac{k_y^2 c_s^2 x^2}{\omega^2 L_s^2}\right)\varphi = 0 \qquad (20)$$

where we have neglected the imaginary part and the last term is from $k_{\parallel}^2 c_s^2/\omega^2$ with $c_s^2 = T_e/M$. Note that the last two terms are of the opposite sign so that, for sufficiently strong magnetic shear, the "potential" proportional to x^2 for the Schrödinger equation becomes a "hump" instead of a "well" and it can no longer trap waves, i.e. localized mode no longer exists. This is the criterion for shear stabilization given by Krall and Rosenbluth

$$\frac{L_n^2}{L_s \rho_i} > 1 \tag{21}$$

This condition, though easy to satisfy, is not sufficient because even with potential hump, unstable modes can still exist if it has an outgoing group velocity. From now on we shall simply neglect the density gradient variation and take into account only the spatial variation of $k_\parallel$ due to magnetic shear.

To do the stability analysis correctly, one must include the complete electron response and the eigenvalue equation is

$$\frac{d^2\varphi}{dx^2} + \left\{ \frac{L_s^2}{L_n^2}\left(\frac{\omega_*}{\omega}\right)^2 x^2 - b - \left(1 - \frac{\omega_*}{\omega}\right)\left[1 + \frac{\omega}{|k_\parallel(x)| v_e} Z\left(\frac{\omega}{|k_\parallel| v_e}\right)\right]\right\}\varphi = 0 \tag{22}$$

where $b = k_y^2 \rho_s^2$ and x is normalized to ρ_s.

Quite surprisingly, it was first shown by numerical analysis that there is no absolute instability, i.e. exponentially growing eigenmodes, even for a very small shear. Then Antonsen proved that in fact Eq. (22) has no absolute instability at all. He transformed the coordinate x by rotating it 90° in the complex plane to $\xi = -i\omega x/\omega_*$.

In this new coordinate,

$$g(\xi) \equiv 1 + \frac{\omega}{|k_{\parallel}(\xi)| v_e} Z\left(\frac{\omega}{|k_{\parallel}(\xi)| v_e}\right) = 1 + y^2 \int_0^{\infty} \frac{dy' \, e^{-y'^2}}{(y'^2 + y^2)} \qquad (23)$$

is real and greater than unity, where $iy = \omega / k_{\parallel}(\xi)\, v_e$ $= -i\,(L_s\, \omega^2 / \omega_* k\, v_e\, \xi)$. With the boundary conditions for localized modes : $\varphi \to 0$ as $\xi \to \infty$, which corresponds to outgoing mode condition (group velocity in x be away from the singular surface) and φ or $\varphi' = 0$ at $\xi = 0$, a quadratic form can be obtained from Eq. (22) by multiplying it with φ^* and integrating over ξ. Then assuming $\omega = \omega_r + i\gamma$ with $\gamma > 0$ for instability, we set real and imaginary part of Eq. (22) to zero to find

$$\gamma \int_0^{\infty} d\xi \left[\frac{\omega_r \omega_*^2}{|\omega^2|^2} \left|\frac{d\varphi}{d\xi}\right|^2 + g(\xi)\, |\varphi|^2 \frac{\omega_*}{\omega} \right] = 0$$

Since $g > 0$ and $\omega_r \omega_* > 0$ the integrand is positive and γ must vanish, a contradiction. Thus we proved the absence of absolute instability. Chang and Liu extended this analysis to include the effects of electron temperature gradient and again no absolute instability exists for both signs of $d\ln T / d\ln n$, even though this makes an important contribution to the growth rate in local analysis.

