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J. Vaclavik and K. Appert

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J. Vaclavik and K. Appert
Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne
21, Av. des Bains, CH-1007 Lausanne, Switzerland

ABSTRACT

The method of correlation functions together with a Fourier-transform technique is used to construct self-consistent equations describing the adiabatic interaction of coherent waves with weakly turbulent plasmas. The formalism is applied to study the propagation of a low-frequency ion-acoustic wave in the presence of high-frequency ion-acoustic turbulence.

It is found that the turbulent waves increase the phase velocity of the coherent wave in the case where they all propagate so that each has a velocity component in a given direction. In the opposite case, the phase velocity of the coherent wave increases or decreases depending on whether the turbulence peaks at low or high values of $k\lambda_D$.

I. INTRODUCTION

In laboratory as well as in astrophysical plasmas, one can frequently encounter situations where coherent and turbulent waves are present at the same time. Such a wave coexistence may arise owing to different causes: coherent waves are excited as a result of the condensation of the turbulent wave energy in the wavenumber space (modulational instability), they are injected into a turbulent plasma for diagnostic or heating purposes, and both kinds of waves appear spontaneously due to the simultaneous operation of different instabilities. As the coherent waves evolve in the turbulent system, they produce modulations in the spectral distribution of the turbulent waves. In turn, the turbulent system affects the behaviour of the coherent waves: their dispersion relations, amplitudes and phases. Consequently, a self-consistent description of the interaction between both kinds of waves should consist of a set of coupled equations: dynamic equations for the coherent waves and a kinetic equation for the turbulence. The formulation of such equations is a difficult problem in general. However, it can be simplified considerably if the interacting waves are far apart in frequency-wavenumber space and the turbulence is weak. One then deals with what is known as adiabatic wave interaction with weak turbulence.[1]

A problem of this type was first considered by Vedenov and Rudakov.[2] On making use of the Lagrangian formalism and semi-quantum mechanical approach, they derived self-consistent equations describing the adiabatic interaction of ion-acoustic waves with the electron Langmuir turbulence. Since then a number of papers have appeared treating similar problems in different contexts. However, in most of them a rather heuristic approach is used: a turbulence is described in an ad hoc manner by means of the Liouville equation for the distribution function of turbulent waves, while coherent waves are described by means of equations of the hydrodynamic type in which a ponderomotive force associated with the turbulence is also introduced in an ad hoc manner (see e.g. Ref. 3). A more systematic approach was adopted by Sakai et al.[4], and Kono and Yajima [5] who have shown how the reductive perturbation method[6] can be extended to treat the problem in question.

An objective of the present paper is to show that the method of correlation functions together with a Fourier-transform technique offers a simple systematic alternative to the approaches cited above. In Sec. II the method is illustrated on an example where the master dynamical equation describing the behaviour of a physical system is a general three-wave interaction equation. The formalism developed is then applied in Sec. III to study the propagation of a low-frequency ion-acoustic wave in the presence of high-frequency ion-acoustic turbulence.

II. THREE-WAVE INTERACTIONS

Let us consider a weakly-nonlinear conservative system whose state is described by a variable Ψ which obeys the following dynamical equation in wavenumber space:

$$\left(i\frac{\partial}{\partial t} - \omega_p\right)\Psi_p = \sum_{p'} V_{p,p',p-p'}\Psi_{p'}\,\Psi_{p-p'} \quad . \tag{1}$$

Here ω_p is an eigenfrequency, $V_{p,p',p''}$ is an interaction kernel, and the index p signifies the wavenumber $\vec{p}$. Eq. (1) is in dimensionless units chosen such that $|\Psi_p| << 1$. Moreover, the quantities Ψ_p, ω_p and $V_{p,p',p''}$ satisfy the symmetry relations (see e.g. Ref. 3)

$$\Psi_{-p} = \Psi_p^* \text{ (complex conjugate)}, \; \omega_{-p} = -\omega_p \quad , \tag{2}$$

$$V_{p,p',p''} = V_{p,p'',p'} = -V_{-p,-p',-p''} = V_{p'',-p',p}\, sign(\omega_p \omega_{p''}) \; .$$

We now assume that Ψ_p consists of two parts: a coherent one $\overline{\Psi}_p \equiv \langle\Psi_p\rangle$ and a fluctuating one $\tilde{\Psi}_p = \Psi_p - \overline{\Psi}_p$, where $\langle\;\rangle$ denotes an ensemble average. Our objective is to obtain a set of equations for the quantity $\overline{\Psi}_p$ and the correlation function $g_{p,p'} \equiv \langle\tilde{\Psi}_p\tilde{\Psi}_{p'}\rangle$.

