Juin 1970

LRP 44/70

CENTRE DE RECHERCHES EN PHYSIQUE DES PLASMAS
FINANCÉ PAR LE FONDS NATIONAL SUISSE DE LA RECHERCHE SCIENTIFIQUE

NUMERICAL COMPUTATION TO DESIGN A ROTATING FIELD PINCH EXPERIMENT

E.S. Weibel and A. Lietti

LAUSANNE

Juin 1970 LRP 44/70

NUMERICAL COMPUTATION TO DESIGN A ROTATING FIELD PINCH EXPERIMENT

E.S. Weibel and A. Lietti

A b s t r a c t

The model proposed is a simple one, describing the dynamics of a rotating field pinch in term of the macroscopic quantities : temperature, pressure, plasma radius and magnetic field strength. The field is obtained by producing oscillating currents in resonant circuits, which are fed by line r.f. pulse generators.

The computation takes into account the equations of the electrical system and of the gas dynamics. The result of this computation is compared with the result of an experiment which has been performed in a 5 x 50 cm. discharge tube.

The amplitude programme which is necessary in order to confine the plasma in a new experiment on larger scale is established.

1. INTRODUCTION

The theory of the dynamics of a rotating field pinch has been reported in a previous paper (ref. 1). The plasma was assumed to be uniform both in density and in temperature at all times. Inertia and viscosity were neglected and the rate of heating was calculated in accordance with the theory of the skin effect, both normal and anomalous (ref. 2).

In the present paper, the model is improved by considering inertia and viscosity. On the other hand, measurements show that the plasma heats faster than the theory predicts. Clearly this anomaly needs further investigation. In the present paper we merely take into account this fact by assuming an experimental value of the surface resistance.

The pinch dynamics can be considered to be produced by the interaction of three systems : the gas discharge, the resonant circuits and the r.f. generators (fig. 1).

The r.f. line generators can be "programmed", to produce various shapes of wave trains.

The calculation proceeds as follows : First an arbitrary line programme is chosen, then the set of equations of the electrical circuits and of the plasma dynamics are solved.

One thus obtains the radius of the plasma column as a function of time. The line programme is subsequentely varied and the computation performed again, in order to obtain the longest confinement time of the plasma column.

2. THE R.F. GENERATORS EQUATIONS

The line generators have been extensively described elsewhere (ref. 3). They are essentially made of capacitors that are assembled with inductors to form a lumped element transmission line.

These capacitors are initially isolated by means of spark-gaps and charged. By properly choosing the time of switching and the charging voltage, it is possible to obtain a wave train wich is "programmed" both in amplitude and in frequency. In this computation, only the amplitude programme is used; the time of switching $T_s = t_m - t_{m+1}$ (fig. 2) is therefore constant and has been calculated, like the other line parameters, by means of the tables reported in ref. (3).

Let n be the total number of the line meshes, the 2 n equations to be used in the computation are :

$$(1) \qquad 0 = C_m \dot{V}_m + \varepsilon(t-t_m)\, i_m - \varepsilon(t-t_{m+1})\, i_{m+1}$$

$$0 = \varepsilon(t-t_m)\left\{V_m - i_m\left[(1+\beta_m) r_m + \beta_{m-1} r_{m-1}\right] - \dot{i}_m\left[(1+\alpha_m) L_m + \alpha_{m-1} L_{m-1}\right]\right\} +$$

$$+ \varepsilon(t-t_{m+1})\left\{\beta_m r_m i_{m+1} + \alpha_m L_m \dot{i}_{m+1}\right\} + \varepsilon(t-t_{m-1})\left\{r_{m-1} i_{m-1} + \alpha_{m-1} L_{m-1} \dot{i}_{m+1}\right\}$$

In order to obtain the currents in the plasma circuits one can solve the system of circuit equations plus the line equations (1). It seems more convenient, however, to split the problem into two parts. First we solve equations (1) considering open circuit termination of the line. We obtain the open circuit voltage of the generator, Vg, that is independent of the circuit which couples the generator to the plasma. The operation is repeated twice because both the I_θ and the I_Z circuits have to be fed. The switching times of the θ and Z lines are adequately shifted in order to produce the 90° shift of the ouptut voltages.

The values of the Vg (t) function are then utilized to solve the plasma circuits equations.

3. PLASMA DYNAMICS

3.1 - Circuit equations

Fig. 3 shows the schematic diagram of the circuits. Taking into account the variation of the elements R and L, the following equation can be written :

$$(2) \qquad \frac{V_g}{R_g} = \ddot{I} L C + \dot{I}\left\{RC + \frac{L}{R_g} + C\dot{L}\right\} + I\left\{1 + \frac{R}{R_g} + C\dot{R}\right\}$$

Let l be the length of the discharge tube and R = 0,0236 Ω the experimental value of the surface resistance. (The "classical" skin depth $\delta = \frac{2\mathcal{R}}{\mu_0 \omega}$ is then 2 mm.)

The values of the circuit elements are as follows :

for the θ - circuit

(3)
$$R = \mathcal{R}\frac{2\pi a}{\ell} \qquad L = \frac{\pi\mu_0}{\ell}(b^2 - a^2)$$
$$\dot{R} = R\frac{\dot{a}}{a} \qquad -\dot{L} = \frac{2\pi\mu_0 a\dot{a}}{\ell}$$

and for the Z - circuit :

(4)
$$R = \mathcal{R}\frac{\ell}{2\pi a} \qquad L = \frac{\mu_0}{2\pi}\ell\log(b/a)$$
$$-\dot{R} = R\frac{\dot{a}}{a} \qquad -\dot{L} = \frac{\mu_0 \ell \dot{a}}{2\pi a}$$

The capacitor C is determined by the resonance condition :

(5)
$$C = 1/L\omega^2$$

The dimension b must be chosen so as to take account of all external self-inductance of the circuit, while a is an "equivalent" radius, which is determined by the current distribution.

Two extreme hypotheses can be proposed :

(i) The whole plasma is contained in a column of radius a and the current flows at the surface of this column.

(ii) The current flows near the walls of the discharge tube.

If the first hypothesis is considered, the value of a will be determined by the equations of the gas dynamics, which will be discussed later; otherwise the radius a must be considered constant, and no variation of L occurs.

Experiments until now have not been sufficient to give a definite answer to the question of how well the current follows the pinch.

Considering the question open, two computations will be done according to both the extreme hypotheses, which will henceforth referred to as "variable L" and "constant L" hypotheses.

3.2 - The magnetic and the kinetic pressure.

The components of the field are established by the geometry of the experiment, the resulting rotating field can consequentely be expressed as follows :

(6)
$$B = \left[\left(\frac{\mu_0 I_\theta}{\ell}\right)^2 + \left(\frac{\mu_0 I_z}{2\pi a}\right)^2\right]^{1/2}$$

It produces a magnetic pressure :

(7) $$P_B = \frac{B^2}{2\mu_0}$$

Let n_e be the electron density, T the temperature, Z the atomic number and K the Boltzmann constant, the kinetic pressure is :

(8) $$P_K = K T n_e (1 + 1/Z)$$

3.3 - The inertial model.

