NON-COOPERATIVE BEHAVIOR
IN WIRELESS NETWORKS

PAR

Márk FÉLEGYHÁZI

Ingénieur Electricien Diplômé
Budapest University of Technology, Hongrie
et de nationalité hongroise

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

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Abstract

Existing cellular networks are centrally managed and require a tremendous initial investment. With the advancement of new wireless technologies, the operators of traditional networks have to face new competition. New technologies make it possible to provide wireless services with substantially less investment. There are two interdependent trends that enable this progress: the evolution of technology and the evolution of spectrum policy. These two trends point towards a more colorful landscape of wireless communication technologies. In particular, new wireless technologies enable users and small operators to deploy their own networks and to compete with the large network operators that run traditional wireless networks. Because the participants have an increased control over their devices, they might be tempted to adjust their devices in order to benefit more from the network. This selfish (i.e., non-cooperative) behavior can dramatically decrease the efficiency of the operation of the network or even paralyze it completely.

In this thesis, we consider various aspects of non-cooperative resource management in wireless networks using the methods provided by game theory. In the first part of the thesis, we present a comprehensive tutorial on game theory to facilitate the understanding of this theory as a tool. To emphasize the appropriateness of game theory in wireless networking, we present a set of selected examples along with their game-theoretic formalization.

In the second part of the thesis, we are concerned with the non-cooperative behavior of users. More precisely, we focus on ad hoc wireless networks and assume that users can alter the default programming of their communication devices to improve performance. First, we consider the problem of multi-radio channel allocation and show that a Nash equilibrium driven by the selfish behavior of users achieves load-balancing. We propose two algorithms, each based on a different set of available information, to achieve the characterized Nash equilibria. Furthermore, we discuss other properties such as efficiency, fairness and coalition-proofness. In the remainder of this second part, we focus on the fundamental problem of packet forwarding in ad hoc networks. In static networks, we prove that cooperative Nash equilibria exist, but the set of conditions that enable selfish participants to mutually forward each others' packets are very restrictive. We also show that in dynamic networks, mobility promotes cooperation.

In the third part of the thesis, we model the non-cooperative behavior of wireless network operators. We assume that they make strategic decisions to maximize the performance of their network. We also assume that the networks of different operators mutually affect each other, hence we model the decisions as network operator games. First, we study the scenario of co-located sensor networks and show that the network operators can mutually increase the lifetime of their network by cooperating. We also show that cooperation in terms of sharing sinks is more beneficial than providing packet forwarding of their sensors only. Second, we focus on the pilot power control of cellular networks assumed to operate in a shared spectrum. We identify Nash equilibria for different parameter values in a single-stage game and show that the cooperative solution can be enforced in a repeated game. Third, we consider the co-existence of cellular networks along national borders. We show that the operators have an incentive to be

strategic at their borders, or, in other words, to adjust the transmit power of their pilot signals. We show the efficiency of the Nash equilibria for different user densities. Finally, we extend the payoff function to include the cost of using high pilot powers and relate this extended game to the well-known Prisoner's Dilemma.

Résumé

Les réseaux cellulaires existants sont gérés de façon centralisée et leur déploiement demande un investissement initial très important. Avec l'évolution des nouvelles technologies sans-fil, les opérateurs des réseaux traditionnels doivent faire face à une nouvelle concurrence. En effet, les nouvelles technologies permettent de fournir des services sans-fil avec un investissement considérablement moindre. Il y a deux tendances interdépendantes qui permettent cette amélioration de service: l'évolution de la gestion du spectre de fréquences et l'évolution de la technologie sans-fil. Ces deux tendances mènent à une nouvelle ère pour les communications sans-fil. En particulier, les nouvelles technologies sans-fil permettent aux utilisateurs et aux petits opérateurs de déployer leurs propres réseaux et ainsi de concurrencer les grands opérateurs qui gèrent habituellement les réseaux sans-fil. Puisque ces nouveaux participants ont un contrle accru de leurs appareils, ils pourraient tre tentés de reprogrammer ces appareils afin de bénéficier plus des ressources communes. Ce comportement égoïste (également apellé non-coopératif) peut nettement diminuer l'efficacité du réseau ou mme le paralyser complètement.

Dans cette thèse de doctorat, nous utilisons différentes méthodes proposées par la théorie des jeux pour analyser divers aspects de la gestion non-coopérative des ressources dans les réseaux sans-fil. Dans la première partie de la thèse, nous présentons une introduction à la théorie des jeux pour faciliter la compréhension de cet outil. Nous présentons également quelques exemples de problèmes avec leur formalisation en jeu pour montrer que la théorie des jeux est parfaitement adaptée à l'analyse des réseaux sans-fil.

Dans la deuxième partie de la thèse, nous considérons le comportement non-coopératif des utilisateurs. Plus précisément, nous nous concentrons sur les réseaux ad hoc sans fil et supposons qu'afin d'améliorer leur performance, les utilisateurs peuvent changer la programmation par défaut de leurs appareils de communication. D'abord, nous considérons le problème de l'attribution du canal de communication et montrons que le comportement égoïste des utilisateurs mène à un équilibre de Nash qui réalise un équilibre le la charge dans le réseau. Nous proposons deux algorithmes, chacun basé sur un ensemble différent d'informations disponibles, pour arriver à ces équilibres de Nash. En outre, nous discutons d'autres propriétés telles que l'efficacité, l'équité et la résistance à la coalition. Dans le reste de cette deuxième partie, nous nous concentrons sur le problème fondamental de la volonté des utilisateurs à participer au fonctionnement du réseau ad hoc en relayant les paquets des autres. Dans les réseaux statiques, nous montrons que les équilibres de Nash coopératifs existent, mais que l'ensemble de conditions qui permettent aux participants égoïstes de relayer les paquets des autres sont très restrictives. Nous montrons également que dans les réseaux dynamiques, la mobilité encourage la coopération.

Dans la troisième partie de la thèse, nous modelons le comportement non-coopératif des opérateurs de réseau sans-fil. Nous supposons qu'ils prennent des décisions stratégiques pour maximiser la performance de leur réseaux. Nous supposons également que les réseaux contrlés par différents opérateurs s'influencent mutuellement, par conséquent nous modelons les décisions en tant que jeu entre les opérateurs. D'abord, nous étudions le scénario des réseaux de capteurs qui coexistent et montrons que les opérateurs

peuvent augmenter la vie de leur réseaux en coopérant. Nous montrons également que la coopération qui consiste à partager les collecteurs de données est plus bénéfique que cette qui consiste à uniquement relayer les paquets. Deuxièmement, nous nous concentrons sur le contrle de la puissance du signal pilote des réseaux cellulaires supposés opérer dans un spectre partagé. Nous identifions des équilibres de Nash pour différentes valeurs de paramètres dans un jeu à une itération et montrons que, dans un jeu répété, l'utilisation de certaines stratégies rend optimale la solution coopérative. Troisièmement, nous considérons la coexistence des réseaux cellulaires le long des frontières nationales. Nous montrons que les opérateurs sont encouragés à tre stratégiques à leurs frontières, et donc, en d'autres termes, à ajuster la puissance de transmission de leurs signaux pilotes. Nous montrons l'efficacité des équilibres de Nash pour différentes densités d'utilisateurs. En conclusion, nous étendons la fonction de profit pour inclure le coût d'utilisation de puissances pilotes élevées et rapportons ce jeu étendu au fameux Dilemme du Prisonnier.

Mots-clés: réseaux sans-fil, théorie des jeux, coopération, égoïsme, équilibre de Nash, optimalité de Pareto, prix d'anarchie, partage de spectre, relais de paquets, jeux d'opérateurs.

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Contents

Introduction

Existing cellular networks are centrally managed and typically provide services to a large number of users. The network operators invest substantial effort and money in the design and supervision of their networks. This implies that only a few companies can afford the tremendous investment of bootstrapping such a business.

With the advancement of new wireless technologies [BH07], the operators of cellular networks face new competition. New technologies, such as WiFi, make it possible to provide wireless services with substantially less investment. This is clearly shown in the spread of (affordable) WiFi access that threatens the services provided by cellular networks. This competition is favorable for the users, because it increases their choice. At the same time, it undermines the quasi-monopolistic situation of the traditional network operators, motivating them to adapt their services (and their business model) to the new challenge.

There are two interdependent trends that enable this progress: the evolution of technology and the evolution of spectrum policy [Ben04, FF03]. On the one hand, the development of new wireless technologies, notably WiFi, revolutionize the area of wireless communication. On the other hand, government agencies are relaxing the rigid spectrum licensing process that prevented the entering of new technologies and operators. The first step toward a more flexible spectrum policy was to provide given spectrum bands as *unlicensed bands* available for all technologies. These two changes reinforce each other and could result in an open and flexible spectrum management.

There are several new technologies that enable networking in unlicensed bands. Because of the popularity of these new technologies and the lack of regulation on unlicensed bands, the quality-of-service (QoS) provided by networks operating in unlicensed bands is a serious issue. Meanwhile, there exist large parts of the frequency spectrum that were previously licensed to certain operators, but are largely unused. In line with the progress, both in policy and technology in wireless networking, the government agencies consider adopting a new concept called *cognitive radio communication* [III01, Hay05]. The cognitive radio architecture enables wireless devices to discover unused frequencies – on a licensed or unlicensed frequency band – and to operate on them.

In the next section, we highlight some of the prospective wireless networks and the challenges at their deployment.

Prospective Wireless Networks

Small operators in unlicensed bands: The need for more efficient spectrum usage led to the creation of unlicensed bands. Unlicensed bands enable anyone to operate his own wireless network. One example is the WiFi technology: WiFi access points are inexpensive and easily deployable, hence a flexible wireless access infrastructure can be built up quickly without substantial initial investment. The lack of substantial initial investments and ease of management opens the door for *small operators* to enter the

1

competition for users. Clearly, as the number of network operators increases, each of them has an incentive to fine-tune the protocol parameters of his access points in order to increase his profit. This is not too difficult as the access points are flexible and – compared to cellular network base stations – relatively easy to program. As the adjustments of the access points in a given network affect other networks in the same unlicensed band, the operators are likely to engage in a strategic optimization problem. Due to the potentially large number of network operators, settling these conflicts via the traditional process of mutual agreements can be far from obvious to do.

Community and ad hoc networks: In contrast to small operators, users with wireless access points can organize themselves into a *community network* to mutually provide wireless access to each other. Community networks can be an additional threat to existing wireless operators, because they can provide wireless access to their members for free. However, the basic principle of altruism that enables the existence of community networks might be to their disadvantage as well. Because they rely on the service offered by individuals, selfish individuals can exploit the service offered by others without contributing to the community. This issue of free-riding can be a serious disadvantage of community networks [EFP06].

Taking the idea of community networks one step further, it is envisioned that users can form a local communication network among themselves solely relying on their wireless devices. This vision is summarized in the idea of *ad hoc networks*, an idea that generated an extensive set of research contributions. The basic principle of ad hoc networks is that users establish the connections among each other using multi-hop communication. This means that they have to provide all the networking services themselves without the involvement of an external entity (i.e., the network operator). Similarly to community networks, the issue of free-riding might be a serious problem in ad hoc networks as well [BH07].

Cognitive radio: *Cognitive radio* communication is a new concept that is being considered by government agencies to utilize the available frequency spectrum more efficiently. Cognitive radios are able to sense a wide band of frequency spectrum in order to select the frequency that is not or sparsely utilized to communicate. This policy change on the part of the government agencies is in parallel with a technology change towards *software-defined radio (SDR)* [III95]. The evolution of SDR is fueled by a set of new techniques: smart antennas, multiband antennas and wideband RF devices. More and more functionalities are implemented in general-purpose programmable processors. The purpose of SDR is to implement an architecture that is able to accommodate any future technology. Even if this objective is not reached completely, SDRs will be able to provide an appropriate platform for cognitive radio communication.

One key property of the cognitive radio architecture is that the programming of the devices makes them highly adaptive. This means that they implement a strategic (i.e., cognitive) behavior by definition. Hence, this architecture provides a fertile ground for users to re-program their devices in order to increase their utility in the network. Unlike in traditional networks, where there is a need for a substantial effort to re-program wireless devices, in cognitive radio networks the impact of strategic behavior has to be taken into account from the very beginning of the network design.

Problem Formulation and Methodology

Existing cellular networks are centrally managed and thus each participant is assumed to follow the pre-defined network protocols. This assumption is backed by the fact that existing wireless devices are difficult to re-program. Prospective wireless networks are envisioned to operate with more flexibility based on new technologies and protocols.

In this thesis, we consider various aspects of non-cooperative resource management in prospective wireless networks. We study two important areas: *spectrum sharing* on the physical and medium access layer and *packet forwarding* in the networking layer. The major difference between the problems in these areas is that spectrum sharing concerns the splitting of an available resource (i.e., the frequency spectrum), whereas the core of the packet forwarding problem is providing a reliable service for multi-hop, end-to-end packet transmissions. Consequently, the instantiation of non-cooperative behavior is different as well. In spectrum sharing, selfish participants have the objective to obtain the largest possible share from the available spectrum. In packet forwarding, however, selfish participants want to exploit the packet forwarding service provided by others without contributing to it themselves.

Motivated by the potential selfishness of network participants, we seek the answer for the following fundamental question in this thesis: **What is the effect of selfish behavior in wireless networks?** In particular, we are interested in the existence of cooperation in networks with selfish participants and the efficiency of such a non-cooperative, self-organizing solution with respect to a centralized method. We study the effect of selfish behavior in two main settings: (a) on a microscopic level, we consider the non-cooperative behavior of user devices in decentralized networks, whereas (b) on a macroscopic level, we assume that networks are centralized, but their operators confront each other in strategic situations.

We study the problem of selfishness in different wireless networks using *game theory* [FH06a, FT91, Gib92, OR94]. Game theory is an analytical tool that has been extensively applied to various problems in different disciplines in the past: to model strategic decision problems in economics or to analyze the rational behavior of different species in biology. In recent years, game theory has been applied to computer networks as well, including wireless communication networks. We survey some of these works in the next section.

Game theory in networking

In the following, we highlight a subset of research contribution in computer networking that rely on game theory.

Peer-to-peer networks: Game theory has been used to solve various problems in peer-to-peer (P2P) networks. In [GLBM01], Golle *et al.* consider the free-rider problem in peer-to-peer file sharing networks, notably in Napster. They construct a game-theoretic model and analyze equilibria of user strategies under several novel payment mechanisms. Qiu and Srikant [QS04] propose a fluid model to study the performance and the incentives of the BitTorrent file-sharing protocol. In [LFSC03], Lai *et al.* model the free-riding problem using the Evolutionary Prisoners Dilemma (EPD) game. They investigate incentive techniques, such as reputation systems, to promote cooperation between P2P nodes. Feldman *et al.* [FPCS06] consider both free-riding and white-washing in P2P systems. They propose a game-theoretic model based on users' types that express their willingness to contribute resources to the system. They propose reputation-based penalization mechanism to prevent free-riding and initial penalization to mitigate white-washing.

In [CFSK04], Chun *et al.* analyze the characteristics of overlay routing networks generated by selfish nodes playing competitive network construction games. They highlight a fundamental tradeoff between performance and resilience, and show that limiting the degree of nodes is of great importance in controlling this balance. Aberer *et al.* [ADGK06] study trust and reputation systems to promote cooperative behavior in P2P networks. They argue that the simplifications made in existing models limit their applicability and propose some guidelines to develop more realistic models.

Wired networks: Game theory has also been used to solve various problems in wired networks. In [CSEZ93], Cocchi *et al.* study the role of pricing policies in multiple service class networks. They present an abstract formulation of service disciplines and pricing policies based on the Nash implementation paradigm.

Korilis, Lazar and Orda [KLO95] address the problem of allocating link capacities in routing decisions; in [KO99], Korilis and Orda suggest a congestion-based pricing scheme. Roughgarden [Rou05] quantifies the worst-possible loss in network performance arising from non-cooperative routing behavior. In [HYR00], Yaïche, Mazumdar and Rosenberg present a game-theoretic framework for bandwidth allocation; they study the centralized problem and show that the solution can be distributed in a way that leads to a system-wide optimum. Jin and Kesidis [JK03] propose a generic mechanism for rate control and study Nash equilibria in a networking game. Johari and Tsitsiklis [JT04] study the price of anarchy in congestion games. Their analysis extends to a wide class of resource allocation systems where end users simultaneously require multiple scarce resources.

He and Walrand [HW06] focus on revenue sharing of Internet service providers. They present a pricing model and show that noncooperative pricing strategies may lead to unfair distribution of profit. Then, they propose a fair revenue-sharing policy and show that this fair allocation policy encourages collaboration among providers and hence it results in higher profits for them.

Wireless networks: In [GM01], Goodman and Mandayam introduce the concept of network-assisted power control to equalize signal-to-interference ratio between the users. In [XSC03], Xiao, Schroff and Chong describe a utility-based power control framework for a cellular system. In [SMG02], Saraydar, Mandayam and Goodman present a power control mechanism in wireless network based on pricing. They analyze this problem in a non-cooperative game and show that pricing can improve the inefficient Nash equilibria to achieve a Pareto-optimal strategy profile. Alpcan *et al.* [ABSA02] apply game theory for uplink power control in cellular networks.

Marbach and Berry [MB02] downlink resource allocation in wireless networks with variable channel quality. They propose a two pricing schemes: in the first one, the base stations know the utility functions of the users and in the second scheme, the base stations do not know these utilities. In both cases, they propose schemes that are system optimal. Qiu and Marbach [QM03] define a price-based approach for bandwidth allocation in wireless ad hoc networks.

MacKenzie and Wicker [MW03] investigate the effect of selfishness in the slotted Aloha medium access protocol. They establish the Nash equilibria and the stability regions in the game for some well-known channel models. Yuen and Marbach [MQ05b] study a price-based rate control mechanism for random access networks based on the slotted Aloha protocol. They propose a mechanism that uses channel feedback information to control the aggregate packet arrival rate. They characterize the throughput and delay at the operating point of the system and show how the operating point can be set a priori by appropriately choosing the control parameters. In [CGAH05], Čagalj *et al.* use a game-theoretic approach to investigate the problem of the selfish behavior of nodes in CSMA/CA networks, notably in IEEE 802.11. They characterize two families of Nash equilibria in a single-stage game, one of which always results in a network collapse. In a repeated game, they establish a Pareto-optimal Nash equilibrium and develop a distributed protocol to achieve it.

Musacchio and Walrand [MW04] study the economic interests of a wireless access point owner and his paying client, and model their interaction as a dynamic game with asymmetric information. In particular, they present a two player game in which the access point does not know whether he faces a web browser or file transferor type client, and show conditions for which it is not a Nash equilibrium for the access point to maintain a constant price.
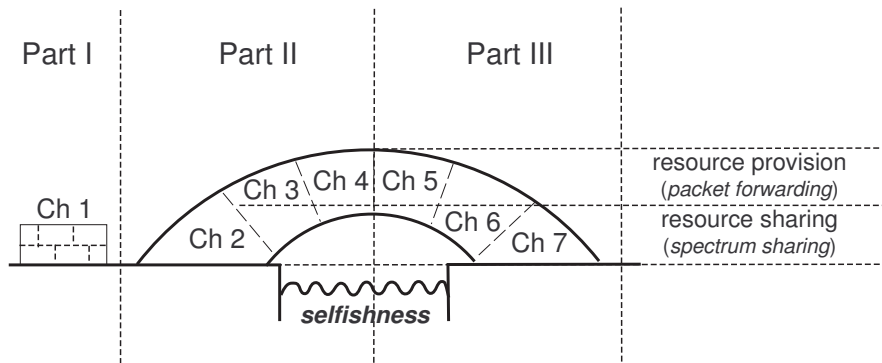
Figure 1: Illustration of the constitution of the thesis.

Outline of the Thesis

In this thesis, our objective is to bridge the potential gap between the efficiency of non-cooperative and centrally managed wireless networks using game-theoretic techniques. We illustrate the constitution of the thesis in Figure 1.

Part I: Introduction to Game Theory

In the first part of the thesis, which consists of Chapter 1, we provide a *comprehensive introduction to non-cooperative game theory* in order to facilitate the understanding of this theory. Game theory provides the basic theoretical building blocks for this thesis. To emphasize the appropriateness of game theory in wireless networking, we present a set of selected examples, along with their game-theoretic formalization. The trend towards distributed computing and more programmable user devices inspired us to focus on non-cooperative game theory.

Part II: Non-Cooperative Behavior of Users

In the second part of the thesis, we are concerned with the non-cooperative behavior of users. More precisely, we assume that they can alter the default programming of their communication devices to improve their performance. Often, this improvement comes at the expense of other users.

In Chapter 2, we study the problem of *multi-radio channel allocation in competitive wireless networks*. We assume that there is a wireless network with communicating pairs of user devices, i.e. that devices send data to their peers via a single hop. Using a static non-cooperative game, we analyze the scenario of a single collision domain, i.e., that each of the devices can interfere with a transmission of every other device. We derive the Nash equilibria in this game and show that they achieve load-balancing by distributing the radios in use over the available channels. We also study fairness issues and the problem of coalition formation in the channel allocation problem. We show that a Nash equilibrium that resists coalitions of users is necessarily fair as well. Furthermore, we propose two algorithms to achieve the system-efficient Nash equilibrium solutions. The first is a sequential algorithm that needs global coordination and the second is a distributed algorithm that is based on imperfect local information. We provide the proof for the convergence properties of these algorithms.