On taking the ensemble average of Eq. (1) we immediately find

$$\left(i\frac{\partial}{\partial t}-\omega_p\right)\overline{\Psi}_p=\sum_{p'}V_{p,p',p-p'}\left(\overline{\Psi}_{p'}\,\overline{\Psi}_{p-p'}+g_{p',p-p'}\right). \tag{3}$$

Further, subtracting Eq. (3) from Eq. (1) we obtain

$$\left(i\frac{\partial}{\partial t}-\omega_p\right)\tilde{\Psi}_p=\sum_{p'}V_{p,p',p-p'}\left(2\overline{\Psi}_{p'}\,\tilde{\Psi}_{p-p'}+\tilde{\Psi}_{p'}\,\tilde{\Psi}_{p-p'}-g_{p',p-p'}\right). \tag{4}$$

Next, we multiply Eq. (4) by $\tilde{\Psi}_{p''}$ and the corresponding equation for $\tilde{\Psi}_{p''}$ by $\tilde{\Psi}_p$. Adding and averaging the resulting equations we find

$$\left(i\frac{\partial}{\partial t}-\omega_p-\omega_{p''}\right)g_{p,p''}=\sum_{p'}\Big\{V_{p,p',p-p'}\left(2\overline{\Psi}_{p'}\,g_{p'',p-p'}+h_{p',p'',p-p'}\right)+\left(p\longleftrightarrow p''\right)\Big\}, \tag{5}$$

where $h_{p,p',p''}\equiv\langle\tilde{\Psi}_p\tilde{\Psi}_{p'}\tilde{\Psi}_{p''}\rangle$ is the triple correlation function. The same procedure as that used above can now be applied to construct an equation for the triple correlation function. It yields

$$\left(i\frac{\partial}{\partial t}-\omega_p-\omega_{p''}-\omega_{p'''}\right)h_{p,p'',p'''}=\sum_{p'}\Big\{V_{p,p',p-p'}\Big(f_{p',p'',p''',p-p'}$$

$$-g_{p',p-p'}\,g_{p'',p'''}\Big)+(p\longleftrightarrow p'')+(p\longleftrightarrow p''')\Big\}\,, \tag{6}$$

where $f_{p,p',p'',p'''}=\langle\tilde{\Psi}_p\tilde{\Psi}_{p'}\tilde{\Psi}_{p''}\tilde{\Psi}_{p'''}\rangle$ is the quadruple correlation function. We have discarded the terms of the order $O(\overline{\Psi}h)$ in Eq. (6) since for our purposes it is sufficient to obtain the triple correlation function correct to $O(\Psi^4)$.

It is obvious that the equations (3), (5) and (6) do not constitute a closed set. In order to achieve a closure we assume that the ensemble considered is a quasi-Gaussian one. We then have (see e.g. Ref. 7)

$$f_{p,p',p'',p'''}=g_{p,p'}\,g_{p'',p'''}+g_{p,p''}\,g_{p',p'''}+g_{p,p'''}\,g_{p',p''}\,. \tag{7}$$

On substituting f from (7) into Eq. (6) we obtain

$$\left(i\frac{\partial}{\partial t}-\omega_p-\omega_{p''}-\omega_{p'''}\right)h_{p,p'',p'''}=R_{p,p'',p'''}\equiv\sum_{p'}\Big\{V_{p,p',p-p'}\Big(g_{p',p''}\,g_{p''',p-p'}$$

$$+g_{p',p'''}\,g_{p'',p-p'}\Big)+(p\longleftrightarrow p'')+(p\longleftrightarrow p''')\Big\}. \tag{8}$$

As will be verified a posteriori, the functions h and g vary on different time scales such that

$$\left|\frac{1}{h}\frac{\partial h}{\partial t}\right| \gg \left|\frac{1}{g}\frac{\partial g}{\partial t}\right| . \tag{9}$$

On making use of this property, Eq. (8) can be integrated on the fast time scale to yield

$$\begin{aligned} h_{p,p',p''} &= R_{p,p',p''}\,\Delta_{p,p',p''} \;, \\ \Delta_{p,p',p''} &= \left.\frac{1}{i\nu-(\omega_p+\omega_{p'}+\omega_{p''})}\right|_{\nu\to 0+} \;, \end{aligned} \tag{10}$$

where we have assumed that h → o for t → - ∞.