This model considers a gas column of variable radius. The gas particles are assumed to be uniformly compressed and to have radial velocity which is proportional to the distance from the axis of the column.

If r is the radial distance of the particle, the radial velocity is :

$$\dot{r} = r_0 \frac{\dot{a}(t)}{a_0}$$

where, as usual, the index o means initial value. The differential expression for the kinetic energy is then :

$$dT = \frac{1}{2} dm \left(\frac{r_0}{a_0} \dot{a}\right)^2 = \pi \rho_0 r_0^3 \left(\frac{\dot{a}}{a_0}\right)^2 dr_0$$

Integration gives the energy :

$$T = \frac{\pi}{4} \rho_0 a_0^2 \left[\dot{a}(t)\right]^2$$

The inertial pressure P_{in} is given by the Lagrange equations :

(9) $$\frac{d}{dt} \frac{\partial T}{\partial \dot{a}} = \frac{\pi}{2} \rho_0 a_0^2 \ddot{a} = -2\pi a P_{in}$$

$$P_{in} = -\frac{1}{4} \rho_0 a_0^2 \frac{\ddot{a}}{a} = -H \frac{\ddot{a}}{a}$$

where

$$H = \frac{1}{4} \rho_0 a_0^2 = \frac{1}{4\pi} \left(\frac{N}{\ell}\right) \frac{A m_H}{Z}$$

(N being the total number of particles,
A the atomic mass number and
m_H the hydrogen mass).

3.4 - The "bounce" model.

The viscosity increases proportionally with the mean free path. For infinite mean free path this viscosity can be simulated by the bounce model which assumes that each particles is reflected at the sharp pinch boundary.

This reflection gives rises to a pressure

$$P_v = - \sqrt{\frac{8}{\pi}} \, P_i \, \frac{\dot{a}}{u_i}$$

where

$$P_i = \kappa T n_e / Z = P_K / (1+Z)$$

$$u_i = \sqrt{\frac{\kappa T}{m_i}} = \sqrt{\frac{P_K}{A \, m_H (1 + 1/Z)}} \cdot \sqrt{\frac{\pi}{(N/\ell)}} \cdot a$$

Thus

(12) $$P_v = - 2 \, K \sqrt{P_K} \, \frac{\dot{a}}{a}$$

where

$$K = \frac{1}{\pi} \sqrt{\frac{2 A \, m_H}{Z (1+Z)} \, \frac{N}{\ell}}$$

3.5 - Equation of motion.

An inertial and a bounce model have been considered, according to the short and long mean free path limits. It is not clear how one should interpolate these extremes for intermediate cases.

We have simply used the following device: Part of the mass $(\varepsilon-1) \, m_H$ is responsible for viscosity and the remainder $\varepsilon \, m_H$ for inertia.

The fraction ε is chosen quite arbitrarly as the following function of the mean free path :

(13) $$\varepsilon = \frac{1}{1 + \lambda/a}$$

From equations (7,8,9,12 and 13) the pressure balance equation can be written :

(14) $$P_B = P_K + \varepsilon \, P_{in} + \sqrt{1-\varepsilon} \; P_v$$

The equation of motion is then :

(15)

$$P_B = P_K - \varepsilon K \sqrt{P_K}\,\frac{\dot{a}}{a} - \sqrt{1-\varepsilon}\; H \frac{\ddot{a}}{a}$$

3.6 - Energy balance.

Let W be the total energy, $\Omega = \pi l a^2$ the volume of the plasma column, and Q the rate of heating due to the alternating current in the plasma boundary layer. Conservation of energy requires :

(16)
$$\dot{W} = - P_B \dot{\Omega} + Q$$

Then the following relation holds

(17)
$$W = \frac{f}{2} \Omega P_K$$

where f is the number of degrees of freedom per particle. In this computation f = 3.

The rate of heating can be expressed as a function of the surface $S = 2\pi a l$ of the plasma column and of the current density j :

(18)
$$Q = S \mathcal{R} j^2 = S \mathcal{R} \left(\frac{B}{\mu_0}\right)^2$$

From equations (7), (16), (17), (18) it follows :

(19)
$$\dot{P}_K = \frac{1}{af}\left[\frac{8\mathcal{R} P_B}{\mu_0} - 2\left(f P_K + 2 P_B\right)\dot{a}\right]$$

4. THE NUMERICAL COMPUTATION.

The integration of equations (1), (2), (15) and (19) gives the following quantities as a function of time :

$$V_g,\ a,\ \dot{a},\ T,\ B,\ P_B,\ P_K$$

The pinch dynamics is then completely defined.

One assumes a fully ionized plasma and an initial temperature of $10^4 K^o$.

4.1 - Comparaison with experiment

A rotating field pinch experiment has already been described (ref. 4-5). Helium gas is contained in a discharge tube of 5 cm diameter and 50 cm length and is preionized by means of an axial current pulse of 9,5 KA amplitude and 32 µsec duration. A 3 MH_z rotating field is then applied during 2,3 microseconds. The time evolution of the generator voltage is shown in fig. 4, the values of Vg are normalized.

Vg = 1 corresponds with a power of 46 MW delivered to a matched load.

Measured and computed fields are shown in fig. 5. Computations following both the variable and the constant L hypothesis have been performed.

Temperature evolution is shown in fig. 6. A streak photography of the discharge can be compared with the calculated dynamics, as shown in fig. 7. The confinement time which can be deduced from the experiment agrees reasonably well with the computed one.

In conclusion, the measured quantities seem to take values between the calculated ones, the calculation having been done following the two extreme hypotheses, variable and constant L.

4.2 - Design of a new experiment.

A new experiment on larger scale, namely with a discharge tube of cm. 15 x 80 has been designed with a field duration of 4 µsec. The available power to be transferred to the plasma can reach the value of 380 MW.

The goal of the present computation is to choose a suitable line programme, in order to obtain the longest containement time. Once again both variable and constant L conditions have been examinated.

An important technical problem arises if the inductivity changes : the coupling circuit goes out of resonance and it may be possible that the confining field can no longer be maintained. This danger can partly be overcome by resonating the circuit on a radius which is somewhat smaller than the wall radius.

In the examples we have computed, when variable L has been considered, the resonance occurs at a = 6,8 cm. (wall radius 7,5 cm).

Fully charged lines (no amplitude programme) were assumed in the computation which is reported in fig. 8. Confinement times of about 4 μsec. are obtained.

By employing a simple line programme, the duration of the containement can be lengthened by 1 μsec, as shown in fig. 9.

The field produced in accordance with this programme is shown in fig. 10 c,v.

It is worthwhile to note that the confinement seems to be possible with either of the two hypotheses, variable and constant L, if the line programme has been adequately predisposed. The final adjustement will be done by experiment. The aim of this computation is merely to show that this adjustement is possible.

We want to discuss briefly some of the approximations which are involved in the model which is proposed in this paper.

The rate of heating of the plasma is clearly an important problem of the r.f. confinement. In fact the resulting kinetic pressure can eventually overcome the magnetic pressure, thus destroying the confinement. From this point of view, the design of an experiment will be safer if the rate of heating is overestimated. We are just in this position by neglecting energy losses of the plasma, and by assuming a fully ionized gas.

