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LUMPED ELEMENT LINE GENERATORS TO PRODUCE PULSED SINUSOIDAL OSCILLATIONS IN THE GIGAWATT RANGE

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LAUSANNE

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A b s t r a c t

After reviewing the present state of knowledge concerning the technique of making high power sinusoidal pulses, an improved kind of line generator is proposed. An analysis of wave propagation in a periodic structure together with the numerical solution of the network differential equations shows that it is possible to utilize the recurrent triggering system (hitherto applied only to homogeneous lines) with lumped element lines. A method for designing such lines is presented. Numerical tables are included and some selected examples are discussed.

The theory agrees with an experiment made with a prototype line. The construction of a 720 MW, 3 MHz installation for use in plasma physics experiments is already underway. This large power is achieved by paralleling a number of lumped element line generators.

Lausanne

1. Introduction

In order to perform certain experiments in plasma physics, pulsed rf magnetic fields of a few kilogauss amplitude and of some microseconds duration have been required (1-5). Real power of hundreds of Megawatts is necessary.

Owing to the short duration of the pulses, and to the fact that only one such pulse is needed for every experiment, one can estimate that the order of magnitude of the total energy involved is in the kilojoule range. Charged capacitors can easily store this energy. The problem of obtaining rf current pulses of constant (or even increasing) amplitude from these capacitors has been solved by means of a special technique developed in this laboratory.

The fundamental idea underlying this technique can briefly be explained as follows: the capacitors are assembled with inductors to form a lumped element transmission line. These capacitors, however, are isolated from the inductor and from each other by means of fast switches (sparkgaps). Initially the capacitors are charged. By properly choosing the charging voltages and the time of switching, it is possible to obtain the desired pulse.

Besides the lumped elements line generators, other generators exist made up of homogeneous line sections. It is obvious that this second category of lines represents the limit of the first one, when the number of meshes per wavelength becomes very large.

These considerations lead us to classify the line generators into two classes which will be henceforth referred to as "lumped lines" and "homogeneous lines". Another classification can be done, which is related with the timing of the switches.

If the switches are triggered simultaneously we can speak of "synchronous lines". Alternatively if they are triggered one after the other, we have "recurrent lines".

We can further speak of "ideal lines", if instantaneous switching and no losses are assumed, and of "real lines" if the behaviour of a real generator is considered.

2. Efficiency of the lines

The first lines were designed following Weibel (6) (ideal lumped synchronous line), and Keller (7) (ideal homogeneous recurrent line). The experimental results generally confirmed the theory (8). The fundamental characteristics of the two kind of generators can briefly be examined. Let η be the efficiency, defined as the ratio of the useful output energy at the desired frequency, to the electrostatic energy stored in the capacitors. The lumped synchronous line mentioned above and shown in Fig. 1 utilizes both the progressive and the regressive waves. When this line is made up of six sections per wavelength, in the ideal case, an efficiency of $\eta = 0.90$ is reached. Such a line produces a constant amplitude pulse. If we wish this amplitude to increase with time, the regressive wave cannot be utilized, and the efficiency is reduced to $\eta = 0.45$.

The same limitation has to be considered even if constant output is required from a line so long, that losses cannot be neglected. In fact, in order to compensate for loss, the capacitors have to be charged at different voltages, and only the progressive wave can be utilized.

Otherwise, let us consider the ideal homogeneous recurrent line (7). In its original form, as it is shown in Fig. 2, this generator consists of two parts, the "active line" which includes the switches, and the "charged delay line" which has no switches.

The electrical length of the two parts is the same. This generator produces square pulses. Since only the fundamental component of these pulses is required, the ideal theoretical efficiency can be shown to be $\eta = 0.81$.

The theory of the homogeneous line can be applied correctly only to lines whose cut-off is infinite. In practice this condition cannot be fulfilled exactly. From this evidence we conclude that the lumped lines can be produced more readily and that the recurrent lines have a better efficiency, when a long pulse of constant amplitude, or a pulse of increasing amplitude, is required.

One is naturally led to think that the advantages of the two lines can be acquired by means of some kind of "lumped recurrent line". The purpose of this paper is to show that it is possible to make these lines. A mathematical investigation has been made and one generator has been constructed. Theoretical and practical results are compared.

3. The lumped recurrent line

A recurrent line is equivalent to a series of charged sections of line, which are connected one after the other following a prescribed timing. As in the case of the homogeneous line, we consider the recurrent line to be composed of two parts. The first part is made up of a number of "active sections", which are charged to negative voltages. A delay line, charged to a positive voltage, is the second part. The structure of both parts is the same; they are made up of identical lumped elementary meshes. The active sections are connected one after the other at T_s intervals.

3.1 Wave propagation on the line

Let us admit (and the validity of this assumption will be discussed later) that a periodic impulsion is generated and that its fundamental component propagates along the line according to the expression

$$V(n, t) = \mathrm{Re}\left\{ Ve^{2\pi i \left[(t/T) - (n/\lambda)\right]} \right\} \tag{1}$$

where V(n, t) is the voltage at the time t and at the input of the n^{th} mesh, λ is the number of meshes per wavelength, and T is the period of the generated wave.

Let each active section be composed of k elementary meshes (Fig. 3). If we consider the time kT/λ which the wave takes to propagate along the section, we have

$$T = T_s + (kT/\lambda)$$

it follows:

$$T_s/T = 1 - (k/\lambda) \tag{2}$$

Let N_a be the total number of meshes of the active lines and N_d the number of meshes of the delay line.

Since it is not always possible to ensure perfect matching of the load and generator, we consider that part of the pulse will be reflected from the load. If we want a regular pulse, it is essential that the reflected signal does not reach the switches before they are closed, in order to avoid perturbation in the formation of the pulse. As a consequence, if N_a is given, N_d cannot take a lower value than the one resulting from the following equation:

$$(N_a T_s/k) = (T/\lambda)\,(2\,N_d + N_a) \tag{3}$$

and using the relation (2) we obtain

$$(N_d/N_a) = \left[\lambda/(2k)\right] - 1 \qquad (4)$$

Equations (2) and (4) permit us to calculate the timing of the switches and the length of the delay line. The consideration made above can easily be extended to the case of the homogeneous line.

The same equations are valid; N_d/N_a significantly becomes the ratio between the lengths of the delay and the active line, and k/λ is the length of a single active section expressed as a fraction of the wavelength. Putting $k/\lambda = 1/4$ we find again the conditions for the Keller line we have discussed above

$$T_s/T = 3/4 \quad ; \quad N_d/N_a = 1 \ .$$

3.2 Structure of the lumped line

In order to proceed further on, it is necessary to decide how the elementary mesh is composed. We consider a mesh made up of a series arm consisting of an inductance L in series with a resistance r, and of a shunt arm consisting of a capacitance C in series with an inductance αL and with a resistance βr. Of course the inductance in the shunt arm can simulate the self-inductivity of real capacitors and the resistances can simulate the losses.

The parameter k (number of meshes for active section) can in general take any value from 1 to ∞ . We will henceforth limit our treatment to the case k = 1. In fact, we already know the performance of the ideal homogeneous line that corresponds to the case $k = \infty$: as will be shown later, it does not appear that the consideration of intermediate values of k has any special interest, nevertheless the treatment that follows can easily be extended to such a case.

3.3 The termination of the line

We consider a load consisting of a series resonant circuit whose resistance has the value as the characteristic impedance of the line, ensuring perfect matching in the steady state condition. The line is supposed to end in a T terminating half section. Fig. 4 shows the schematic diagram of the line. The active sections are charged to negative voltages and the delay line is charged to a positive voltage through the charging resistors R_{ch}, where $R_{ch} \gg R$.

3.4 The dispersion relation

Let us now consider the ideal line, which corresponds to the diagram of fig. 4 with r = o.

Let $R_o^2 = L/C$; and $T_o^2 = (2\pi)^2 LC$

Following the classical theory of transmission lines, it is easy to obtain the dispersion relation

$$2 \sin (\pi/\lambda) = (T_o/T) \left[1 - (1/4)(T_o/T)^2\right]^{-\frac{1}{2}} \qquad (5)$$

The characteristic impedance R must satisfy the following equation:

$$(R/R_o)^2 = 1 - (1/4)(T_o/T)^2 - \alpha (T_o/T)^2 \qquad (6)$$

The ratio of the phase and the group velocity is found to be

$$\frac{U_p}{U_g} = \frac{\lambda}{2\pi} \frac{R_o}{R} \left[\frac{T_o}{T} + \frac{(T_o/T)^3}{1 - (T_o/T)^2} \right] \qquad (7)$$

Equations (5), (6), (7) together with (2) and (4), where $k = 1$, completely determine the line parameters.

The tables joined to this report show the values of T_o/T, λ, R/R_o, U_p/U_g, T_s/T and N_d/N_a for a series of values of α, which satisfy the above-mentioned equations.

3.5 The differential equations of the line

It is now necessary to test the validity of the basic assumption which was made in paragraph 3.1. We postulated that a periodic pulse whose fundamental component satisfies (1), was generated. To test this we can begin by writing down the differential equations of the line. The symbols generally agree to those shown in Fig. 4, one has to note however that for the sake of generality the cases of lines with non identical meshes are included. The number of equations is twice the number of meshes. The equations of the mesh m are

$$C_m \frac{dV_m}{dt} + \varepsilon(t - t_m)i_m - \varepsilon(t - t_{m+1})i_{m+1} = o \qquad (9)$$

$$\begin{aligned} &\varepsilon(t - t_m)\left\{V_m - i_m\left[(1+\beta_m)r_m + \beta_{m-1}\, r_{m-1}\right] - \frac{di_m}{dt}\left[(1+\alpha_m)L_m + \alpha_{m-1}\, L_{m-1}\right]\right\} + \\ &+ \varepsilon(t - t_{m+1})\left\{\beta_m\, r_m\, i_{m+1} + \alpha_m L_m \frac{di_{m+1}}{dt}\right\} + \\ &+ \varepsilon(t - t_{m-1})\left\{r_{m-1}\, i_{m-1} + \alpha_{m-1}\, L_{m-1} \frac{di_{m-1}}{dt}\right\} = o \qquad (10) \end{aligned}$$

where $\varepsilon(x)$ is the unit step function. The initial conditions are determined by the charging voltages. We note that the validity of the system is quite general. For example if we make all the times of switching equal and properly choose the initial charging voltage, we can deal with the case of the synchronous lines. The solution of the system with ordinary transient analysis by means of Laplace transform is convenient only when the line is made of very few sections. When more complex structures are involved, a direct numerical solution is suitable. From a mathematical point of view the problem is characterized by the fact that the number of equations increases with time. This corresponds to the physical fact that the number of sections constituting the entire network increases by one unity each time a spark gap is triggered. A program of integration especially suited to this problem has been written by F. Troyon for the IBM 7040 computer of the "Ecole Polytechnique de l'Université de Lausanne", and a number of line generators have been analysed. A few of the more representative solutions are reported here. We begin to examine the simplest cases.

3.6 Numerical solutions

By reference to Fig. 4, we assume that the N_a active sections are charged to the same negative voltage V_a, and that the N_d delay sections are charged to a positive voltage V_d. L, C, r, α and β are the same in each section of the line and are supposed to be given.

In order to achieve the best efficiency, a balance of charges is required so that at the end of the operation the network is completely discharged. We have

$$(N_d/N_a) = -(V_a/V_d) \qquad (11)$$

In order to do the computation we proceed as follows. The quantities $R_o = (L/C)^{1/2}$ and $T_o = 2\pi(L/C)^{1/2}$ are calculated. A value of λ is assumed. Then, if T is the period of the oscillation required, R/R_o, T_s/T_o and N_d/N_a are found in the tables. The switching time of the delay sections is zero. The times of the active sections are chosen according to : $t_{m+1} - t_m = T_s$.