We now combine (10), the right-hand side of Eq. (8) and Eq. (5). After some algebra we finally find

$$\begin{aligned} &\left(i\frac{\partial}{\partial t}-\omega_p-\omega_{p'}\right)g_{p,p'} - 2\sum_{p''}\overline{\Psi}_{p''}\left(V_{p,p'',p-p''}\,g_{p',p-p''}\right. \\ &\left. + V_{p',p'',p'-p''}\,g_{p,p'-p''}\right) = S_{p,p'} \;, \end{aligned} \tag{11}$$

where

$$S_{p,p'} = 2\sum_{p'',p'''} \{ V_{p,p'',p-p''} V_{p',p''',p'-p'''} g_{p'',p'''} g_{p-p'',p'-p'''}$$

$$\times (\Delta_{p',p'',p-p''} + \Delta_{p,p''',p'-p'''}) + 2 V_{p'',p''',p''-p'''} (V_{p,p'',p-p''} \Delta_{p',p'',p-p''} \tag{12}$$

$$\times g_{p',p'''} g_{p-p'',p''-p'''} + V_{p',p'',p'-p''} \Delta_{p,p'',p'-p''} g_{p,p'''}$$

$$\times g_{p'-p'',p''-p'''})\} .$$

In what follows, the wavenumbers $\vec{q}$, $\vec{q}'$ etc. refer to the coherent field while the wavenumbers $\vec{k}$, $\vec{k}'$ etc. to the fluctuating one. We now assume

$$|\vec{q}| \ll |\vec{k}| \, , \; |\omega_q| \ll |\omega_k| \tag{13}$$

and

$$\overline{\Psi}_k = \tilde{\Psi}_q = 0 \; . \tag{14}$$

On applying (14) to Eq. (3) we have

$$\left(i\frac{\partial}{\partial t}-\omega_q\right)\overline{\Psi}_q-\sum_{q'}V_{q,q',q-q'}\overline{\Psi}_{q'}\,\overline{\Psi}_{q-q'}=\sum_{k}V_{q,k,q-k}\,N_{k-q/2\,,\,q}\ , \tag{15}$$

where we have defined

$$N_{k,q}=g_{q/2+k\,,\,q/2-k}\ . \tag{16}$$

The reason for this definition will be discussed shortly. Expanding the right-hand side of Eq. (15) in $|\vec{q}|/|\vec{k}|$ and keeping only the lowest order terms we arrive at

$$\left(i\frac{\partial}{\partial t}-\omega_q\right)\overline{\Psi}_q-\sum_{q'}V_{q,q',q-q'}\overline{\Psi}_{q'}\,\overline{\Psi}_{q-q'}=\sum_{k}V_{q,k,-k}\,N_{k,q}\ . \tag{17}$$

The application of (14) to Eq. (11) yields

$$\begin{aligned}&\left(i\frac{\partial}{\partial t}-\omega_{k+q/2}+\omega_{k-q/2}\right)N_{k,q}-2\sum_{q'}\overline{\Psi}_{q'}\Big(V_{q/2+k,\,q',\,q/2-q'+k}\\&\times N_{k-q'/2\,,\,q-q'}+V_{q/2-k,\,q',\,q/2-q'-k}\,N_{k+q'/2\,,\,q-q'}\Big)\\&=S_{q/2+k\,,\,q/2-k}\ .\end{aligned} \tag{18}$$

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On expanding again in $|\vec{q}|/|\vec{k}|$ and keeping only the lowest-order non-vanishing terms we finally have

$$\frac{\partial N_{k,q}}{\partial t} + i\vec{q}\cdot\frac{\partial \omega_k}{\partial \vec{k}} N_{k,q} + 2i\sum_{q'} \overline{\Psi}_{q'} \Big\{ N_{k,q-q'} (\vec{q}-\vec{q}')\cdot\frac{\partial}{\partial \vec{p}} \Big(V_{p,q',k} + V_{k,q',p} \Big)\Big|_{p=k} - V_{k,q',k}\, \vec{q}'\cdot \frac{\partial N_{k,q-q'}}{\partial \vec{k}} \Big\} = 4\pi \sum_{k',q'} V^2_{k,k',k-k'} \times \delta(\omega_k - \omega_{k'} - \omega_{k-k'}) \Big\{ N_{k',q'} N_{k-k',q-q'} - 2\,\mathrm{sign}(\omega_k \omega_{k'}) N_{k,q'} N_{k-k',q-q'} \Big\} , \tag{19}$$

where the symmetry relations (2) have been used. Equations (17) and (19) constitute the desired set for the unknowns $\overline{\Psi}_q$ and $N_{k,q}$. From the structure of this set it is easily seen that the condition (9) is equivalent to the assumption (13).

Let us now justify the definition (16). It is obvious that the correlation function $g_{q/2 + k,q/2-k}$ is related to the two-point spatial correlation function $C \equiv \langle\tilde{\Psi}(\vec{x})\tilde{\Psi}(\vec{x}')\rangle$ by the following relation

$$C = \sum_{p,p'} g_{p,p'}\, e^{i(\vec{p}\cdot\vec{x} + \vec{p}'\cdot\vec{x}')} = \sum_{k,q} g_{q/2+k,\,q/2-k}\, e^{i\vec{q}\cdot\frac{\vec{x}+\vec{x}'}{2}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} . \tag{20}$$

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On the other hand, it is well known that for a homogenous turbulence (in the absence of a coherent wave) the two-point spatial correlation function is related to the spectrum N_k by

$$C_{hom} = \sum_k N_k \, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \, . \tag{21}$$