5. EQUILIBRIUM CONFINEMENT

The result of the computation can be compared with the result one obtains, if an ideal equilibrium confinement is considered. In effect the equilibrium confinement time has to be considered as the maximum one which can be achieved.

If : $\dot{a} = o$, $f = 3$, $P_K = P_B$,

equation (19) gives :

$$\frac{dT}{T} = \frac{8 \mathcal{R}}{3 a \mu_0} dt$$

and integration leads to the following expressions, which give the time evolution of the temperature and field, in accordance with the equilibrium condition.

(20) $$T = T_0 \, e^{2\alpha t}$$

(21) $$B = B_0 \, e^{\alpha t}$$

where

(22) $$\alpha = \frac{4\mathcal{R}}{3a\mu_0}$$

Examination of these formulae leads to the conclusion that, if R and B are given, the maximum confinement time grows at the same rate as the radius.

The physical meaning of this statement lies in this, that the rate of energy transfer is proportional to the surface area of the plasma column, while the rate of heating of the gas is proportional to the volume.

The temperature and field evolution of the two pinches we have discussed has been evaluated and is shown in fig. 11.

The dotted lines which are drawn in the graphs (8), (9) and (10) represent the same functions and allows a comparison to be made with the result of the calculations, we have discussed before.

The approximation of the computed values to the ideal ones can be estimated as satifactory.

REFERENCES

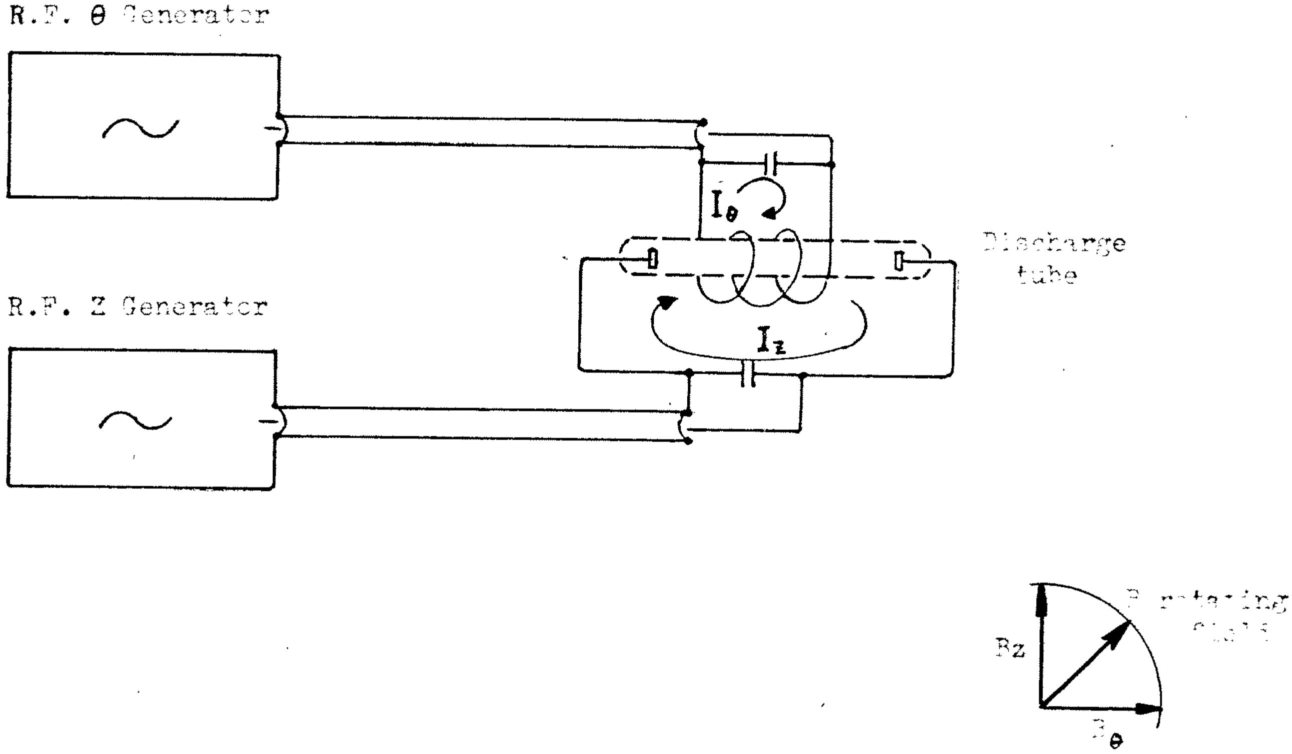
1) E.S. Weibel and R. Keller,
Plasma Physics 9, 401, 1967
"On the design of a rotating field pinched discharge".

2) E.S. Weibel, Physics of Fluids, 1967
"The anomalous skin effect in a plasma".

3) A. Lietti, Rev. Sci. Instruments 40, 473, 1969
"Lumped Element Line Generators to Produce Sinusoidal Oscillations of Very Large Power".

4) I.R. Jones, A. Lietti and J.-M. Peiry
Plasma Physics 10, 213, 1968
"A Rotating Magnetic Field Pinch".

5) A. Berney, H. Heym, F. Hofmann and I.R. Jones,
Third European Conference on Controlled Fusion and Plasma Physics, Utrecht, 1969
"Recent Measurements on a Rotating Magnetic Field Pinch".