The charging voltages must satisfy (11). The elements of the output resonant circuit are calculated, for a given Q factor, according to the values of R and T. Then the system of equations (9), (10) is solved.

The performance of the line is hence analysed by considering quantities such as the voltage across the load resistor and the efficiency. In addition the quantity Δ^{-2} has been calculated. Δ is defined by

$$\Delta = (V_{max} - V_{min}) / (V_d - V_a) \qquad (12)$$

where $(V_{max} - V_{min})$ is the greatest voltage difference which develops across the capacitors of the line. The significance of Δ^{-2} becomes evident if we consider two lines with different Δ and made up of the same capacitors. To satisfy (12) the charging voltages of the two lines must be different, and consequently the output power of each line is proportional to Δ^{-2}.

Lines without losses and series inductivity are first examined. Also the following values have been chosen: $Q = 6$ and $N_a + N_d = 38.5$ (38 sections and a half terminating section). Substantially sinusoidal waves have been obtained, whose frequency 1/T agrees to the required value. Results of the calculation, with four values of λ are shown in Fig. 5. It appears that higher values of λ, corresponding to lower dispersion, allows better performances. In fact Δ^{-2} increases and η approaches unity at the end of the pulse.

3.7 Variations of optimum design

It is worth while to check what happens if we design some parameters which are not in agreement with the given optimum design rules. With reference to the example given above, we resume our investigations.

I) Variations of the load

Consider the resistance of the output circuit (Fig. 4). Variations of R by $\pm$ 50 % results in power loss of 4 %.

II) Variations of the charging voltages

The best performances are achieved when the charging voltages satisfy condition (11). We refer to the example (a) of Fig. 5. If we give to V_a/V_d the value 1 instead of the correct value (23.5/15), the useful power goes down to 5 %.

III) Variations of N_d/N_a

Still referring to the above mentioned example, the optimum ratio between the number of sections of the delay and of the active line, following (4) is $N_d/N_a = 1.5641$. Four examples of variations in such a ratio are shown in Fig. 6. As before $N_a + N_d$ has been kept constant.

It is worthy of note that the number of cycles approaches N_a when the value of N_d/N_a approaches the correct one.

If N_d/N_a increases, this number approaches $N_a + 1$. If N_d/N_a decreases, it becomes lower than N_a.

3.8 Lines with inductivity in the shunt arms

Hitherto lines have been considered whose shunt arms were made up of pure capacity, namely $\alpha = 0$.

The case of $\alpha = 0,3$ has been calculated corresponding to three values of λ and is shown in Fig. 7. We note that the shape of the wave tends to become irregular, the factor Λ^{-2} decreases and the dispersion increases. All this agrees to the fact that, when α increases, the cutoff frequency of the line decreases, as the equation (8) shows.

3.9 Rise time of the oscillation

The rise time is a function both of the Q value of the resonant output circuit and the value of the ratio U_p/U_g.

In the examples hitherto considered, we assumed Q = 6. The rise time increases when U_p/U_g, namely the dispersion, increases. This statement agrees with our knowledge of the laws of energy propagation in a dispersive system.

4. Effect of losses

Even if high losses occur in the generator, it is possible to a certain degree to obtain oscillations whose amplitude is constant or increasing with time, if the charging voltages of the capacitors of the active line are properly chosen.

High loss lines are interesting because they can be made up of low price capacitors.

One has to note that this solution is suitable only in a limited frequency range and for short pulses. Outside these limits the efficiency falls very quickly. An example has been reported in Fig. 8. A line made up of better quality capacitors will be considered later.

5. The cut-off frequency

We have hitherto considered lines made up of one mesh every active section, namely k = 1.

The examples we have examined show that these lines can reach high efficiency, sometimes higher than the efficiency of the homogeneous line, partly because the production of harmonics is strongly limited. For verification, it is easy to derive the following relation for the cut-off frequency $1/T_c$:

$$T_c = T_o \left(\alpha + \frac{1}{4} \right)^{1/2}$$

It is clear that the configuration with $k > 1$ will increase the cut-off frequency and will improve the harmonic content.

In addition to the presumed diminution of the efficiency, an increased harmonic content can have some other indesirable effects in plasma physics applications. For these reasons, it does not seem to be of any particular interest to pursue the analysis of the case $k > 1$ any further.

6. An experimental line generator

The method described in these pages has been used to design the rf power installation, now in construction at Lausanne, for research in plasma physics. A total power of 720 MW is required, to be obtained from 24 lumped recurrent lines, each of them producing 30 MW. The configuration with one mesh ($k = 1$) has been chosen (Fig. 4), the C consisting of a stacks of 3 nF ceramic capacitors and the L consisting of a copper connecting rod, whose length can be varied. Pressurized spark gaps have been used as switches. The gaps are triggered by means of a special synchronizing device, which ensures correct timing and which has been designed to trigger the 24 lines. This "synchronizer" has already been described (9). The mechanical assembly of the line is shown in fig. 9. Its performances have been computed at the lower and the higher frequency of operation (2 and 3 MHz). In the computation the following values have been assumed:

a) operation at 2 MHz

$L = 0{,}702\ \mu H$; $\alpha = 0{,}05$

$\lambda = 4{,}9297$; $T_a = 0{,}398\ \mu sec$; $N_a = 8$; $N_d = 11{,}5$.

b) operation at 3 MHz

$L = 0{,}367\ \mu H$; $\alpha = 0{,}05$

$\lambda = 4{,}4321$; $T_a = 0{,}258\ \mu sec$; $N_a = 9$; $N_d = 10{,}5$

for both frequencies

$C = 0{,}012\ \mu F$

Q of the output resonant circuit = 3

Active line: $r = 0{,}085\ \Omega$, $\beta = 0{,}17$

Delay line : $r = 0{,}015\ \Omega$, $\beta = 1$

6.1 Measurements

Before comparing the calculated results with the measured values, it is necessary to consider the approximations involved in applying the proposed method.

a) The real network is more complex than the simplified one shown in Fig. 4. The error increases with the frequency.

b) To simulate the sparkgaps losses we have considered an "equivalent" resistor, chosen on the basis of previous measurements, and of energetic equivalence.

c) In writing equations (9) and (10) it is assumed that the switches are ideal. In reality the commutation is not instantaneous but requires a finite time. Furthermore a certain amount of jitter is unavoidable. The error introduced increases with frequency.

One can compare the computed and measured values by looking at Fig. 10.

In practice it has been possible to improve the regularity of the oscillation by means of a careful experimental adjustment. The adjustment needed is found by a trial-and-error method varying:

a) time of gaps triggering

b) L and C of some sections

c) charging voltage of some active sections.

Figure 11 shows the oscillograph trace of the 3 MHz pulse which has been obtained after the adjustment of the generator.

The superposition of many oscillograms proves the reproducibility of the waveform.

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Figure Captions

Fig. 1 The Weibel lumped synchronous line

The capacitors are charged to positive and negative voltages and discharged altogether. One progressive and one regressive wave are generated. The regressive wave is reflected at the short circuit. The two waves give rise to the output pulse.

Fig. 2 The Keller homogeneous recurrent line

It produces square waves of period T. At time $t = o$ the first spark gap is switched. The wavefront reaches the second spark gap at the time $t = (1/4)\ T$. This wave front reflects at the second spark gap which remains open until the time $t = (3/4)\ T$. All the other gaps are successively triggered at intervals of $(3/4)\ T$.

Fig. 3 The recurrent line in its general form

k = number of meshes in each active section

N_a = total number of meshes in all active sections

N_d = number of meshes in the delay line

$t_o = o$; $t_1 = T_s$; $t_2 = 2T_s$ switching times

Fig. 4 Schematic diagram of the lumped line

The switch S_m is supposed to be closed at time t_m.

Fig. 5 Calculated amplitude V_z across the load and efficiency as a function of time for the ideal, $\alpha = o$, $k = 1$, lumped recurrent lines. Normalized units V_z/V_d and t/T are used, where V_d is the charging voltage of the delay line and T the calculated period of the oscillation. The characteristic parameters of the lines are:

	λ	N_a	N_d	Δ^{-2}	U_p/U_g
a)	5.1283	15	23,5	0,59	1,1461
b)	4.0516	19	19,5	0,53	1,2629
c)	3.3124	23	15,5	0,46	1,4681
d)	2.8056	27	11,5	0,40	1,8421

Fig. 6 Number of sections of the delay and of the active line $V_a/V_d = - N_d/N_a$. $\lambda = 5.1283$, and $U_p/U_g = 1.1461$

	N_a	N_d	N_d/N_a	Δ^{-2}
e)	11	27,5	2,5	0,59
f)	14	24,5	1,75	0,62
g)	16	22,5	1,41	0,62
h)	19	19,5	1,025	0,62

Fig. 7 Lines with α = 0.3

The other parameters have been designed according to the given rules. $N_a + N_d = 38.5$

	λ	N_a	N_d	Δ^{-2}	U_p/U_g
i)	5.7794	13	25.5	0.51	1.4672
j)	5.1119	15	23	0.48	1.6049
k)	4.3184	18	20.5	0.40	1.8716

Fig. 8 Line with losses

$N_a = 6$; $N_d = 7$; $\alpha = 0.1$

$\lambda = 3.9575$; $U_p/U_g = 1.5402$; $\Delta^{-2} = 0.87$

Charging voltage of the delay line $-V_d$.
The sections of the active line are charged to different voltages in order to compensate losses.
V_a/V_d of the six section are respectively

0.4 ; 0.66 ; 0.95 ; 1.30 ; 1.72 ; 1.72

Q of the active line meshes = 9
Q of the delay line meshes = 11
Q of the output resonant circuit = 6

Fig. 9 Construction of the line

(1) spark gaps , (2) copper rod, (3) ceramic disc capacitors

Fig. 10 Comparison of the calculated and measured values of voltages across the R load resistor.
Charging voltages $\pm$ 36 k V

Fig. 11 10 superimposed oscillograms of the voltage across the 4.83 load resistor after final adjustment.
Vertical scale 20 kV/div, horizontal scale 0.5 μsec/div.