In Chapter 3, we study the problem of multi-hop *packet forwarding in selfish ad hoc networks*. In particular, we address the problem of whether cooperation can exist without incentive mechanisms. We propose a model, based on game theory and graph theory, to investigate the conditions of Nash equilibria of packet forwarding strategies. We formulate a set of theorems to characterize the Nash equilibria based on non-cooperative and cooperative strategies. We perform simulations to estimate the probability that the conditions for a cooperative equilibrium hold in randomly generated network scenarios. As the problem is rather involved, we deliberately restrict ourselves to a static configuration. We conclude that, in static ad hoc networks, cooperation needs to be encouraged.

In Chapter 4, we study *how mobility affects the cooperative packet forwarding behavior* of user devices. We prove several theorems about the existence of Nash equilibria in a simple scenario and we investigate a more realistic scenario by simulations. We show that in the realistic model the level of contribution of the nodes to reach cooperation is much higher than in the theoretical model, and we quantify the relationship between mobility and cooperation. We conclude that spontaneous cooperation is easier to reach when mobility is higher.

Part III: Non-Cooperative Behavior of Network Operators

In the third part of the thesis, we model the selfish behavior of wireless network operators. We assume that they make strategic decisions to maximize the performance of their network. We assume that the networks of different operators mutually affect each other, hence we model their decisions as non-cooperative games.

In Chapter 5, we introduce the concept of *multi-domain sensor networks* that, to the best of our knowledge, has never been considered before. We propose a game-theoretic model to investigate the effect of cooperation in joint packet forwarding and power control. Our results show that energy saving by cooperation provides a "natural incentive" for the authorities. The advantage of cooperation is twofold: (a) the authorities can largely benefit by providing service of their sinks for other's sensor networks and (b) if sinks are common resources, then cooperative packet forwarding is beneficial for sparse networks or in hostile environments.

In Chapter 6, we investigate the problem of *co-existing wireless network operators in a shared spectrum*. We assume that the operators apply power control at the base stations to mitigate interference, while providing a permanent service to the users. Furthermore, we assume that mobile devices can freely roam among the networks of various operators. Free roaming means that the mobile devices measure the signal strength of the pilot signals (i.e., beacon signals) of the base stations and attach to the base station with the strongest pilot signal. We model the behavior of the network operators in a game theoretic setting in which each operator decides about the power of the pilot signal of his base stations. We first identify possible Nash equilibria in the theoretical setting in which all base stations are located on the vertices of a two-dimensional lattice. We then prove that a socially optimal Nash equilibrium exists and that it can be enforced by using punishments. Finally, we relax the topological assumption and show that, in the more general case, finding the Nash equilibria is an NP-complete problem.

In Chapter 7, we investigate the *strategic interaction of the operators of 3G wireless networks on the border of two countries*. More precisely, we focus on the scenario of two operators, who want to fine-tune the emitting power of the pilot signals of their base stations. This operation is crucial, because the pilot signal power determines the number of users the operators can attract and hence the revenue they obtain. Due to the complexity of the problem, we restrict ourselves to the scenario of two competing base stations. We show that the operators have an incentive to be strategic at their borders, i.e., to adjust the transmit power of their pilot signals. In addition, we study Nash equilibrium conditions in an empirical

model and show the efficiency of the Nash equilibria for different user densities. We provide a distributed convergence algorithm to achieve the identified Nash equilibria. Finally, we extend the payoff function to include the cost of using high pilot powers. The game with power costs corresponds to the well-known Prisoner's Dilemma: The players still have the incentive to adjust their pilot powers, but their strategic behavior leads to a Nash equilibrium that is sub-optimal from the system's point of view.

Part I

Introduction to Non-Cooperative Game Theory

Chapter 1

A Tutorial on Game Theory

1.1 Introduction

Game theory [FT91, Gib92, OR94] is a discipline aimed at modeling situations in which decision makers have to make specific actions that have mutual, possibly conflicting, consequences. It has been used primarily in economics, in order to model competition between companies: for example, should a given company enter a new market, considering that its competitors could make similar (or different) moves? Game theory has also been applied to other areas, including politics and biology.[1]

The first textbook in this area was written by von Neumann and Morgenstern, in 1944 [vNM44]. A few years later, John Nash made a number of additional contributions [Nas50, Nas51], the cornerstone of which is the famous *Nash equilibrium*. Since then, many other researchers have contributed to the field, and in a few decades game theory has become a very active discipline; it is routinely taught in economics curricula. An amazingly large number of game theory textbooks have been produced, but almost all of them consider economics as the premier application area (and all their concrete examples are inspired by that field). Our tutorial is inspired by three basic textbooks and we mention them in the ascending order of complexity. Gibbons [Gib92] provides a very nice, easy-to-read introduction to non-cooperative game theory with many examples using economics. Osborne and Rubinstein [OR94] introduce the game-theoretic concepts very precisely, although this book is more difficult to read because of the more formal development. This is the only book out of the three that covers cooperative game theory as well. Finally, Fudenberg and Tirole's [FT91] book covers many advanced topics, in addition to the basic concepts.[2]

Not surprisingly, game theory has also been applied to networking, in most cases to solve routing and resource allocation problems in a competitive environment. The references are so numerous that we cannot list them due to space constraints. A subset of these papers is included in [ABA+06]. Recently, game theory was also applied to wireless communication: the decision makers in the game are rational users or networks operators who control their communication devices. These devices have to cope with a limited transmission resource (i.e., the radio spectrum) that imposes a conflict of interests. In an attempt to resolve this conflict, they can make certain moves such as transmitting now or later, changing their transmission channel, or adapting their transmission rate.

There is a significant amount of work in wired and wireless networking that make use of game theory.

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[1] The name itself of "game theory" can be slightly misleading, as it could be associated with parlor games such as chess and checkers. Yet, this connection is not completely erroneous, as parlor games do have the notion of players, payoffs, and strategies - concepts that we will introduce shortly.

[2] Note that there exist books on the application of game theory to specific topics in wireless networking, such as on routing [Rou05].

11

Oddly enough, there exists no comprehensive tutorial specifically written for wireless networkers.[3] We have wrote this tutorial with the hope of contributing to fill this void. As game theory is still rarely taught in engineering and computer science curricula, we assume that the reader has no (or very little) background in this field; therefore, we take a basic and intuitive approach. Because in most of the strategic situations in wireless networking the players have to agree on sharing or providing a common resource in a distributed way, our approach focuses on the theory of *non-cooperative games*. Cooperative games require additional signalization or agreements between the decision makers and hence a solution based on them might be more difficult to realize.

In a non-cooperative game, there exist a number of decision makers, called *players*,[4] who have potentially conflicting interests. In the wireless networking context, the players are the *users* or *network operators* controlling their devices. In compliance with the practice of game theory, we assume that the players are *rational*, which means that they try to maximize their *payoffs* (or utilities). This assumption of rationality is often questionable, given, for example, the altruistic behavior of some animals. Herbert A. Simon was the first one was to question this assumption and introduced the notion of *bounded rationality* [Sim69]. But, we believe that in computer networks, most of the interactions can be captured using the concept of rationality, with the appropriate adjustment of the payoff function. In order to maximize their payoff, the players act according to their *strategies*. The strategy of a player can be a single *move* (as we will see in Section 1.2) or a set of moves during the game (as we present in Section 1.4).

In this tutorial, we devote particular attention to the selection of the examples so that they match our focus on wireless networks. For the sake of clarity, and similarly to classic examples, we define these examples for two decision makers, hence the corresponding games are two-player games. For each of the examples, we highlight a corresponding research contribution that models the same problem in a more complex scenario. As we demonstrate in this thesis, the application of game theory extends far beyond two-person games. Indeed, in most networking problems, there are several participants.

We take an intuitive top-down approach in the protocol stack to select the examples in wireless networking as follows. Let us first assume that the time is split into *time steps* and each device can make one move in each time step.

1. In the first game called the *Forwarder's Dilemma*,[5] we assume that there exist two devices as players, p_1 and p_2. Each of them wants to send a packet to his destination[6], dst_1 and dst_2 respectively, in each time step using the other player as a forwarder. We assume that the communication between a player and his receiver is possible only if the other player forwards the packet. We show the Forwarder's Dilemma scenario in Figure 1.1. If player p_1 forwards the packet of p_2, it costs player p_1 a fixed *cost* $0 < C << 1$, which represents the energy and computation spent for the forwarding action. By doing so, he enables the communication between p_2 and dst_2, which gives p_2 a *benefit*[7] of 1. The payoff is the difference of the benefit and the cost. We assume that the game is symmetric and the same reasoning applies to the forwarding move of player p_2. The dilemma

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[3]To the best of our knowledge, there exist only two references: a monograph [MDT06] that provides a synthesis of lectures on the topic, and a survey [ABA+06] that focuses mostly on wired networks.

[4]Note that there exists no convention to refer to the players with a particular gender. In this work, we will use the male pronouns following the basic textbooks [FT91, OR94].

[5]We have chosen this name as a tribute to the famous Prisoner's Dilemma game in the classic literature [Axe84, Gib92, FT91, OR94].

[6]Note that in the networking context, wireless engineers use the terms source and destination to refer to the endpoints to a given network flow. In case of a direct transmission between two devices, the terms sender and receiver are used.

[7]Note that we introduce the concept of benefit to express the positive results for a given player in the game. Hence, the concept of benefit should be interpreted as "something that contributes to or increases one's well-being."
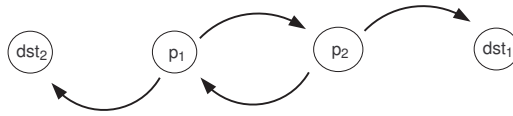
Figure 1.1: The network scenario in the Forwarder's Dilemma game.

Figure 1.2: The Joint Packet Forwarding Game.

is the following: Each player is tempted to drop the packet he should forward, as this would save some of his resources; but if the other player reasons in the same way, then the packet that the first player wanted to send will also be dropped. They could, however, do better by mutually forwarding each other's packet. Hence the dilemma. The Forwarder's Dilemma serves as a basis for our packet forwarding games in Chapters 3 and 4.

2. In the second example, we present a scenario, in which a source src wants to send a packet to his destination dst in each time step. To this end, he needs *both devices* p_1 and p_2 to forward for him. Thus, we call this game the *Joint Packet Forwarding Game*. Similarly to the previous example, there is a forwarding cost $0 < C << 1$ if a player forwards the packet of the sender. If both players forward, then they each receive a benefit of 1 (e.g., from the sender or the receiver). We show this packet forwarding scenario in Figure 1.2. The packet forwarding games in Chapters 3 and 4 are also inspired by the Joint Packet Forwarding Game.

3. The third example, called *Multiple Access Game*,[8] introduces the problem of medium access. Suppose that there are two players p_1 and p_2 who want to access a shared communication channel to send some packets to their receivers re_1 and re_2. We assume that each player has one packet to send in each time step and he can decide to access the channel to transmit it or to wait. Furthermore, let us assume that p_1, p_2, re_1 and re_2 are in the power range of each other, hence their transmissions mutually interfere. If player p_1 transmits his packet, it incurs a sending cost of $0 < C << 1$. The packet is successfully transmitted if p_2 waits in that given time step (i.e., he does not transmit), otherwise there is a collision. If there is no collision, player p_1 gets a benefit of 1 from the successful packet transmission. The framework presented by Čagalj *et al.* in [CGAH05] is a generalized version of the Multiple Access Game.

4. In the last example, we assume that player p_1 wants to send a packet in each time step to a receiver re_1. In this example, we assume that the wireless medium is split into two channels x and y according to the Frequency Division Multiple Access (FDMA) principle [Rap02, Sch05]. The objective of the *malicious* player p_2 is to prevent player p_1 from a successful transmission by transmitting on the same channel in the given time step. In wireless communication, this is called

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[8]In the classic game theory textbooks, this type of game is referred to as the "Hawk-Dove" game, or sometimes the "Chicken" game.
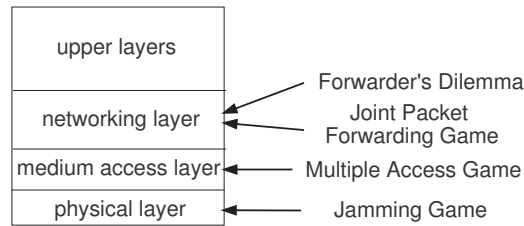
Figure 1.3: The classification of the examples according to protocol layers.

jamming, hence we refer to this game as the *Jamming Game*.[9] Clearly, the objective of p_1 is to succeed in spite of the presence of p_2. Accordingly, he receives a payoff of 1 if the attacker cannot jam his transmission and he receives a payoff of -1 if the attacker jams his packet. The payoffs for the attacker p_2 are the opposite of those of player p_1. We assume that p_1 and re_1 are synchronized, which means that re_1 can always receive the packet, unless it is destroyed by the malicious player p_2. Note that we neglect the transmission cost C, since it applies to each payoff (i.e., the payoffs would be $1 - C$ and $-1 - C$) and does not change the conclusions drawn from this game. The Jamming Game models the simplified version of a game-theoretic problem presented by Zander in [Zan91].

We deliberately chose these examples to represent a wide range of problems over different protocol layers (as shown in Figure 1.3). There are indeed fundamental differences between these games as follows. The Forwarder's Dilemma is a symmetric *nonzero-sum game*, because the players can mutually increase their payoffs by cooperating (i.e., from zero to $1 - C$). The conflict of interest is that they have to provide the packet forwarding service for each other. Similarly, the players have to establish the packet forwarding service in the Joint Packet Forwarding Game, but they are not in a symmetric situation anymore. The Multiple Access Game is also a nonzero-sum game, but the players have to share a common resource, the wireless medium, instead of providing it. Finally, the Jamming Game is a *zero-sum game* because the gain of one player represents the loss of the other player. These properties lead to different games and hence to different strategic analyses, as we will demonstrate in the next section.

1.2 Static Games

In this section, we assume that there exists only one time step, which means that the players have only one move as a strategy. In game-theoretic terms, this is called a *single-stage game*. In the game-theoretic terms this is called a *static game*. We will demonstrate how game theory can be used to analyze the games introduced before and to identify the possible outcomes of the strategic interactions of the players.

1.2.1 Static Games in Strategic Form

We define a game $\mathbf{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$ in *strategic form* (or normal form) by the following three elements. $\mathcal{N}$ is the set of *players*. Note that in this chapter we have two players, $p_1, p_2 \in \mathcal{N}$, but we present each definition such that it holds for any number of players. For convenience, we will designate by subscript $-i$ all the players belonging to $\mathcal{N}$ except i himself. These players are often designated as being the

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[9]In the classic game theory literature, this game corresponds to the game of "Matching Pennies."

opponents of i. In our games, player i has one opponent referred to as j. $\mathcal{S}_i$ corresponds to the *pure strategy space* of player i. This means that the strategy assigns zero probability to all moves, except one (i.e., it clearly determines the move to make). We will see in Section 1.2.4, that the players can also use *mixed strategies*, meaning that they choose different moves with different probabilities. We designate the joint set of the strategy spaces of all players as follows $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_{|\mathcal{N}|}$. We will represent the pure strategy space of the opponents of player i by $\mathcal{S}_{-i} = \mathcal{S} \backslash \mathcal{S}_i$. The set of chosen strategies constitutes a *strategy profile* $s = \{s_1, s_2\}$. In this tutorial, we have the same strategy space for both players, thus $\mathcal{S}_1 = \mathcal{S}_2$. Note that our examples have two players and thus we refer to the strategy profile of the opponents as $s_{-i} = s_j \in \mathcal{S}$. The *payoff*[10] or utility $u_i(s)$ quantifies the outcome of the game for player i given the strategy profile s. In our examples, we have $\mathcal{U} = \{u_1(s), u_2(s)\}$. Note that the objectives (i.e., payoff functions) might be different for the two players, as for example in the Jamming Game.

At this point of the discussion, it is very important to explicitly state that we consider the game to be with *complete information*.

Definition 1.1. *A game with* complete information *is a game in which each player has full knowledge of all aspects of the game.*

In particular, complete information means that the players know each element in the game definition: (i) who the other players are, (ii) what their possible strategies are and (iii) what payoff will result for each player for any combination of moves. The concept of complete information should not be confused with the concept of *perfect information*, another concept we present in detail in Section 1.3.3.

Let us first study the Forwarder's Dilemma in a static game. As mentioned before, in the static game we have only one time step. The players can decide to forward (F) the packet of the other player or to drop it (D); this decision represents the strategy of the player. As mentioned earlier, this is a nonzero-sum game, because by helping each other to forward, they can achieve an outcome that is better for both players than mutual dropping.

Matrices provide a convenient representation of strategic-form games with two players. We can represent the Forwarder's Dilemma game as shown in Table 1.1. In this table, p_1 is the row player and p_2 is the column player. Each cell of the matrix corresponds to a possible combination of the strategies of the players and contains a pair of values representing the payoffs of players p_1 and p_2, respectively.

		p_2	
		F	D
p_1	F	(1-C,1-C)	(-C,1)
	D	(1,-C)	(0,0)

Table 1.1: The Forwarder's Dilemma game in strategic form, where p_1 is the row player and p_2 is the column player. Each of the players has two strategies: to forward (F) or to drop (D) the packet of the other player. In each cell, the first value is the payoff of player p_1, whereas the second is the payoff of player p_2.

1.2.2 Iterated Dominance

Once the game is expressed in strategic form, it is usually interesting to *solve* it. Solving a game means predicting the strategy of each player, considering the information the game offers and assuming that the

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[10]The notation we use to denote payoffs / utilities corresponds to the conventional notation of many game theory textbooks [FT91, Gib92, OR94].

players are rational. There are several possible ways to solve a game; the simplest one consists in relying on *strict dominance*.

Definition 1.2. *Strategy* $s_i^{'}$ *of player* i *is said to be* strictly dominated *by his strategy* s_i *if,*

$$u_i(s_i^{'}, s_{-i}) < u_i(s_i, s_{-i}), \forall s_{-i} \in S_{-i} \tag{1.1}$$

Coming back to the example of Table 1.1, we solve the game by *iterated strict dominance* (i.e., by iteratively eliminating strictly dominated strategies). If we consider the situation from the point of view of player p_1, then it appears that for him the F strategy is strictly dominated by the D strategy. This means that we can eliminate the first row of the matrix, since a rational player p_1 will never choose this strategy. A similar reasoning, now from the point of view of player p_2, leads to the elimination of the first column of the matrix. As a result, the solution of the game is (D, D) and the payoffs are $(0, 0)$. This can seem quite paradoxical, as the pair (F, F) would have led to a better payoff for each of the players. It is the lack of trust between the players that leads to this suboptimal solution.

The technique of iterated strict dominance cannot be used to solve every game. Let us now study the Joint Packet Forwarding Game. The two devices have to decide whether to forward the packet simultaneously, before the source actually sends it.[11] Table 1.2 shows the strategic form.

		p_2	
		F	*D*
p_1	*F*	(1-C,1-C)	(-C,0)
	D	(0,0)	(0,0)

Table 1.2: The Joint Packet Forwarding Game in strategic form. The players have two strategies: to forward (F) or to drop (D) the packet sent by the sender. Both players p_1 and p_2 get a benefit, but only if each of them forwards the packet.

In the Joint Packet Forwarding Game, none of the strategies of a certain player strictly dominates the other. If player p_1 drops the packet, then the move of player p_2 is indifferent and thus we cannot eliminate his strategy D based on strict dominance. To overcome the requirements defined by strict dominance, we define the concept of *weak dominance*.

Definition 1.3. *Strategy* $s_i^{'}$ *of player* i *is said to be* weakly dominated *by his strategy* s_i *if,*

$$u_i(s_i^{'}, s_{-i}) \leq u_i(s_i, s_{-i}), \forall s_{-i} \in \mathcal{S}_{-i} \tag{1.2}$$

with strict inequality for at least one $s_{-i} \in \mathcal{S}_{-i}$.

Using the concept of weak dominance, one can notice that the strategy D of player p_2 is weakly dominated by the strategy F. One can perform an elimination based on *iterated weak dominance*, which results in the strategy profile (F, F). Note, however, that the solution of the iterated strict dominance technique is unique, whereas the solution of the iterated weak dominance technique might depend on the sequence of eliminating weakly dominated strategies, as explained at the end of Section 1.2.3.

It is also important to emphasize that the iterated elimination techniques are very useful, even if they do not result in a single strategy profile. These techniques can be used to reduce the size of the strategy space (i.e., the size of the strategic-form matrix) and thus to ease the solution process.

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[11]In Section 1.3, we will show that the game-theoretic model and its solution changes if we consider a sequential move of the players (i.e., if player p_2 knows the move of player p_1 at the moment he makes a move).

1.2.3 Nash Equilibrium

In general, the majority of the games cannot be solved by the iterated dominance techniques. As an example, let us consider the Multiple Access Game introduced at the beginning. Each of the players has two possible strategies: either access the channel (i.e., to transmit) (A) or wait (W). As the channel is shared, a simultaneous transmission of both players leads to a collision. The game is represented in strategic form in Table 1.3.

		p_2	
		W	A
p_1	W	(0,0)	(0,1-C)
	A	(1-C,0)	(-C,-C)

Table 1.3: The Multiple Access Game in strategic form. The two moves for each player are: access (A) or wait (W).