This result is surprising because for a number of years the analysis of drift wave stability in a sheared field has followed the prescription of Berk and Pearlstein in which the resonant electron response in Eq. (22) is approximated by setting Z to $i\sqrt{\pi}\, \omega / |k_{\parallel}| v_e$, which is then treated as a perturbation for modes satisfying outgoing boundary conditions. This leads to a shear stabilizing condition $L_n/L_s > (m/M)^{1/3}$. The complete stabilization of absolute modes by small amount of shear shows the importance of $Z(\omega/|k_\parallel|v_e)$ near $\omega \approx k_\parallel(x)v_e$ in affecting the stability. An interesting conjecture which has received considerable publicity recently due to Molvig and Hirshman is the possible destabilizing effect of resonance broadening in a sheared magnetic field. The argument goes as follows. It is seen that the complete stabilization is due to the dynamic effects of those electrons near the rational surface $x = 0$ or more precisely between $x = 0$ where $k_\parallel = 0$ and $x = x_e$ where $\omega_* \approx k_\parallel(x_e)v_e$. So if some anomalous process can diffuse these electrons out of this region to the region where $x > x_e$ or $\omega_*/k_\parallel(x) \ll v_e$ then one would expect the usual resonant approximation to be valid and the drift wave again become unstable. In a sheared magnetic field, the resonance broadening is enhanced because of the coupling between parallel and perpendicular diffusion. The usual decorrelation time due to electron flow along the magnetic field $(k_\parallel v_e)^{-1}$ is now coupled to motion across the field because $k_\parallel^2(x) = k_y^2 x^2/L_s^2$ $= k_y^2 D t/L_s$ so that

$$k_\parallel^2(x)\, v_e^2 t^2 \approx k_y^2 v_e^2 D t^3/L_s^3$$

or the decorrelation time

$$\tau_s = \left(L_s^2/k_y^2 v_e^2 D\right)^{1/3}$$

much smaller than $(k^2 D)^{-1}$. Adding this to the electron dynamics, they found that the drift wave becomes absolutely unstable for relatively small value of D. In fact the growth rate as a function of D first becomes positive for $D > D_{min}$, reaches a maximum then decreases again to zero for $D > D_{max}$. They interprete D_{max} as the saturation value for the anomalous diffusion. The work, though interesting, suffers from the criticism that "resonance broadening" is important only for resonance particles which in this case have $v_{\parallel} > \omega_*/k_{\parallel}$ located beyond x_e in the first place. Thus they applied the broadening mostly to the nonresonant particles $v_{\parallel} \lesssim \omega_*/k_{\parallel}$ for which one has little justification to use resonance broaden.

Making use of the Antonsen transformation, Lee and Chen developed an elegant S-matrix scattering method to prove that, quite generally for $T_i \neq 0$ and even including the electromagnetic perturbation of shear Alfvén type, there is no absolute instability of drift wave type in a plasma slab with sheared magnetic field. Then only remaining instabilities in a slab plasma with sheared magnetic field are (i) current-driven drift waves. This, however, requries a large parallel drift velocity between electrons and ions to become unstable

$$\frac{u}{v_e} > L_n/L_s$$

that it is not likely to occur in present Tokamaks. There may also be electromagnetic instability driven by parallel current.

(ii) Ion temperature gradient instability. For sufficiently large ion temperature gradient $\eta_i = \frac{d\ln T_i}{d\ln n} > 1$, local theory predicts an instability. In a sheared field, this instability has a very narrow localization width and requires the integral equation for φ as one can no longer expand $\varphi(x) = \varphi(0) + \varphi' x + \varphi''(0)\, x^2/2$ with $x^2 \approx \rho_i^2$ and an integral equation must be solved.

(iii) Trapped particle modes. In particular the trapped electron mode is important for most of the present Tokamak devices because the effective trapped electron collision frequency $\nu_{eff} = \nu_{ei}/\epsilon$ ($\epsilon = a/R$ the inverse aspect ratio) is already comparable to or smaller than their bounce frequence $\omega_{be} = \sqrt{\epsilon}\, v_e/qR$. In this case because the ion is untrapped, i.e. $\nu_{ii} > \omega_{bi}$ (except PLT at the highest temperature of 4.5 Kev), we may resort to the slab-like analysis for ions and untrapped electrons but replace the resonant electrons by the trapped electrons including their collisional contributions to find the dispersion relation as

$$1 - \frac{\omega_*}{\omega} + \frac{k_y^2 \rho_i^2}{2}\left(\frac{\omega_*}{\omega} + \tau\right) = \sqrt{2\epsilon}\left\langle \frac{\omega - \omega_{*e}^T}{\omega + i\nu_{eff}} \right\rangle - i(2n+1)2\frac{L_n}{L_s}$$