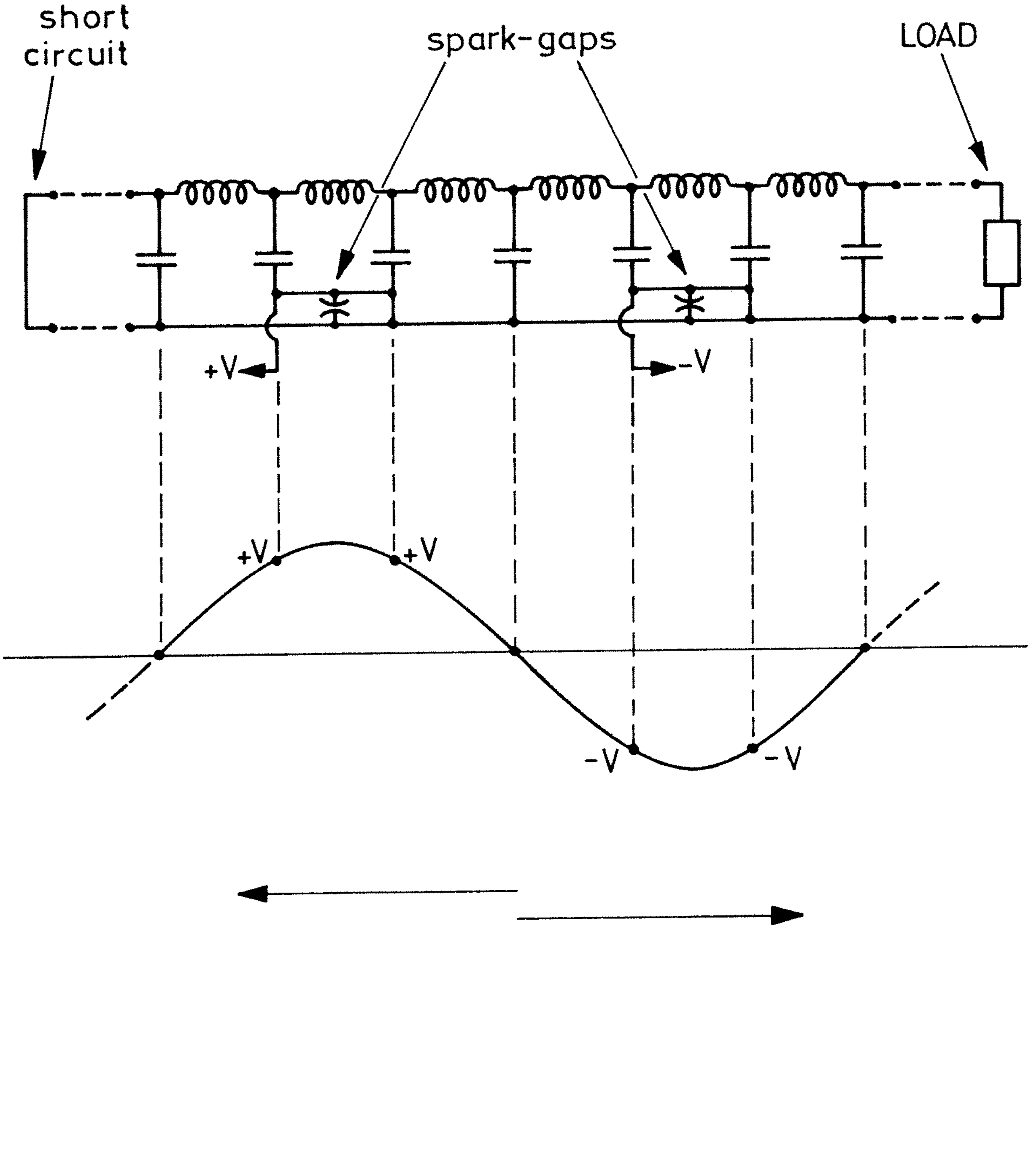


Fig. 1

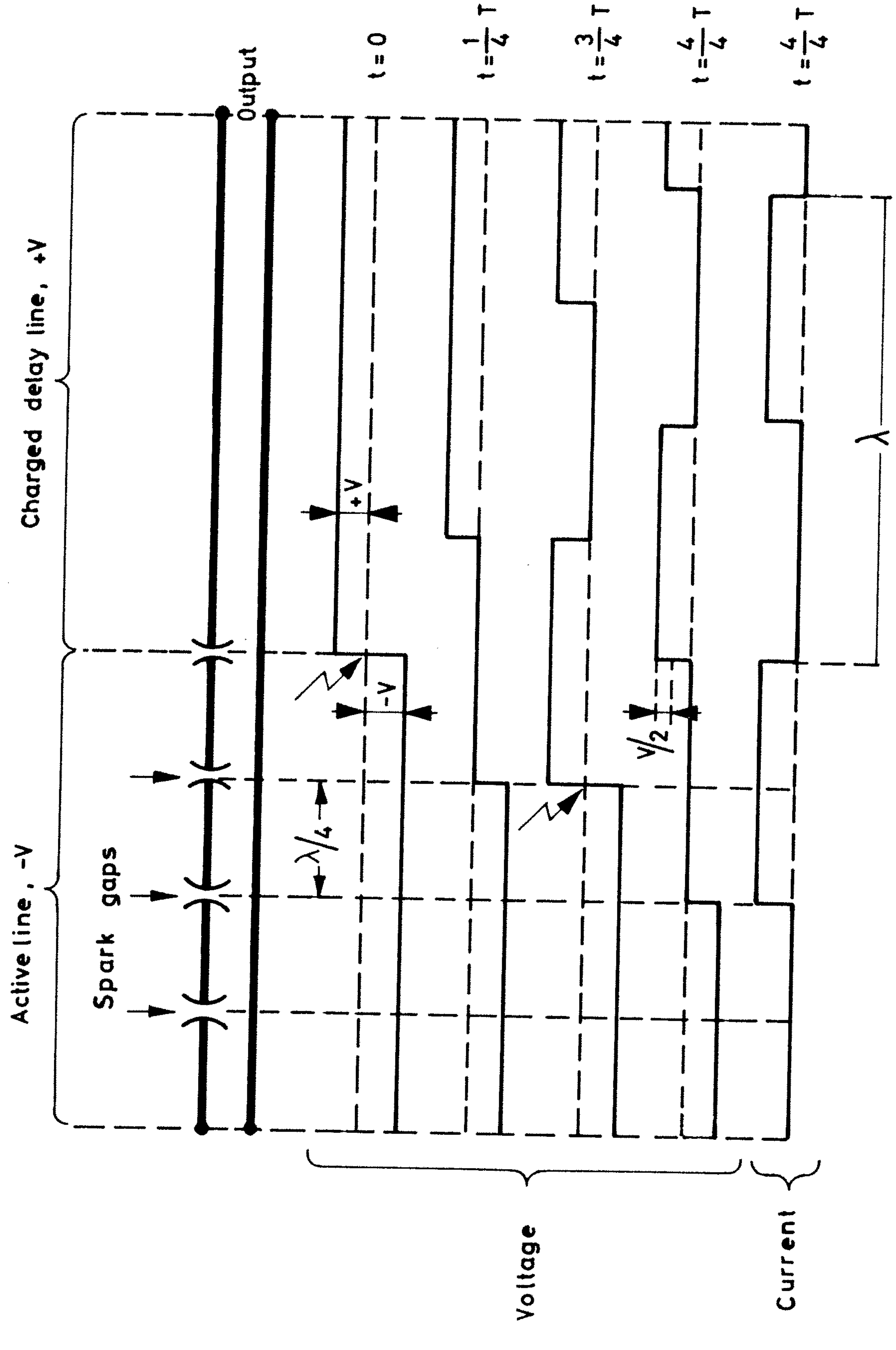
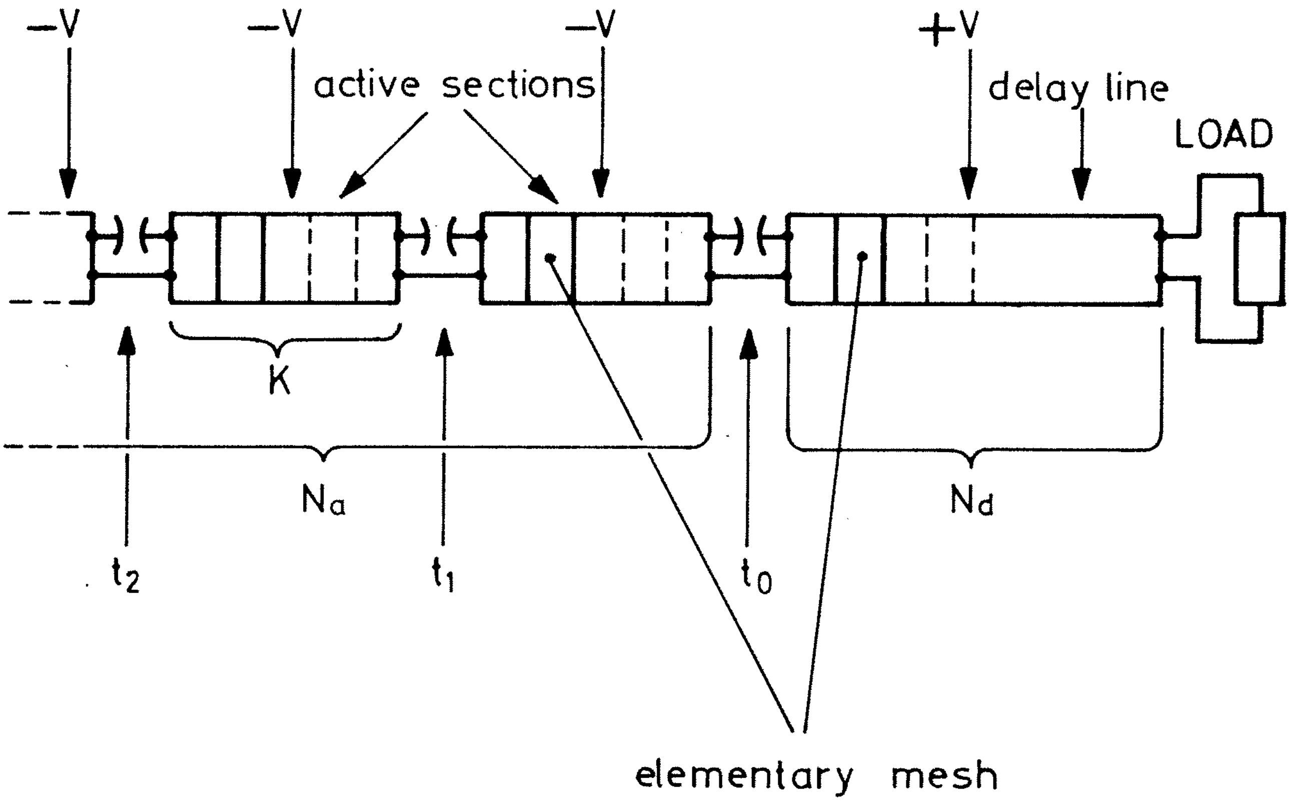

Active line, -V
Charged delay line, +V
Spark gaps
Output
t = 0
t = 1/4 T
t = 3/4 T
t = 4/4 T
t = 4/4 T
+V
-V
λ/4
V/2
λ
Voltage
Current

Fig. 2



−V
−V
−V
+V
active sections
delay line
LOAD
K
N_a
N_d
t_2
t_1
t_0
elementary mesh