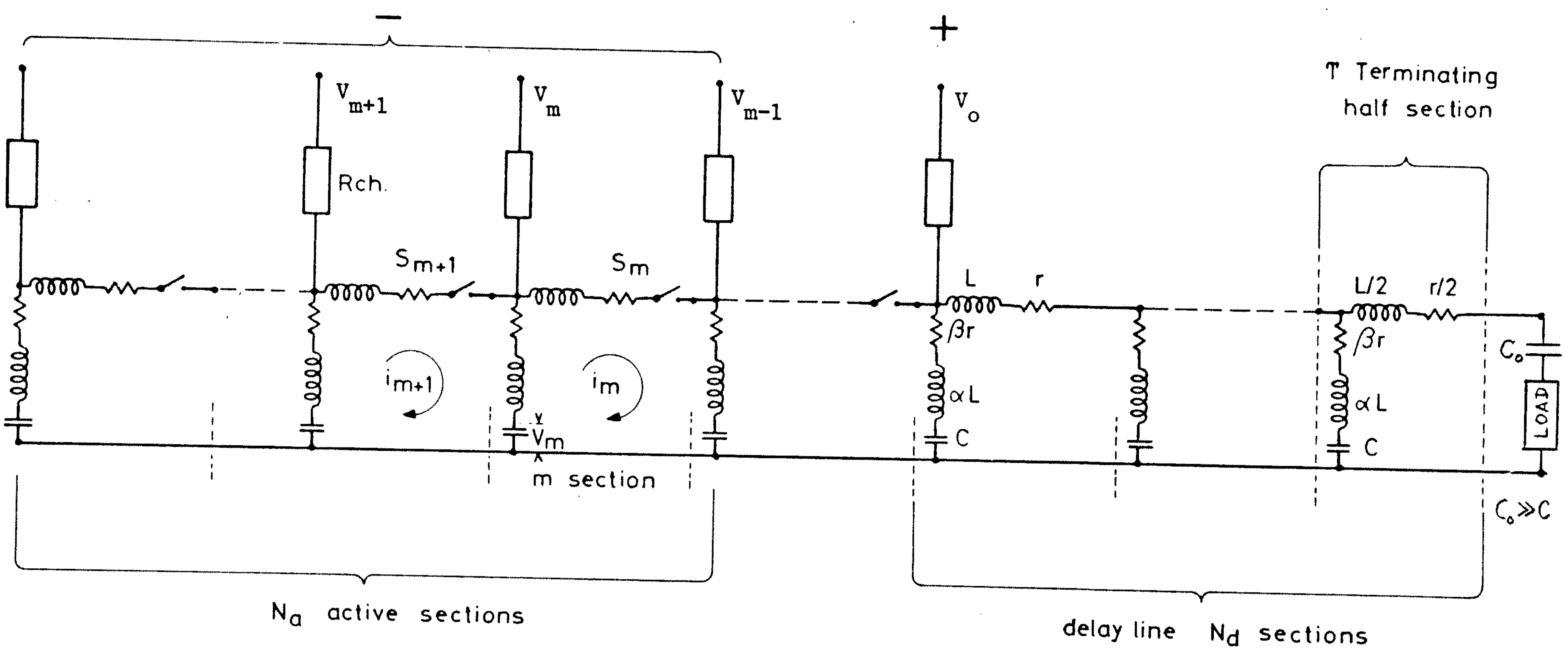


The dephasing between I_θ and Iz is 90°, and the amplitude of the associated Bz and B_θ fields are equal at the inner wall of the discharge tube.

Schematic diagram of the experimental R.F. installation.

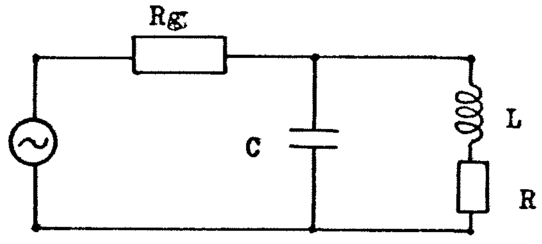
Fig. 1

Fig. 2 The line r.f. pulse generator



The switch S_m is supposed to be closed at time t_m.

Fig. 2



The circuit coupling the
generators to the plasma.

Fig. 3

The 50 cm. pinch

Amplitude of the generator voltage, normalized values.

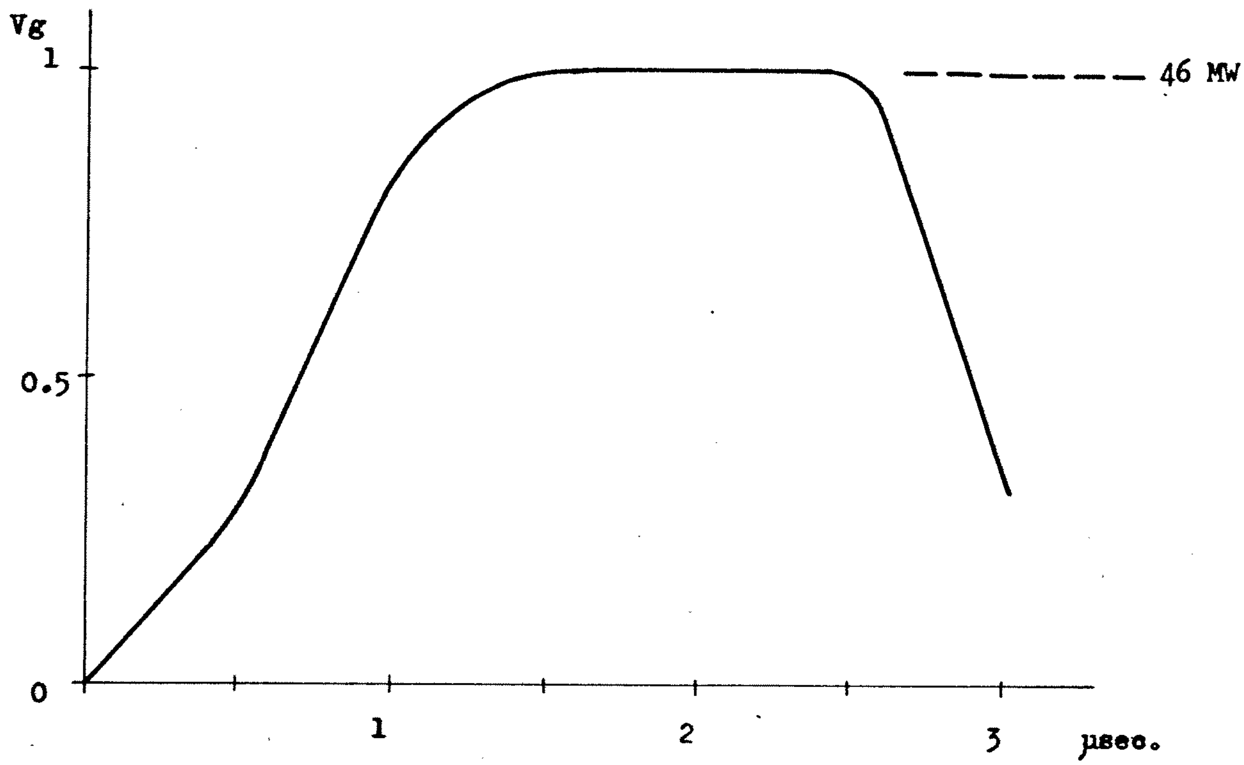


Fig.4

The 50 cm. pinch. Magnetic field.