where $\tau = \dfrac{T_e}{T_i}$. The shear stabilizes the mode if

$$\frac{L_n}{L_s} > 0.3\, \epsilon^{1/2} \frac{\tau}{\left(1 + \tau \frac{k^2 \rho_i^2}{2}\right)^2}\left[\eta_e \frac{\left[1 + (\tau+1)\frac{k^2\rho_i^2}{2}\right]}{1+\tau} + \frac{k^2\rho_i^2}{2}\right]$$

c) Convective Amplification

Although there is no absolute universal instability, there is still the possibility of convective amplification of a source such as the thermal noise. The wave packet initially grows in time, but unlike absolute instability which grows indefinitely in time, here the growth would saturate by convection. If the exponential amplification factor is much greater than unity, then the mode is convectively unstable. The condition for convective amplification is

III NONLINEAR THEORY OF DRIFT WAVES

Once the linear theory predicts instability, unstable modes would grow quickly into nonlinear region where the nonlinear processes become important. Nonlinear theory for drift waves have been developed to take into account (1) the quasilinear effects, (2) nonlinear ion Landau damping, (3) three-wave interaction, both among drift modes and between drift waves and convection cells, (4) strong turbulence effects and finally (5) the anomalous cross-field transport due to drift waves. Computer simulation has also been a valuable tool for studying effects of nonlinear drift waves. Since quasilinear theory and weak turbulence theory of drift waves have been adequately reviewed, we consider here only the more recent development of the nonlinear mode-coupling theory because of its simplicity, and its ready extension into strong turbulence with contact to two-dimensional hydrodynamic turbulence.

As we discussed before, the electrons with Boltzmann distribution do not contribute to the nonlinear mode coupling. If we neglect the resonant electron effects and assume cold ions, then the only effect contributing to mode-coupling is the usual hydrodynamic $V \cdot \nabla V$ term which in this case make its presence in the ion polarization drift, the last term in Eq. (4). Substituting Eq. (4) into Eq. (5) and using the quasi-neutrality condition : $\delta n_i = \delta n_e = n_0 e\varphi/T_e$, Hasegawa and Mima found the following nonlinear equation for drift wave by neglecting resonant electrons and ion temperature effects :

$$\frac{\partial}{\partial t}(\varphi - \rho^2 \nabla^2 \varphi) + V_d \frac{\partial \varphi}{\partial y} = \rho^2 D_B (\nabla\varphi \times \hat{z}) \nabla \nabla^2 \varphi \tag{2-1}$$

where $\varphi = e\phi/T_e$ and $D_B = cT/eB$, $\rho^2 = \rho_s^2 = T_e/M\Omega_i$.

In the frame moving with V_d , the third term on the left vanishes and we may rewrite Eq. (2-1) by letting $\nabla^2 \phi = u$ and Fourier analyse

$$\left(1 + \frac{1}{k^2\rho^2}\right) \frac{dU(k)}{dt} = \sum_{p+r=k} M(p,r)\, U(p)\, U(r) \tag{2-2}$$

where $M(p,r) = \frac{1}{2}\left[\hat{e}_z \cdot (\vec{r} \times \vec{p})\right](p^{-2} - r^{-2})$

If one set $k\rho \to \infty$, this equation is identical to the two-dimension hydrodynamics. There are two constants of motion : the energy W and the square vortricity (enstrophy) ω

$$W = \sum W_k = \sum_k \left[\rho^{-2}|\phi(k)|^2 + k^2 |\varphi_k|^2\right] = \tag{2-3}$$

$$= \sum \left[\lambda^2 |u_k|^2 k^{-2} + |u_k|^2\right]$$

$$\omega = \sum \omega_k = \sum_k \left[k^2 \rho^{-2} |\varphi(k)|^2 + k^4 |\varphi(k)|^2\right] = \sum k^2 w_k^2 \tag{2-4}$$