Fig. 3

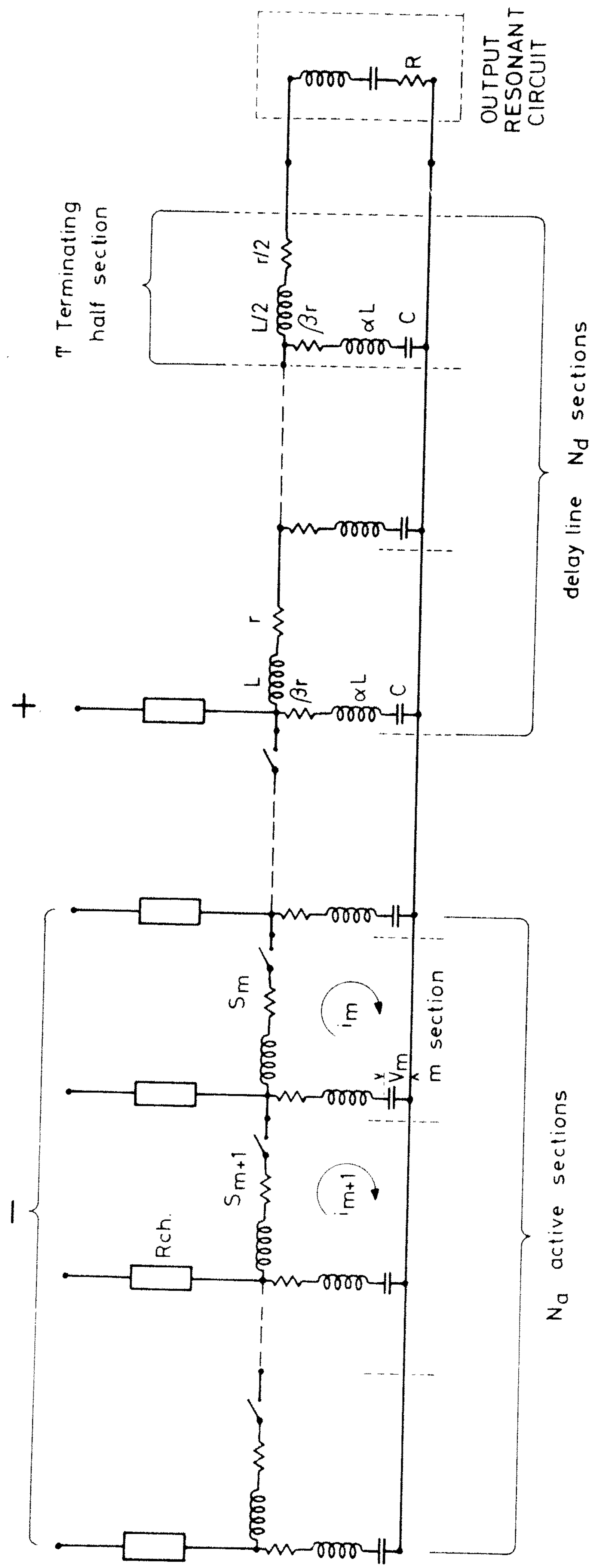
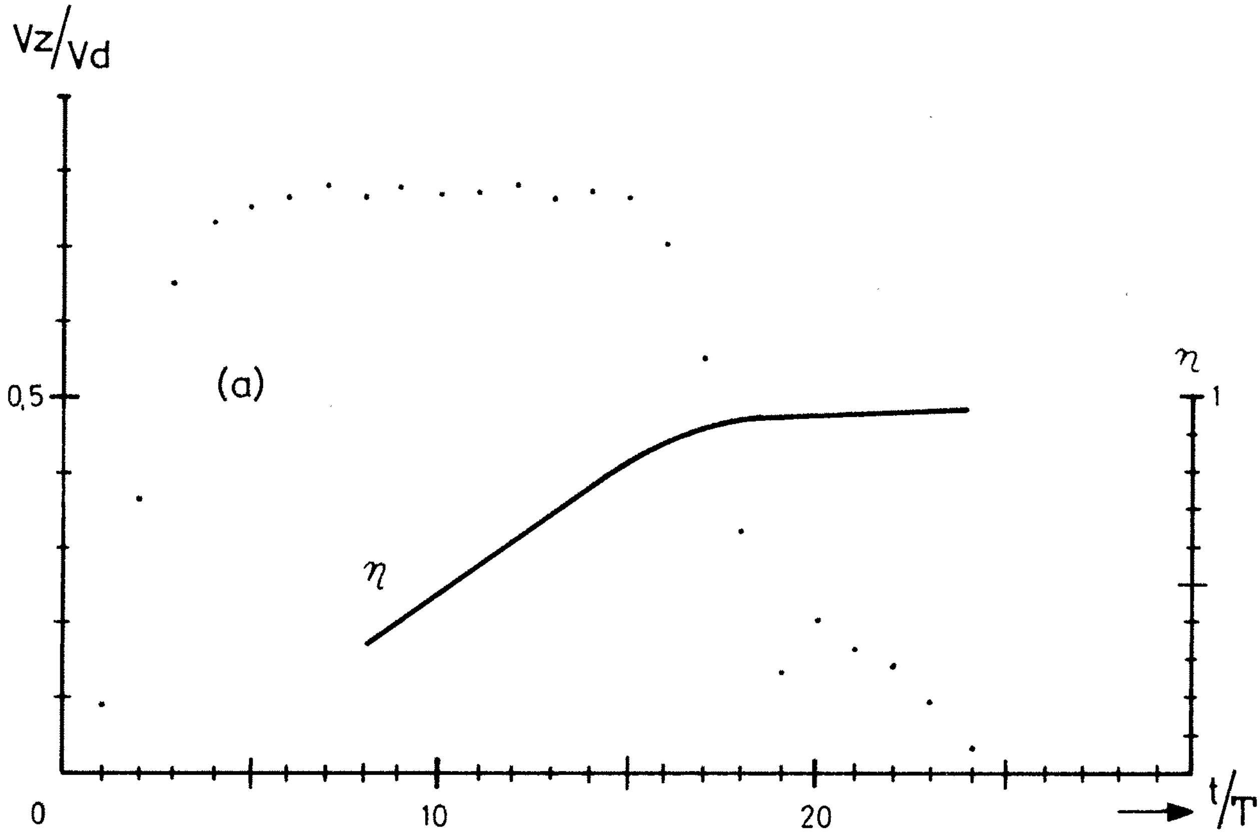

OUTPUT RESONANT CIRCUIT
R
T Terminating half section
r/2
L/2
βr
αL
C
delay line Nd sections
+
L
r
βr
αL
C
−
Rch.
Sm
Sm+1
im
im+1
Vm
m section
Na active sections

Fig. 4



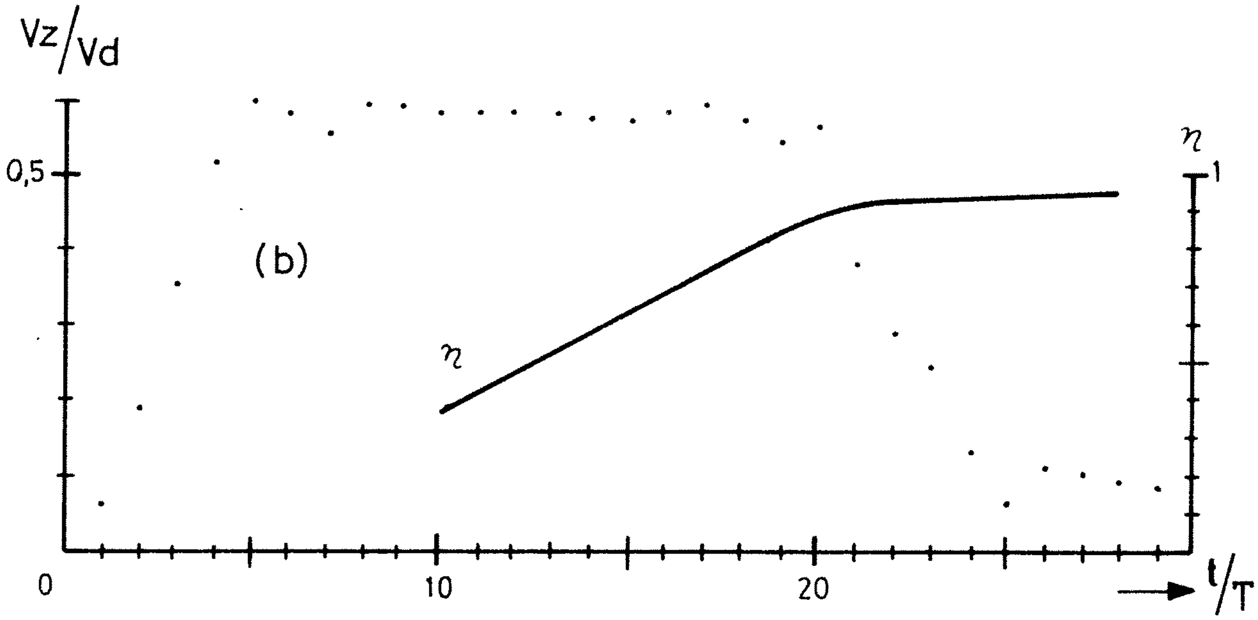


Fig. 5/ab

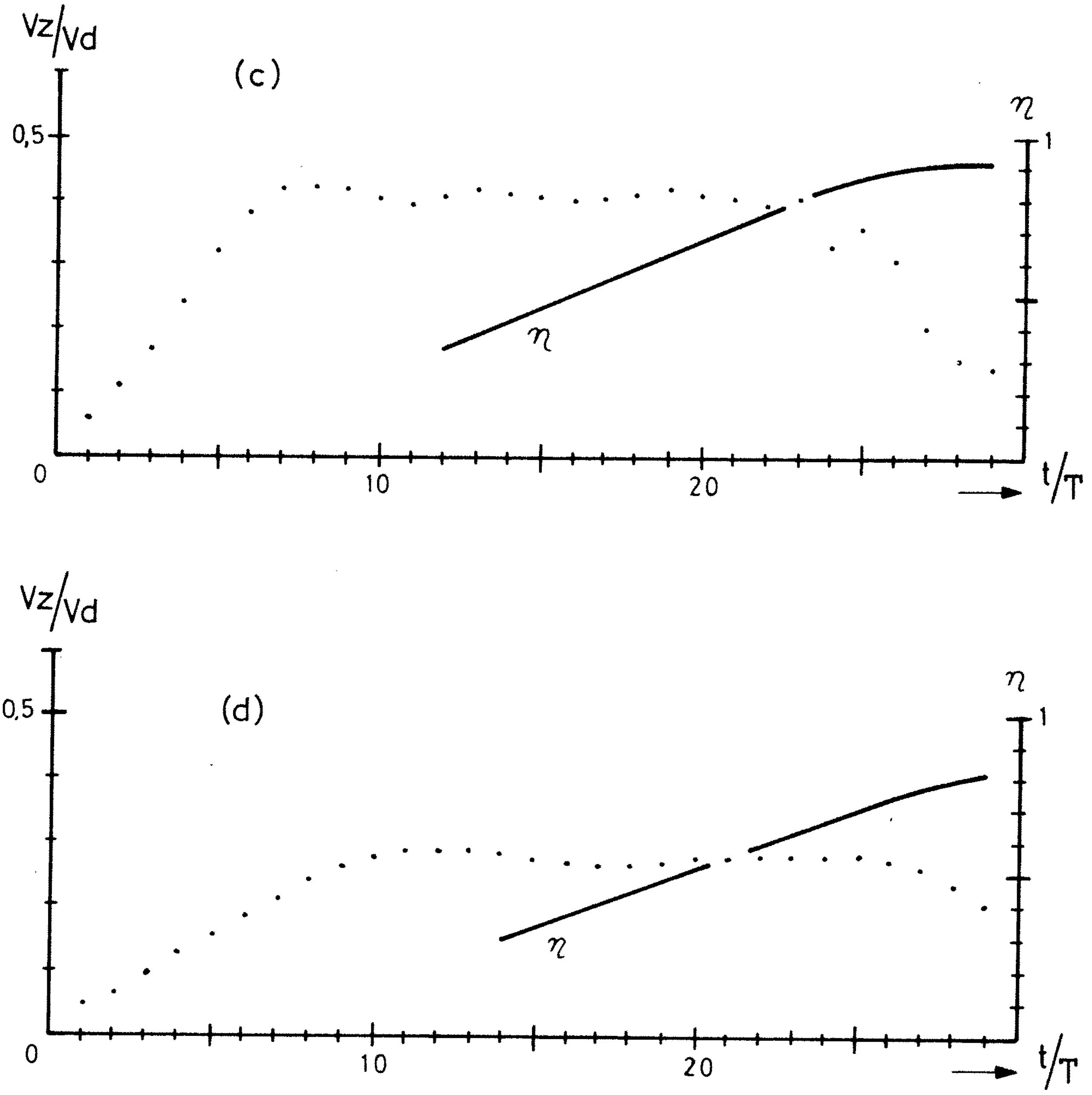
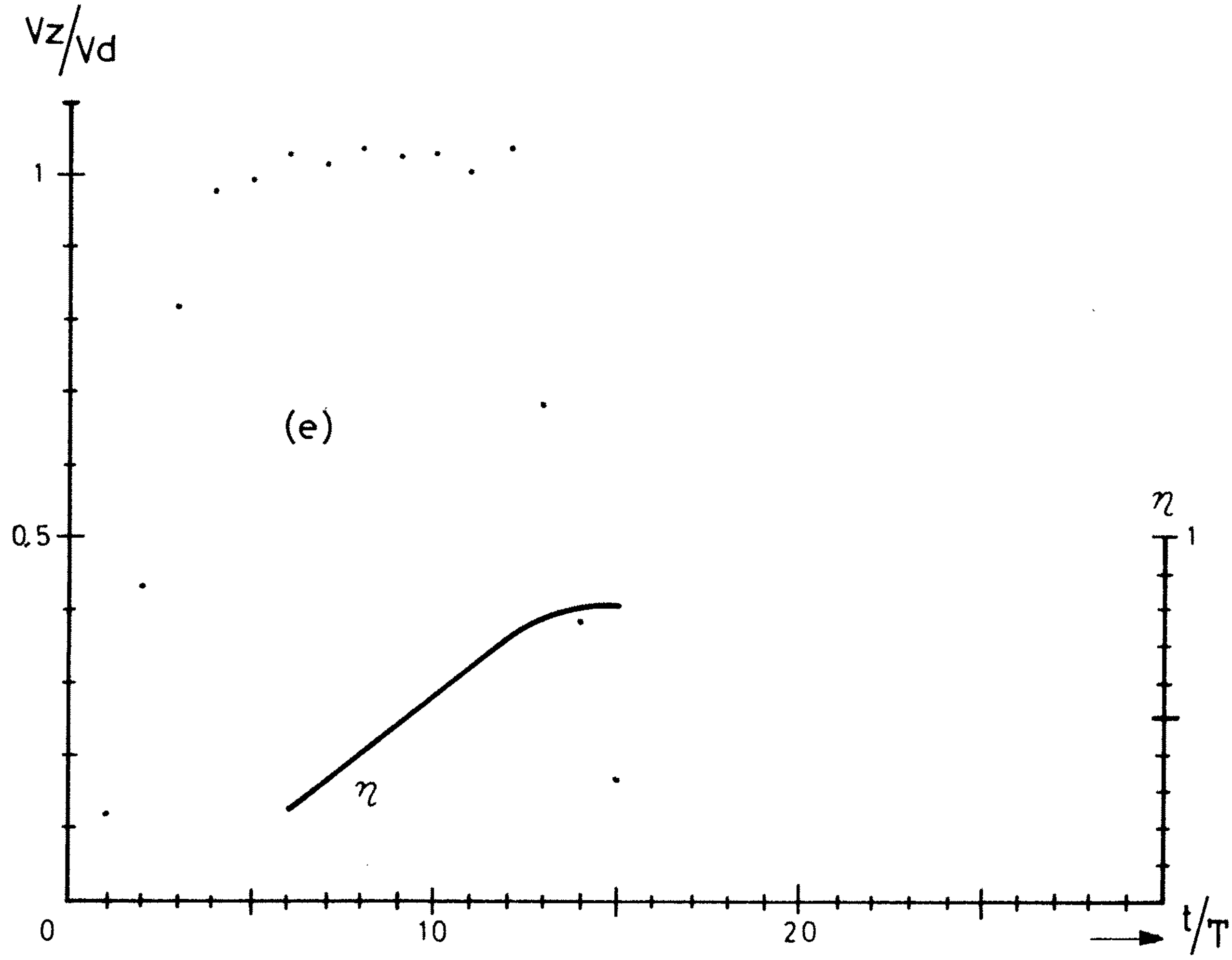