It can immediately be seen that no strategy is dominated in this game. To solve the game, let us introduce the concept of *best response*. If player p_1 accesses the channel, then the best response of player p_2 is to wait. Conversely, if player p_2 waits, then p_1 is better off transmitting a packet. We can write $br_i(s_{-i})$, the best response of player i to an opponents' strategy vector s_{-i} as follows.

Definition 1.4. *The best response $br_i(s_{-i})$ of player i to the profile of strategies s_{-i} is a strategy s_i such that:*

$$br_i(s_{-i}) = \arg\max_{s_i \in \mathcal{S}_i} u_i(s_i, s_{-i}) \tag{1.3}$$

One can see, that if two strategies are mutual best responses to each other, then no player would have a reason to deviate from the given strategy profile. In the Multiple Access Game, two strategy profiles exist with the above property: (W, A) and (A, W). To identify such strategy profiles in general, Nash introduced the concept of Nash equilibrium in his seminal paper [Nas50]. We can formally define the concept of *Nash equilibrium (NE)* as follows.

Definition 1.5. *The pure strategy profile s^* constitutes a Nash equilibrium if, for each player i,*

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in \mathcal{S}_i \tag{1.4}$$

This means that in a Nash equilibrium, none of the users can unilaterally change his strategy to increase his payoff. Alternatively, a Nash equilibrium is a strategy profile comprised of mutual best responses of the players.

A Nash equilibrium is *strict* [HS88] if we have:

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*), \forall s_i \in \mathcal{S}_i \tag{1.5}$$

It is easy to check that (D, D) is a Nash equilibrium in the Forwarder's Dilemma game represented in Table 1.1. This corresponds to the solution obtained by iterated strict dominance. This result is true in general: Any solution derived by iterated strict dominance is a Nash equilibrium. The proof of this statement is presented in [FT91]. In the Multiple Access Game, however, the iterated dominance techniques do not help us derive the solutions. Fortunately, using the concept of Nash equilibrium, we can identify the two pure-strategy Nash equilibria: (W, A) and (A, W). Note that the best response $br_i(s_{-i})$ is not necessarily unique. For example in the Joint Packet Forwarding Game presented in Table 1.2,

player p_2 has two best responses (D or F) to the move D of player p_1. Multiple best responses are the reason that the solutions of the iterated weak dominance technique in a given game might depend on the order of elimination.

1.2.4 Mixed Strategies

In the examples so far, we have considered only pure strategies, meaning that the players clearly decide on one behavior or another. But in general, a player can decide to play each of these pure strategies with some probabilities; in game-theoretic terms such a behavior is called a *mixed strategy*.

Definition 1.6. *The* mixed strategy $\sigma_i(s_i)$, *or shortly* σ_i, *of player i is a probability distribution over his pure strategies* $s_i \in \mathcal{S}_i$.

Accordingly, we will denote the mixed strategy space of player i by Σ_i, where $\sigma_i \in \Sigma_i$. Hence, the notion of profile, which we defined earlier for pure strategies, is now characterized by the probability distribution assigned by each player to his pure strategies: $\sigma = \sigma_1, ..., \sigma_{|\mathcal{N}|}$, where $|\mathcal{N}|$ is the cardinality of $\mathcal{N}$. As in the case of pure strategies, we denote the strategy profile of the opponents by σ_{-i}. For a finite strategy space, i.e. for so called *finite games*[12] [FT91] for each player, player i's payoff to profile σ is then given by:

$$u_i(\sigma) = \sum_{s_i \in |\mathcal{S}_i|} \sigma_i(s_i) u_i(s_i, \sigma_{-i}) \tag{1.6}$$

Each of the concepts that we have considered so far for pure strategies can be also defined for mixed strategies. As there is no significant difference in these definitions, we refrain from repeating them for mixed strategies.

Let us first study the Multiple Access Game. We call q_1 the probability with which player p_1 decides to access the channel, and q_2 the equivalent probability for p_2 (this means that p_1 and p_2 wait with probability $1 - q_1$ and $1 - q_2$, respectively).

The payoff of player p_1 is:

$$u_1 = q_1(1 - q_2)(1 - C) - q_1 q_2 C = q_1(1 - C - q_2) \tag{1.7}$$

Likewise, we have:

$$u_2 = q_2(1 - C - q_1) \tag{1.8}$$

As usual, the players want to maximize their payoffs. Let us first derive the best response of p_2 for each strategy of p_1. In (1.8), if $q_1 < 1 - C$, then $(1 - C - q_1)$ is positive, and u_2 is maximized by setting q_2 to the highest possible value, namely $q_2 = 1$. Conversely, if $q_1 > 1 - C$, u_2 is maximized by setting $q_2 = 0$ (these two cases will bring us back to the two pure-strategy Nash equilibria that we have already identified). More interesting is the last case, namely $q_1 = 1 - C$, because here u_2 does not depend on q_2 anymore (and is always equal to 0); hence, *any* strategy of p_2 (meaning any value of q_2) is a best response. The game being symmetric, reversing the roles of the two players leads of course to the same result. This means that $(q_1 = 1 - C, q_2 = 1 - C)$ is a mixed-strategy Nash equilibrium for the Multiple Access Game.

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[12]The general formula for infinite strategy space is slightly more complicated. The reader can find it in [FT91] or [OR94].
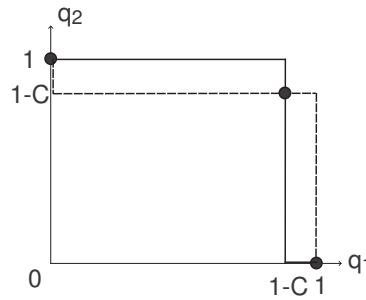
Figure 1.4: Best response functions in the Multiple Access Game. The best response function of player p_1 (q_1 as a function of q_2) is represented by the dashed line; that of player p_2 (q_2 as a function of q_1) is represented by the solid one. The two dots at the edges indicate the two pure-strategy Nash equilibria and the one in the middle shows the mixed-strategy Nash equilibrium.

We can graphically represent the best responses of the two players as shown in Figure 1.4. In the graphical representation, we refer to the set of best response values as the *best response function*.[13] Relying on the concept of mutual best responses, one can identify the Nash equilibria as the crossing points of these best response functions.

Note that the number of Nash equilibria varies from game to game. There are games with no pure-strategy Nash equilibrium, such as the Jamming Game. We show the strategic form of this game in Table 1.4.

		p_2 (jammer)	
		x	y
p_1 (sender)	x	(-1,1)	(1,-1)
	y	(1,-1)	(-1,1)

Table 1.4: The Jamming Game in strategic form.

The reader can easily verify that the Jamming Game cannot be solved by iterated strict or weak dominance. Moreover, this game does not even admit a pure-strategy Nash equilibrium. In fact, there exists only a mixed-strategy Nash equilibrium in this game that dictates each player to play a uniformly random distribution strategy (i.e., select one of the channels with probability 0.5).

The importance of mixed strategies is further reinforced by the following theorem of Nash [Nas50, Nas51]. This theorem is a crucial existence result in game theory. The proof uses the Brouwer-Kakutani fixed-point theorem and is provided in [Gib92].

Theorem 1.1 (Nash, 1950)**.** *Every finite strategic-form game has a mixed-strategy Nash equilibrium.*

1.2.5 Efficiency of Nash Equilibria and Equilibrium Selection

As we have seen so far, the first step in solving a game is to investigate the *existence* of Nash equilibria. Theorem 1.1 states that in a broad class of games there always exists at least a mixed-strategy Nash equilibrium. However, in some cases, such as in the Jamming Game, there exists no pure-strategy Nash

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[13]Let us emphasize that, according to the classic definition of a function in calculus, the set of best response values does not correspond to a function, because there might be several best responses to a given opponent strategy profile.

equilibrium. Once we have verified that a Nash equilibrium exists, we have to determine if it is a *unique* equilibrium point. If there is a unique Nash equilibrium, then we have to study its *efficiency*. One specific concept that characterizes the efficiency of Nash equilibria is the *price of anarchy (POA)* [KP99, Rou05]. The price of anarchy of a game is the ratio between the sum of the payoff of all players in a globally optimal solution compared to the those achieved in a worst-case Nash equilibrium (or alternatively, how much more cost does the Nash equilibrium bear with respect to the globally optimal solution). If we consider the best-case Nash equilibria instead of the worst-case Nash equilibria, then we refer to the *price of stability* [Rou05].

Efficiency can also be used to compare different Nash equilibria and to select the most appropriate one. *Equilibrium selection* means that the users have identified the desired Nash equilibrium profiles, but they also have to coordinate which one to choose. For example, in the Multiple Access Game both players are aware that there exist three Nash equilibria with different payoffs, but each of them tries to be "the winner" by deciding to access the channel (in the expectation that the other player will wait). Hence, their actions result in a profile that is not a Nash equilibrium. The topic of equilibrium selection is one of the active research fields in game theory [FL98, Sam97].

1.2.6   Pareto-Optimality

So far, we have seen how to identify Nash equilibria. We have also seen that there might be several Nash equilibria, as in the Joint Packet Forwarding Game. One method for identifying the desired Nash equilibrium point in a game is to compare strategy profiles using the concept of Pareto-optimality. To introduce this concept, let us first define Pareto-superiority.

Definition 1.7. *The strategy profile s is* Pareto-superior *to the strategy profile $s^{'}$ if for any player $i \in N$:*

$$u_i(s_i, s_{-i}) \geq u_i(s_i^{'}, s_{-i}^{'}) \tag{1.9}$$

with strict inequality for at least one player.

In other words, the strategy profile s is *Pareto-superior* to the strategy profile $s^{'}$, if the payoff of a player i can be increased by changing from $s^{'}$ to s without decreasing the payoff of other players. The strategy profile $s^{'}$ is defined as *Pareto-inferior* to the strategy profile s. Note that the players might need to change their strategies simultaneously to reach the Pareto-superior strategy profile s.

Based on the concept of Pareto-superiority, we can identify the most efficient strategy profile or profiles.

Definition 1.8. *The strategy profile s^{po} is* Pareto-optimal *(or Pareto-efficient) if there exists no other strategy profile that is Pareto-superior to s^{po}.*

In a Pareto-optimal strategy profile, one cannot increase the payoff of player i without decreasing the payoff of at least one other player. Using the concept of Pareto-optimality, we can eliminate the Nash equilibria that can be improved by changing to a more efficient (i.e. Pareto-superior) strategy profile. The game can have several Pareto-optimal strategy profiles and the set of these profiles is called the *Pareto-frontier*. It is important to stress that a Pareto-optimal strategy profile is not necessarily a Nash equilibrium.

We can now use the concept of Pareto-optimality to study the efficiency of pure-strategy Nash equilibria in our running examples.

- In the Forwarder's Dilemma game, the Nash equilibrium (D, D) is not Pareto-optimal. The strategy profile (F, F) is Pareto-optimal, but not a Nash equilibrium.

- In the Joint Packet Forwarding game, both strategy profiles (F, F) and (D, D) are Nash equilibria, but only (F, F) is Pareto-optimal.

- In the Multiple Access Game, both pure-strategy profiles (A, W) and (W, A) are Nash equilibria and Pareto-optimal.

- In the Jamming game, there exists no pure-strategy Nash equilibrium, and all strategy profiles are Pareto-optimal (because it is a zero-sum game).

We have seen that the Multiple Access Game has three Nash equilibria. We can notice that the mixed-strategy Nash equilibrium $\sigma = (q_1 = 1 - C, q_2 = 1 - C)$ results in the expected payoffs $(0, 0)$. Hence, this mixed-strategy Nash equilibrium is Pareto-inferior to the two pure-strategy Nash equilibria. In fact, it can be shown in general that there exists no mixed strategy profile that is Pareto-superior to all pure strategy profiles, because any mixed strategy of a player i is a linear combination of his pure-strategies with positive coefficients that sum up to one.

1.3 Dynamic Games

In the strategic-form representation it is usually assumed that the players make their moves *simultaneously* without knowing what the other players do. This might be a reasonable assumption in some problems, for example in the Multiple Access Game. In most of the games, however, the players might have a *sequential* interaction, meaning that the move of one player is conditioned by the move of the other player (i.e., the second mover knows the move of the first mover before making his decision). These games are called *dynamic games* and we can represent them in an *extensive form*. We refer to a *game with perfect information*, if the players have a perfect knowledge of all previous moves in the game at any moment they have to make a new move.

1.3.1 Extensive Form with Perfect Information

In the *extensive form*, the game is represented as a tree, where the root of the tree is the start of the game and shown with an empty circle. We refer to one level of the tree as a *stage*. The nodes of a tree, denoted by n and a filled circle, show the possible unfolding of the game, meaning that they represent the sequence relation of the moves of the players. This sequence of moves defines a *path* on the tree and is referred to as the *history* h of the game. It is generally assumed that a single player can move when the game is at a given node.[14] This player is represented as a label on the node. Note that this is a tree, thus each node is a complete description of the path preceding it (i.e., each node has a unique history). The moves that lead to a given node are represented on each branch of the tree. Each terminal node (i.e., leaf) of the tree defines a potential end of the game called *outcome* and it is assigned the corresponding payoffs. In addition, we consider *finite-horizon games*, which means that there exist a finite number of stages. Otherwise, we call it an *infinite-horizon game*.

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[14]Osborne and Rubinstein [OR94] define a game where a set of players can move in one node. Also, there exist specific examples in [FT91], in which different players move in the same stage. For the clarity of presentation, we do not discuss these specific examples in this tutorial.

Note that the extensive form is a more convenient representation, but basically every extensive form can be transformed to a strategic form and vice versa. However, extensive-form games can be used to describe *sequential* interactions more easily than strategic-form games. In extensive form, the *strategy* of player i assigns a move $m_i(h(n))$ to every non-terminal node in the game tree with the history h. The strategies define a Nash equilibrium, which definition is basically the same as the one provided in Definition 1.5. For simplicity, we use pure strategies in this section.

To illustrate these concepts, let us consider the *Sequential Multiple Access Game*. This is a modified version of the Multiple Access Game supposing that the two transmitters p_1 and p_2 are not perfectly synchronized, which means that p_1 always moves first (i.e., accesses or not) and p_2 observes the move of p_1 before making his own move.[15] We show this extensive form game with perfect information in Figure 1.5. In this game, the strategy of player p_1 is to access the channel (A) or to wait (W). But the strategy of player p_2 has to define a move given the previous move for player p_1. Thus, the possible strategies of p_2 are AA, AW, WA and WW, where for example AW means that player p_2 accesses if p_1 does the same and he waits if p_1 waits as well. Thus, we can identify the pure-strategy Nash equilibria in the Sequential Multiple Access Game. It turns out that there exist three pure-strategy Nash equilibria: (A, WA), (A, WW) and (W, AA).
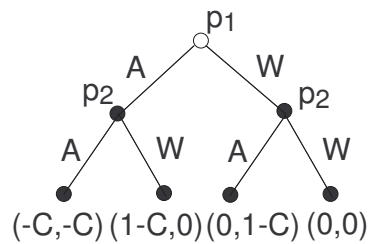
Figure 1.5: The Sequential Multiple Access Game in extensive form.

Kuhn formulated a relevant existence theorem about Nash equilibria in finite extensive-form games in [Kuh53]. The intuition of the proof is provided in [FT91].

Theorem 1.2 (Kuhn, 1953). *Every finite extensive-form game of perfect information has a pure-strategy Nash equilibrium.*

The proof relies on the concept of backward induction, which we introduce in the following.

1.3.2 Backward Induction

We have seen that there exist three Nash equilibria in the Sequential Multiple Access Game. For example, if player p_2 plays the strategy AA, then the best response of player p_1 is to play W. Let us notice, however, that the claim of player p_2 to play AA is an *incredible (or empty) threat*. Indeed, AA is not the best strategy of player p_2 if player p_1 chooses A in the first round.

In games with perfect information, we can eliminate equilibria based on such incredible threats using the technique of *backward induction*. Let us first solve the Sequential Multiple Access Game presented in Figure 1.5 with the backward induction method as shown in Figure 1.6.

The Sequential Multiple Access Game is a finite game with complete information. Hence, player p_2 knows that he is the player that has the last move. For each possible history, he predicts his best move.

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[15]In fact, this is called the *carrier sense* and it is the basic technique in the CSMA/CA protocols [Rap02, Sch05].
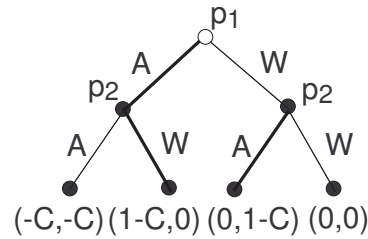
Figure 1.6: The backward induction solution of the Sequential Multiple Access Game in extensive form.

For example, if the history is $h = A$ in the game, then player p_2 concludes that the move W results in the best payoff for him in the last stage. Similarly, player p_2 defines A as his best move following the move W of player p_1. In Figure 1.6, similarly to the examples in [Gib92], we represent these best choices with thick solid lines in the last game row. Given all the best moves of player p_2 in the last stage, player p_1 calculates his best moves as well. In fact, each reasoning step reduces the extensive form game by one stage. Following this backward reasoning, we arrive at the beginning of the game (the root of the extensive-form tree). The continuous thick line from the root to one of the leaves in the tree gives us the backward induction solution. In the Sequential Multiple Access Game, we can identify the backward induction solution as $h = \{A, W\}$. Backward induction can be applied to any finite game of perfect information. This technique assumes that the players can reliably forecast the behavior of other players and that they believe that the other can do the same. Note, however, that this argument might be less compelling for longer extensive-form games due to the complexity of prediction. Note that the technique of backward induction is analogous to the technique of iterated strict dominance in strategic-form games. It is an elimination method for reducing the game.

Let us now derive the backward induction solution in the Sequential Multiple Access Game by considering how player p_1 argues. If p_1 chooses A, then the best response for p_2 is to play WW or WA, which results in the payoff of $1 - C$ for p_1. However, if p_1 chooses Q, then the best response of p_2 is WA or AA, which results in the payoff of zero for p_1. Hence, p_1 will choose A and (A, WA) or (A, WW) are the backward induction solutions in the Sequential Multiple Access Game.

Let us now briefly discuss the extensive form of the other three wireless networking examples with sequential moves. In the extensive-form version of the Forwarder's Dilemma, the conclusions do not change. Both players will drop each others' packets. In the extensive form of the Joint Packet Forwarding Game, if player p_1 chooses D, then the move of player p_2 is irrelevant. Hence by induction, we deduce that the backward induction solution is (F, F). Finally, in the Jamming Game, let us assume that p_1 is the leader and the jammer p_2 is the follower. In this case, the jammer can easily observe the move of p_1 and jam.

1.3.3 Extensive Form with Imperfect Information

In this section, we will extend the notions of history and information. As we have seen, in the game with perfect information, the players always know the moves of all the other players when they have to make their moves. However, in the examples with simultaneous moves (e.g., the static games in Section 1.2), the players have an *imperfect information* about the unfolding of the game. To define perfect information more precisely, let us first introduce the notion of *information set* $h(n)$, i.e. the amount of information the players have at the moment they choose their moves in a given node n. The information set $h(n)$ is a

partition of the nodes in the game tree. The intuition of the information set is that a player moving in n is uncertain if he is in node n or in some other node $n^{'} \in h(n)$. We can now formally define the concept of *perfect information.*[16]

Definition 1.9. *The players have a* perfect information *in the game if every information set is a singleton (meaning that each player always knows the previous moves of all players when he has to make his move).*

It is not a coincidence that we use the same notation for the information set as for the history. In fact, the concept of information set is a generalized version of the concept of history.

To illustrate these concepts, let us first consider the extensive form of the original Multiple Access Game shown in Figure 1.7. Recall that this is a game with imperfect information. The dashed line represents the information set of player p_2 at the time he has to make his move. The set of nodes in the game tree circumvented by the dashed line means that player p_2 does not know whether player p_1 is going to access the channel or not at the time he makes his own move, i.e. that they make simultaneous moves.
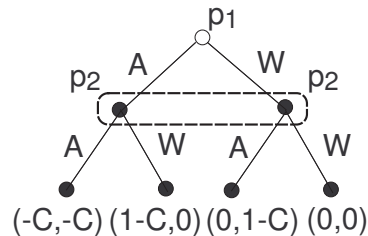
Figure 1.7: The original Multiple Access Game in extensive form. It is a game with imperfect information.

The *strategy* of player i assigns a move $m_i(h(n))$ to every non-terminal node n in the game tree with the information set $h(n)$. Again, we deliberately restrict the strategy space of the players to pure strategies, but the reasoning holds for mixed strategies as well [FT91, OR94]. The possible strategies of each player in the Multiple Access Game are to access the channel (A) or to wait (W). As we have seen before, both (A, W) and (W, A) are pure-strategy Nash equilibria. Note that in this game, player p_2 cannot condition his move on the move of player p_1.