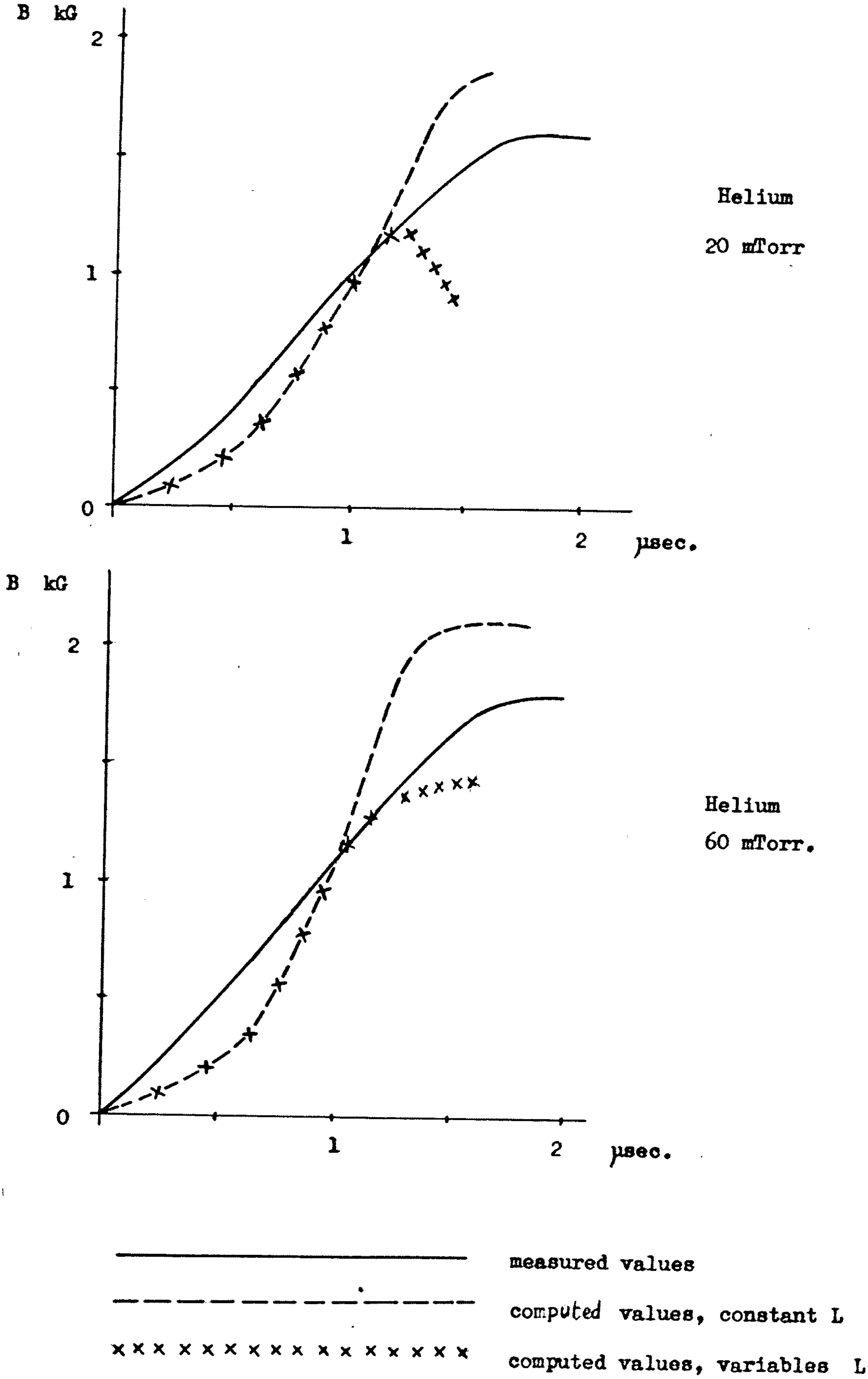
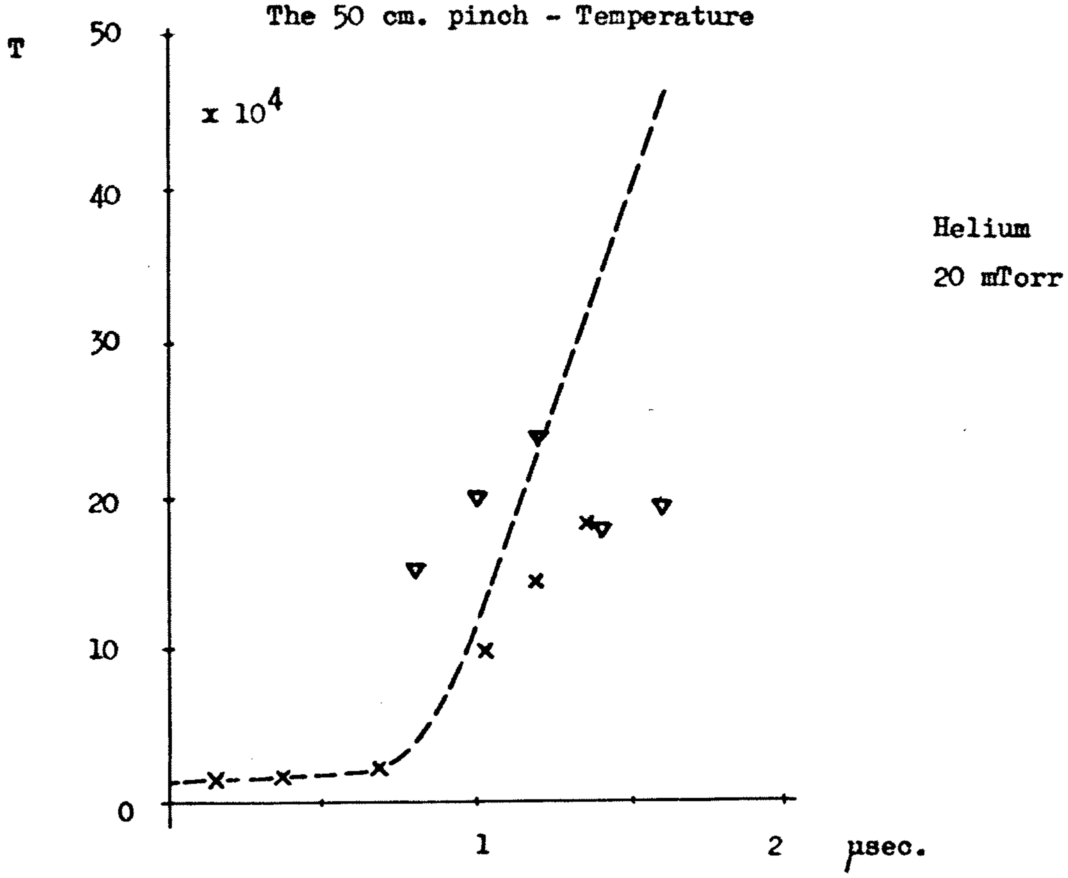
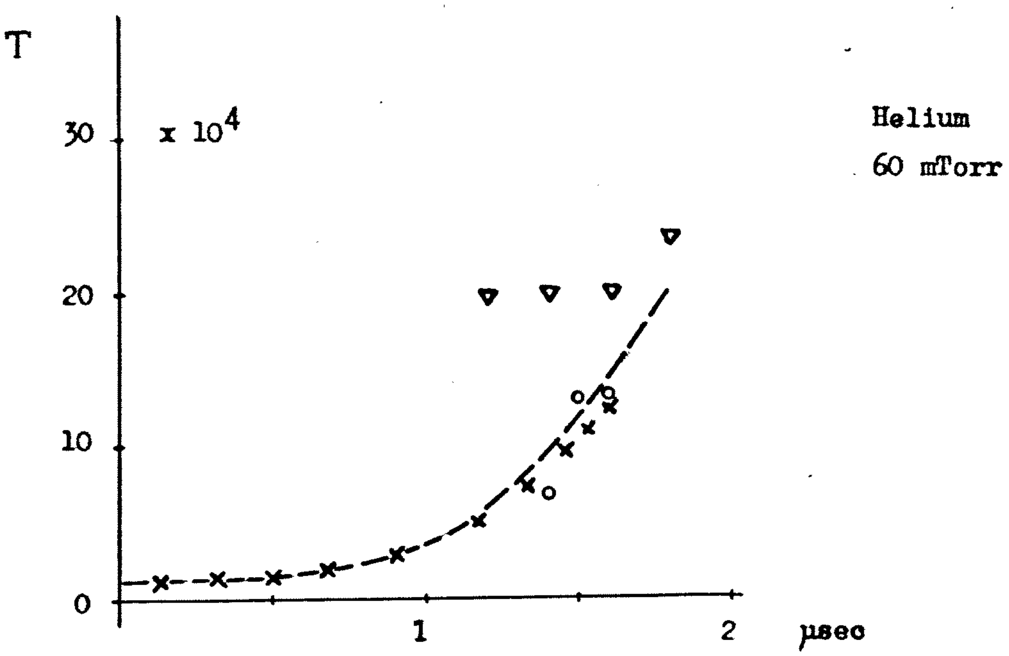


Fig. 5

The 50 cm. pinch - Temperature





— — — — — — — — computed values- constant L

× × × × × × × × × computed values- variable L

o o o o o o electronic temperature- } measured

▽ ▽ ▽ ▽ ▽ ▽ ionic temperature- } values

Fig. 6

The 50 cm. pinch.

