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ABSTRACT

A parametric study of the loading resistance of antennae for Alfvén wave excitation has been performed using a numerical code based on ideal MHD equation. It is found that the resistance exhibits a resonant enhancement if a collective mode is excited. It is shown that this feature can be used to optimize the antenna and rf generator in such a way that an efficient energy absorption takes place at the innermost plasma surface.

1. INTRODUCTION

It is widely recognized that supplementary heating, in addition to the basic ohmic heating, will be necessary to bring a tokamak reactor into the ignition regime. One of the many schemes proposed for this purpose is resonant absorption of Alfvén waves in a nonuniform plasma {1,2}. It has the basic merit of using low-frequency rf fields for which high-power sources are readily available.

The basic theory for the rate of energy absorption using this scheme has been given by Chen and Hasegawa {3} using a simple slab geometry. They found that the absorption rate is strongly enhanced when the nonuniformity of equilibrium is sharp and the driving frequency is close to the frequency of the weakly-damped surface eigenmode {4}. To some extent, this feature was also indicated in the calculations of Tataronis and Grossmann for a cylindrical geometry {5}.

An objective of the present paper is to show that a similar phenomenon takes place in an equilibrium with an arbitrary nonuniformity. In particular, the energy absorption rate always increases whenever the applied frequency approaches the real part of the frequency of a collective mode of the plasma response {4}. Essentially, the collective mode is a remnant of a global mode of the system, which disappeared within the Alfvén continuum, and consequently its frequency became complex. The weakly-damped surface eigenmode is just a special type of collective mode. The fact that the absorption rate exhibits a resonant enhancement due to the excitation of a collective mode can be used to optimize the antenna and rf generator in such a way that optimum coupling to the plasma is achieved.

In Section 2 the computational model used is briefly described, while the results of a parametric study of the loading resistance and Q-factor for Alfvén wave excitation are presented in Section 3.

2. COMPUTATIONAL MODEL

For the sake of simplicity, the unperturbed plasma is described by a cylindrically-symmetric equilibrium with the following characteristics : $B_z = 1$, $j_z = j_o(1-r^2)^\alpha$ and $\rho = 1.01 - r^2$ in dimensionless units, where the plasma radius is unity. Here B_z is the axial component of magnetic field, j_z is the plasma current density and ρ is the plasma mass density. The free parameters j_o and α are varied within certain ranges such that zero plasma pressure at the plasma boundary results in β-values of a few per cent on the axis.

The Alfvén waves are excited by an ideal antenna consisting of 2m helical current sheets (of vanishingly small thickness) which are located at the radius $r_A = 1.2$ in the vacuum region between the plasma column and the perfectly conducting wall of radius 1.5. The sheets are arranged in such a manner that the resulting current density is of the form $\{J_\Theta, J_z\} = \{k,-m/r_A\} \cos(\omega t)\cos(m\Theta+kz)/2$, where Θ is the azimuthal angle, $2\pi/k$ is the axial wavelength of winding and ω is the frequency of the rf generator. The normalization is chosen such that the total current in each sheet is equal to unity. Toroidal geometry can be simulated by setting $k = n/R$, where n is the toroidal mode number and R is the major radius of a torus.

The plasma motion is described by linearized ideal MHD equations which are complemented by a source term respresenting the excitation. The equations are solved by means of the spectral code THALIA {6}. In fact, since we seek a stationary plasma response the code was slightly modified by including a small artificial damping in the equations of plasma motion. Once the stationary response is found we can compute the loading resistance and Q-factor (the ratio of reactance and resistance) by a standard procedure. At the same time, however, the magnitude of the artificial damping must be chosen sufficiently small that the results do not depend on it. The problem treated here is analogous to the case of a damped harmonic oscillator when acted on by a force with a continuous spectrum {7}. The total energy absorbed by the oscillator does not depend on the attenuation decrement when the attenuation is weak. Thus, we calculate the "true" resonant absorption without specifying a dissipation mechanism.