1.3.4 Subgame-Perfect Equilibria

As we have seen in Section 1.3.2, backward induction can be used to eliminate incredible threats. Unfortunately, the elimination technique based on backward induction cannot always be used. To illustrate this, let us construct the game called *Multiple Access Game with Retransmissions* and solve it in the pure strategy space. In this game, the players play the Sequential Multiple Access Game, and they play the Multiple Access Game if there is a collision (i.e., they both try to access the channel). We show the extensive form in Figure 1.8.

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[16]Note that two well-established textbooks on game theory, [FT91] and [OR94], have different definitions of perfect information. We use the interpretation of [FT91], which we believe is more intuitive. The authors of [OR94] define, in Chapter 6 of their book, a game with simultaneous moves also as a game with perfect information, where the players are substituted with a set of players, who make their moves. Indeed, there seems to be no consensus in the research community either.
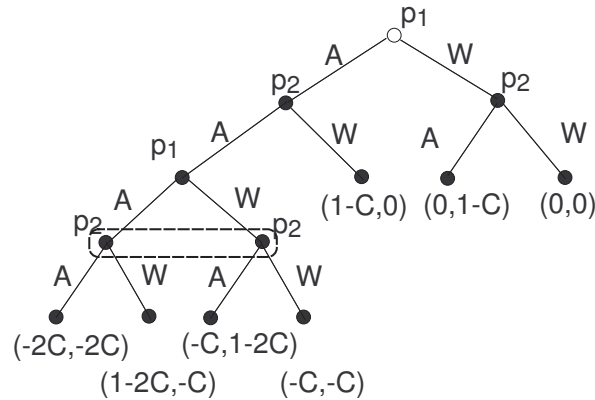
Figure 1.8: The Multiple Access Game with Retransmissions in extensive form. It is also a game with imperfect information.

Note that the players have many more strategies than before. Player p_1 has four strategies, because there exist two information sets, where he has to move; and he has two possible moves at each of these information sets. For example, the strategy $s_1 = AW$ means that player p_1 accesses the channel at the beginning and he waits in the second Multiple Access Game. Similarly, player p_2 has $2^3 = 8$ strategies, but they are less trivial to identify. For example, each move in the strategy $s_2 = WAA$ means the following: (i) the first move means that player p_2 waits if player p_1 accessed, or (ii) p_2 accesses if p_1 waited and (iii) p_2 accesses in the last stage if they both accessed in the first two stages. This example highlights an important point: *The strategy defines the moves for a player for every information set in the game, even for those information sets that are not reached if the strategy is played.* The common interpretation of this property is that the players may not be able to perfectly observe the moves of each other and thus the game may evolve along a path that was not expected. Alternatively, the players may have incomplete information, meaning that they have certain beliefs about the payoffs of other players and hence, they may try to solve the game on this basis. These beliefs may not be precise and so the unfolding of the game may be different from the predicted unfolding. Game theory covers these concepts in the notion of Bayesian games [FT91], but we do not present this topic in our tutorial due to space constraints.

It is easy to see that the Multiple Access Game with Retransmissions cannot be analyzed using backward induction, because the Multiple Access Game in the second stage is of imperfect information. To overcome this problem, Selten suggested the concept called *subgame perfection* in [Sel65, HS88]. In Figure 1.8, the Multiple Access Game in the second stage is a *proper subgame* of the Multiple Access Game with Retransmissions. Let us now give the formal definition of a proper subgame.

Definition 1.10. *The game $\mathbf{G}'$ is a* proper subgame *of an extensive-form game $\mathbf{G}$ if it consists of a single node in the extensive-form tree and all of its successors down to the leaves. Formally, if a node $n \in \mathbf{G}'$ and $n' \in h(n)$, then $n' \in \mathbf{G}'$. The information sets and payoffs of the subgame $\mathbf{G}'$ are inherited from the original game $\mathbf{G}$: this means that n and n' are in the same information set in $\mathbf{G}'$ if they are in the same information set in $\mathbf{G}$; and the payoff function of $\mathbf{G}'$ is the restriction of the original payoff function to $\mathbf{G}'$.*

Now let us formally define the concept of subgame perfection. This definition reduces to backward induction in finite games with perfect information.

Definition 1.11. *The strategy profile s is a* subgame-perfect equilibrium *of a finite extensive-form game* **G** *if it is a Nash equilibrium of any proper subgame* **G**$'$ *of the original game* **G**.

One can check the existence of subgame perfect equilibria by applying the *one-deviation property*.

Definition 1.12. *The* one-deviation property *requires that there must not exist any information set, in which a player i can gain by deviating from his subgame perfect equilibrium strategy and conforming to it in other information sets.*

A reader somewhat familiar with dynamic programming may wonder about the analogy between the optimization in game theory and in dynamic programming [Bel57]. Indeed, the one-deviation property corresponds to the *principle of optimality* in dynamic programming, which is based on backward induction. Hence, strategy profile s is a *subgame-perfect equilibrium* of a finite extensive-form game **G** if the one-deviation property holds.

Subgame perfection provides a method for solving the Multiple Access Game with Retransmissions. We can simply replace the Multiple Access Game subgame (the second one with simultaneous moves) with one of his pure-strategy Nash equilibria. Hence, we can obtain one of the game trees presented in Figure 1.9. Solving the reduced games with backward induction, we can derive the following solutions. In the game shown in Figure 1.9a, we have the subgame perfect equilibrium (WW, AAA). In Figure 1.9b we obtain the subgame perfect equilibria $(AA, W*W)$, where $*$ means any move from $\{A, W\}$.
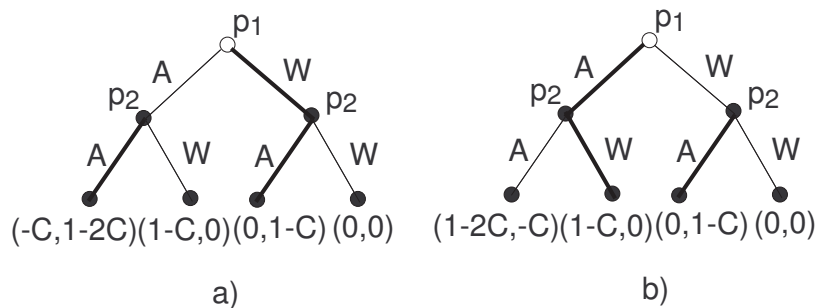
Figure 1.9: Application of subgame perfection to the Multiple Access Game with Retransmissions. In a) the proper subgame is replaced by one of the Nash equilibria of that game, namely (W, A). Solution b) represents the case, where the subgame is replaced by the other Nash equilibrium (A, W). The thick lines show the result of the backward induction procedure on the reduced game trees.

Because any game is a proper subgame of itself, a subgame-perfect equilibrium is necessarily a Nash equilibrium, but there might be Nash equilibria in **G** that are not subgame-perfect. In fact, the concept of Nash equilibrium does not require that the one-deviation property holds. We leave it to the reader as an exercise to verify that there are more Nash equilibria than subgame-perfect equilibria in the Multiple Access Game with Retransmissions.

The concept of subgame perfection has often been criticized with arguments based on equilibrium selection (recall the issue from Section 1.2.5). Many researchers point out that the players might not be able to determine how to play if several Nash equilibria exist in a given subgame. As an example, they might both play A in the Multiple Access Game with Retransmissions in the second subgame as well. This disagreement can result in an outcome that is not an equilibrium according to the definitions considered so far.

1.4 Repeated games

So far, we have assumed that the players interact only once and we modeled this interaction in a static game in strategic form in Section 1.2 and partially in Section 1.3. Furthermore, we have seen the Multiple Access Game with Retransmissions, which was a first example to illustrate repeated games, although the number of stages was quite limited. As we have seen in Section 1.3, the extensive form provides a more convenient representation for sequential interactions. In this section, we assume that the players interact several times and hence we model their interaction using a *repeated game*. The analysis of repeated games in extensive form is basically the same as presented in Section 1.3, hence we focus on the strategic form in this section. To be more precise, we consider repeated games with *observable actions* and *perfect recall*: this means that each player knows all the moves of others, and that each player knows his own previous moves at each stage in the repeated game.

1.4.1 Basic Concepts

In repeated games, the players interact several times. Each interaction is called a *stage*. Note that the concept of stage is similar to the one in extensive form, but here we assume that the players make their moves simultaneously in each stage. The set of players is defined similarly to the static game presented in Section 1.2.1.

As a running example, let us consider the *Repeated Forwarder's Dilemma*, which consists of the repetition of the Forwarder's Dilemma stage game. In such a repeated game, all past moves are common knowledge at each stage t. The set of the past moves at stage t is commonly referred to as the history $h(t)$ of the game. We call it a history (and not an information set), because it is uniquely defined at the beginning of each stage. Let us denote the move of player i in stage t by $m_i(t)$. We can formally write the history $h(t)$ as follows:

$$h(t) = \{(m_1(t), \dots, m_{|\mathcal{N}|}(t)), \dots, (m_1(0), \dots, m_{|\mathcal{N}|}(0))\} \tag{1.10}$$

For example, at the beginning of the third stage of the Repeated Forwarder's Dilemma, if both players always cooperate, the history is $h(2) = \{(F, F), (F, F)\}$.

The *strategy* s_i defines a move for player i in the next stage $t+1$ for each history $h(t)$ of the game.[17]

$$m_i(t+1) = s_i(h(t)) \tag{1.11}$$

Note that the initial history $h(0)$ is an empty set. The strategy s_i of player i must define a move $m_i(0)$ for the initially empty history, which is called the *initial move*. For a moment, suppose that the Repeated Forwarder's Dilemma has two stages. Then one example strategy of each player is $FFFFF$, where the entries of the strategy define the forwarding behavior for the following cases: (i) in the first stage, i.e. as an initial move, (ii) if the history was $h(1) = \{(F, F)\}$, (iii) if the history was $h(1) = \{(F, D)\}$, etc. As we can notice, the strategy space grows very quickly as the number of stages increases: In the two-stage Repeated Forwarder's Dilemma, we have $|\mathcal{S}_i| = 2^5 = 32$ strategies for each player i. Hence in repeated games, it is typically infeasible to make an exhaustive search for the best strategy and hence for Nash equilibria.

The payoff in the repeated game might change as well. In repeated games, the users typically want to maximize their payoff for the whole duration $\mathbf{T}$ of the game. Hence, they maximize:

$$u_i = \sum_{t=0}^{\mathbf{T}} u_i(t, s) \tag{1.12}$$

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[17]Recall that in the static game, the strategy was a single move.

where $u_i(t, s)$ denotes the *stage payoff*, i.e., the payoff player i receives in stage t.

In some cases, the objective of the players in the repeated game can be to maximize their payoffs only for the next stage (i.e., as if they played a static game). We refer to these games as *myopic games* as the players are short-sighted optimizers. If the players maximize their total payoff during the game, we call it a *long-sighted game*.

Recall that we refer to a finite-horizon game if the number of stages **T** is finite. Otherwise, we refer to an infinite-horizon game. We will see in Section 1.4.3 that we can also model finite-horizon games with an unpredictable end as an infinite-horizon game with specific conditions.

1.4.2   Nash Equilibria in Finite-Horizon Games

Let us first solve the finite Repeated Forwarder's Dilemma using the concept of Nash equilibrium. Assume that the players are long-sighted and want to maximize their total payoff (the outcome of the game). As we have seen, it is computationally infeasible to calculate the Nash equilibria based on strategies that are mutual best responses to each other as the number of stages increases. Nevertheless, we can apply the concept of subgame perfection as we have learned in Section 1.3.4. Because the game is of complete information, the players know the end of it. Now, in the last stage game, they both conclude that their dominant strategy is to drop the opponent's packet (i.e., to play D). Given this argument, their best strategy is to play D in the penultimate stage. Following the same argument, this technique of backward induction dictates that the players should choose a strategy that plays D in every stage. Note that many strategies exist with this property.

In repeated games in general, it is computationally infeasible to consider all possible strategies for every possibly history, because the strategy space increases exponentially with the length of the game. Hence, one usually restricts the strategy space to a reasonable subset. One widely-used family of strategies is the *strategies of history-1*. These strategies take only the moves of the opponents in the *previous stage* into account (meaning that they are "forgetful" strategies, because they "forget" the past behavior of the opponent). In the games we have considered thus far, we have two players and hence the history-1 strategy of player i in the repeated game can be expressed by the *initial move* $m_i(0)$ and the following strategy function:

$$m_i(t + 1) = s_i(m_j(t)) \tag{1.13}$$

Accordingly, we can define the strategies in the Repeated Forwarder's Dilemma as detailed in Table 1.5. Note that these strategies might enable a feasible analysis in general, i.e., if there exists a large number of stages.

We can observe that in the case of some strategies, such as All-D or All-C, the players do not condition their next move on the previous move of the opponents. One refers to these strategies as *non-reactive strategies*. Analogously, the strategies that take the opponents' behavior into account are called *reactive strategies* (for example, TFT or STFT).

Let us now analyze the Repeated Forwarder's Dilemma assuming that the players use the history-1 strategies. We can conclude the same result as with the previous analysis.

Theorem 1.3. *In the Repeated Forwarder's Dilemma, the strategy profile (All-D, All-D) is a Nash equilibrium.*

Although not proven formally, the justification of the above theorem is provided in [Axe84].

Let us now consider the finite repeated version of the Multiple Access Game. Suppose that accessing the channel A corresponds to defection (competitive behavior) and waiting W corresponds to cooperation. Then, All-D for example means to access the channel independently of the move of the other player.

$m_1(0)$	$m_1(t)\|m_2(t) = F$	$m_1(t)\|m_2(t) = D$	strategy function s_1	name of the strategy
D	D	D	$m_1(t+1) = D$	Always Defect (All-D)
D	F	D	$m_1(t+1) = m_2(t)$	Suspicious Tit-For-Tat (STFT)
D	D	F	$m_1(t+1) = \overline{m_2(t)}$	Suspicious Anti Tit-For-Tat (SATFT)
D	F	F	$m_1(t+1) = F$	Suspicious Always Cooperate (S-All-C)
F	D	D	$m_1(t+1) = D$	Nice Always Defect (Nice-All-D)
F	F	D	$m_1(t+1) = m_2(t)$	Tit-For-Tat (TFT)
F	D	F	$m_1(t+1) = \overline{m_2(t)}$	Anti Tit-For-Tat (ATFT)
F	F	F	$m_1(t+1) = F$	Always Cooperate (All-C)

Table 1.5: History-1 strategies of player 1 in the Repeated Forwarder's Dilemma. The entries in the first three columns represent: the initial move of player p_1, a move of player p_1 to a previous move $m_2(t) = F$ of player p_2, and the move of p_1 as a response to $m_2(t) = D$. The bar represents the alternative move (e.g., $\overline{F} = D$). As an example, let us highlight the TFT strategy, which begins the game with forwarding (i.e., cooperation) and then copies the behavior of the opponent in the previous stage.

As opposed to the two-stage Repeated Forwarder's Dilemma, this game admits several Nash equilibria in the history-1 strategy space: (All-D, All-C), (All-C, All-D), (STFT, TFT) and (TFT, STFT). We leave the verification of this claims as an exercise for the interested reader.

1.4.3 Infinite-Horizon Games with Discounting

In the game theory literature, infinite-horizon games with discounting are used to model a finite-horizon game in which the players are not aware of the duration of the game. Clearly, this is often the case in strategic interactions, in particular in networking operations. In order to model the unpredictable end of the game, one decreases the value of future stage payoffs. This technique is called *discounting*. In such a game, the players maximize their *discounted total payoff*:

$$u_i = \sum_{t=0}^{\infty} u_i(t, s) \cdot \delta^t \tag{1.14}$$

where δ denotes the *discounting factor*. The *discounting factor* δ determines the decrease of the value for future payoffs, where $0 < \delta < 1$ (although in general, we can assume that δ is close to one). The discounted total payoff expressed in (1.14) is often normalized, and thus we call it the *normalized payoff*:

$$u_i = (1 - \delta) \cdot \sum_{t=0}^{\infty} u_i(t, s) \cdot \delta^t \tag{1.15}$$

The role of the factor $1 - \delta$ is to let the normalized payoff of the repeated game be expressed in the same unit as the static game. Indeed, with this definition, if the stage payoff $u_i(t, s) = 1$ for all $t = 0, 1, ...$, then the normalized payoff is equal to 1, because $\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$.

We have seen that the Nash equilibrium in the finite Repeated Forwarder's Dilemma was a non-cooperative one. Yet, this rather negative conclusion should not affect our morale: in most networking problems, it is reasonable to assume that the number of iterations (e.g., of packet transmissions) is very large and a priori unknown to the players. Therefore, as discussed above, games are usually assumed to have an infinite number of repetitions. And, as we will see, infinitely repeated games can lead to more cooperative behavior.

Consider the history-1 strategies All-C and All-D for the players in the Repeated Forwarder's Dilemma. Thanks to the normalization in (1.15), the corresponding normalized payoffs are exactly those presented in Table 1.1. A conclusion similar to the one we drew in Section 1.4.2 can be directly derived at this time. The strategy profile (All-D, All-D) is a Nash equilibrium: If the opponent always defects, the best response is All-D. A sketch of proof is provided (for the Prisoner's Dilemma) in [FT91].

To show other Nash equilibria, let us first define the *Trigger* strategy. If a player i plays Trigger, then he forwards in the first stage and continues to forward as long as the other player j does not drop. As soon as the opponent j drops his packet, player i drops all packets for the rest of the game. Note that Trigger is not a history-1 strategy. The Trigger strategy applies the general technique of *punishments*.

If no players drops a packet, the payoffs corresponds to (F, F) in Table 1.1, meaning that it is equal to $1 - C$ for each player. If a player i plays $m_i(t) = D$ at stage t, his payoff will be higher at this stage (because he will not have to face the cost of forwarding), but it will be zero for all the subsequent stages, as player j will then always drop. The normalized payoff of player i will be equal to:

$$(1 - \delta) \left[(1 + \delta + ... + \delta^{t-1})(1 - C) + \delta^t \cdot 1 \right] = 1 - C + \delta^t(C - \delta) \tag{1.16}$$

As $C < \delta$ (remember that, in general, C is very close to zero, whereas δ is very close to one), the last term is negative and the payoff is therefore smaller than $1 - C$. In other words, even a single defection leads to a payoff that is smaller than the one provided by All-C. Hence, a player is better off always forwarding in this infinite-horizon game, in spite of the fact that, as we have seen, the stage game only has (D, D) as an equilibrium point. It can be easily proven that (Trigger, Trigger) is a Nash equilibrium and that it is also Pareto-optimal (the intuition for the latter is the following: there is no way for a player to go above his normalized payoff of $1 - C$ without hurting his opponent's payoff). Note that by similar arguments, one can show that (TFT, TFT) is also a Pareto-optimal Nash equilibrium, because it results in the payoff $1 - C$ for each of the players.

It is important to mention that the players cannot predict the end of the game and hence they cannot exploit this information. As mentioned in [FT91], reducing the information or the strategic options (i.e., decreasing his own payoff) of a player might lead to a better outcome in the game. This uncertainty is the real reason that the cooperative equilibrium appears in the repeated version of the Forwarder's Dilemma game.

1.4.4 The Folk Theorem

We will now explore further the mutual influence of the players' strategies on their payoffs. We will start by defining the notion of *minmax value* (sometimes called the *reservation utility*). The minmax value is the lowest stage payoff that the opponents of player i can force him to obtain with punishments, provided that i plays the best response against them. More formally, it is defined as follows:

$$\underline{u_i} = \min_{s_{-i}} \left[\max_{s_i} u_i(s_i, s_{-i}) \right] \tag{1.17}$$

This is the lowest stage payoff that the opponents can enforce on player i. Let us denote by $s_{min} = \{s_{i,min}, s_{-i,min}\}$ the strategy profile for which the minimum is reached in (1.17). We call the $s_{-i,min}$ the *minmax profile* against player i within the stage game.

It is easy to see that player p_i can obtain at least his minmax value $\underline{u_i}$ in any stage and hence we call *feasible payoffs* the payoffs higher than the minmax payoff. In the Repeated Forwarder's Dilemma, the feasible payoffs for any player p_1 are higher than 0. Indeed, by playing s_1=All-D, he is assured to obtain at least that value, no matter what the strategy of p_2 can be. Similar argument applies to player p_2. Let

us graphically represent the feasible payoffs in Figure 1.10. We highlight the convex hull of payoffs that are strictly non-negative for both players as the set of feasible payoffs.
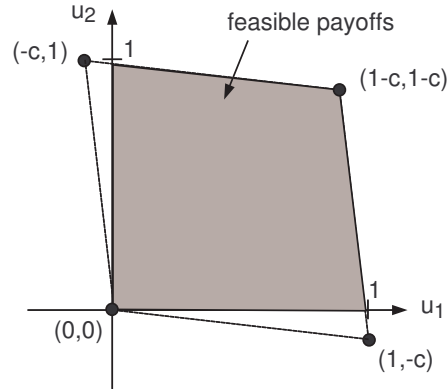
Figure 1.10: The feasible payoffs in the Repeated Forwarder's Dilemma.

The notion of minmax what we have just defined refers to the stage game, but it has a very interesting application in the repeated game, as the following theorem shows.

Theorem 1.4. *Player i's normalized payoff is at least equal to $\underline{u_i}$ in any equilibrium of the infinitely repeated game, regardless of the level of the discount factor.*

The intuition can be obtained again from the Repeated Forwarder's Dilemma: a player playing All-D will obtain a (normalized) payoff of at least 0. The theorem is proven in [FT91].