In the presence of a longwavelength coherent wave (a weakly inhomogeneous turbulence) the two-point correlation function can be represented in the form $C = C(\vec{x}-\vec{x}', \frac{\vec{x}+\vec{x}'}{2})$, where the dependence on the second argument is weak. Transforming into the Fourier representation we thus have

$$\begin{aligned} C &= \sum_k N_k\left(\frac{\vec{x}+\vec{x}'}{2}\right) e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \\ &= \sum_{k,q} N_{k,q}\, e^{i\vec{q}\cdot\frac{\vec{x}+\vec{x}'}{2}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \, . \end{aligned} \tag{22}$$

Combining (22) with (20) we obtain (16). Evidently, in the absence of a coherent wave we have $N_{k,q} = N_k \delta_{q,o}$, and Eq. (19) is reduced to the well-known kinetic equation for three-wave interactions (see e.g. Ref. 3).

Concluding this section, we would like to point out that the method described above can easily be extended to include four-wave, wave-particle and wave-particle-wave interactions.

III. INTERACTION OF ION-ACOUSTIC WAVES WITH ION-ACOUSTIC TURBULENCE

Let us now apply the formalism developed in the previous section to study the propagation of a low-frequency ion-acoustic wave in the presence of high-frequency ion-acoustic turbulence. First, we need to establish an appropriate master dynamical equation. We shall consider a collisionless, nonmagnetized plasma composed of a cold ion fluid and a warm electron fluid. The nonlinear evolution of ion-acoustic perturbations about a uniform, velocity-free equilibrium may then be described by the equations of continuity and momentum transfer for the ion fluid, the Boltzmann distribution for the electron fluid and the Poisson equation. For our purpose it is sufficient to keep only quadratic and cubic nonlinearities in these equations. It can then be shown by means of standard technique[8] that they are equivalent to the following equation for the wave variable C_p

$$\left(i\frac{\partial}{\partial t}-\omega_p\right)C_p = \sum_{p'} V_{p,p',p-p'}\, C_{p'}\, C_{p-p'} \tag{23}$$

$$+\sum_{p',p''} V_{p,p',p'',p-p'-p''}\, C_{p'}\, C_{p''}\, C_{p-p'-p''} \quad ,$$

where

$$V_{p,p',p''} = \frac{e}{4T_e}\frac{\omega_p}{\beta_p^2}\left\{c_s^2\left(\beta_p^2\frac{\vec{p}'\cdot\vec{p}''}{\omega_{p'}\omega_{p''}}+\beta_{p'}^2\frac{\vec{p}\cdot\vec{p}''}{\omega_p\omega_{p''}}+\beta_{p''}^2\frac{\vec{p}\cdot\vec{p}'}{\omega_p\omega_{p'}}\right)-1\right\} , \tag{24}$$

$$V_{p,p',p'',p'''} = \frac{1}{12}\left(\frac{e}{T_e}\right)^2 \frac{\omega_p}{\beta_p^2}\left(\frac{1}{\beta^2_{p-p'}} + \frac{1}{\beta^2_{p-p''}} + \frac{1}{\beta^2_{p-p'''}} - 1\right) , \qquad (25)$$

$$\omega_p = \frac{c_s p}{\beta_p} \operatorname{sign}(p_z) , \quad \beta_p = \left(1 + (p\lambda_D)^2\right)^{1/2} . \qquad (26)$$

Here e, T_e and λ_D are the electron charge, temperature and Debye length, respectively, and c_s is the ion sound speed. The normalization of the wave variable is chosen in such a way that in the linear approximation $c_p = \varphi_p$, where φ is the electrostatic potential. In Eq. (23) we have dispensed with the mode corresponding to the eigenfrequency $-\omega_p$. The reason for this approximation will be seen soon. It is worth mentioning that Eq. (23) can also be derived from the Hamiltonian formalism.[9]

The coupled equations for the coherent wave and turbulence can now be written down by analogy with the case treated in the previous section.

Confining ourselves to the lowest order terms in $\bar{C}_q$ and $I_{k,q} \equiv \langle \tilde{C}_{q/2+k} \tilde{C}_{q/2-k} \rangle$ we obtain

$$\left(i\frac{\partial}{\partial t}-\omega_q\right)\bar{C}_q = \sum_k V_{q,k,-k}\, I_{k,q} + 3\sum_{k,q'} V_{q,q',k,-k}\, \bar{C}_{q'}\, I_{k,q-q'} \,, \tag{27}$$

$$\left(i\frac{\partial}{\partial t}-\frac{\partial\omega_k}{\partial\vec{k}}\cdot\vec{q}\right)I_{k,q} = 2\sum_{q'}\bar{C}_{q'}\left\{I_{k,q-q'}\left(\vec{q}\cdot\left.\frac{\partial V_{p,q',k}}{\partial\vec{p}}\right|_{p=k} + (\vec{q}-2\vec{q}')\cdot\left.\frac{\partial V_{k,q',p}}{\partial\vec{p}}\right|_{p=k}\right) - V_{k,q',k}\,\vec{q}'\cdot\frac{\partial I_{k,q-q'}}{\partial\vec{k}}\right\}, \tag{28}$$

where the symmetry relation $V_{-p,p',-p''} = -V_{p,p',p''}$ has been used.