3. RESULTS AND DISCUSSION

In the first study we investigated the excitation of the m = 1 mode in the equilibrium with $j_o = 0.6$ and $\alpha = 2$. The frequency and the axial wavenumber k were varied within a domain defined by $\min\omega_A(r) \leqslant \omega \leqslant \min(\omega_A(1), \omega_{max})$, where ω_{max} was chosen such that $\omega_{max} < \min\omega_{2F}(k)$. Here $\omega_A(r) = |k + mB_\Theta/r|/\rho^{\frac{1}{2}}$ is the Alfvén continuum, B_Θ is the azimuthal component of equilibrium magnetic field and $\omega_{2F}(k)$ is the frequency of the second eigenmode of the fast magnetoacoustic branch. The computed values of the loading resistance R (per unit length of plasma) were then plotted in the ωk-plane. The resulting altitude chart of $R(\omega,k)$ for $k > o$ is shown in Fig. 1. One can observe that the resistance peaks along a certain line $\omega = \omega_c(k)$. We argue that this line roughly represents the real part of the frequency of a collective mode. There are two reasons for this assertion. Firstly, we repeated the same computations with the equilibrium plasma current switched off. In this case one can determine approximately the behaviour of the line $\omega = \omega_c(k)$ by a semi-quantitative analysis. This behaviour agrees well with that obtained from the computations and is qualitatively similar to the case with the current included. Secondly, we can see by means of the spectral code that when the line emerges from the continuum it represents a discrete (global) mode - kink. The chart of R for $k < o$ (not shown) is similar to that for $k > o$ except for the location of the $\omega_c(k)$-line which is shifted towards the plasma boundary.

Fig. 1 can now be used to determine R as a function of k or ω for a fixed resonance surface. To this end, we simply draw a line $\omega_A(r = \text{const})$ on the chart (in Fig. 1 the position of the $r = .8$ surface is indicated by the dashed line) and infer the corresponding values of R and k. Fig. 2 shows R and the Q-factor versus k for three different resonance surfaces: $r = .3$, $r = .5$ and $r = .8$. It is easily seen that for each surface the loading resistance has a maximum M(r) at a definite value of k. At the same time, the corresponding Q-factor is reasonably small, which implies a good coupling. Moreover, the plot indicates that for a given equilibrium and fixed value of m there exists an optimal resonance surface associated with a maximal resistance maxM(r). In order to find the position of this surface we plot M(r) versus the radius as shown in Fig. 3. We see that for the case considered the optimal surface is located at $r = .42$.

Next, we tried to establish how the optimal resistance depends on the characteristics of the plasma equilibrium. For this purpose, we repeated the above-described computations with different values of the parameters j_o and α. The dependence of the quantity M(r) upon the value of j_o is demonstrated in Fig. 3 for the case where $\alpha = 2$. We notice that for a fixed current profile, the position of the optimal resonance surface is shifted towards the plasma axis when the current increases. At the same time the optimal resistance is enhanced. Thus, in plasmas with higher β-values one can expect an efficient energy absorption at the innermost surfaces. A peaking of the current profile, the total current being fixed, can have a similar effect. As can be seen from Fig. 4, the optimal resonance surface is shifted towards the plasma axis when the current profile is steeper. For a very peaked current the energy absorption seems to be equally good for all inner surfaces. Also we varied the boundary value of the plasma density. It turned out that the absorption is not very sensitive to this value. A variation within the range .01 - .1 resulted in a variation of the resistance by a few per cent.

In the last study we tried to obtain some insight into Canobbio's heating scheme {8} in which a strong absorption is predicted to occur near the singular surface defined by $k = -mB_{\Theta}/r$. We investigated the excitation of the $m = -2$ mode. In general, we found that typical values of the loading resistance are not very different from those of the case with $m = 1$. However, the position of the optimal surface is shifted towards the plasma boundary. For the Canobbio case we considered two singular surfaces $r = .8$ ($k = .3$) and $r = .4$ ($k = .5$). The excitation frequencies were chosen in such a manner that the resulting resonant surfaces were close to the singular surface. The results are shown in Fig. 5. We see that in both cases the resistance rapidly decreases with the frequency (resonant surfaces approach the singular one) and the Q-factor increases. This can be understood if we invoke the foregoing arguments about the collective mode. The applied frequencies are very low, and consequently far from the frequency of the collective mode. Thus, the heating scheme considered does not seem to be an optimal one.