We are now in a position to introduce a fundamental result, which is of high relevance to our framework: the Folk Theorem.[18]

Theorem 1.5 (Folk Theorem). *For every feasible payoff vector $u = \{u_i\}_i$ with $u_i > \underline{u_i}$, there exists a discounting factor $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$ there is a Nash equilibrium with payoffs u.*

The intuition is that if the game is long enough (meaning that δ is sufficiently close to 1), the gain obtained by a player by deviating once is outweighed by the loss in every subsequent period, when loss is due to the punishment (minmax) strategy of the other players.

We have seen the application of this theorem in the infinite Repeated Forwarder's Dilemma. A player is deterred from deviating, because the short term gain obtained by the deviation (1 instead of $1 - C$) is outweighed by the risk of being minmaxed (for example using the Trigger strategy) by the other player (provided that $C < \delta$).

1.5 Discussion

One of the criticisms of game theory, as applied to the modeling of human decisions, is that human beings are, in practice, rarely fully rational. Therefore, modeling the decision process by means of a few equations and parameters is questionable. In wireless networks, the users do not interact with each other

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[18]This denomination of "folk" stems from the fact that this theorem was part of the oral tradition of game theorists, before it was formalized. Strictly speaking, we present the folk theorem for the discounting criterion. There exist different versions of the folk theorem, each of them is proved by different authors (as they are listed in [OR94] at end of Chapter 8).

on such a fine-grained basis as forwarding one packet or access the channel once. Typically, they (or the device manufacturer, or the network operator, if any) program their devices to follow a protocol (i.e., a strategy) and it is reasonable to assume that they rarely reprogram their devices. Hence, such a device can be modeled as a rational decision maker. Yet, there are several reasons the application of game theory to wireless networks can be criticized. We detail them here, as they are usually never mentioned, for understandable reasons, in research papers.

Payoff function and cost   The first issue is the notion of payoff function: How important is it for a given user that a given packet is properly sent or received? This very much depends on the situation: the packet can be a crucial message, or could just convey a tiny portion of a figure appearing in a game. Likewise, the sensitivity to delay can also vary dramatically from situation to situation.

Similarly, the definition of cost might be a complex issue as well. In our examples (and in the state of the art in the application of game theory to wireless networks), the cost represents the energy consumption of the devices. In some cases, however, a device can be power-plugged, thus its "cost" could be neglected. Likewise, a device whose battery is almost depleted probably has a different evaluation of cost than when his battery is full. Furthermore, cost can include other considerations than energy, such as the previously mentioned delay or the consumed bandwidth. Expressing this diversity properly in the game models is still an open research issue.

Pricing and mechanism design   Mechanism design is concerned with the question of how to lead the players to a desirable equilibrium by changing (designing) some parameters of the game. In particular, pricing is considered to be a good technique for regulating the usage of a scarce resource by adjusting the costs of the players. Many network researchers have contributed to this field. These contributions provide a better understanding of specific networking mechanisms.

Yet it is not clear today, even for wired networks how relevant these contributions are going to be in practice. Usually the pricing schemes used in reality by operators are very coarse-grained, because operators tend to charge based on investment and personnel costs and on the pricing strategy of their competitors, and not on the instantaneous congestion of the network. If a part of the network is frequently congested, they will increase the capacity (deploy more base stations, more optical fibers, more switches) rather than throttle the user consumption by pricing.

Hence, the only area where pricing has practical relevance is probably for service provisioning *among* operators (e.g., renting transmission capacity); but very little has been published so far on this topic.

Infinite-horizon games   As mentioned, games in networking are usually assumed to be of infinite horizon, in order to capture the idea that a given player does not know when the interaction with another player will stop. This is, however, not perfectly true. For example, a given player/device could "know" that he/it is about to be turned off and moved away (e.g., his/its owner is about to finish a given session for which the player has been attached at a given access point). Yet we believe this not to be a real problem: indeed, the required "knowledge" is clearly related to the application layer, whereas the games we are considering involve networking mechanisms (and thus are typically related to the MAC and network layers).

Discounting factor   As we have seen, in the case of infinitely repeated games, it is common practice to make use of the discounting factor. This notion comes from the application of game theory to economics: a given capital at time t_0 has "more value" than the same amount at a later time t_1 because, between t_0

and t_1, this capital can generate some (hopefully positive) interest. At first sight, transposing this notion into the realm of networking makes sense: a user wants to send (or to receive) information as soon as he expresses the wish to do so.

But this may be a very rough approximation, and the comment we made about the payoff function can be applied here as well: The willingness to wait before transmitting a packet heavily depends on the current situation of the user and on the content of the packet. In addition, in some applications such as audio or video streaming, the network can *forecast* how the demand will evolve.

A more satisfactory interpretation of the discounting factor in our framework is related to the uncertainty that there will be a subsequent iteration of the stage game, for example, connectivity to an access point can be lost. With this interpretation in mind, the discounting factor represents the probability that the current round is not the last one.

It is important to emphasize that the average discounted payoff is not the only way to express the payoff in an infinitely repeated game. Osborne and Rubinstein [OR94] discuss other techniques, such as "Limit of Means" and "Overtaking". But, none of them captures the notion of users' impatience, and hence we believe that they are therefore less appropriate for our purpose.

Reputation   In some cases, a player can include the reputation of another player in order to anticipate his moves. For example, a player observed to be non-cooperative frequently in the past is likely to continue to be so in the future. If the game models individual packet transmissions, this attitude would correspond to the suspicion that another player has been programmed in a highly "selfish" way. These issues go beyond the scope of this tutorial. For a discussion of these aspects, the reader is referred to [FT91], Chapter 9.

Cooperative vs. non-cooperative players   In this tutorial, we assume that each player is a selfish individual, who is engaged in a non-cooperative game with other players. We do not cover the concept of cooperative games, where the players might have an agreement on how to play the game. Cooperative games include the issues of bargaining and coalition formation. These topics are very interesting and some of our problems could be modeled using these concepts. Due to space limitation, the reader interested in cooperative games is referred to [OR94].

Information   In this thesis, we mostly study games with *complete information*. This means that each player knows the identity of other players, their strategy functions and the resulting payoffs or outcomes. In addition, we consider games with *observable actions* and *perfect recall*. In wireless networking, these assumptions might not hold: For example, due to the unexpected changes of the radio channel, a given player may erroneously reach the conclusion that another player is behaving selfishly. This can trigger a punishment (assuming there is one), leading to the risk of further retaliation, and so on. This means that, for any design of a self-enforcement protocol, special care must be devoted to the assessment of the amount and accuracy of the information that each player can obtain. The application of games with incomplete and imperfect information is an emerging field in wireless networking, with very few papers published so far.

Publication: [FH06a]

Part II

Non-Cooperative Behavior of Users

Chapter 2

Multi-Radio Channel Allocation in Wireless Networks

2.1 Introduction

Wireless networks provide a flexible and cost-efficient method for establishing connections between communication devices. Each wireless network operates in a frequency band assigned by the authorities that regulate the frequency spectrum in a given country. In general, the communication medium assigned to a given network is shared among the communication devices using a *multiple access* technique.

Frequency Division Multiple Access (FDMA) is one of the widely used techniques that enables several users to share a communication medium that consists of a given frequency band [Rap02, Sch05]. The basic principle of FDMA is to split up the available bandwidth to distinct sub-bands called *channels*. Assigning the radio transceivers to these channels is commonly referred to as the *channel allocation* problem.[1] Not surprisingly, an efficient channel allocation is a cornerstone of the design of wireless networks.

In this chapter, we present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios. Using a static non-cooperative game, we analyze the scenario of a single collision domain, i.e., where each of the devices can interfere with a transmission of every other device. We derive the Nash equilibria in this game and show that they result in load balancing over the channels. Our main results show that there exist two types of Nash equilibria: in the first type, each user distributes his radios over the available channels, whereas in the second type, some users allocate multiple radios on certain channels. We also study fairness issues and the problem of coalition formation in the channel allocation problem. We show that a Nash equilibrium that resists coalitions of users is necessarily fair as well. Furthermore, we propose three algorithms to achieve the Nash equilibria. The first is a sequential algorithm that needs global coordination, the second is a distributed algorithm that needs perfect information and the third is a distributed algorithm that is based on imperfect information. We provide the proof for the convergence properties of these algorithms.

This work is a first step towards a deeper understanding of the non-cooperative behavior of such devices and is applicable in particular in the emerging field of cognitive radio systems [Hay05].

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[1]In the literature, the terms channel assignment and frequency assignment are also used for the channel allocation problem.

2.2   System Model and Concepts

We assume that the available frequency band is divided into orthogonal channels of the same bandwidth using the FDMA method (e.g., 8 orthogonal channels in case of the IEEE 802.11a protocol). We denote the set of available orthogonal channels by $\mathcal{C}$.

In our model, *pairs of users* want to communicate with each other over a single hop. We assume that each user participates in only one such communication session, hence we denote the set of such communication links by $\mathcal{N}$. Each user owns a device equipped with $k \leq |\mathcal{C}|$ radio transmitters, all having the same communication capabilities. The communication between two devices is bidirectional and they always have some packets to exchange. Due to the bidirectional links, the sender and the receiver are able to coordinate and thus to select the same channels to communicate. Accordingly, we assume that the sender controls the communication in a certain pair and we refer to him as a *selfish player*. The objective of each player is to maximize his total throughput or channel utilization. We assume that there is a finite number of players. We further assume that each device can hear the transmissions of every other device if they are using the same channel. This means that the players reside in a *single collision domain*. We make this assumption to avoid the hidden terminal problem described for example in [Sch05]. Because the devices reside in a single collision domain, it is reasonable to assume that the channels have roughly the same expected channel characteristics.

We assume that there is a mechanism that enables the players to use multiple channels to communicate at the same time (as it is implemented in [ABP+04] for example). We denote the number of radios of player i using channel x by $k_{i,x}$ for every $x \in \mathcal{C}$. For the simplicity of presentation, let us denote the set of channels used by player i by $\mathcal{C}_i$, where $\mathcal{C}_i \subset \mathcal{C}$ and $0 \leq |\mathcal{C}_i| \leq k$. We further assume that there is no limitation on the number of radios per channel.

We formulate the multi-radio channel allocation problem as a non-cooperative game as follows. We define the *strategy* of player i as his channel allocation vector:

$$s_i = \{k_{i,1}, \ldots, k_{i,|\mathcal{C}|}\} \tag{2.1}$$

Hence, his strategy consists in defining the number of radios on each of the channels.[2] The strategy vectors of all players defines the strategy matrix S (i.e., the strategy profile), where the row i of the matrix corresponds to the strategy vector of player i:

$$S = \begin{pmatrix} s_1 \\ \ldots \\ s_{|\mathcal{N}|} \end{pmatrix} \tag{2.2}$$

Furthermore, we denote the strategy matrix except for the strategy of player i by S_{-i} as shown in (2.3):

$$S_{-i} = \begin{pmatrix} s_1 \\ \ldots \\ s_{i-1} \\ s_{i+1} \\ \ldots \\ s_{|\mathcal{N}|} \end{pmatrix} \tag{2.3}$$

Figure 2.1 presents an example channel allocation with six available channels ($|\mathcal{C}| = 6$), four players ($|\mathcal{N}| = 4$) and each user device equipped by four radios ($k = 4$). Figure 2.2 presents the strategy matrix that corresponds to this example.
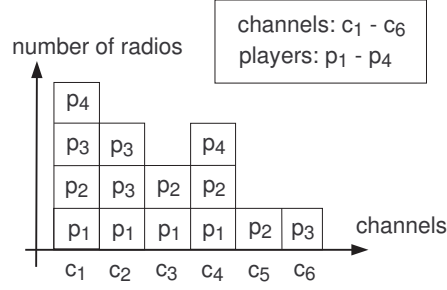
Figure 2.1: An example for a channel allocation, where $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

Figure 2.2: Strategy matrix of the example in Figure 2.1.

The total number of radios employed by player i can be written as $k_i = \sum_{x \in \mathcal{C}} k_{i,x}$. Similarly, we can obtain the number of radios using a particular channel $k_x = \sum_{i \in \mathcal{N}} k_{i,x}$. In Figure 2.1, each player has a radio on channel c_1, but channel c_5 is occupied only by player p_2. Player p_3 employs two radios on channel c_2 to get more bandwidth on that particular channel. Regarding the number of radios per player, we have $k_{p_1} = k_{p_2} = k_{p_3} = 4$ and $k_{p_4} = 2$, meaning that player p_4 is not using all of his radios.

We assume that the players are rational and their objective is to maximize their *payoff* in the network. We denote the payoff of player i by u_i. For simplicity, we assume that each player i wants to *maximize his aggregated throughput τ_i in the system* and this corresponds to his payoff. We leave the study of other payoff functions for future work.

We assume that the total throughput on channel x is shared equally among the radio transmitters using that channel. We denote the *throughput of one radio* on channel x by $\tau_x(k_x)$. As we assume that channels have the same bandwidth and channel characteristics, the achieved throughput does not depend on the channel and thus we can write $\tau(k_x)$ for any channel $x \in \mathcal{C}$. The fair throughput allocation is achieved, for example, by using a reservation-based TDMA schedule on a given channel. A similar result was reported by Bianchi in [Bia00] for the CSMA/CA protocol. Even if the radio transmitters are controlled by selfish users in the CSMA/CA protocol, they can achieve this fair sharing as shown in [CGAH05]. We further assume that the *total throughput* $\tau^t(k_x) = k_x \cdot \tau(k_x)$ on a channel x (i.e., the sum of the achieved throughput of all players on channel x) is a non-increasing function of the number of radios k_x deployed on this channel. In fact $\tau^t(k_x)$ is independent of k_x for a TDMA protocol and for the CSMA/CA protocol using optimal backoff window values [Bia00]. In practice, the backoff window values used in the CSMA/CA protocol implementation (e.g., in the 802.11 standard) are not optimal; and due to packet collisions $\tau^t(k_x)$ becomes a decreasing function of k_x for $k_x > 1$. If $k_x = 0$, we define $\tau^t(0) = 0$; note however that this case has no relevance in our model. We emphasize that our system model is general enough to incorporate many multiple access techniques, such as TDMA or CSMA/CA.

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[2]Note that this number can be zero.

Figure 2.3 presents the total throughput $\tau^t(k_x)$ as a function of the number of radios using channel x.
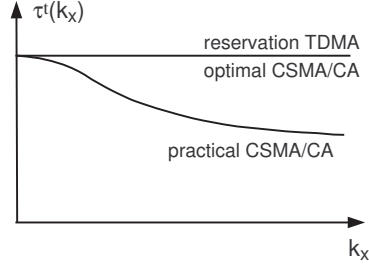
Figure 2.3: The total available throughput $\tau^t(k_x)$ for different MAC protocols.

If player i chooses to operate $k_{i,x}$ radios in a given channel, his throughput on this channel can be written as $\tau_{i,x} = k_{i,x} \cdot \tau(k_x)$. We assume that the players do not cheat at the MAC layer as opposed to the model for example in [CGAH05]. Thus, we can write that $\tau_{i,x} > 0$ for all $x \in \mathcal{C}$, where $k_{i,x} > 0$. Recall that in Figure 2.1, the higher the number of radios in a given channel is, the lower the throughput per radio is. Hence in Figure 2.1, for player p_2 the following conditions hold $\tau_{p_2,c_1} < \tau_{p_2,c_4} < \tau_{p_2,c_3} < \tau_{p_2,c_5}$. We can obtain the aggregated throughput τ_i for player i by $\tau_i = \sum_x \tau_{i,x}$.

In summary, we can write the payoff function for player i as:

$$u_i(S) = \tau_i = \sum_{x \in \mathcal{C}} \tau_{i,x} = \sum_{x \in \mathcal{C}} k_{i,x} \cdot \tau(k_x) \tag{2.4}$$

We model the channel allocation problem with a *single stage game*, which corresponds to a fixed channel allocation among the players.

In order to study the strategic interaction of the players, we rely on the concept of Nash equilibrium we introduced in Definition 1.5. Recall that in a NE none of the players can unilaterally change his strategy to increase his payoff. A NE solution is often inefficient from the system point of view. We characterize the efficiency of the solution by the concept of Pareto-optimality (see Definition 1.8).

2.3 Nash Equilibria

In this section, we study the existence of Nash equilibria in the single collision domain channel allocation game. It is straightforward to see that if the total number of radios is smaller than or equal to the number of channels, then a flat channel allocation, in which the number of radios per channel does not exceed one, is a Nash equilibrium.

Fact 2.1. *If $|\mathcal{N}| \cdot k \leq |\mathcal{C}|$, then any channel allocation, in which $k_x \leq 1, \forall x \in \mathcal{C}$ is a Pareto-optimal NE.*

For the remainder of this chapter, we assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$, hence the devices have a conflict during the channel allocation process.

In the following, we consider a NE strategy matrix in the multi-radio channel allocation game denoted by S^*, where $s_i^* \in S^*$ is the NE strategy of player i (i.e., the i-th row of the matrix).

We first show the following intuitive result: a selfish player should use all of his radios in order to maximize his total throughput. This is a necessary condition for a Nash equilibria.

Lemma 2.2. *If S^* is a NE of the multi-radio channel allocation game, then $k_i = k, \forall i$.*

In the example presented in Figure 2.1, Lemma 2.2 does not hold for player p_4, because he uses only two radios. Hence, the example cannot be a NE.

Proof. We can prove the lemma by contradiction. Assume that there exists a NE, in which player i uses only $k_i < k$ radios. As mentioned previously, in our model we assume that $k \leq |\mathcal{C}|$ and also that in the NE $|\mathcal{C}_i| \leq k_i < k$, thus we necessarily have $|\mathcal{C}_i| < |\mathcal{C}|$. This implies that there always exists a channel $x \notin \mathcal{C}_i$. If the player deploys an additional radio on this channel x, then he increases his payoff due to the fact that $\tau_{i,x} > 0$ for $k_{i,x} = 1$. Hence, we arrive at a contradiction and S^* cannot be a NE. $\square$

Let us now divide the channels in a certain channel allocation S into three sets. We define the set of channels $\mathcal{C}^+$ with the maximum number of radios, i.e., where $x \in \mathcal{C}^+$ has $k_x = \max_{l \in \mathcal{C}} k_l$. Similarly, let us define the set of channels $\mathcal{C}^-$ for which $k_y = k_x - 1$. We denote the set of the remaining channels by $\mathcal{C}^{--}$. In Figure 2.1, $\mathcal{C}^+ = \{c_1\}$, $\mathcal{C}^- = \{c_2, c_4\}$ and $\mathcal{C}^{--} = \{c_3, c_5, c_6\}$.

Let us now consider two arbitrary channels x and y. Without loss of generality, we assume that the number of radios using channel x is higher than or equal to the number of radios using channel y, meaning that $k_x \geq k_y$. We denote their difference by:

$$d_{x,y} = k_x - k_y \tag{2.5}$$

Assume that player i moves one of his radios from channel x to y. Let us define the *payoff difference* (Δ) of player i, i.e. the change of his payoff due to the move of the radio, as follows:

$$
\begin{aligned}
\Delta &= u_i(s_i', S_{-i}^*) - u_i(s_i^*, S_{-i}^*) \\
&= (k_{i,x} - 1) \cdot \tau(k_x - 1) + (k_{i,y} + 1) \cdot \tau(k_y + 1) - k_{i,x} \cdot \tau(k_x) - k_{i,y} \cdot \tau(k_y) \tag{2.6}
\end{aligned}
$$

We can show a second necessary condition for a NE, namely that player i can increase his payoff by moving one radio to a channel where he has no radios if the difference of the number of radios deployed on the two channels exceeds one.

Lemma 2.3. *If $k_{i,x} > 0$, $k_{i,z} = 0$ and $d_{x,z} \geq 2$ for any player i and channels $x, z \in \mathcal{C}$, then S^* is not a NE channel allocation.*

In the example presented in Figure 2.1, Lemma 2.3 holds e.g. for player p_1 and the channels $x = c_1$ and $z = c_5$. Hence, the example cannot be a NE.

Proof. Assume that S^* is a NE channel allocation. Suppose that player i moves one of his radios from channel x to z. Using the conditions in the lemma, we can write the payoff difference defined in (2.6) as:

$$
\begin{aligned}
\Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_z + 1) - k_{i,x} \cdot \tau(k_x) \\
&= k_{i,x} \cdot \tau(k_x - 1) - \tau(k_x - 1) + \tau(k_x - d_{x,z} + 1) - k_{i,x} \cdot \tau(k_x) \tag{2.7}
\end{aligned}
$$

Let us notice that the sum of the first and last terms is always positive, because $d_{x,z} \geq 2$ implies that $k_x > 1$. The sum of the two other terms is non-negative, because $d_{x,z} \geq 2$. Hence, the payoff difference is positive and thus S^* cannot be a NE. This contradiction concludes the proof. $\square$

Let us now derive the third necessary condition. This condition shows that player i should again change the position of one of his radios if he has at least two radios more on channel x than on another channel y and if the total number of radios on channel x is more than the total number of radios on channel y.