Fig. 5/cd



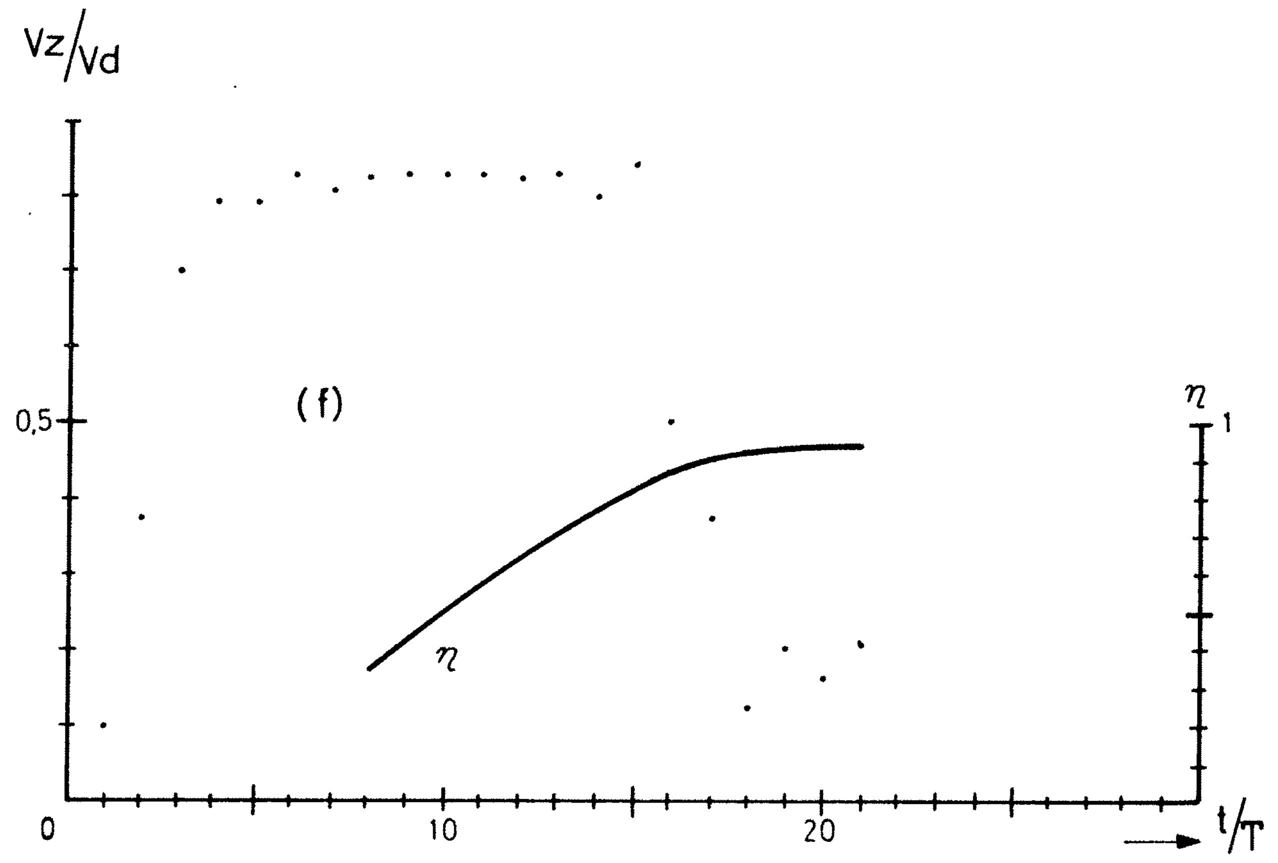
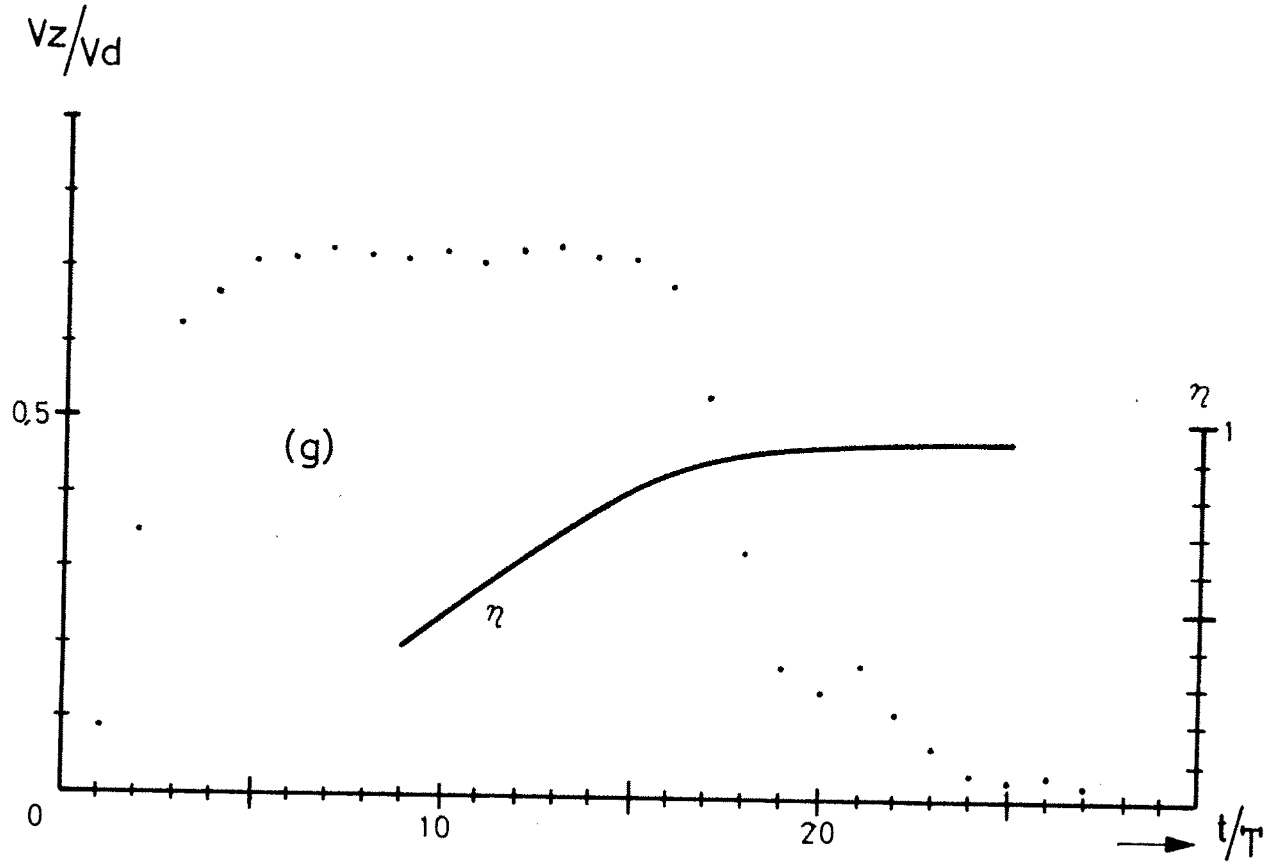