In conclusion, we have shown that the structure of the antenna and the frequency of the rf generator used for Alfvén wave excitation can be optimized in such a way that an efficient energy absorption takes place at the innermost plasma surface.

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FIGURE CAPTIONS

Fig. 1 Altitude chart of the loading resistance $R(\omega,k)$ (divided by 2π) for $m = 1$, $j_o = .6$, $\alpha = 2$.

Fig. 2 li Loading resistance (solid lines) and Q-factor (dashed lines) versus the axial wavenumber k for three different resonance surfaces. The parameters used are the same as in Fig. 1.

Fig. 3 Maximal loading resistance versus the position of resonance surface r for three different equilibrium currents of the same profile. The parameters used are: $m = 1$, $\alpha = 2$.

Fig. 4 Maximal loading resistance versus the position of resonance surface r for three different profiles of the same equilibrium current; $m = 1$.

Fig. 5 Loading resistance (solid lines) and Q-factor (dashed lines) versus the frequency in the vicinity of two different singular surfaces. The parameters used are: $m = -2$, $j_o = .6$, $\alpha = 2$.

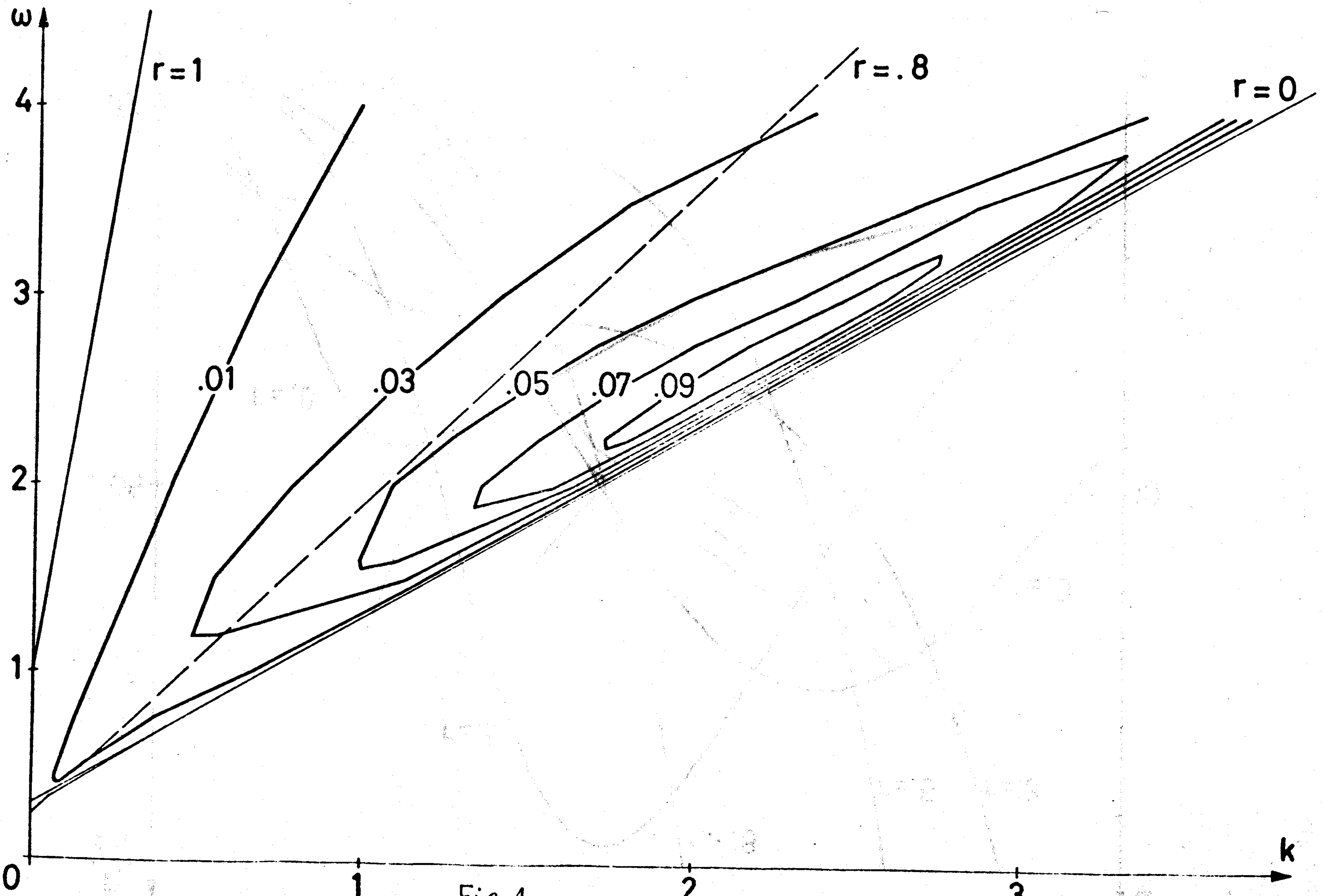


Fig. 1