Lemma 2.4. *If $k_{i,x} \geq 2$, $k_{i,y} = 0$ and $d_{x,y} = 1$ for any player i, then S^* is not a NE.*

In the example presented in Figure 2.1, the conditions of Lemma 2.4 hold for player p_3 and the channels $x = c_2$ and $y = c_3$. Hence, the example cannot be a NE.

Proof. We prove the lemma by contradiction. Assume that S^* is a NE and there exists a player i with $k_{i,x} \geq 2$, $k_{i,y} = 0$ and $d_{x,y} = 1$. Assume that this player moves one of his radios from channel x to y. Let us express the payoff difference (2.6) given the conditions of the lemma:

$$\Delta = (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_y + 1) - k_{i,x} \cdot \tau(k_x) \tag{2.8}$$

Using $d_{x,y} = 1$, we can reformulate (2.8) as follows:

$$\begin{aligned} \Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_x) - k_{i,x} \cdot \tau(k_x) \\ &= (k_{i,x} - 1) \left[\tau(k_x - 1) - \tau(k_x) \right] \end{aligned} \tag{2.9}$$

Note that the expression in (2.9) is always positive, because $k_{i,x} \geq 2$. Hence moving one radio is beneficial to player i and S^* cannot be a NE. $\square$

Using Lemmas 2.3 and 2.4, we conclude on another necessary condition. This shows that in a Nash equilibrium, the difference in the total number of radios between any two channels cannot exceed one.

Proposition 2.5. *If S^* is a NE in the multi-radio channel allocation game, then $d_{x,y} \leq 1$ for all $x, y \in \mathcal{C}$.*

Proof. We prove the proposition by contradiction. As introduced before, let us divide the set of channels $\mathcal{C}$ into three subsets: $\mathcal{C}^+$, $\mathcal{C}^-$ (with $k_y = k_x - 1$) and $\mathcal{C}^{--}$ (with $k_z \leq k_x - 2$). Suppose then that there exists a NE channel allocation S^* such that $d_{x,z} \geq 2$ for two channels $x, z \in \mathcal{C}$.

In the following, we will show a procedure that should result in the NE channel allocation S^*. Let us consider the set of players $\mathcal{N}^+$ who have at least one radio on one of the channels $x \in \mathcal{C}^+$. We will assign the radios of the players in $\mathcal{N}^+$ in a sequential order; we start with a player $i \in \mathcal{N}^+$. We derived in Lemma 2.3 that player i, who has a radio in any channel in $x \in \mathcal{C}^+$, must have at least radio on each of the channels $z \in \mathcal{C}^{--}$. In addition, his channel allocation regarding the channels in $\mathcal{C}^+$ can be categorized into two cases as shown in Figure 2.4: (a) player i has at most one radio on each channel $x \in \mathcal{C}^+$ or (b) player i has multiple radios on at least one channel $x \in \mathcal{C}^+$. In the second case (b), player i must have at least one radio on each channel $y \in \mathcal{C}^-$ as well, because of the conditions in Lemma 2.4 Player i has $k \leq |\mathcal{C}| = |\mathcal{C}^+| + |\mathcal{C}^-| + |\mathcal{C}^{--}|$ radios. In case (b), he allocates at least $|\mathcal{C}^-| + |\mathcal{C}^{--}|$ radios on the channels in $\mathcal{C}^-$ and $\mathcal{C}^{--}$. Thus, he has $k - |\mathcal{C}^-| + |\mathcal{C}^{--}| \leq |\mathcal{C}^+|$ radios left for the channels $x \in \mathcal{C}^+$. This implies, that in both of the aforementioned cases, it is true that $\sum_{x \in \mathcal{C}^+} k_{i,x} \leq |\mathcal{C}^+|$.

Let us denote the average number of radios on the channels in $\mathcal{C}^+$ and in $\mathcal{C}^{--}$ by K^+ and K^{--}, respectively. Here $K^+ = \frac{\sum_{x \in \mathcal{C}^+} k_x}{|\mathcal{C}^+|}$ and $K^{--} = \frac{\sum_{z \in \mathcal{C}^{--}} k_z}{|\mathcal{C}^{--}|}$. Based on the above derivation, we know that after the introduction of the radios of the first player $i \in \mathcal{N}^+$, we obtain $K^+ \leq 1$ and $K^{--} \geq 1$.

One can continue the procedure by allocating the radios of the other players in $\mathcal{N}^+$. But, for each player, K^+ increases by *at most* by 1 and K^{--} increases by *at least* by 1. Upon finishing this procedure, we obtain $K^+ \leq K^{--}$. But, we defined the set of channels $\mathcal{C}^+$ such that for any two channels $x \in \mathcal{C}^+$ and $z \in \mathcal{C}^{--}$, $d_{x,z} \geq 2$ holds. This implies that $K^+ \geq K^{--} + 2$. This is in a contradiction with the result of the allocation procedure. $\square$

Proposition 2.5 shows that in a NE only $\mathcal{C}^+$ and $\mathcal{C}^-$ exist. This establishes an interesting property about NE: *In fact, all NE channel allocations achieve load-balancing over the channels in $\mathcal{C}$.* Based on Proposition 2.5, we express a set of sufficient conditions for the NE.
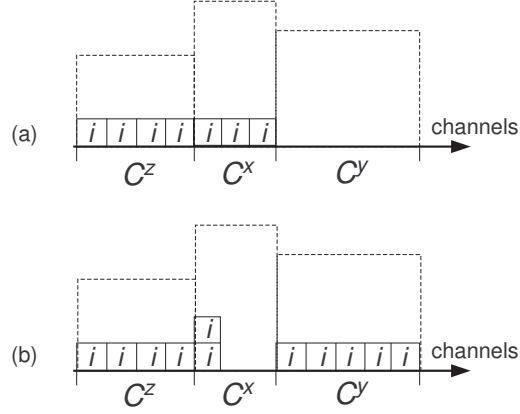
Figure 2.4: An illustration for the proof of Proposition 2.5.

Theorem 2.6. *Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then a channel allocation S^* is a NE if the two following conditions hold:*

> ▷ $d_{x,y} \leq 1$ *for any $x, y \in \mathcal{C}$ and*

> ▷ $k_{i,x} \leq 1$ *for any $x \in \mathcal{C}$.*

Proof. The proof is straightforward. Assume that player i moves one radio from x to y. This is obviously not beneficial if both $x, y \in \mathcal{C}^+$ or both $x, y \in \mathcal{C}^-$. Furthermore, it results in the decrease of the payoff of i if $x \in \mathcal{C}^-$ and $y \in \mathcal{C}^+$. Finally, if $x \in \mathcal{C}^+$ and $y \in \mathcal{C}^-$, then the payoff of player i is unchanged. $\square$

An example of a NE channel allocation corresponding to Theorem 2.6 is shown in Figure 2.5.

Figure 2.5: A NE channel allocation corresponding to Theorem 2.6. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$. Each player distributes his radios over the channels (i.e., $k_{i,x} \leq 1, \forall i, \forall c$).

Theorem 2.6 suggests that players should distribute their radios over the set of available channels. Surprisingly, there exist another type of Nash equilibria in which some players use multiple radios in some channels. We characterize these Nash equilibria in the following theorem.

Theorem 2.7. *Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then a channel allocation S^* is a NE if the following conditions hold:*

> ▷ $d_{x,y} \leq 1$ *for any $x, y \in \mathcal{C}$ and*

> ▷ *for any player i that has $k_{i,x} \geq 2$, $k_{i,x} \leq \frac{\tau(k_x-1)-\tau(k_x+1)}{\tau(k_x-1)-\tau(k_x)}$ also holds; and*

> ▷ *for any player i that has $k_{i,x} \geq 2$ and $x \in \mathcal{C}^+$, it is also true that $k_{i,y} \geq k_{i,x} - 1, \forall y \in \mathcal{C}^-$*

Proof. Let us consider a player i who has $k_{i,x} \geq 2$ on a channel $x \in \mathcal{C}$. We know that $k \leq \mathcal{C}$. These two conditions ensure that there exists another channel $y \in \mathcal{C}$ such that $k_{i,y} = 0$. Consider the following four cases as shown in Figure 2.6:
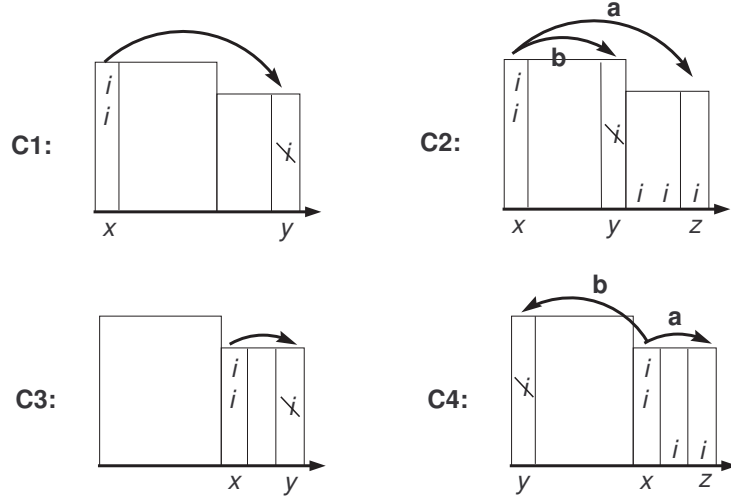
Figure 2.6: An illustration for the proof of Theorem 2.7.

C1: $x \in \mathcal{C}^+$ and $y \in \mathcal{C}^-$

In this case, there exists a channel that i is not using and it is in $\mathcal{C}^-$. Based on Lemma 2.4, $\Delta \geq 0$. Hence, this cannot be a NE.

C2: $x \in \mathcal{C}^+$, $y \in \mathcal{C}^+$ and $\forall z \in \mathcal{C}^-$ it holds that $k_{i,z} > 0$

In this case, player i uses each channel in $\mathcal{C}^-$. Let us consider two subcases:

C2a: Player i attempts to move one radio from $x \in \mathcal{C}^+$ to $z \in \mathcal{C}^-$. Then, the payoff difference is:

$$\Delta = (k_{i,x} - 1) \cdot \tau(k_x - 1) + (k_{i,z} + 1) \cdot \tau(k_z + 1) - k_{i,x} \cdot \tau(k_x) - k_{i,z} \cdot \tau(k_z)$$

Using that $k_z = k_x - 1$, we obtain:

$$
\begin{aligned}
\Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + (k_{i,z} + 1) \cdot \tau(k_x) - k_{i,x} \cdot \tau(k_x) - k_{i,z} \cdot \tau(k_x - 1) \\
&= (k_{i,x} - k_{i,z} - 1) \cdot [\tau(k_x - 1) - \tau(k_x)]
\end{aligned}
\tag{2.10}
$$

The second factor in (2.10) is always positive. Thus, $\Delta \leq 0$ for $k_{i,z} \geq k_{i,x} - 1$. This results in the third condition of Theorem 2.7.

C2b: Player i attempts to move one radio from $x \in \mathcal{C}^+$ to $y \in \mathcal{C}^+$, where $k_{i,y} = 0$. Since $k_x = k_y$, we can rewrite the payoff difference as:

$$
\begin{aligned}
\Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_x + 1) - k_{i,x} \cdot \tau(k_x) \\
&= k_{i,x} \cdot [\tau(k_x - 1) - \tau(k_x)] - [\tau(k_x - 1) - \tau(k_x + 1)]
\end{aligned}
$$

In a NE, we should have $\Delta \leq 0$, which is true for:

$$k_{i,x} \leq \frac{\tau(k_x - 1) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)} \tag{2.11}$$

This implies the second condition in Theorem 2.7 for $k_{i,x} \geq 2$ and $x \in \mathcal{C}^+$.

C3: $x \in \mathcal{C}^-$ and $y \in \mathcal{C}^-$

In this case, player i has multiple radios on one of the channels in $\mathcal{C}^-$, but there is another channel in the same set, where he has no radios. Then, we consider the payoff difference, when player i moves one radio from x to y. In this case, we obtain the same condition as for **C2b** as expressed in (2.11), but let us recall that the value of k_x is smaller because $x \in \mathcal{C}^-$:

$$k_{i,x} \leq \frac{\tau(k_x - 1) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)} \tag{2.12}$$

This implies the second condition in Theorem 2.7 for $k_{i,x} \geq 2$ and $x \in \mathcal{C}^-$.

C4: $x \in \mathcal{C}^-$ and $y \in \mathcal{C}^+$ and $\forall z \in \mathcal{C}^-$ it holds that $k_{i,z} > 0$

Then, we distinguish two subcases:

C4a: Player i attempts to move one radio from $x \in \mathcal{C}^-$ to $z \in \mathcal{C}^-$, where $k_{i,z} > 0$. Then, the payoff difference is:

$$\Delta = (k_{i,x} - 1) \cdot \tau(k_x - 1) + (k_{i,z} + 1) \cdot \tau(k_z + 1) - k_{i,x} \cdot \tau(k_x) - k_{i,z} \cdot \tau(k_z)$$

Using that $k_z = k_x$, we obtain:

$$\begin{aligned} \Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + (k_{i,z} + 1) \cdot \tau(k_x + 1) - k_{i,x} \cdot \tau(k_x) - k_{i,z} \cdot \tau(k_x) \\ &= k_{i,x} \cdot [\tau(k_x - 1) - \tau(k_x)] - \\ &\quad - k_{i,z} \cdot [\tau(k_x) - \tau(k_x + 1)] - [\tau(k_x - 1) - \tau(k_x + 1)] \end{aligned} \tag{2.13}$$

From (2.13), we can see that $\Delta \geq 0$ for:

$$k_{i,x} \leq k_{i,z} \cdot \frac{\tau(k_x) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)} + \frac{\tau(k_x - 1) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)} \tag{2.14}$$

We can observe that the condition for case **C3** in (2.12) is always smaller than the one for case **C4a** in (2.14). Hence the condition of **C3** is more restrictive.

C4b: Player i attempts to move one radio from $x \in \mathcal{C}^-$ to $y \in \mathcal{C}^+$, where $k_{i,y} = 0$. Since $k_y = k_x + 1$, we can rewrite the payoff difference as:

$$\begin{aligned} \Delta &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_y + 1) - k_{i,x} \cdot \tau(k_x) \\ &= (k_{i,x} - 1) \cdot \tau(k_x - 1) + \tau(k_x + 2) - k_{i,x} \cdot \tau(k_x) \\ &= k_{i,x} \cdot [\tau(k_x - 1) - \tau(k_x)] - [\tau(k_x - 1) - \tau(k_x + 2)] \end{aligned} \tag{2.15}$$

In a NE, we should have $\Delta \leq 0$, which is true for:

$$\begin{aligned} k_{i,x} &\leq \frac{\tau(k_x - 1) - \tau(k_x + 2)}{\tau(k_x - 1) - \tau(k_x)} \\ &= \frac{\tau(k_x - 1) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)} + \frac{\tau(k_x + 1) - \tau(k_x + 2)}{\tau(k_x - 1) - \tau(k_x)} \end{aligned} \tag{2.16}$$

Again, the condition in (2.16) is always larger than the condition for case **C3** in (2.12). Thus the condition of **C3** is more restrictive.

$\square$

Figure 2.7 presents an example for Theorem 2.7 assuming that the second (numerical) condition of the theorem holds.
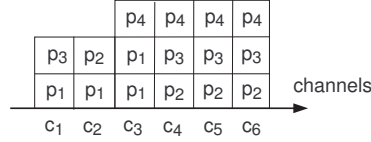
Figure 2.7: An example for a NE channel allocation corresponding to Theorem 2.7. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 7$ and $k = 4$. Note that player p_1 uses multiple radios on channel c_1.

In summary, Theorems 2.6 and 2.7 characterize two types of Nash equilibria. In the first type, each player distributes his radios such that he has at most one radio per channel. Intuitively, this results in load balancing. Note, however, the existence of a second type of Nash equilibria, in which some players allocate multiple radios on certain channels. We mention that there could be a small set of other Nash equilibria that are not covered by these theorems, but they exist for very specific conditions on the total throughput function $\tau^t(k_x)$. These Nash equilibria can be derived from the proof of Theorem 2.7.

2.4   Efficiency

In this section, we study the efficiency of the previously identified Nash equilibria. In the next theorem, we will show that selfish channel allocation results in an efficient bandwidth utilization if the total throughput function is independent of the number of radios on a given channel.

Theorem 2.8. *Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$ and the total throughput function $\tau^t(k_x)$ is independent of k_x on any channel x. Then any NE channel allocation S^* is Pareto-optimal.*

Proof. The proof is straightforward. If S^* is a NE channel allocation, then $k_x > 0$ for each $x \in \mathcal{C}$. If the total throughput function $\tau^t(k_x)$ is independent of k_x, then in S^* the sum of the payoff of all players $u_{total} = \max_i \sum_i u_i$ which implies Pareto-optimality. $\square$

Note that this property might not hold for decreasing total throughput functions, because the players might remove some of their radios to decrease the total number of radios on certain channels. If they do this mutually, they could mutually increase each others' payoff, because they decrease the amount of throughput loss due to collisions. A simple example is shown in Figure 2.8.

From Figure 2.8, we can conclude on the following property of Pareto-optimal channel allocations.

Fact 2.9. *If $|\mathcal{N}| \cdot k > |\mathcal{C}|$ and $\tau^t(k_x)$ is a decreasing function of k_x, then every Pareto-optimal channel allocation has the property $k_x = 1, \forall x \in \mathcal{C}$.*

Using the concept of the *price of anarchy (POA)* introduced in Section 1.2.5, we can express the inefficiency caused by selfish behavior.
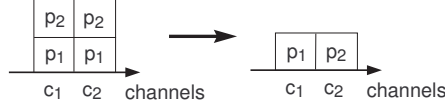
Figure 2.8: A simple example to show that Nash equilibria are not necessarily Pareto-optimal if $\tau^t(k_x)$ decreases with k_x. Here $|\mathcal{C}| = 2$, $|\mathcal{N}| = 2$ and $k = 2$. If both players remove one radio each, then they achieve a higher payoff.

Theorem 2.10. *If $\tau^t(k_x)$ is a decreasing function of k_x, then the price of anarchy (POA) is given by:*

$$POA = \frac{\tau^t(1)}{(k_x + 1 - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|}) \cdot (\tau^t(k_x) - \tau^t(k_x + 1)) + \tau^t(k_x + 1)} \tag{2.17}$$

where $k_x = \left\lfloor \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rfloor$ (i.e., $k_x + 1 = \left\lceil \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rceil$).

Proof. The price of anarchy is the ratio between the sum of the payoffs of the players in a system optimal solution (in our case, the Pareto-optimal solution) and the sum of their payoffs achieved in a Nash equilibrium. Hence, we can write the inverse of the *POA* as follows.

$$
\begin{aligned}
\frac{1}{POA} &= \frac{|\mathcal{C}^-| \cdot \tau^t(k_x) + |\mathcal{C}^+| \cdot \tau^t(k_x + 1)}{|\mathcal{C}| \cdot \tau^t(1)} \\
&= \frac{|\mathcal{C}^-|}{|\mathcal{C}|} \frac{\tau^t(k_x)}{\tau^t(1)} + \frac{|\mathcal{C}^+|}{|\mathcal{C}|} \frac{\tau^t(k_x + 1)}{\tau^t(1)} \\
&= \frac{|\mathcal{C}| - |\mathcal{C}^+|}{|\mathcal{C}|} \frac{\tau^t(k_x)}{\tau^t(1)} + \frac{|\mathcal{C}^+|}{|\mathcal{C}|} \frac{\tau^t(k_x + 1)}{\tau^t(1)} \\
&= \frac{\tau^t(k_x)}{\tau^t(1)} - \frac{|\mathcal{C}^+|}{|\mathcal{C}|} \left(\frac{\tau^t(k_x)}{\tau^t(1)} - \frac{\tau^t(k_x + 1)}{\tau^t(1)} \right)
\end{aligned} \tag{2.18}
$$

We know that $|\mathcal{C}^+| = |\mathcal{N}| \cdot k - |\mathcal{C}| \cdot k_x$.

$$
\begin{aligned}
\frac{1}{POA} &= \frac{\tau^t(k_x)}{\tau^t(1)} - \frac{|\mathcal{N}| \cdot k - |\mathcal{C}| \cdot k_x}{|\mathcal{C}|} \left(\frac{\tau^t(k_x)}{\tau^t(1)} - \frac{\tau^t(k_x + 1)}{\tau^t(1)} \right) \\
&= \frac{\tau^t(k_x)}{\tau^t(1)} + k_x \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \cdot \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} \\
&= \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} + \frac{\tau^t(k_x + 1)}{\tau^t(1)} + k_x \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \cdot \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} \\
&= \left(k_x + 1 - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right) \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} + \frac{\tau^t(k_x + 1)}{\tau^t(1)}
\end{aligned} \tag{2.19}
$$

The result of the theorem follows from (2.19). $\square$

Let us now further examine the expression obtained for the price of anarchy. The first factor in the first term in (2.19) can be written as follows:

$$k_x + 1 - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} = \left\lceil \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rceil - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \geq \frac{1}{|\mathcal{C}|} \tag{2.20}$$

Thus, we can give a lower bound on (2.19):

$$\frac{1}{POA} \geq \frac{1}{|\mathcal{C}|} \cdot \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} + \frac{\tau^t(k_x + 1)}{\tau^t(1)} \tag{2.21}$$

Hence, we obtain that

$$POA \leq \frac{\tau^t(1)}{\frac{\tau^t(k_x) - \tau^t(k_x+1)}{|\mathcal{C}|} + \tau^t(k_x + 1)} \tag{2.22}$$

In the remainder of this section, we will further develop (2.22) for specific assumptions on the total throughput function $\tau^t(k_x)$.