He. 60 mTorr. filling pressure.

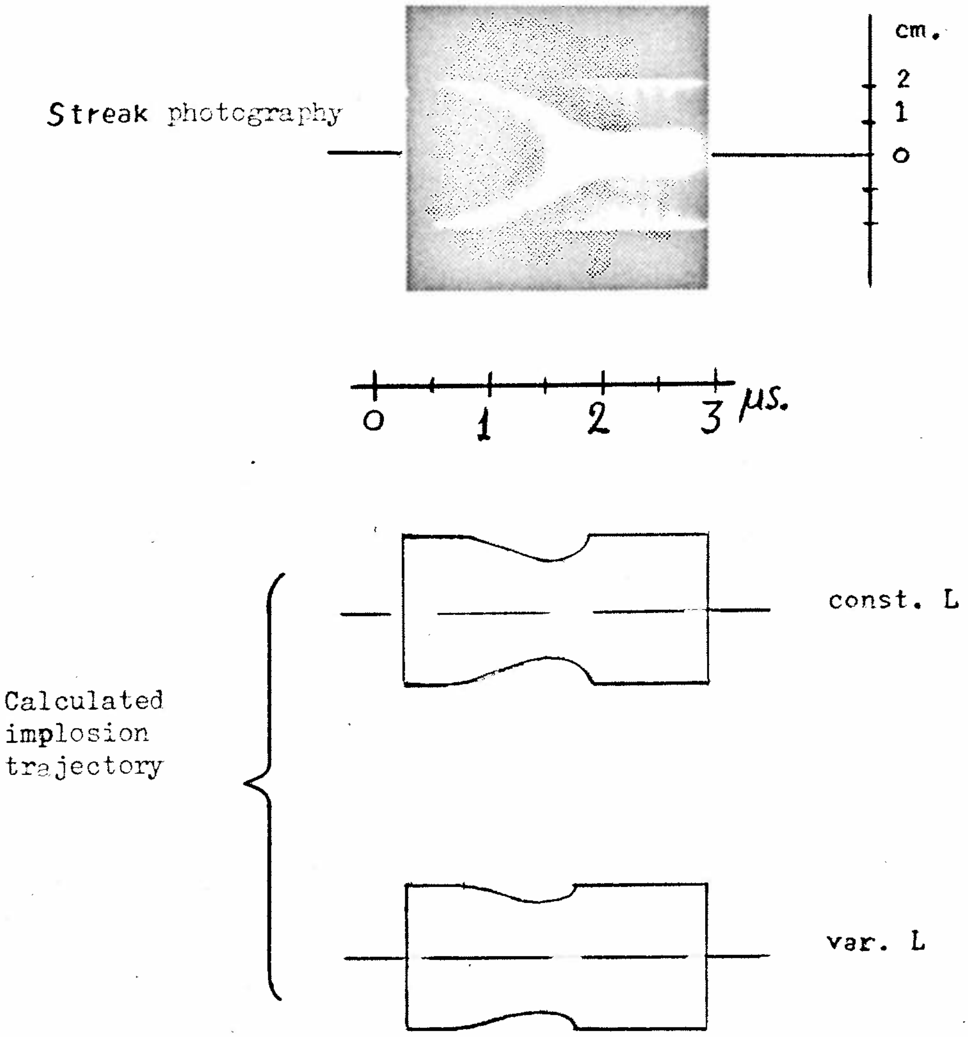


Fig. 7

The 80 cm. pinch

Fully charged line

H_2 45 mTorr - variable L - ▼