Fig. 6/ef



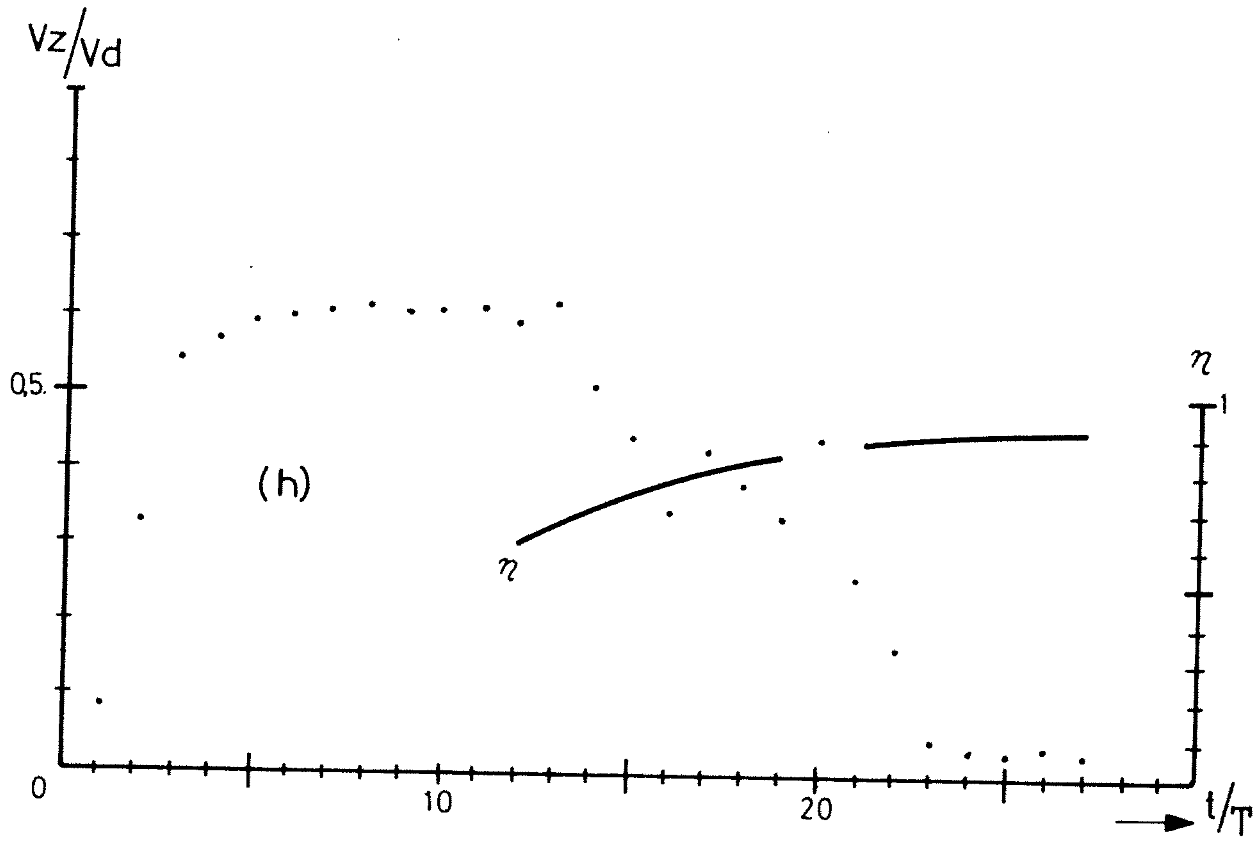


Fig. 6/gh

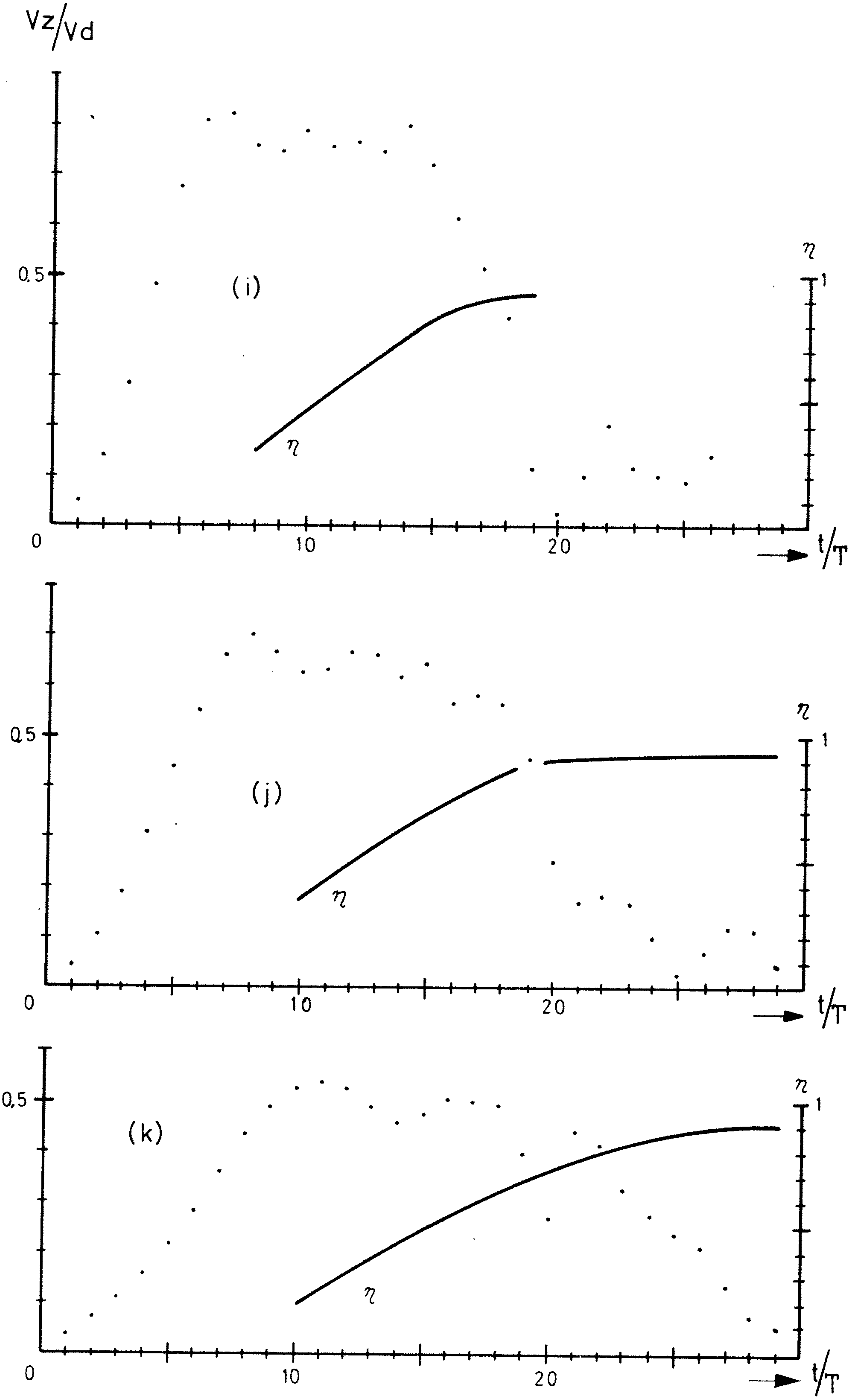


Fig 7/ i j k

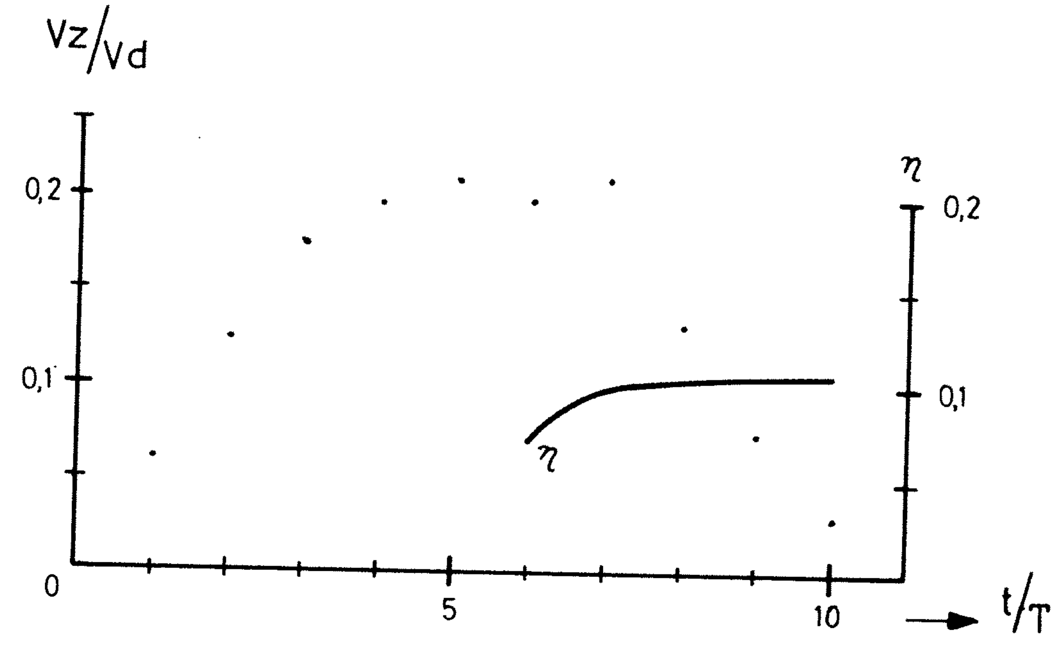


Fig. 8

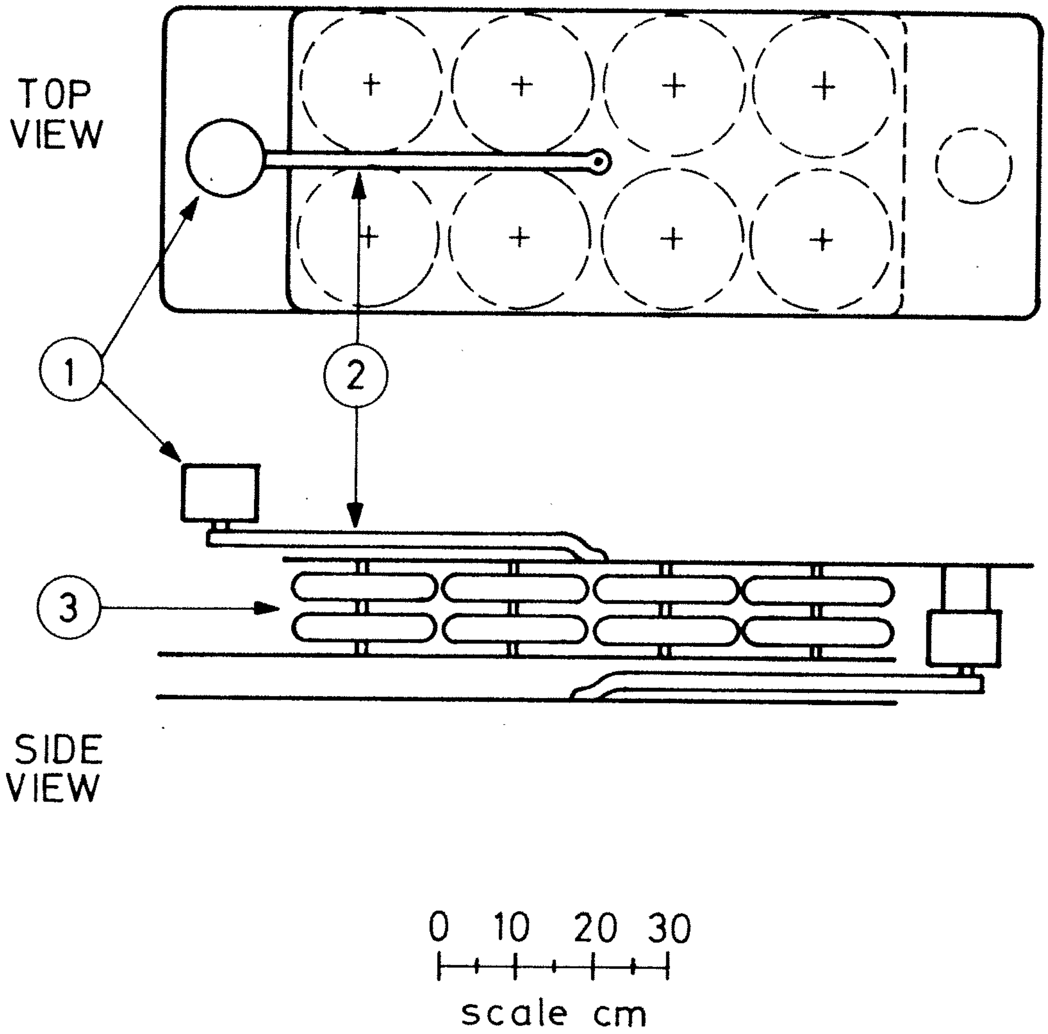
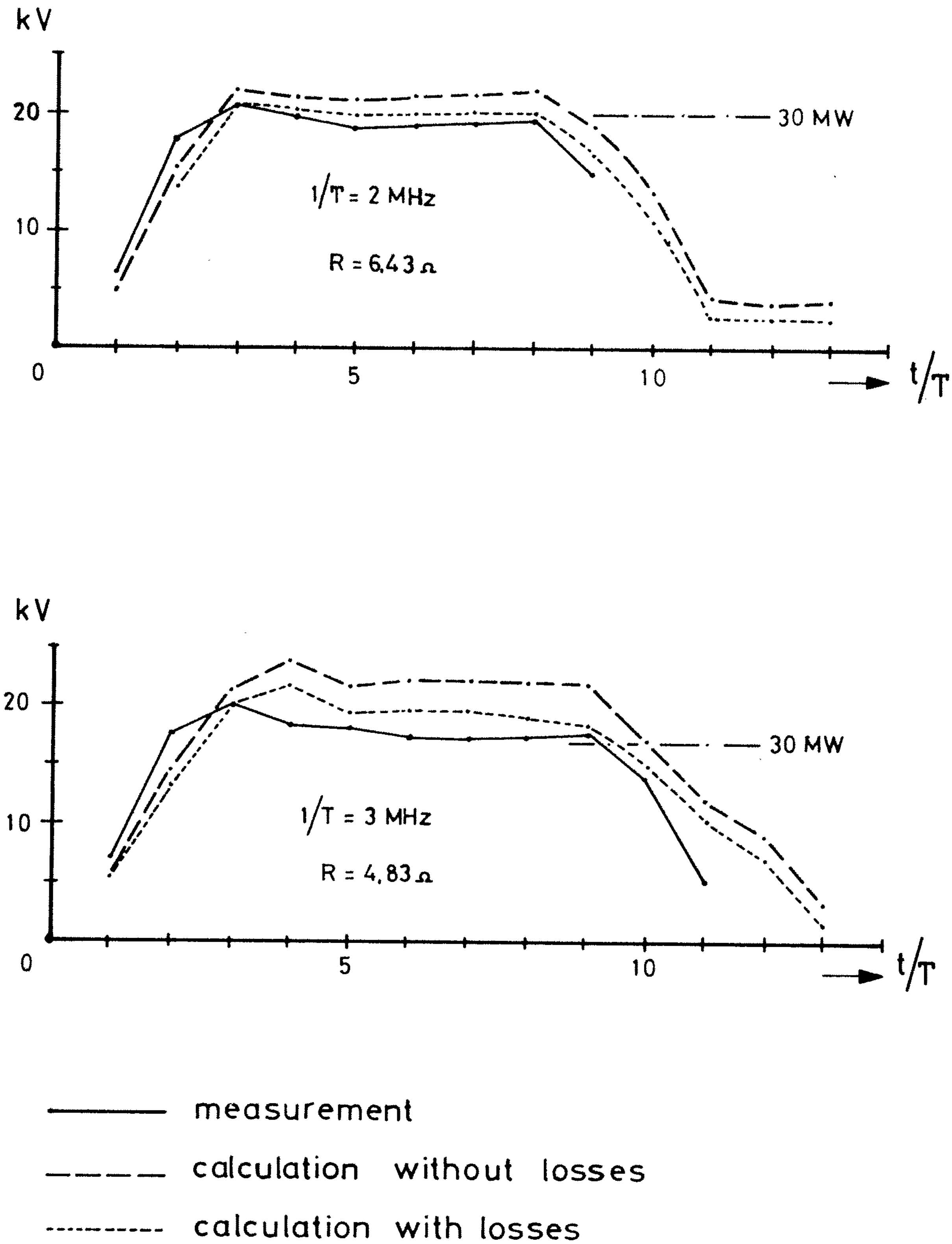


Fig. 9



kV
20
10
0
5
10
t/T
30 MW
1/T = 2 MHz
R = 6,43 Ω
kV
20
10
0
5
10
t/T
30 MW
1/T = 3 MHz
R = 4,83 Ω
measurement
calculation without losses
calculation with losses

Fig. 10

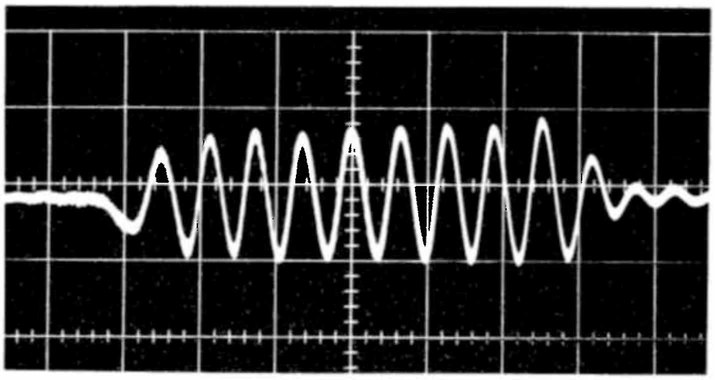


Fig. 11

$\alpha = 0$

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
0.7500	8.1728	0.9270	1.0513	0.8776	3.0864
0.7750	7.8951	0.9219	1.0553	0.8733	2.9475
0.8000	7.6342	0.9165	1.0595	0.8690	2.8171
0.8250	7.3886	0.9110	1.0639	0.8647	2.6943
0.8500	7.1569	0.9052	1.0686	0.8603	2.5784
0.8750	6.9379	0.8992	1.0734	0.8559	2.4689
0.9000	6.7306	0.8930	1.0785	0.8514	2.3653
0.9250	6.5339	0.8866	1.0839	0.8470	2.2670
0.9500	6.3471	0.8800	1.0895	0.8424	2.1736
0.9750	6.1694	0.8731	1.0954	0.8379	2.0847
1.0000	6.0000	0.8660	1.1016	0.8333	2.0000
1.0250	5.8384	0.8587	1.1081	0.8287	1.9192
1.0500	5.6839	0.8511	1.1149	0.8241	1.8420
1.0750	5.5361	0.8433	1.1221	0.8194	1.7681
1.1000	5.3945	0.8352	1.1297	0.8146	1.6973
1.1250	5.2587	0.8268	1.1377	0.8098	1.6294
1.1500	5.1283	0.8182	1.1461	0.8050	1.5641
1.1750	5.0028	0.8092	1.1550	0.8001	1.5014
1.2000	4.8820	0.8000	1.1644	0.7952	1.4410
1.2250	4.7656	0.7905	1.1743	0.7902	1.3828
1.2500	4.6533	0.7806	1.1847	0.7851	1.3267
1.2750	4.5448	0.7705	1.1959	0.7800	1.2724
1.3000	4.4399	0.7599	1.2076	0.7748	1.2199
1.3250	4.3383	0.7491	1.2202	0.7695	1.1692
1.3500	4.2399	0.7378	1.2335	0.7641	1.1199
1.3750	4.1444	0.7262	1.2477	0.7587	1.0722
1.4000	4.0516	0.7141	1.2629	0.7532	1.0258
1.4250	3.9614	0.7017	1.2792	0.7476	0.9807
1.4500	3.8736	0.6887	1.2966	0.7418	0.9368
1.4750	3.7880	0.6753	1.3154	0.7360	0.8940
1.5000	3.7044	0.6614	1.3357	0.7301	0.8522
1.5250	3.6228	0.6470	1.3577	0.7240	0.8114
1.5500	3.5430	0.6320	1.3817	0.7177	0.7715
1.5750	3.4647	0.6163	1.4078	0.7114	0.7324
1.6000	3.3879	0.6000	1.4365	0.7048	0.6940
1.6250	3.3124	0.5830	1.4681	0.6981	0.6562
1.6500	3.2381	0.5651	1.5032	0.6912	0.6190
1.6750	3.1647	0.5464	1.5424	0.6840	0.5824
1.7000	3.0922	0.5268	1.5866	0.6766	0.5461
1.7250	3.0202	0.5061	1.6369	0.6689	0.5101
1.7500	2.9486	0.4841	1.6947	0.6609	0.4743
1.7750	2.8772	0.4608	1.7622	0.6524	0.4386