We now assume that in the absence of the coherent wave the plasma is in a stationary state in which only the turbulent waves with positive phase velocities along the z-axis are excited. This is a typical situation encountered in experiments. Next, we assume that a small amplitude coherent wave with a positive phase velocity along the z-axis is injected into the plasma. The interaction of the coherent and turbulent waves is then described by Eqs. (27) and (28). We put $I_{k,q}(t) = I_k\delta_{q,o} + I^{(1)}_{k,q}(t)$, where $|I_{k,q}^{(1)}| \ll I_k$, and linearize Eqs. (27) and (28) to obtain

$$\left(i\frac{\partial}{\partial t}-\omega_q\right)\bar{C}_q = \sum_k V_{q,k,-k}\, I_{k,q}^{(1)} + 3\bar{C}_q \sum_k V_{q,q,k,-k}\, I_k \quad , \tag{29}$$

$$\left(i\frac{\partial}{\partial t}-\frac{\partial\omega_k}{\partial\vec{k}}\cdot\vec{q}\right) I_{k,q}^{(1)} = -2\bar{C}_q V_{k,q,k}\left\{\frac{\partial I_k}{\partial\vec{k}} + \frac{I_k}{\omega_k}\frac{\partial\omega_k}{\partial\vec{k}}\left(2\beta_k^2-3\right)\right\}\cdot\vec{q} \quad , \tag{30}$$

where the symmetry relation

$$V_{p'',p',p} = \frac{\omega_{p''}}{\omega_p}\frac{\beta_p^2}{\beta_{p''}^2} V_{p,p',p''}$$

has been used. On assuming the time dependence of $\bar{C}_q$ and $I_{k,q}^{(1)}$ to be of the form $\exp(-i\Omega t)$ we find the following dispersion relation for the coherent wave from Eqs. (29) and (30)

$$\Omega-\omega_q = \sum_k \left\{3V_{q,q,k,-k}\, I_k - 2\frac{V_{q,k,-k}\, V_{k,q,k}}{\omega_q - \frac{\partial\omega_k}{\partial\vec{k}}\cdot\vec{q}} \times\left\{\frac{\partial I_k}{\partial\vec{k}} + \frac{I_k}{\omega_k}\frac{\partial\omega_k}{\partial\vec{k}}\left(2\beta_k^2-3\right)\right\}\cdot\vec{q}\right\} \quad , \tag{31}$$

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where on the right-hand side, which is small, we have replaced Ω by ω_q. It should be noted that the term with the denominator in Eq. (31) is regular since $\omega_q - \frac{\partial\omega_k}{\partial\vec{k}} \cdot \vec{q} \neq 0$ for all finite $\vec{q}$ and $\vec{k}$. Thus, there is no "Landau damping" of the coherent wave due to the turbulence.

To proceed further, we evaluate the interaction kernels V's. On combining Eqs. (24) and (26) we have immediately

$$V_{k,q,k} = \frac{e}{4T_e} \frac{\omega_k}{\beta_k^2} \left\{ \beta_k^2 \left(1 + 2\beta_k \frac{\vec{k}\cdot\vec{q}}{kq} \operatorname{sign}(k_z q_z) \right) - 1 \right\} = \frac{\omega_k}{\omega_q \beta_k^2} V_{q,k,-k} \quad (32)$$

while Eq. (25) yields

$$V_{q,q,k,-k} = \frac{1}{6} \left(\frac{e}{T_e} \right)^2 \frac{\omega_q}{\beta_k^2} \quad . \quad (33)$$

We have approximated everywhere $\beta_q \approx 1$. On substituting V's from Eqs. (32) and (33) into Eq. (31) and integrating the term proportional to $\frac{\partial I_k}{\partial \vec{k}}$ by parts we finally obtain

$$\Omega-\omega_q=\frac{\omega_q}{4}\left(\frac{e}{T_e}\right)^2\sum_k I_k\left\{2\cos^2\theta\,\beta_k^4\left(3-4\beta_k^2\right)+4|\cos\theta|\beta_k^3\right.$$
$$\times\left(-4\beta_k^6+3\beta_k^4-2\beta_k^2+2\right)+2\left(-12\beta_k^{12}+9\beta_k^{10}-8\beta_k^8+10\beta_k^6\right. \quad (34)$$
$$\left.\left.-4\beta_k^4\right)+\frac{5}{2}\beta_k^2+1+\frac{1}{2}\beta_k^{-2}+\frac{\alpha_k}{1-|\cos\theta|\beta_k^{-3}}+\frac{\gamma_k}{(1-|\cos\theta|\beta_k^{-3})^2}\right\},$$