H_2 100 mTorr - constant L - o

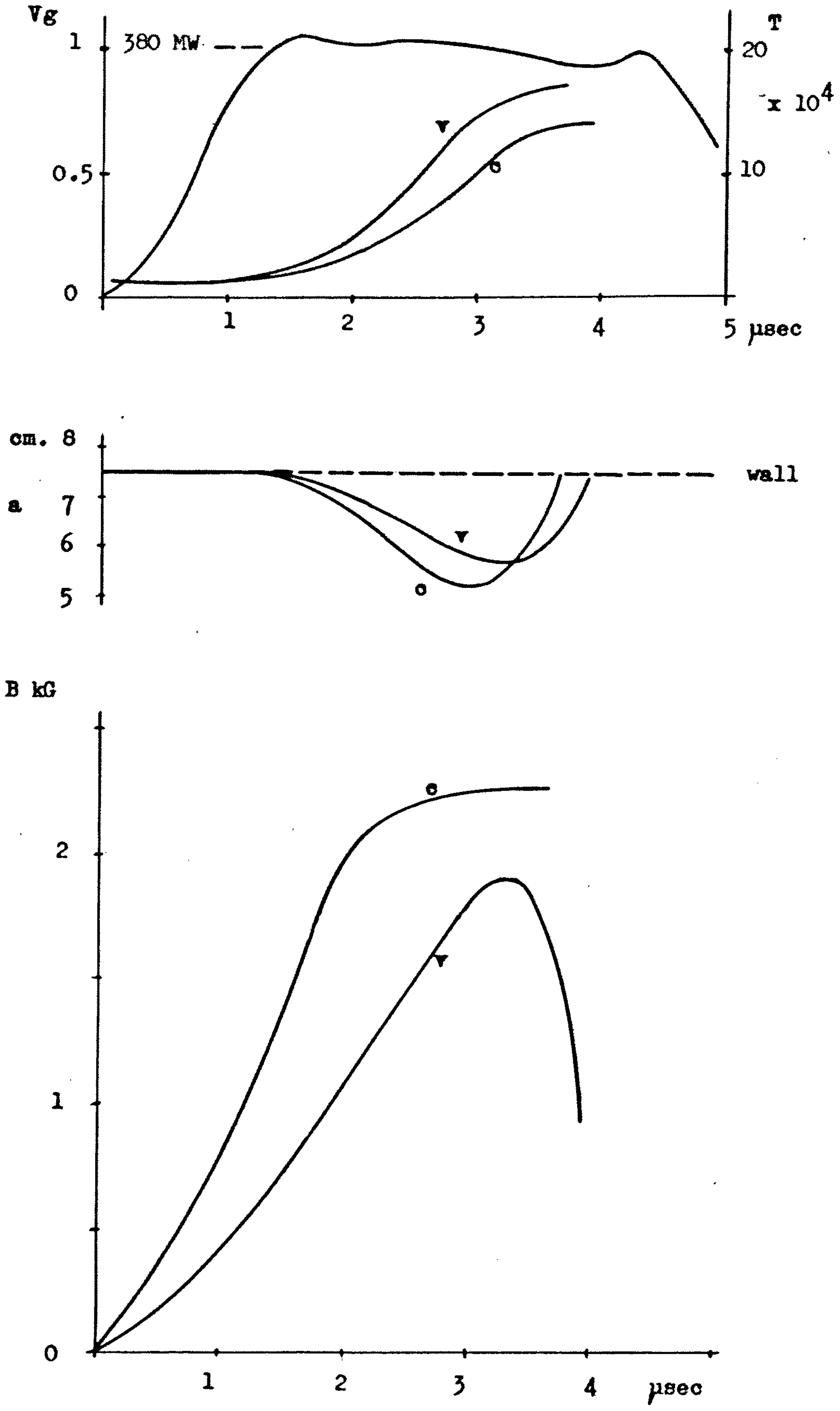
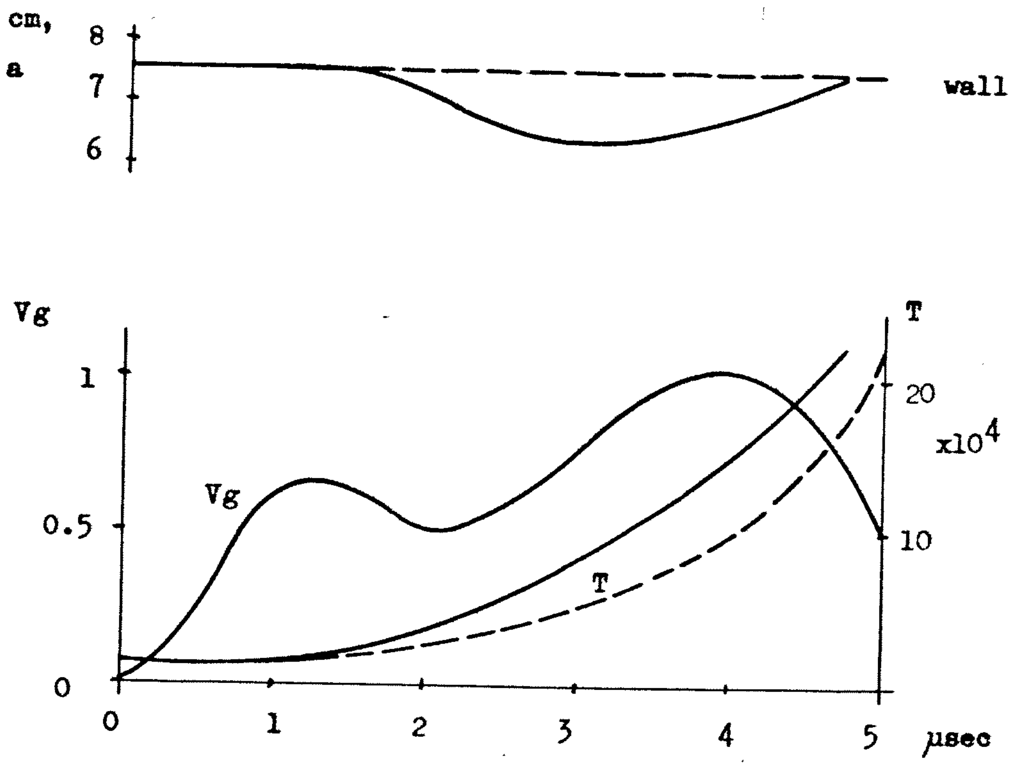
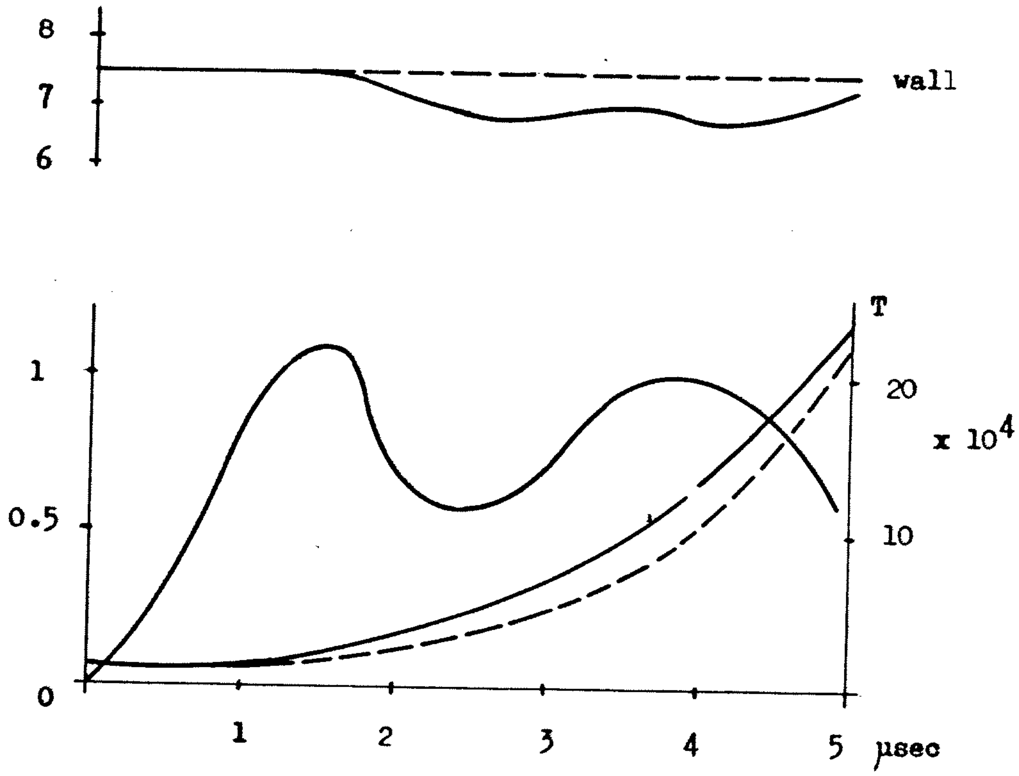