In the limit of $k\rho \to \infty$, they reduce to the well-known constants in 2-d hydrodynamics. In the non dissipative 2-d hydrodynamics it is well-known from equilibrium statistical mechanics that the canonical distribution of $U(k)$ in the phase-space defined by the real and imaginary parts of $\mathcal{U}(k)$ is

$$D \propto e^{-\alpha W_k - \beta \omega_k} = e^{-(\alpha+\beta k^2) W_k} \tag{2-5}$$

from which we find the standard Gibb's spectrum for the expectation value for the energy for the k^{th} mode :

$$\langle |\mathcal{U}(k)|^2 \rangle = (\alpha+\beta k^2)^{-1} \tag{2-6}$$

where α, β are constants determined by the total energy and enstrophy :

$$\begin{aligned} W &= \sum_k \frac{1}{\alpha+\beta k^2} \\ \omega &= \sum_k \frac{k^2}{\alpha+\beta k^2} \end{aligned} \tag{2-7}$$

For a negative - temperature system, α and β are of the opposite sign and it is possible to have "condensation" in which practically all energy is concentrated in the mode with $k_0^2 = |\alpha/\beta|$. If $k_0 = k_{min}$, then the mode with longest wavelength has the most energy. This indicates the possibility of energy cascade towards the longest wave, i.e. inverse cascade, opposite to the usual mode coupling process in 3-d hydrodynamic turbulence where the energy is cascaded towards short wavelength. With dissipation, Kraichman has shown that in fact there is a <u>dual cascade</u> in which the energy is cascaded towards long wavelength and enstrophy is cascaded towards short wavelength, mutually exclusive of each other in a given inertial range. We may obtain the spectrum in the inertial range from the following considerations. Suppose there is a source for the k_f mode where energy and enstrophy are injected into the system. Introducing the definitions of omnidirectional energy $\mathcal{E}_k = 2\pi k W_k$ such that $\int \mathcal{E}_k dk$ = total energy. Then for $k < k_f$ we expect the energy cascade with the energy flux in k-space conserved in the inertial range where there is no dissipation

$$\mathcal{E}_k \dot{k} = \frac{\mathcal{E}_k k}{\tau_k} = \text{Const.} \tag{2-8}$$

where $\tau_k^{-1} = kv$ is the nonlinear rate of eddy turnover, which is also the rate of energy transfer to different modes. Since $Mnv^2 \approx \mathcal{E}_k k$, we have from Eq. (2-8)

$$\mathcal{E}_k = \text{Const}\ k^{-5/3} \quad \text{or} \quad W_k = \text{Const}\ k^{-8/3} \tag{2-9}$$

which is the well-known Kolmogoroff spectrum. For $k > k_f$, we expect that there is enstrophy cascade but no energy cascade so that the enstrophy flux in k-space is conserved in the inertial range

$$\mathcal{E}_k k^2 \dot{k} = \frac{\mathcal{E}_k k^3}{\tau_k} = \text{Const.} \tag{2-10}$$

and

$$\mathcal{E}_k \propto k^{-11/3} \tag{2-11}$$

The corresponding enstrophy spectrum is $\omega_k = k^2 w_k \propto k^{-8/3}$. Compare this with the equilibrium enstrophy spectrum $k^2/(\alpha+\beta k^2)$ which approaches a constant for large k, we can see that the enstrophy must cascade toward high k to approach equilibrium. These spectra have been verified numerically by Fife and Montegomery for $k\rho \geq 1$ (hydrodynamic region).

Despite its relatively simple form, Hasegawa-Mima equation is not expected to be relevant to the present Tokamaks because it neglects the growth term and ion thermal effect, both are expected to have important modifications.

Coherent solution

The Hasegawa-Mima equation

$$\frac{\partial}{\partial t}\left(\Delta\phi - \phi\right) + v_d \frac{\partial \phi}{\partial y} - \left(\frac{\partial \phi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \phi}{\partial y}\frac{\partial}{\partial x}\right)\Delta\phi = 0 \qquad (2\text{-}11)$$

also has the following vortex solution

$$\varphi = \frac{u+v_d}{K^2}\left[a\,\frac{J_1(kr)}{J_1(ka)} - r\left(1+\frac{K^2}{\rho^2}\right)\right]\sin\theta \qquad \text{for } r<a$$

$$= -\frac{u+v_d}{\rho^2}\, a\, \frac{K_1(\rho r)}{K_1(\rho a)} \sin\theta \qquad \text{for } r>a$$