where

$$\alpha_k=4\left(8\beta_k^{12}-6\beta_k^{10}+6\beta_k^8-9\beta_k^6+4\beta_k^4-2\beta_k^2+1\right), \quad (35)$$

$$\gamma_k=-8\beta_k^{12}+6\beta_k^{10}-8\beta_k^8+16\beta_k^6-8\beta_k^4+\frac{15}{2}\beta_k^2-7 \quad (36)$$
$$+2\beta_k^{-2}-\beta_k^{-4}+\frac{1}{2}\beta_k^{-6},$$

and θ is the angle between $\vec{k}$ and $\vec{q}$.

The dispersion relation (34) may be further simplified if we assume that the spectrum I_k is isotropic. On performing the integration over the angle θ in Eq. (34), and introducing the dimensionless wave energy density by means of the relation

$$\tilde{W}_k = \frac{W_k}{n T_e} = I_k \beta_k^2 \left(\frac{e}{T_e}\right)^2 , \tag{37}$$

where n is the particle density, we find

$$\Omega = \omega_q \left(1 + \sum_k \tilde{W}_k F(\beta_k)\right) . \tag{38}$$

Here $F(\beta_k)$ is a form-factor defined by

$$\begin{aligned} F(\beta_k) = {} & \beta_k^2 \left(-8\beta_k^8 + 6\beta_k^6 - 6\beta_k^4 + \frac{19}{3}\beta_k^2 - 2\right) \\ & + \beta_k \left(-4\beta_k^6 + 3\beta_k^4 - 3\beta_k^2 + 3\right) + \frac{1}{2}\left(1 - \beta_k^{-1} + \beta_k^{-2}\right) + \frac{1}{8}\beta_k^{-4} \\ & + \beta_k \left(-8\beta_k^{12} + 6\beta_k^{10} - 6\beta_k^8 + 9\beta_k^6 - 4\beta_k^4 + 2\beta_k^2 - 1\right) \ln\left(1 - \beta_k^{-3}\right) \\ & + \frac{1}{4}\left(-\frac{1}{2} + \beta_k^{-2} - \beta_k^{-3} + \frac{1}{2}\beta_k^{-5}\right) \frac{1}{\beta_k^3 - 1} . \end{aligned} \tag{39}$$

Formula (39) may be simplified in two limiting cases: $k\lambda_D \ll 1$ and $k\lambda_D \gg 1$. We have

$$F(\beta_k) \approx \begin{cases} -\left(\frac{25}{6} + 2\ln\left(\frac{3}{2}k^2\lambda_D^2\right)\right), & k\lambda_D \ll 1, \\ \frac{1}{2}(\beta_k + 1), & k\lambda_D \gg 1. \end{cases} \tag{40}$$

The behaviour of the form-factor for intermediary values of $k\lambda_D$ is plotted in Fig. 1, where the broken lines represent the approximate expressions (40). We observe that the form-factor is positive over the whole range of $k\lambda_D$-values. Thus, in the case where the coherent and turbulent waves propagate with positive phase velocities along the z-axis, the phase velocity of the former increases due to the interaction with the latter. In order to obtain a quantitative result the integral in Eq. (38) must be calculated numerically for a given spectrum I_k.

Let us now turn to the case where the z-components of the phase velocities of the coherent and turbulent waves have the opposite sign. If we include the mode with the eigenfrequency $-\omega_p$ in Eq. (23), write down a similar equation for this mode, and repeat the foregoing analysis we find that the dispersion relation for the case in question is obtained

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from the previous one just by changing the sign of β_k. Thus, we have

$$\Omega = \omega_q \left(1 + \sum_k \tilde{W}_k F(-\beta_k)\right) . \tag{41}$$

It is straightforward to show that

$$F(-\beta_k) \approx \begin{cases} 2 \ln 2 - \frac{7}{6} , & k\lambda_D \ll 1 , \\ \frac{1}{2}(1-\beta_k) , & k\lambda_D \gg 1 . \end{cases} \tag{42}$$

The behaviour of the form-factor $F(-\beta_k)$ for intermediary values of $k\lambda_D$ is plotted in Fig. 2, where the broken lines represent the approximate expressions (42). We see that the form-factor changes its sign at $k\lambda_D \approx .65$ approximately. Consequently, if the spectrum peaks in the region $k\lambda_D < .65$ the phase velocity of the coherent wave increases due to the interaction with the turbulence. On the other hand, if the spectrum peaks in the region $k\lambda_D > .65$ the interaction results in a decrease of the phase velocity of the coherent wave. Once again quantitative results can be obtained numerically from Eq. (41).