2.4.1 Price of Anarchy for Linear $\tau^t(k_x)$

Let us now approximate the total throughput function with a linear function. We can express the change of the total throughput as:

$$\tau^t(k_x + 1) = \tau^t(k_x) - H = \tau^t(1) - H \cdot (k_x - 1) \tag{2.23}$$

where $0 < H < \tau^t(1)$ is a constant value.

In this case, we can reformulate (2.21) as follows:

$$\begin{aligned}
\frac{1}{POA} &\geq \frac{1}{|\mathcal{C}|} \cdot \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} + \frac{\tau^t(k_x + 1)}{\tau^t(1)} \\
&= \frac{1}{|\mathcal{C}|} \cdot \frac{H}{\tau^t(1)} + \frac{\tau^t(1) - H \cdot (k_x - 1)}{\tau^t(1)} \\
&= \frac{\frac{H}{|\mathcal{C}|} + \tau^t(1) - H \cdot (k_x - 1)}{\tau^t(1)} \\
&= 1 + \frac{H}{\tau^t(1)} \left(\frac{1}{|\mathcal{C}|} - k_x + 1 \right)
\end{aligned} \tag{2.24}$$

Let us take a closer look at (2.24).

▷ If $H \to 0$, we obtain that the right-hand side of (2.24) approaches 1. Hence, $POA \to 1$. Intuitively, if the total throughput function $\tau^t(k_x)$ differs very little from a constant function, then the Nash equilibria are close to being Pareto-optimal.

▷ If $H \to \tau^t(1)$, then we obtain from (2.24) that

$$\frac{1}{POA} \geq 2 + \frac{1}{|\mathcal{C}|} - k_x \tag{2.25}$$

There exist two cases, depending on the sign of the right-hand side of (2.25).

– The right-hand side expression is non-positive:

$$\begin{aligned}
2 + \frac{1}{|\mathcal{C}|} - k_x &\leq 0 \\
2 + \frac{1}{|\mathcal{C}|} &\leq k_x \\
2 + \frac{1}{|\mathcal{C}|} &\leq \left\lfloor \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rfloor
\end{aligned} \tag{2.26}$$

This case is true for $|\mathcal{N}| \cdot k \geq 3|\mathcal{C}|$. In this case, the price of anarchy is unbounded, meaning that $POA < \infty$. This suggests that if the total throughput function drops rapidly, then the system falls into the well-know problem of the tragedy of the commons [Har68].

– For $2 + \frac{1}{|\mathcal{C}|} - k_x > 0$, meaning that $|\mathcal{C}| \leq |\mathcal{N}| \cdot k < 3 \cdot |\mathcal{C}|$, we obtain:

$$POA \leq \frac{1}{2 + \frac{1}{|\mathcal{C}|} - k_x} = \frac{|\mathcal{C}|}{2 \cdot |\mathcal{C}| + 1 - \left\lfloor \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rfloor \cdot |\mathcal{C}|} \tag{2.27}$$

2.4.2 Price of Anarchy for Geometric $\tau^t(k_x)$

Second, we approximate the total throughput function with a geometric function. Then, we express the change of the throughput as:

$$\tau^t(k_x + 1) = \tau^t(k_x) \cdot H = \tau^t(1) \cdot H^{(k_x - 1)} \tag{2.28}$$

where $0 < H < 1$ is a constant value.

In this case, we can reformulate (2.21) as follows:

$$
\begin{aligned}
\frac{1}{POA} &\geq& \frac{1}{|\mathcal{C}|} \cdot \frac{\tau^t(k_x) - \tau^t(k_x + 1)}{\tau^t(1)} + \frac{\tau^t(k_x + 1)}{\tau^t(1)} \\
&=& \frac{1}{|\mathcal{C}|} \left(H^{(k_x - 1)} - H^{k_x} \right) + H^{k_x} \\
&=& \frac{1}{|\mathcal{C}|} \left(H^{(k_x - 1)} - H^{k_x} \right) + H^{k_x} \\
&=& H^{k_x} \left(\frac{1}{|\mathcal{C}| \cdot H} - \frac{1}{|\mathcal{C}|} + 1 \right)
\end{aligned}
\tag{2.29}
$$

We can further study (2.29) as follows.

▷ If $H \rightarrow 0$, we obtain from (2.29) that $\frac{1}{POA} \geq 0$. This means that $POA < \infty$, i.e., the price of anarchy is unbounded. The reason is similar to the arguments in case of the linear throughput function.

▷ If $H \rightarrow 1$, then $\frac{1}{POA} \geq 1$ and the price of anarchy converges to one. Analogously to the previous case in Section 2.4.1, this means that the throughput function $\tau^t(k_x)$ differs very little from a constant function, then the Nash equilibria are close to being Pareto-optimal.

In theory, the total throughput function of the CSMA/CA protocol is independent of the number of radios using a particular channel [Bia00]. Note that this independence requires that the users know the total number of radios using the channel to be able to adjust their backoff window accordingly in order to reduce collision to the minimum. In practice, the users do not possess this knowledge and hence they operate their radios according to the pre-defined protocol. Nevertheless, the throughput function declines rather slowly, notably thanks to the RTS/CTS extension of the basic CSMA/CA protocol. This suggests that the price of anarchy is one in theory and it is close to one in practice.

Let us emphasize that neither the Nash equilibria nor the Pareto-optimal strategy profiles ensure fairness among the players. We devote the next section to study the fairness of selfish channel allocation.

2.5 Fairness Issues

In this section, we study the *fairness* properties of the selfish multi-radio channel allocation game. Fairness is an important aspect of resource allocation problems in general, and of computer networks in particular. Very often, a system-efficient resource allocation gives more (or all) resources to a few players while neglecting other players (e.g. in the NE derived for the CSMA/CA protocol in [CGAH05]). Such a solution might be system-efficient, but it is not desired from the network designer's point of view (i.e., neither from the users' point of view). In this section, we will show, which additional properties are required to make a NE a fair channel allocation.

We have seen in Section 2.3 that in the selfish multi-radio channel allocation problem, the NE achieve load balancing. Unfortunately, these NE might be highly unfair by giving advantage to some players and neglecting others. For example, in the channel allocation presented in Figure 2.7 assuming that the total throughput function $\tau^t(k_x)$ is constant and normalized (i.e., $\tau^t(1) = 1$), then player p_1 has the aggregated throughput $u_1 = \frac{19}{20}$ whereas player p_4 has the aggregated throughput $u_4 = \frac{16}{20}$. In order to study the fairness properties of the NE channel allocations, we use a particular metric called *max-min fairness (MMF)* as defined in [BG92]:

Definition 2.1. *(Max-Min Fairness – MMF): The strategy matrix S^{mmf} is max-min fair if the payoff of player i cannot be increased without decreasing the payoff of another player j for which $u_i(S^{mmf}) \geq u_j(S^{mmf})$.*

Using this concept, we identify the max-min fair NE channel allocations as expressed in Theorem 2.11.

Theorem 2.11. *A NE channel allocation S^* is max-min fair if and only if $\sum_{x \in \mathcal{C}^-} k_{i,x} = \sum_{x \in \mathcal{C}^-} k_{j,x}$, for all $i, j \in \mathcal{N}$. This implies that $u_i = u_j, \forall i, j \in \mathcal{N}$.*

In other words, if the total number of radios in the least allocated channels are equal for every player, the NE allocation is max-min fair. For example, the channel allocation in Figure 2.5 is max-min fair, but the one shown in Figure 2.7 is not.

In the proof, we will show that the condition implies an equal payoff for each player.

Proof. Let us express the payoff of any player i in a NE allocation S^*. In the equation below, k_x denotes the number of radios that use channel $x \in \mathcal{C}^-$.

$$
\begin{aligned}
u_i &= \left(\sum_{x \in \mathcal{C}^-} k_{i,x} \right) \cdot \tau(k_x) + \left(k - \sum_{x \in \mathcal{C}^-} k_{i,x} \right) \cdot \tau(k_x + 1) \\
&= \sum_{x \in \mathcal{C}^-} k_{i,x} \cdot (\tau(k_x) - \tau(k_x + 1)) + k \cdot \tau(k_x + 1)
\end{aligned}
$$

Similarly, we can express the payoff of another player j:

$$
u_j = \sum_{x \in \mathcal{C}^-} k_{j,x} \cdot (\tau(k_x) - \tau(k_x + 1)) + k \cdot \tau(k_x + 1)
$$

If and only if every player has an equal number of radios in the channels $x \in \mathcal{C}^-$, meaning that $\sum_{x \in \mathcal{C}^-} k_{i,x} = \sum_{x \in \mathcal{C}^-} k_{j,x}$, does $u_i = u_j$ hold for players $i, j \in \mathcal{N}$.

Second, we prove by contradiction that the equality of the payoffs is necessary and sufficient to max-min fairness. Let us suppose that there exist a max-min fair NE channel allocation S^* in which $u_i < u_j$

for some $i, j \in \mathcal{N}$. We know from Lemma 2.2 that $k_i = k_j = k$, and hence we can interchange the radios of player i with the radios of player j. This results in $u_i > u_j$, but the payoffs of other players do not change. Hence, the original channel allocation S^* is not max-min fair. Conversely, if $u_i = u_j$, this implies max-min fairness by definition. $\qquad\square$

From this theorem, we can immediately see that the perfectly balanced channel allocation is also max-min fair.

Corollary 2.12. *If S^* is a NE such that $\mathcal{C}^- = \mathcal{C}^+$ (i.e., $k_y = k_x$, $\forall x, y \in \mathcal{C}$), then S^* is max-min fair as well.*

2.6 Coalition-Proof Nash Equilibria

The definition of NE expresses the resistance to the deviation of a single player. In a realistic situation, it might be possible that several players collude to increase their payoff at the expense of other players. Such a collusion is called a *coalition*. The problem of how these coalitions are formed is a research topic in itself, thus in this chapter we assume that any group of players can form a coalition. We can generalize the notion of NE for coalitions as defined in [BPW87].

Definition 2.2. *(Coalition-Proof Nash Equilibrium – CPNE): The strategy matrix S^{cpne} defines a coalition-proof Nash equilibrium if there does not exist any coalition $\Gamma \subset \mathcal{N}$ and any strategy of this coalition S'_Γ such that the following set of conditions is true:*

$$u_i(S'_\Gamma, S^{cpne}_{-\Gamma}) \geq u_i(S^{cpne}_\Gamma, S^{cpne}_{-\Gamma}), \forall i \in \Gamma \qquad (2.30)$$

with strict inequality for at least one player $i \in \Gamma$.

This means that no coalition can deviate from S^{cpne} such that the payoff of *at least one* of its members increases and the payoff of other members do not change.[3]

Players in a coalition can help each other in two ways. The first possibility is if a player relocates his radio to improve the payoff of another player he is in a coalition with. This property is expressed for two players in Lemma 2.13.

Lemma 2.13. *If there exist two channels $x \in \mathcal{C}^+$ and $y \in \mathcal{C}^-$ and two players $i, j \in \mathcal{N}$ such that $k_{i,x} > 0$ and $k_{j,x} > 0$ whereas $k_{i,y} = 0$ and $k_{j,y} = 0$, then the NE channel allocation S^{cpne} is not coalition-proof.*

Proof. It is easy to see that if the conditions hold, then i and j can form a coalition and one of them (for example player i) can increase the payoff of the other by moving one radio from x to y. $\qquad\square$

The players in a coalition can also improve their payoff if they mutually remove some radios to reduce the number of radios contending for these channels. This property is shown for two players in Lemma 2.14.

Lemma 2.14. *If there exist two channels $x, y \in \mathcal{C}^+$ or $x, y \in \mathcal{C}^-$ and two players $i, j \in \mathcal{N}$ such that $k_{i,x} > 0$, $k_{j,x} > 0$, $k_{i,y} > 0$ and $k_{j,y} > 0$, then the NE channel allocation S^{cpne} is not coalition-proof.*

---

[3]Note that our definition corresponds to the principle of *weak deviation*. One can define the notion of a *strict deviation* of a coalition which requires that each coalition member increases his payoff by deviating. In the literature of coalition-proof equilibria, both concepts are used.

Proof. If the above conditions hold, then the two players can remove one radio each from the channels x and y (for example, i removes from x and j removes from i). This way, the players mutually increase the payoffs of each other. □

Based on Lemmas 2.13 and 2.14, we can prove the following theorem.

Theorem 2.15. *If in a NE channel allocation S^* it is true that $|\mathcal{C}^+| \geq 2$, then S^* is not coalition-proof.*

Proof. We prove the theorem by contradiction. Let us consider two channels $x \in \mathcal{C}^+$ and $y \in \mathcal{C}^-$. Let us denote the set of players who have at least one radio in channel x and y by $\mathcal{N}^+$ and $\mathcal{N}^-$, respectively.

Assume first that S^* corresponds to Theorem 2.6, thus $k_{i,x} \leq 1$ for any channel. Then, we can state that $\mathcal{N}^- = \mathcal{N}^+ - 1$, otherwise the two players not present in $\mathcal{N}^-$ could form a coalition. The same logic applies to any other channel $x^{'} \in \mathcal{C}^+$. This implies that x and $x^{'}$ must contain the same set of users and hence the conditions of Lemma 2.14 hold.

If S^* corresponds to Theorem 2.7 and $k_{i,y} \geq 2$ for a channel $y \in \mathcal{C}^-$, then the above reasoning applies.

If S^* corresponds to Theorem 2.7 and $k_{i,x} \geq 2$ for a channel $x \in \mathcal{C}^+$, then we know that $k_{i,y} \geq k_{i,x} - 1$ for each channel $y \in \mathcal{C}^-$. If we disregard (remove) $k_{i,y}$ radios from channel x and each channel in $\mathcal{C}^-$, then by the same reasoning, we can conclude that $\mathcal{N}^- = \mathcal{N}^+ - 1$ for any channels $x \in \mathcal{C}^+$ and $y \in \mathcal{C}^-$. Hence, the contradiction applies to this case as well. □

We narrowed down the set of potential coalition-proof Nash equilibria by Theorem 2.15. However, we could not find a set of sufficient conditions for coalition-proof Nash equilibria. Nevertheless, we can show that each coalition-proof NE is max-min fair as well. Note that the converse is not true.

Theorem 2.16. *If NE channel allocation S^* is coalition-proof, then it is max-min fair as well.*

Proof. If $|\mathcal{C}^+| = 1$, then by the same arguments as in the proof of Theorem 2.15, we can conclude that $\mathcal{N}^- = \mathcal{N}^+ - 1$. In other words, each player must have one radio in x. That means that $\sum_{y \in \mathcal{C}^-} k_{i,y} = \sum_{y \in \mathcal{C}^-} k_{j,y}$ and hence the channel allocation S^* is max-min fair. If $|\mathcal{C}^+| = 0$, then the theorem follows from Corollary 2.12. □

As a summary, Figure 2.9 shows all channel allocations by properties.
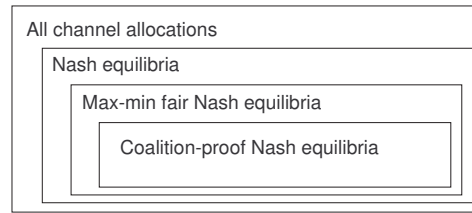
Figure 2.9: Summary of channel allocations with different properties.

2.7   Convergence to a Nash Equilibrium

In Section 2.3, we have demonstrated that the non-cooperative behavior of the selfish players leads to load-balancing Nash equilibria. In this section, we propose two algorithms, each using a different set of

available information, to enable the selfish players to converge to one of these Nash equilibria from an arbitrary initial configuration. The two algorithms are the following: 1) a centralized algorithm using perfect information and 2) a distributed algorithm using imperfect (local) information.

2.7.1 Centralized Algorithm Using Perfect Information

We have proved in Proposition 2.5 that a Nash equilibrium channel allocation has a load-balancing property. In addition, we have shown in Theorem 2.8 that for a constant throughput function the Nash equilibria are Pareto-optimal as well. Here, we present the pseudo-code of Algorithm 2.1, a simple centralized algorithm to achieve one of these efficient Nash equilibria.

Algorithm 2.1 NE channel allocation with global coordination and perfect information

1: **for** $i = 1$ to $|\mathcal{N}|$ **do**
2:      **for** $j = 1$ to k **do**
3:          **if** $k_x = k_y, \forall x, y \in \mathcal{C}$ **then**
4:              use radio j on a channel x, where $k_{i,x} = 0$
5:          **else**
6:              use radio j on a channel x, where $k_x = \min_{y \in \mathcal{C}} k_y$
7:          **end if**
8:      **end for**
9: **end for**

Using the algorithm, the players allocate their radios such that they fill the channels almost equally. Note that the algorithm requires the sequential action of the players and hence it needs global coordination. In addition, the players must have perfect information about the number of radios on each of the channels. This can be achieved by the global coordination mentioned before or by having an extra radio per device for scanning the channels. Global coordination is unlikely to exist in a wireless networking scenario with selfish players. The second assumption about perfect information might not hold either, because selfish players should allocate all of their radios for communication as shown in Lemma 2.2. It is possible to model the cost of scanning with one radio instead of using it for communication. The investigation of this issue is part of our future work.

2.7.2 Distributed Algorithm Using Imperfect Information

In order to overcome the limitations of the centralized algorithm proposed in Section 2.7.1, we suggest a second algorithm that does not require global coordination and uses only imperfect information. In this subsection, we assume that players have imperfect information, meaning that they know the total number of radios on only those channels on which they operate a radio.

We define a *round-based* distributed algorithm that works as follows. First, we assume that there exists a random radio assignment of the players over the channels. For simplicity, we focus on the Nash equilibria that correspond to Theorem 2.6. This means that we assume that no player allocates more than one device on any channel. After the initial channel assignment, each player tries to improve his total throughput by reorganizing his radios. To avoid that all players change together, we leverage the technique of backoff mechanism well known in the IEEE 802.11 medium access technology [Sch05]. We define a *backoff window BW* and each player chooses a random initial value for his *backoff counter* with uniform probability from the set $\{1, \ldots, BW\}$. Then in every round each player decreases his backoff counter by one and applies the re-allocation of his radios only when the backoff counter reaches

zero. After he changes his channel allocation, he resets the backoff counter as described previously. We can notice that using the backoff mechanism, the players play a game in an almost sequential order.

In each round, when player i's backoff counter is equal to zero, he calculates the average number of devices on the channels he knows (recall that we denote this set by C_i). We denote the average number of devices on the channels in C_i by K_i. For each channel $y \in C_i$ with $k_y - K_i \geq 1$ player i *moves his radio to another channel* $x \notin C_i$. The probability to chose a channel $x \notin C_i$ is $\frac{1}{|C \backslash C_i|}$. This is the first property of the algorithm with imperfect information.

We can show that the above procedure reaches a stable state. Unfortunately, the available local information might be insufficient for the players to determine if the achieved stable state is Nash equilibrium. We show an example for such a "false Nash equilibrium" in Figure 2.10.
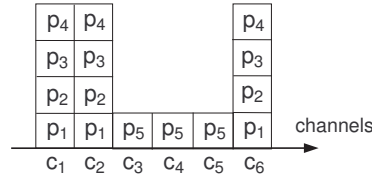
Figure 2.10: An example for a stability state using the distributed algorithm with imperfect information. Here $|C| = 6$, $|\mathcal{N}| = 5$ and $k = 3$. Each player believes that this is a Nash equilibrium due to the insufficient local information.

In order to solve the problem of inefficient stable states, we introduce the following mechanism: player i checks the number of radios for each of the channels $y \in C_i$ as suggested above and *with a small probability ϵ he moves his radio to another channel $x \notin C_i$ even if $0 < k_y - K_i < 1$*. He chooses the new channel x with a probability $\frac{1}{|C \backslash C_i|}$ as presented before. This second property allows us to resolve the inefficient stability states, but at the same time, it will also cause the instability of the Nash equilibria.

We provide the description of our algorithm below. Note that this algorithm now includes both properties: 1) the backoff mechanism and 2) the mechanism to resolve inefficient stable states.

Due to the second property of our algorithm, it does not perfectly converge to the existing Nash equilibria (more precisely, it converges there with high probability, but it does not stay in a Nash equilibrium solution). Nevertheless, we can observe that the algorithm remains in states that are "close" to Nash equilibria in terms of load-balancing. We demonstrate this intuition by the simulations presented in Section 2.7.3.

2.7.3   Simulation Results for Algorithm 2.2

We implemented Algorithm 2.2 in MATLAB and with a special focus on wireless IEEE 802.11a protocol (meaning that we have chosen 8 orthogonal channels as a default value for $|C|$). In this subsection, we present our simulation results showing the convergence time and efficiency of Algorithm 2.2. In each of the simulations, we assume a constant throughput function $\tau(\cdot)$. Note however, that the algorithm shows similar properties for any decreasing throughput function introduced in Section 2.2.