1.8000	2.8056	0.4359	1.8421	0.6436	0.4028

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
		$\alpha = 0.05$			
0.8500	7.0172	0.8850	1.1118	0.8575	2.5086
0.8750	6.7937	0.8777	1.1198	0.8528	2.3968
0.9000	6.5817	0.8701	1.1282	0.8481	2.2909
0.9250	6.3804	0.8622	1.1371	0.8433	2.1902
0.9500	6.1889	0.8540	1.1464	0.8384	2.0945
0.9750	6.0064	0.8455	1.1563	0.8335	2.0032
1.0000	5.8322	0.8367	1.1667	0.8285	1.9161
1.0250	5.6657	0.8275	1.1777	0.8235	1.8328
1.0500	5.5062	0.8181	1.1892	0.8184	1.7531
1.0750	5.3534	0.8083	1.2015	0.8132	1.6767
1.1000	5.2066	0.7981	1.2144	0.8079	1.6033
1.1250	5.0655	0.7876	1.2282	0.8026	1.5328
1.1500	4.9297	0.7767	1.2427	0.7971	1.4649
1.1750	4.7988	0.7654	1.2582	0.7916	1.3994
1.2000	4.6724	0.7537	1.2747	0.7860	1.3362
1.2250	4.5503	0.7415	1.2922	0.7802	1.2751
1.2500	4.4321	0.7289	1.3110	0.7744	1.2160
1.2750	4.3176	0.7158	1.3311	0.7684	1.1588
1.3000	4.2064	0.7021	1.3526	0.7623	1.1032
1.3250	4.0984	0.6880	1.3758	0.7560	1.0492
1.3500	3.9934	0.6732	1.4009	0.7496	0.9967
1.3750	3.8910	0.6579	1.4280	0.7430	0.9455
1.4000	3.7910	0.6419	1.4576	0.7362	0.8955
1.4250	3.6934	0.6252	1.4899	0.7292	0.8467
1.4500	3.5977	0.6077	1.5253	0.7220	0.7989
1.4750	3.5039	0.5893	1.5646	0.7146	0.7519
1.5000	3.4116	0.5701	1.6082	0.7069	0.7058
		$\alpha = 0.1$			
0.8750	6.6463	0.8556	1.1703	0.8495	2.3232
0.9000	6.4294	0.8465	1.1827	0.8445	2.2147
0.9250	6.2231	0.8370	1.1959	0.8393	2.1116
0.9500	6.0265	0.8271	1.2098	0.8341	2.0133
0.9750	5.8388	0.8169	1.2245	0.8287	1.9194
1.0000	5.6593	0.8062	1.2401	0.8233	1.8296
1.0250	5.4873	0.7952	1.2567	0.8178	1.7437
1.0500	5.3224	0.7837	1.2744	0.8121	1.6612
1.0750	5.1639	0.7717	1.2932	0.8063	1.5819
1.1000	5.0113	0.7593	1.3133	0.8005	1.5057
1.1250	4.8643	0.7463	1.3347	0.7944	1.4321
1.1500	4.7224	0.7329	1.3578	0.7882	1.3612
1.1750	4.5851	0.7189	1.3825	0.7819	1.2926
1.2000	4.4522	0.7043	1.4091	0.7754	1.2261
1.2250	4.3234	0.6890	1.4379	0.7687	1.1617
1.2500	4.1982	0.6731	1.4691	0.7618	1.0991
1.2750	4.0763	0.6565	1.5030	0.7547	1.0382
1.3000	3.9575	0.6391	1.5402	0.7473	0.9788
1.3250	3.8416	0.6209	1.5810	0.7397	0.9208
1.3500	3.7280	0.6018	1.6262	0.7318	0.8640
1.3750	3.6167	0.5816	1.6764	0.7235	0.8084
1.4000	3.5072	0.5604	1.7329	0.7149	0.7536
1.4250	3.3993	0.5378	1.7969	0.7058	0.6997
1.4500	3.2926	0.5139	1.8703	0.6963	0.6463
1.4750	3.1866	0.4884	1.9557	0.6862	0.5933
1.5000	3.0809	0.4610	2.0568	0.6754	0.5405