Finally, we comment on the results obtained recently by Kono and Yajima[10], using a method that is different from ours. Their analysis is confined to the case where $k\lambda_D << 1$. On putting $\beta_p = 1$ a priori, they found that the phase velocity of the coherent wave decreases due to the interaction with the turbulence. Our results differ from theirs, essentially due to the presence of the logarithmic terms in Eqs. (40) and (42), which stem from the denominator $\omega_q - \frac{\partial\omega_k}{\partial\vec{k}} \cdot \vec{q}$. In their case this denominator was just $\omega_q - \vec{k} \cdot \vec{q}c_s/k$ which is singular. It is not clear how this singularity was eliminated.

IV. CONCLUSIONS

We have shown that the method of correlation functions together with a Fourier-transform technique can be used to construct self-consistent equations describing the adiabatic interaction of coherent waves with weakly turbulent plasmas. In our opinion, the advantages of the method are to be found in its simplicity and lucidity.

We have applied the formalism developed to study the propagation of a low-frequency ion-acoustic wave in the presence of high frequency ion-acoustic turbulence. We have found that the turbulent waves increase the phase velocity of the coherent wave in the case where they all propagate so that each has a velocity component in a given direction. In the opposite case, the phase velocity of the coherent wave increases or decreases depending on whether the turbulence peaks at low or high values of $k\lambda_D$. In either case there is no "Landau damping" of the coherent wave due to the turbulence.

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FIGURE CAPTIONS

Fig. 1 Form-factor $F(\beta_k)$ versus $k\lambda_D$. The broken lines represent the approximate expressions (40).

Fig. 2 Form-factor $F(-\beta_k)$ versus $k\lambda_D$. The broken lines represent the approximate expressions (42).