Fig. 8



H_2 60 mTorr
const. L

Line programme : sections 2,3,4,5,6 reduced charge



H2 45 mTorr
var. L

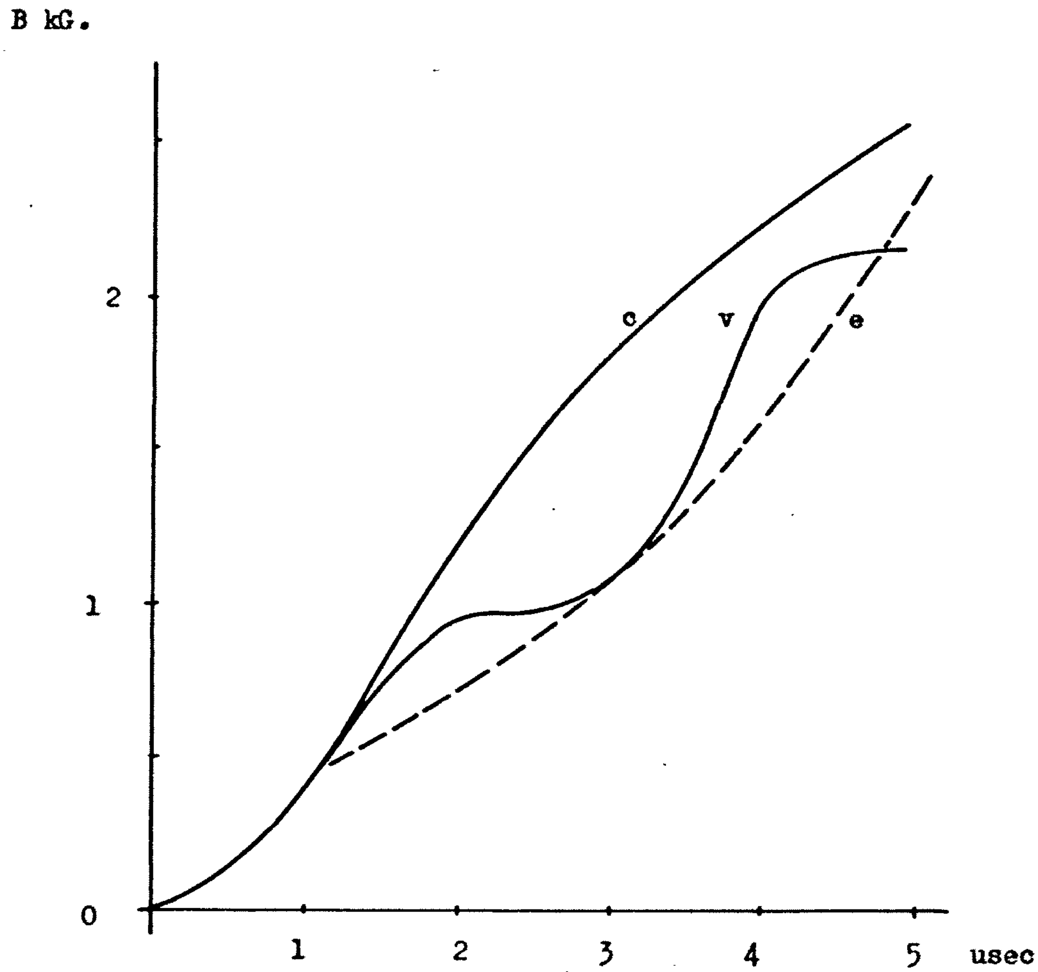
Line programme: sections 4,5,6 reduced charge

— — — — — — equilibrium confinement (eq. 20)

Fig. 9

The 80 cm. pinch.

Magnetic field.

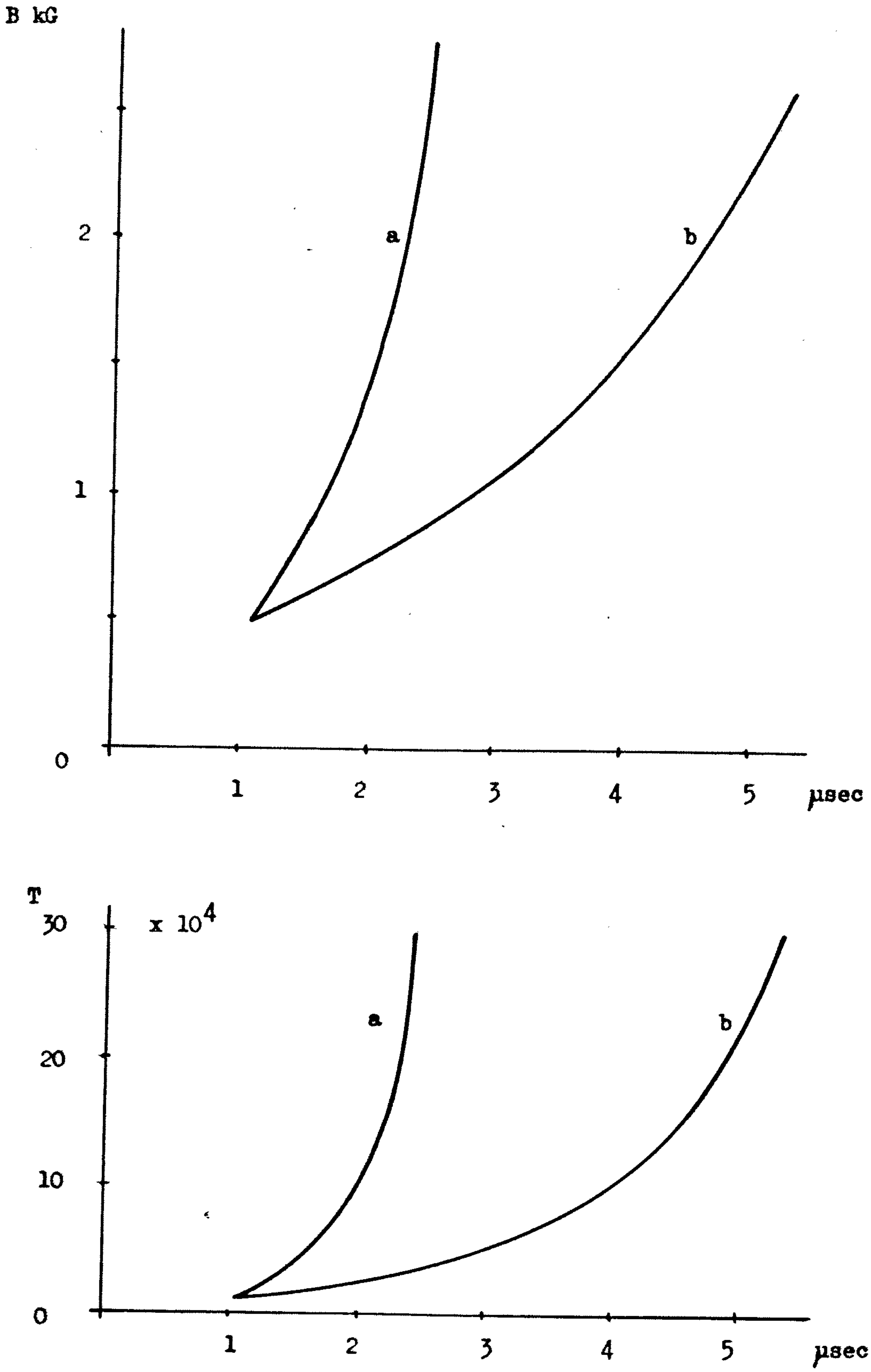


c - H_2 60 mTorr constant L

v - H_2 45 mTorr variable L

e - - - - - - equilibrium field (equation 21)

Fig. 10



Equilibrium confinement (eq. 20,21)

a) 50 cm. pinch

b) 80 cm. pinch

Fig 11