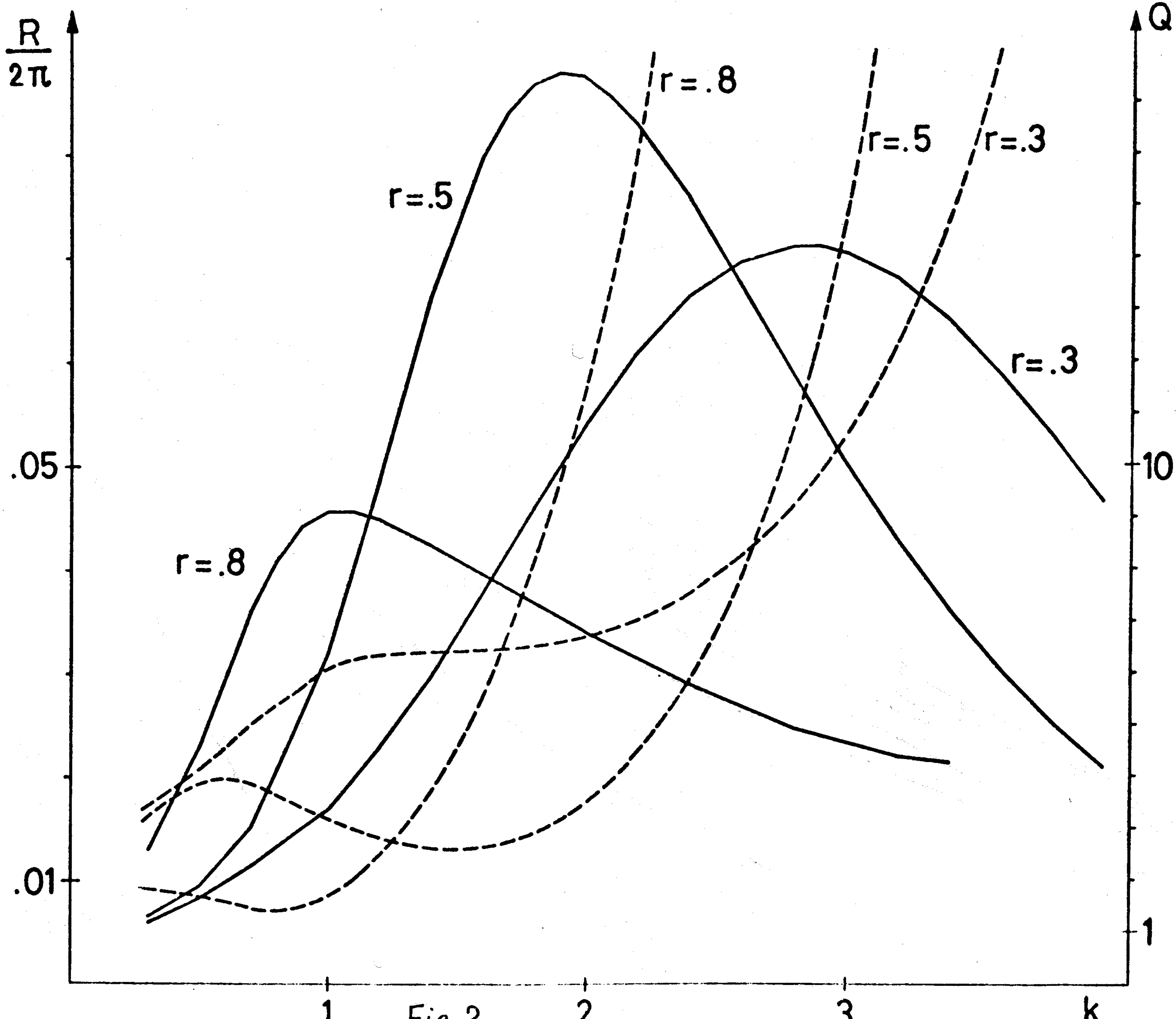


Fig. 2

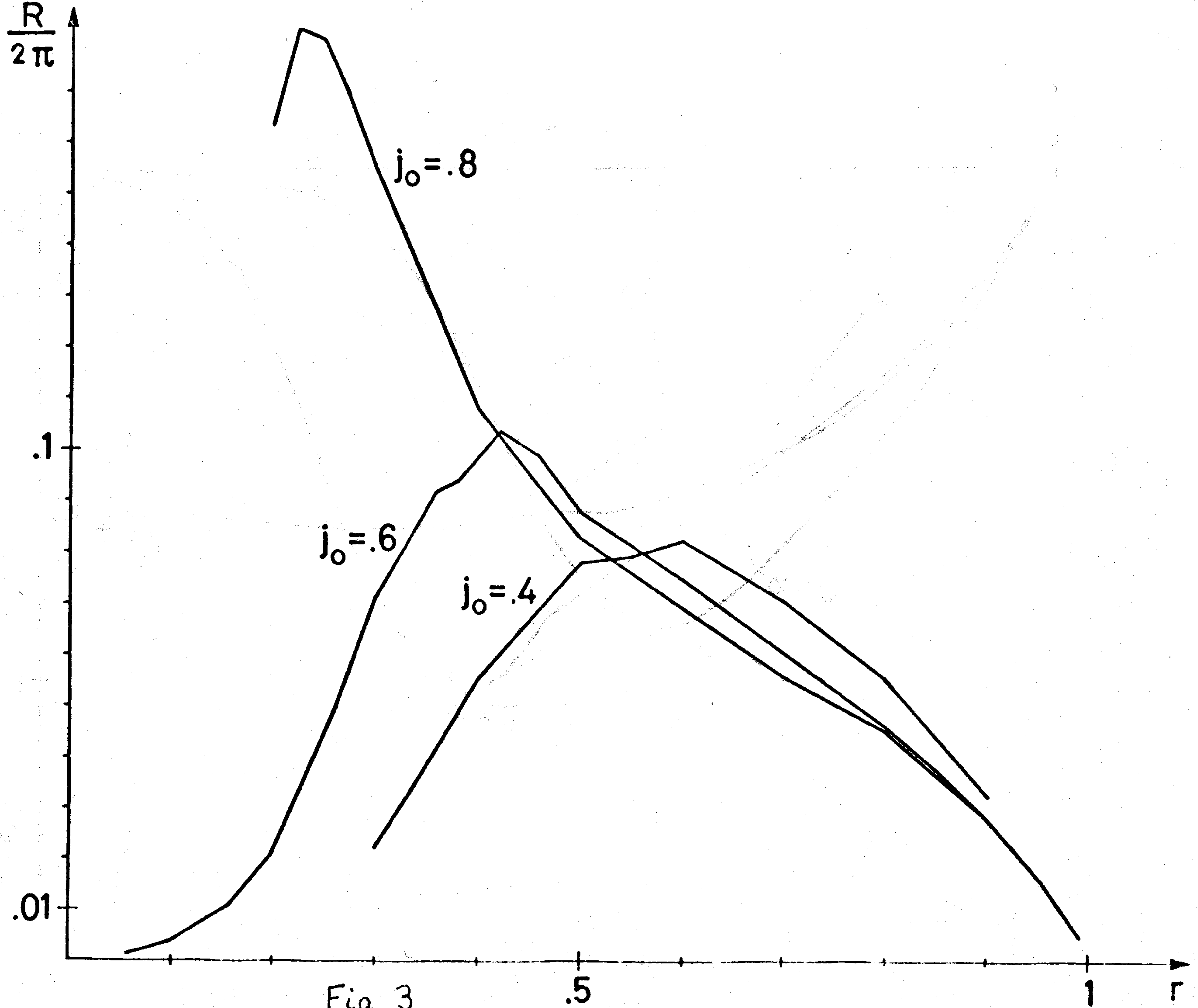


Fig. 3

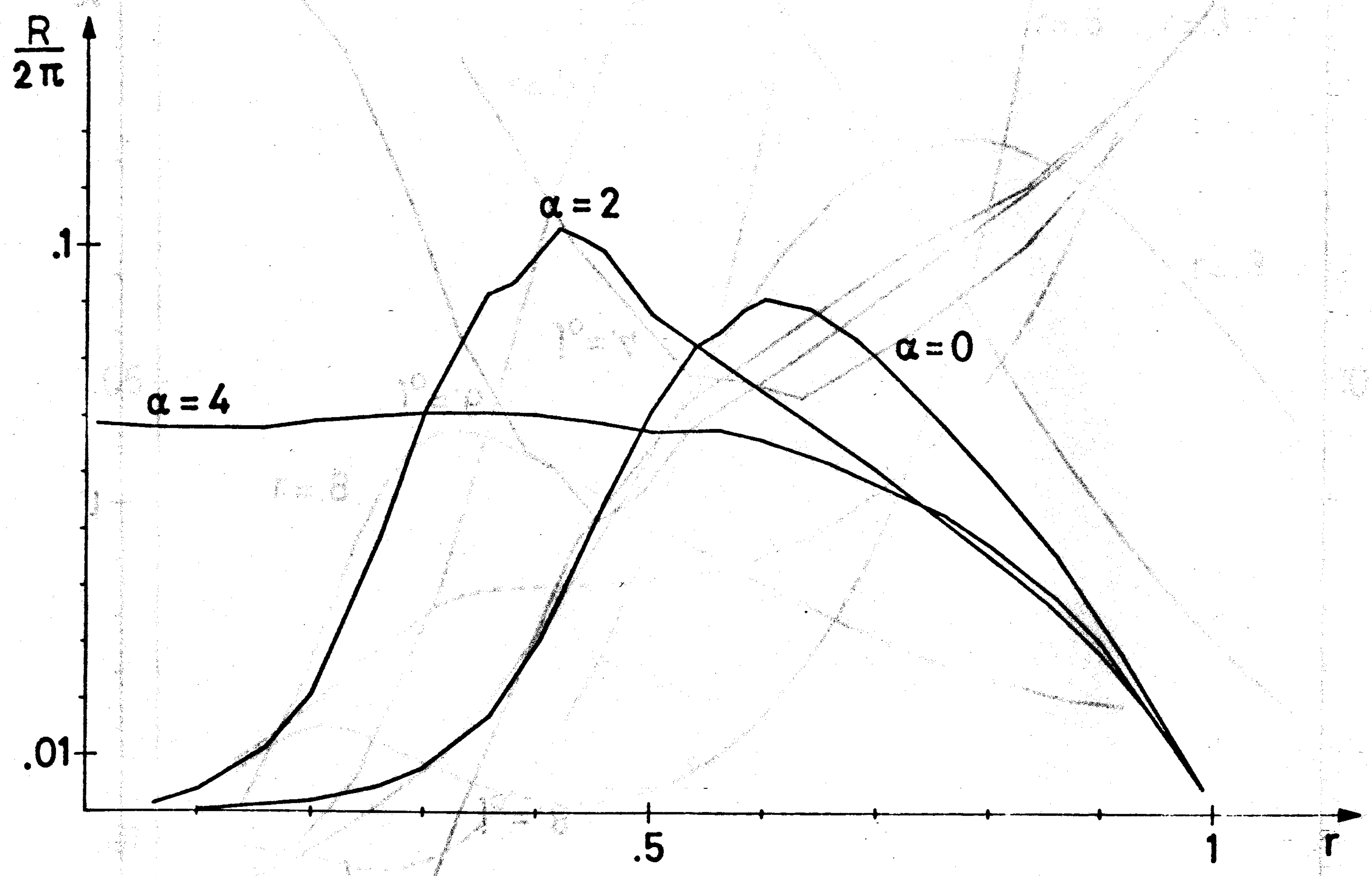
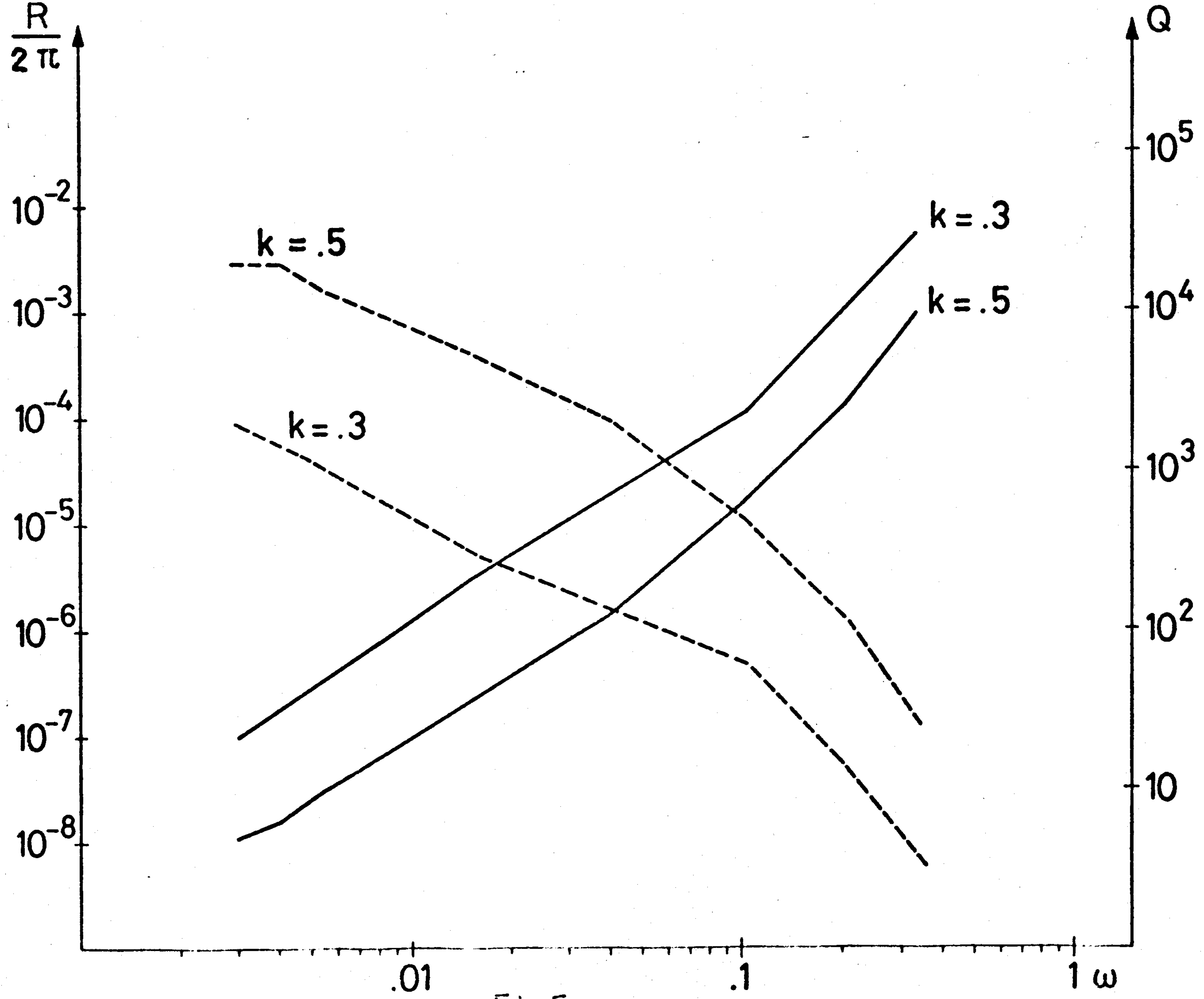


Fig. 4



R/2π
Q
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^5
10^4
10^3
10^2
10
k = .5
k = .3
k = .3
k = .5
.01
.1
1 ω

Fig. 5