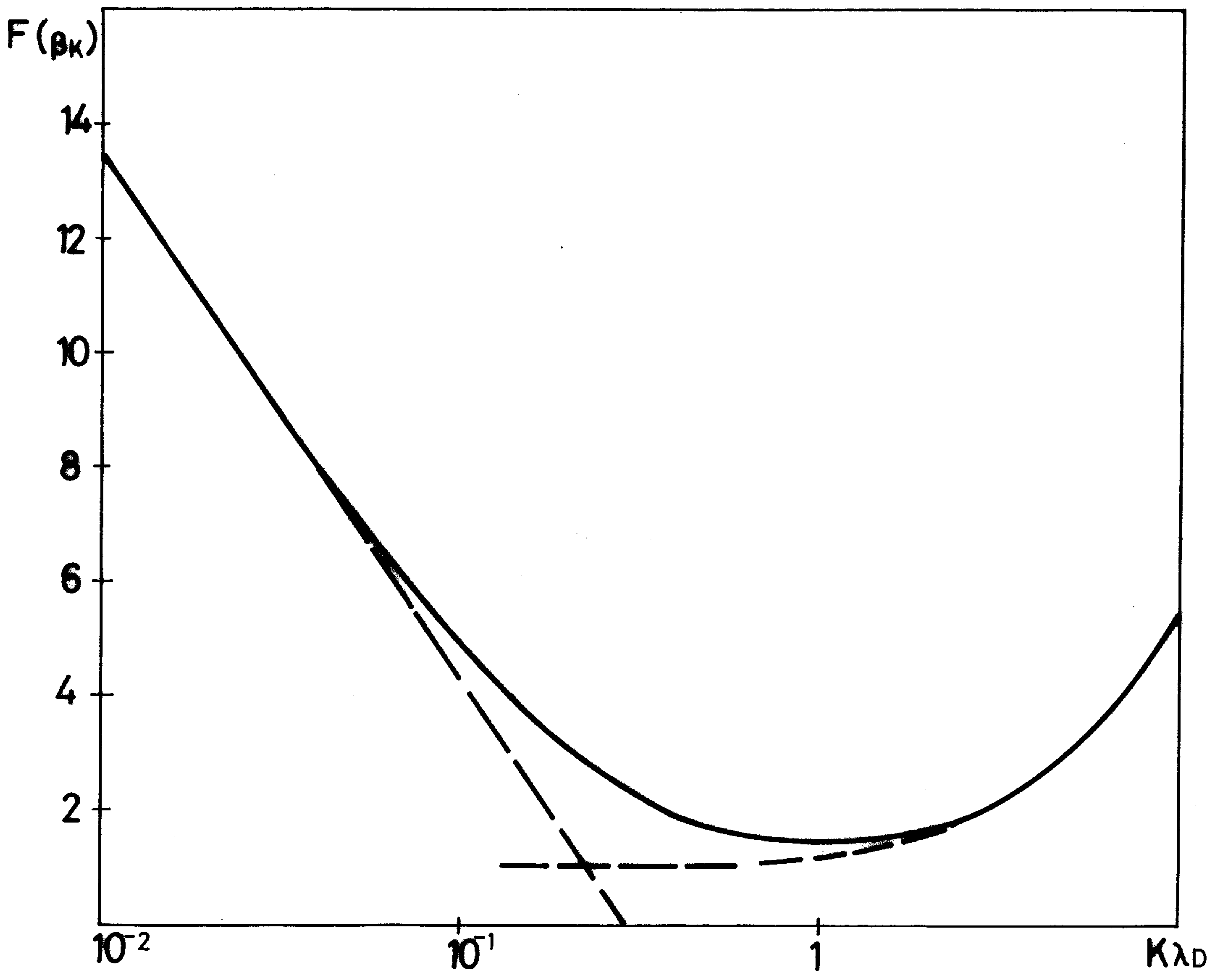

Fig. 1

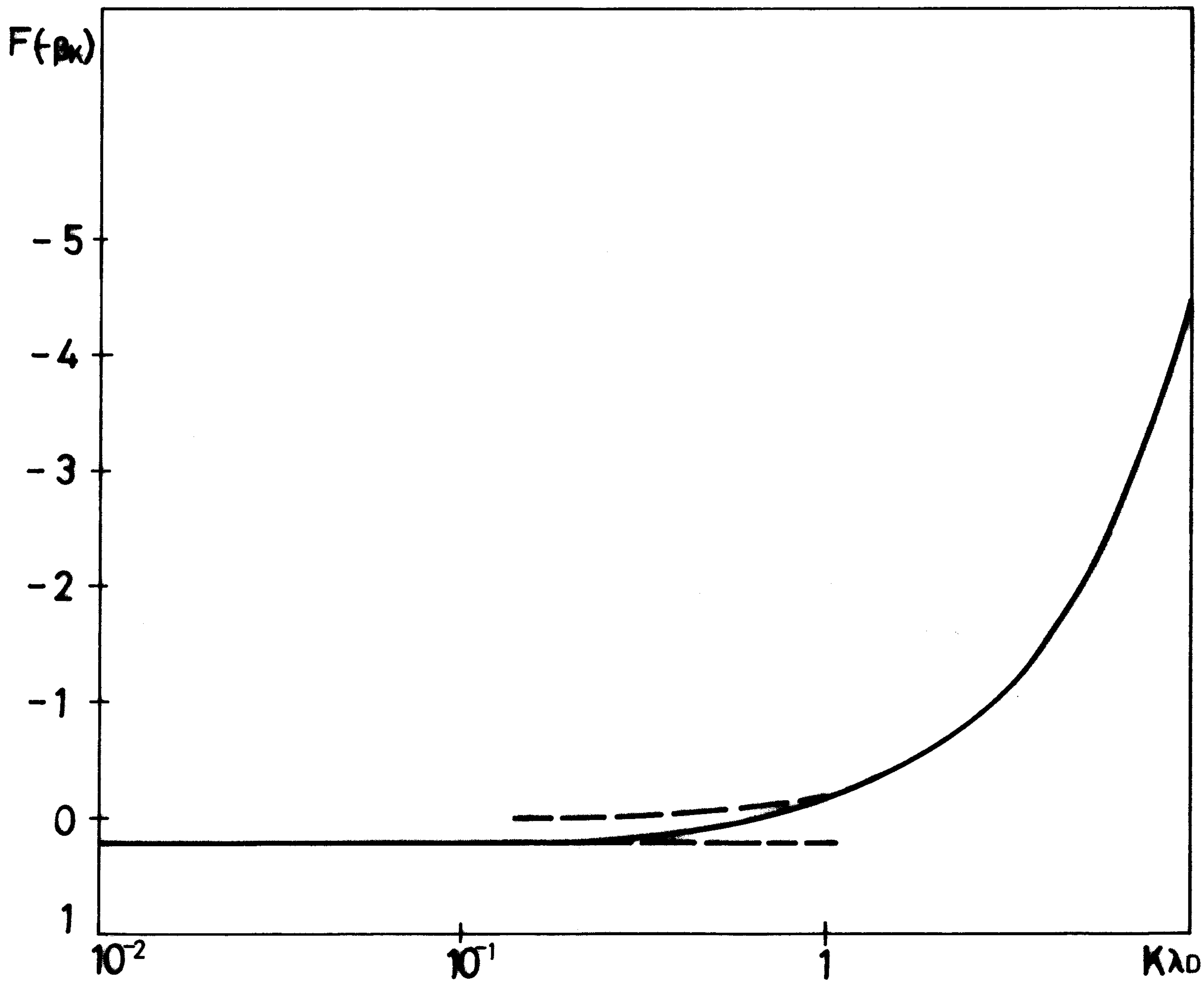

Fig.2