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
		$\alpha = 0.15$			
0.7500	7.8014	0.8803	1.1542	0.8718	2.9007
0.7750	7.5099	0.8716	1.1668	0.8668	2.7550
0.8000	7.2350	0.8626	1.1802	0.8618	2.6175
0.8250	6.9752	0.8531	1.1945	0.8566	2.4876
0.8500	6.7291	0.8432	1.2096	0.8514	2.3646
0.8750	6.4955	0.8329	1.2257	0.8460	2.2478
0.9000	6.2734	0.8222	1.2429	0.8406	2.1367
0.9250	6.0617	0.8110	1.2611	0.8350	2.0308
0.9500	5.8595	0.7994	1.2806	0.8293	1.9297
0.9750	5.6661	0.7872	1.3013	0.8235	1.8330
1.0000	5.4808	0.7746	1.3236	0.8175	1.7404
1.0250	5.3028	0.7614	1.3474	0.8114	1.6514
1.0500	5.1317	0.7477	1.3729	0.8051	1.5658
1.0750	4.9668	0.7333	1.4005	0.7987	1.4834
1.1000	4.8077	0.7183	1.4302	0.7920	1.4038
1.1250	4.6538	0.7027	1.4623	0.7851	1.3269
1.1500	4.5048	0.6863	1.4972	0.7780	1.2524
1.1750	4.3602	0.6691	1.5353	0.7707	1.1801
1.2000	4.2195	0.6512	1.5770	0.7630	1.1098
1.2250	4.0825	0.6323	1.6230	0.7551	1.0412
1.2500	3.9487	0.6124	1.6739	0.7468	0.9743
1.2750	3.8177	0.5914	1.7307	0.7381	0.9089
1.3000	3.6892	0.5692	1.7946	0.7289	0.8446
1.3250	3.5627	0.5457	1.8672	0.7193	0.7813
1.3500	3.4378	0.5206	1.9508	0.7091	0.7189
1.3750	3.3139	0.4937	2.0484	0.6982	0.6569
1.4000	3.1905	0.4648	2.1644	0.6866	0.5952
		$\alpha = 0.20$			
0.7500	7.6737	0.8642	1.1931	0.8697	2.8368
0.7750	7.3770	0.8542	1.2094	0.8644	2.6885
0.8000	7.0969	0.8438	1.2269	0.8591	2.5485
0.8250	6.8318	0.8329	1.2455	0.8536	2.4159
0.8500	6.5803	0.8215	1.2654	0.8480	2.2901
0.8750	6.3411	0.8096	1.2867	0.8423	2.1706
0.9000	6.1133	0.7972	1.3095	0.8364	2.0566
0.9250	5.8956	0.7842	1.3340	0.8304	1.9478
0.9500	5.6874	0.7706	1.3603	0.8242	1.8437
0.9750	5.4878	0.7565	1.3887	0.8178	1.7439
1.0000	5.2960	0.7416	1.4193	0.8112	1.6480
1.0250	5.1114	0.7261	1.4525	0.8044	1.5557
1.0500	4.9333	0.7098	1.4885	0.7973	1.4667
1.0750	4.7612	0.6928	1.5278	0.7900	1.3806
1.1000	4.5945	0.6749	1.5708	0.7823	1.2972
1.1250	4.4327	0.6561	1.6181	0.7744	1.2163
1.1500	4.2752	0.6363	1.6704	0.7661	1.1376
1.1750	4.1217	0.6154	1.7286	0.7574	1.0609
1.2000	3.9716	0.5933	1.7939	0.7482	0.9858
1.2250	3.8244	0.5698	1.8678	0.7385	0.9122
1.2500	3.6796	0.5449	1.9523	0.7282	0.8398
1.2750	3.5366	0.5181	2.0503	0.7172	0.7683
1.3000	3.3947	0.4894	2.1659	0.7054	0.6974
1.3250	3.2532	0.4582	2.3051	0.6926	0.6266
1.3500	3.1111	0.4241	2.4776	0.6786	0.5555
1.3750	2.9669	0.3863	2.7001	0.6629	0.4834
1.4000	2.8186	0.3435	3.0041	0.6452	0.4093

$\alpha=0,30$

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
0.7000	8.0829	0.8547	1.2340	0.8763	3.0414
0.7250	7.7371	0.8432	1.2558	0.8708	2.8685
0.7500	7.4114	0.8310	1.2794	0.8651	2.7057
0.7750	7.1037	0.8183	1.3048	0.8592	2.5519
0.8000	6.8123	0.8050	1.3322	0.8532	2.4061
0.8250	6.5354	0.7910	1.3619	0.8470	2.2677
0.8500	6.2718	0.7763	1.3941	0.8406	2.1359
0.8750	6.0202	0.7609	1.4290	0.8339	2.0101
0.9000	5.7794	0.7446	1.4672	0.8270	1.8897
0.9250	5.5484	0.7276	1.5088	0.8198	1.7742
0.9500	5.3261	0.7097	1.5545	0.8122	1.6631
0.9750	5.1119	0.6908	1.6049	0.8044	1.5559
1.0000	4.9047	0.6708	1.6608	0.7961	1.4524
1.0250	4.7039	0.6497	1.7230	0.7874	1.3520
1.0500	4.5087	0.6274	1.7927	0.7782	1.2544
1.0750	4.3184	0.6037	1.8716	0.7684	1.1592
1.1000	4.1322	0.5784	1.9617	0.7580	1.0661
1.1250	3.9493	0.5513	2.0658	0.7468	0.9747
1.1500	3.7689	0.5221	2.1879	0.7347	0.8845
1.1750	3.5901	0.4906	2.3339	0.7215	0.7950
1.2000	3.4116	0.4561	2.5128	0.7069	0.7058
1.2250	3.2320	0.4179	2.7396	0.6906	0.6160
1.2500	3.0490	0.3750	3.0418	0.6720	0.5245
1.2750	2.8591	0.3254	3.4765	0.6502	0.4296
1.3000	2.6556	0.2655	4.1934	0.6234	0.3278

$\alpha=0,50$

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
0.6000	9.3036	0.8544	1.2668	0.8925	3.6518
0.6250	8.8290	0.8409	1.2967	0.8867	3.4145
0.6500	8.3855	0.8265	1.3294	0.8807	3.1927
0.6750	7.9695	0.8113	1.3652	0.8745	2.9847
0.7000	7.5778	0.7953	1.4046	0.8680	2.7889
0.7250	7.2076	0.7783	1.4481	0.8613	2.6038
0.7500	6.8565	0.7603	1.4961	0.8542	2.4283
0.7750	6.5224	0.7413	1.5495	0.8467	2.2612
0.8000	6.2032	0.7211	1.6091	0.8388	2.1016
0.8250	5.8973	0.6997	1.6760	0.8304	1.9487
0.8500	5.6031	0.6768	1.7515	0.8215	1.8015
0.8750	5.3190	0.6525	1.8375	0.8120	1.6595
0.9000	5.0436	0.6265	1.9362	0.8017	1.5218
0.9250	4.7755	0.5986	2.0507	0.7906	1.3877
0.9500	4.5132	0.5684	2.1855	0.7784	1.2566
0.9750	4.2553	0.5358	2.3467	0.7650	1.1277
1.0000	4.0000	0.5000	2.5440	0.7500	1.0000
1.0250	3.7452	0.4605	2.7925	0.7330	0.8726
1.0500	3.4882	0.4161	3.1190	0.7133	0.7441
1.0750	3.2249	0.3651	3.5763	0.6899	0.6125
1.1000	2.9480	0.3041	4.2919	0.6608	0.4740

T_o/T	λ	R/R_o	U_p/U_g	T_s/T	N_a/N_d
		$\alpha = 0,70$			
0.5000	11.2666	0.8732	1.2433	0.9112	4.6333
0.5250	10.5948	0.8592	1.2755	0.9056	4.2974
0.5500	9.9758	0.8442	1.3110	0.8998	3.9879
0.5750	9.4024	0.8282	1.3505	0.8936	3.7012
0.6000	8.8687	0.8112	1.3944	0.8872	3.4343
0.6250	8.3693	0.7930	1.4434	0.8805	3.1846
0.6500	7.8999	0.7737	1.4984	0.8734	2.9500
0.6750	7.4567	0.7531	1.5603	0.8659	2.7283
0.7000	7.0362	0.7311	1.6304	0.8579	2.5181
0.7250	6.6355	0.7076	1.7103	0.8493	2.3177
0.7500	6.2519	0.6824	1.8022	0.8400	2.1259
0.7750	5.8828	0.6553	1.9088	0.8300	1.9414
0.8000	5.5260	0.6261	2.0338	0.8190	1.7630
0.8250	5.1789	0.5945	2.1827	0.8069	1.5895
0.8500	4.8394	0.5600	2.3629	0.7934	1.4197
0.8750	4.5047	0.5222	2.5863	0.7780	1.2524
0.9000	4.1719	0.4801	2.8717	0.7603	1.0859
0.9250	3.8370	0.4326	3.2525	0.7394	0.9185
0.9500	3.4943	0.3777	3.7952	0.7138	0.7471
0.9750	3.1337	0.3113	4.6646	0.6809	0.5669
1.0000	2.7312	0.2236	6.4736	0.6339	0.3656

$\alpha = 1,00$

0.5000	10.7279	0.8292	1.3715	0.9068	4.3640
0.5250	10.0199	0.8096	1.4262	0.9002	4.0099
0.5500	9.3629	0.7886	1.4886	0.8932	3.6814
0.5750	8.7493	0.7660	1.5601	0.8857	3.3747
0.6000	8.1728	0.7416	1.6427	0.8776	3.0864
0.6250	7.6277	0.7153	1.7389	0.8689	2.8138
0.6500	7.1091	0.6869	1.8521	0.8593	2.5545
0.6750	6.6124	0.6561	1.9870	0.8488	2.3062
0.7000	6.1335	0.6225	2.1503	0.8370	2.0667
0.7250	5.6681	0.5856	2.3519	0.8236	1.8340
0.7500	5.2117	0.5449	2.6072	0.8081	1.6059
0.7750	4.7595	0.4992	2.9416	0.7899	1.3797
0.8000	4.3052	0.4472	3.4014	0.7677	1.1526
0.8250	3.8396	0.3863	4.0825	0.7396	0.9198
0.8500	3.3468	0.3112	5.2369	0.7012	0.6734
0.8750	2.7843	0.2073	7.9732	0.6408	0.3921