(2-12)

where $y - ut = r\cos\theta$, $x = r\sin\theta$, $\rho = \sqrt{(u+v_d)/u}$ and a is the radius from the center. The value of κ is determined by requiring $\partial\phi/\partial r$ to be continuous across a line at $r = a$. J_1 and K_1 are the Bessel function of the first kind and modified Bessel function of the second kind. A remarkable feature of this solution is its rapidly decreasing amplitude with r, i.e. its localized nature : $\varphi \to e^{-\rho r}/\sqrt{r}$ as $r \to \infty$. The collision of two such vortices has been studied numerically by Makino, Kamimura and Taniuti, and found the vortices are remarkably well preserved after collision, thus suggesting two-dimensional recurrence. This problem has also been examined analytically using the reductive perturbation theory by Nozaki et al.

IV EXPERIMENTS ON DRIFT WAVES IN TOROIDAL PLASMAS

Experimental studies of drift wave in Tokamaks have been performed by Mazucatto, Surko and Slusher, Semet et al. A definitive study of drift wave in toroidal plasmas has also been carried out on FM-1 Spherator by Okabayashi and Arunasalam, in which the variation of the drift wave spectrum with magnetic field shear was reported. In the case of high shear, $L_s/L_n \gtrsim 7$, the frequency spectrum for a given k shows sharp peak at $\omega_* = kV_d$. In the case of medium shear $L_s/L_n \simeq 70$, the width of the peak is substantially broaden indicating a larger growth and stronger nonlinear effect. In the case of low shear $L_s/L_n > 70$, the peak at ω_* disappear all together, instead at strong, broad peak at zero frequency appears for all k studied ($0.5 \lesssim k\rho_i \lesssim 3$). The frequency spectrum of $(\delta n/n)^2$ varies as $\omega^{-\alpha}$ with $4 \leq \alpha \leq 6$. This was interpreted as the excitation of convection cell by drift waves. With microwave scattering on ATC plasma, Mazucatto has observed density fluctuations peaked near $k_\theta \rho_i \simeq 1$ in the drift wave frequency range (100 - 200 KHz). The total density fluctuation $\delta n/\bar{n} \simeq 5 \times 10^{-2}$ can account for the anomalous electron heat transport in ATC with the assumption $\gamma_k/\omega_* \simeq 0.1$. More recently he measured the density fluctuation in PLT where the trapped electrons are important as $\nu_{eff} < \omega_{be}$. He found that for a given $k = 10\ cm^{-1}$, the frequency spectrum is sensitive to the plasma density. For $n = 2 \times 10^{13}/cc$, the frequency spectrum is peaked at zero frequency and is symmetric with respect to zero frequency in contrast to previous observations in ATC and TFR where a clear shift of the frequency spectra towards the electron diamagnetic frequency was observed. At higher densities $n \simeq 4 \times 10^{13}/cc$, this shift was also observed in PLT. The fluctuation is observed to be larger on the outer side of the torus consistent with the predication of trapped electron modes. Although $\delta n/n$ appears to remain constant as the plasma density changes, it increases significantly with rising ion temperature due to the neutral beam injection.

Furthermore as ion temperature exceeds 4 Kev, where ion trapping is important , a new mode appears (trapped ion mode ?), strongly located in the outer region (with respect to the major radius). The frequency spectrum for k = 10 cm^{-1} peaks at 50 KHz on both electron and ion diamagnetic directions. This mysterious mode has yet to be explained though the theoretical possibility of trapped ion mode or ion temperature gradient mode are being explored.

Semet et al. recently used absolutely calibrated far-infrared laser scattering and Langmuir probe to study the drift wave turbulence in UCLA Microtor. With a wide range wavenumber measurement from k = 3 to 50 cm^{-1}, they were able to deduce the wave-number dependence of the scattered power as $k^{-3.5}$ between $0.6 \leq k\rho_i \leq 2$. This is the region where ion nonlinear Landau damping is expected to be important and it would be of interest to compute a steady-state spectrum in the region.

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