Let us first highlight the best and worst case in terms of the desired load-balancing for Algorithm 2.2. The best case is one of the NE channel allocations, and the worst case is characterized by the fact that there exist k channels where each of the players have a radio, whereas the rest of the channels have no radios at all. In Figure 2.11, we present an example of the worst case channel allocation that is opposed to the best case NE in Figure 2.5 for $|C| = 6$ and we refer to it as *unbalanced (UB)* channel allocation.

Algorithm 2.2 Distributed NE channel allocation algorithm using local information

---

 1: random channel allocation
 2: **while** () **do**
 3:     get the current channel allocation
 4:     **for** $i = 1$ to $|\mathcal{N}|$ **do**
 5:         **if** backoff counter is 0 **then**
 6:             **if** $(max_{x \in \mathcal{C}_i}(k_x) - min_{x \in \mathcal{C}_i}(k_x) > 1)$ **then**
 7:                 **for** $j = 1$ to k **do**
 8:                     assume that radio j uses channel y
 9:                     **if** $k_y > K_i$ **then**
10:                         move the radio j from y to $x \notin \mathcal{C}_i$, where x is chosen with uniform random probability from the set $\mathcal{C} \backslash \mathcal{C}_i$
11:                     **end if**
12:                 **end for**
13:             **else**
14:                 **for** $j = 1$ to k **do**
15:                     assume that radio j uses channel y
16:                     **if** $k_y \geq K_i$ **then**
17:                         move the radio j from y to $x \notin \mathcal{C}_i$ with probability ϵ, where x is chosen with uniform random probability from the set $\mathcal{C} \backslash \mathcal{C}_i$
18:                     **end if**
19:                 **end for**
20:             **end if**
21:             reset the backoff counter to a new value from the set $\{1, \ldots, BW\}$
22:         **else**
23:             decrease the backoff counter value by one
24:         **end if**
25:     **end for**
26: **end while**
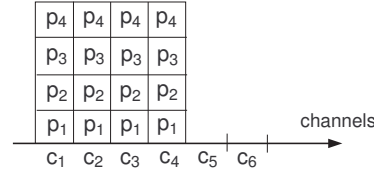
Figure 2.11: An example for a worst case channel allocation for Algorithm 2.2. The channel allocation is completely unbalanced, as opposed to the NE (best case) shown in Figure 2.5. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

We calculate the average number of radios per channel as $K = \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|}$. We can compare the utilization of every channel x to the average to achieve the total balance of the channel allocation S:

Definition 2.3. *(Balance:) The* balance β *of a channel allocation S is defined as the sum* $\beta(S) = \sum_{x \in \mathcal{C}} |k_x - K|$.

The notion of balance allows us to define the efficiency of a given channel allocation as a proportion between the worst case and the best case channel allocations.

Definition 2.4. *(Efficiency:) The* efficiency ϕ *of a channel allocation S is defined as* $\phi(S) = \frac{\beta(S_{\mathrm{UB}}) - \beta(S)}{\beta(S_{\mathrm{UB}}) - \beta(S_{\mathrm{NE}})}$.

Let us emphasize that for any channel allocation S, it is true that $0 \leq \phi(S) \leq 1$. Furthermore, $\phi(S_{\mathrm{NE}}) = 1$ and $\phi(S_{\mathrm{UB}}) = 0$ as desired by this measure.

Let us now define the *average efficiency* over time and the *efficiency ratio*. To this end, we denote the efficiency in round t by $\phi(t, S)$.

Definition 2.5. *(Average efficiency and efficiency ratio:) The* average efficiency $\overline{\phi}$ *at round T is defined as the sum* $\overline{\phi}(T, S) = \frac{\sum_{t=1}^{T} \phi(t, S)}{T}$. *We define the* efficiency ratio *as* $\Phi = \liminf_{T \to \infty} \overline{\phi}(T, S)$.

Note that the efficiency ratio expresses the performance of the distributed channel allocation algorithm per round over a long period of time. In our simulations, we applied a finite simulation time, hence we measured the efficiency ratio for $T = 10000$ rounds.

Finally, let us define the *convergence time* of Algorithm 2.2 as follows.

Definition 2.6. *(Convergence Time): We define the* convergence time *of Algorithm 2.2, as the time when the channel allocation efficiency first reaches the value of one (i.e., the efficiency of a NE, $\phi(S_{\mathrm{NE}})$).*

We assume that the duration of one round in the updating algorithm is 10ms. This duration of one round corresponds roughly to the time needed for all these devices to transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel. As mentioned previously, we run each simulation for 10000 rounds, which corresponds to 100s according to the assumption above. Each figure shows the average values of 100 simulation runs and the corresponding confidence interval of 95 per cent.

Let us first present an example run for our distributed algorithm with imperfect information in Figure 2.12 for 20s. One can notice that the algorithm quickly reaches the NE state and thus the average efficiency converges to one. Also, one can observe that the players sometimes leave the NE state due to the second property (change a radio on a channel $x \in \mathcal{C}^+$ in a stable state with probability ϵ), but they quickly return to it.
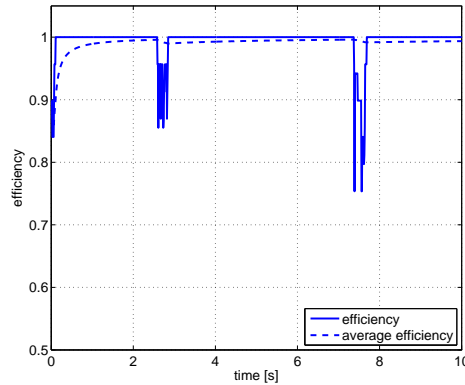
Figure 2.12: One simulation run: Efficiency and averaged efficiency vs. time using the values $|\mathcal{C}| = 8$, $|\mathcal{N}| = 10$, $k = 3$, $\epsilon = 10^{-4}$ and $W = 15$.

Suppose that the total available throughput per channel is $\tau(k_x) = 54Mbps$, for any k_x. In Figure 2.13, we present a snapshot of the total payoff for the players in the first NE reached in the previous simulation. One can observe that the total throughput is very similar for the users, hence we conclude that our algorithm converges to fair channel allocations.
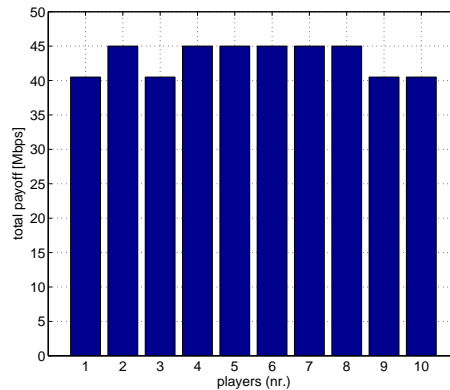
**Figure 2.13:** Total throughput (payoff) received by each device in the first NE channel allocation. The parameter values are those of Figure 2.12.

Next, we investigate the effect of the number of radios per device on the efficiency ratio (shown in Figure 2.14a) and on the convergence time of the algorithm (presented in Figure 2.14b). We can observe on the figures that Algorithm 2.2 converges fast with high efficiency ratio if the number of radios per device is 3 or 5. The higher the number of radios per device, the more channels the players know. Hence, more information helps them making their decisions. This is the reason that the convergence is slower if the number of radios is 2. For two radios per devices, the effect of changing the channel for even one radio has a significant impact that undermines the stability of the NE more easily. If the number of radios is 4, then convergence is slow for another reason: There is only one Nash equilibrium channel allocation, namely the perfectly flat one; thus it takes more time to find it. With longer convergence, the efficiency ratio decreases as well.

(a)   (b)

(c)   (d)
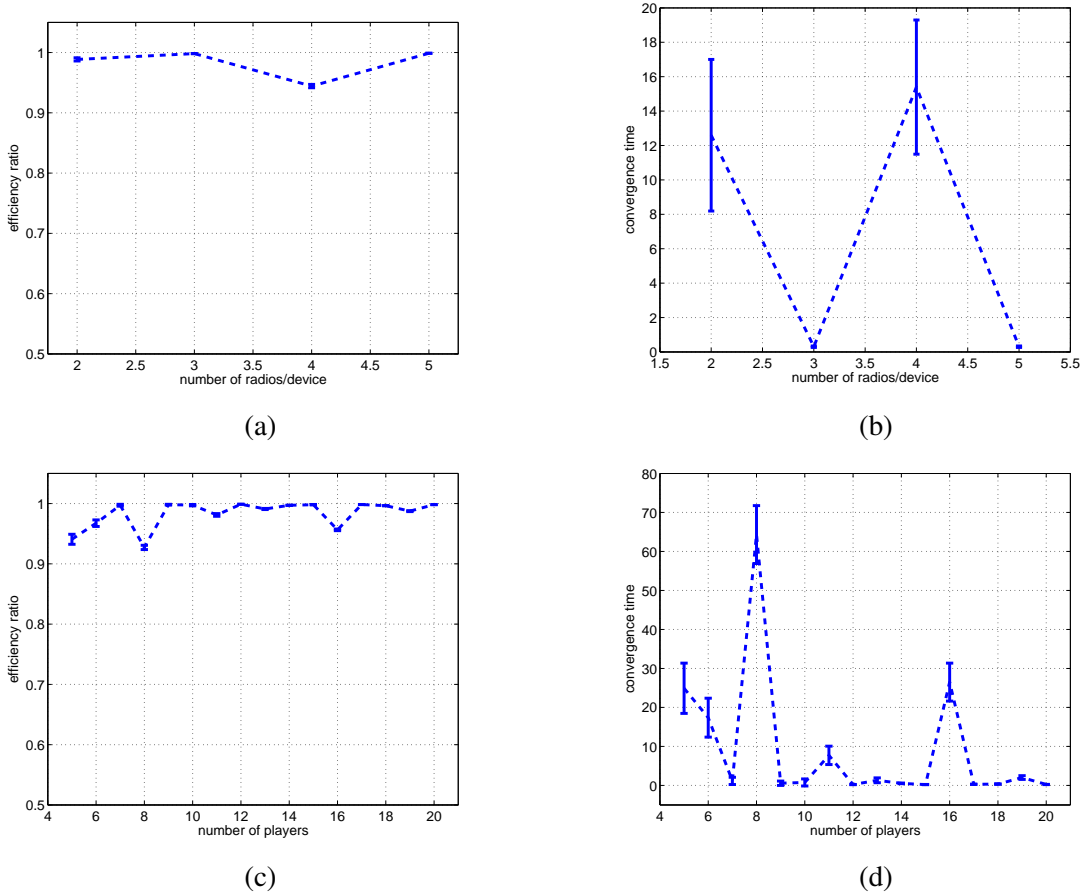
Figure 2.14: The effect of the total number of radios: (a) The efficiency ratio and (b) the convergence time as a function of the number of radios per device k. Similarly, we show (c) the efficiency ratio and (d) the convergence time as a function of the number of players $|\mathcal{N}|$. The simulation parameters are $|\mathcal{C}| = 8$, $\epsilon = 10^{-4}$ and $W = 15$. In addition, we used the following default values $|\mathcal{N}| = 10$ and $k = 3$, where they did not correspond to the measured parameter.

Then, we investigate the effect of the number of players, each device having three radios and present our results in Figures 2.14c and 2.14d. We can see that our distributed algorithm keeps the system in an efficient state, although the efficiency is slightly lower for multiples of $|\mathcal{N}|$ with higher convergence time. As mentioned above, the reason is that in this case, there exists only one NE (the perfectly load-balanced) and thus it is more difficult to converge to.

In the second set of simulations, we study the effect of the two parameters that introduce the randomness to Algorithm 2.2. First, we show the effect of ϵ on the efficiency ratio and the convergence time in Figures 2.15a and 2.15b. One can observe that the efficiency ratio is very high, but slightly decreases as ϵ increases. The reason is that with a higher ϵ value it is more likely that the algorithm does not stay in a NE, once it has reached it.

Finally, we study the effect of the size of the backoff window in Figures 2.15c and 2.15d. Intuitively, efficiency increases with the backoff window size, because of the decreasing number of simultaneous
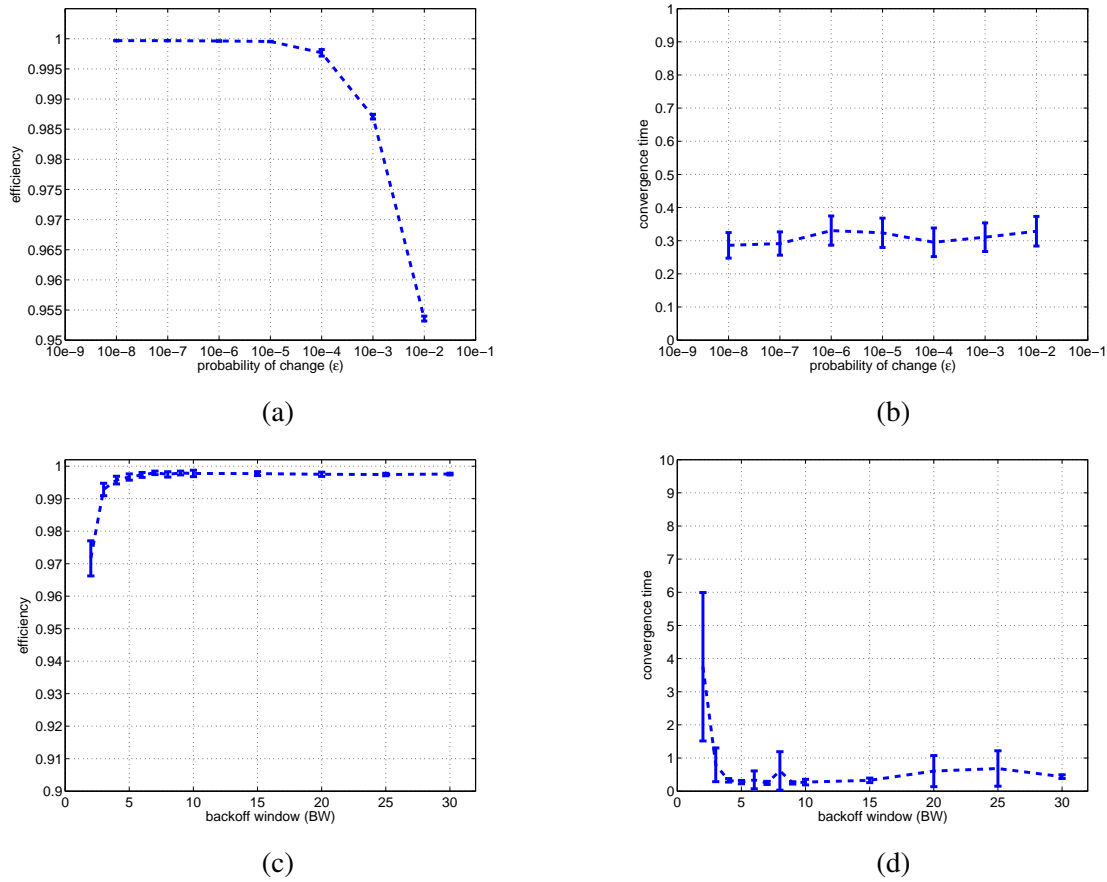
Figure 2.15: The effect of randomness parameters: (a) Efficiency ratio and (b) convergence time as a function of ϵ. Furthermore, we present (c) the efficiency ratio and (d) the convergence time as a function of the backoff window size W. The simulation parameters are $|\mathcal{C}| = 8$, $|\mathcal{N}| = 10$ and $k = 3$. In addition, we used the following default values $\epsilon = 10^{-4}$ and $W = 15$, where they did not correspond to the measured parameter.

channel changes. Interestingly, this increase is very rapid and the algorithm is very efficient for small backoff window values. With a larger backoff window, our algorithm realizes with high probability a sequential procedure similar to the centralized algorithm with perfect information described in Algorithm 2.1. Note, however, that setting a very high backoff window value is not reasonable, because it makes the players wait for an unnecessarily long time. For the same reason, convergence time drops quickly as the backoff window value increases.

In summary, we can observe that, in spite of the fact that convergence is not theoretically ensured, the proposed distributed algorithm based on imperfect information ensures high system performance and good convergence time.

2.8   Related work

There has been a significant amount of work on channel allocation in wireless networks, notably for cellular networks. Channel allocation schemes in cellular networks can be divided into three categories: fixed channel allocation (FCA), dynamic channel allocation (DCA) and hybrid channel allocation (HCA), which combines the two former methods.

In a fixed channel allocation scheme, the same number of channels are permanently allocated to the radios at the base stations. To study fixed channel allocation, most authors used graph coloring / labelling techniques (e.g., in [vdHLS98]). The FCA method performs very well under a high traffic load, but it cannot adapt to changing traffic conditions or user distributions.

To overcome the inflexibility of FCA, many authors propose dynamic channel allocation (DCA) methods (e.g. as presented in [CC96, PW96, ZE93]). In contrast to FCA, there is no constant relationship between the base stations in a cell and their respective channels. All channels are available for each base station and they are assigned dynamically as new users arrive. Typically, the available channels are evaluated according to a cost function and the one with the minimum cost is used [CR72]. Due to its dynamic property, the DCA can adapt to changing traffic demand. Because adaptation implies some cost, it performs worse than FCA in the case of a heavy traffic load. For a comprehensive survey on the topic, we refer the reader to [KN96].

Due to the emergence of alternative communication technologies, channel allocation schemes are becoming a focus of research again. Mishra *et al.* [MBA05] propose a channel allocation method for wireless local area networks (WLANs) based weighted graph coloring. Zheng and Cao [ZC05] present a rule-based spectrum management scheme for cognitive radios.

Recently, several researchers have considered devices using multiple radios, notably in mesh networks (for a survey on mesh networks, see [AWW05]). In the multi-radio communication context, channel allocation and access also became two of the crucial topics. Related work on multi-radio medium access includes, but is not restricted to [ABP[+]04, ABL05, RC05].

In all the related work cited in this chapter so far, the authors assumed that the radio devices cooperate to achieve a high system performance. But this assumption might not hold, as the users of these devices are usually selfish and they want to maximize their own performance without necessarily respecting the system objectives. Game theory provides a straightforward tool to study medium access problems in competitive wireless networks and has been applied to the CSMA/CA protocol [CGAH05, Kon02] and to the Aloha protocol [MW03]. Furthermore, a fixed channel allocation game was presented in [HHLM04] based on graph coloring. Unfortunately, their model does not apply to multi-radio devices. For cognitive radio networks, the authors of [NC05] propose a dynamic channel allocation scheme based on a potential game. In addition, they suggest another technique based on machine learning with different payoff functions. Cao and Zheng [CZ05] propose distributed spectrum allocation in cognitive radio networks based on local bargaining.

2.9   Summary

In this chapter, we considered the problem of competitive channel allocation among devices that use multiple radios. This model applies to wireless communications in general and to mesh networks in particular. In mesh networks, if the access points are operated by home users, then the competition presented in this chapter might threaten the performance of the network. But, we showed in this chapter that selfish players are able to efficiently coordinate their channel allocation.

We studied this problem in a static channel allocation game. Our main conclusion is that, in spite of the non-cooperative behavior of such devices, their Nash equilibrium channel allocations result in load balancing. Furthermore, we investigated the efficiency of these Nash equilibria and showed that they are efficient from the system point of view (i.e., that the price of anarchy is close to one). Because the static game model already identifies efficient Nash equilibria, we refrained from extending the model to a repeated game. We also studied fairness issues and coalition-proof NE. Finally, we provided two algorithms to achieve this efficient Nash equilibrium channel allocation and we studied their convergence properties theoretically or numerically.

The results showed that efficient channel allocation can be reached even if the devices are selfish. This is particulary encouraging because the channel allocation model can be easily extended to model upcoming mesh and cognitive radio networks.

Publications: [FCH06], [FCSH07]

Chapter 3

Packet Forwarding in Wireless Ad Hoc Networks - the Static Case

3.1 Introduction

In multi-hop wireless ad hoc networks, networking services are provided by the nodes themselves. As a fundamental example, the nodes must make a mutual contribution to packet forwarding in order to ensure an operable network. If the network is under the control of a single authority, as is the case for military networks and rescue operations, the nodes cooperate for the critical purpose of the network. However, if each node is his[1] own authority, cooperation between the nodes cannot be taken for granted; on the contrary, it is reasonable to assume that each node has the goal to maximize his own benefits by enjoying network services and at the same time minimizing his cost by contributing to these services. This selfish behavior can significantly damage network performance [BH00, MGLB00].

In recent years, researchers have identified the problem of stimulating cooperation in ad hoc networks and proposed several solutions to give nodes incentive to contribute to common network services. These solutions are based on a reputation system [BB02, MM02] or on a virtual currency [BH03, ZYC03]. All of these solutions are heuristics to provide a reliable cooperation enforcement scheme. However, it has never been formally proven that these techniques are really needed.

Recently, some researchers have claimed that under specific conditions, cooperation may emerge without incentive techniques [SNCR03, UBG03]. However, they have assumed a random connection setup, thus abstracting away the topology of the network. This paper aims at determining under which conditions such cooperation without incentives can exist, while taking the network topology into account. Indeed, in reality, the interactions between nodes are not random, as they are determined by the network topology and the communication pattern in the network.

We focus on the most basic networking mechanism, namely packet forwarding. We define a model in a game-theoretic framework and identify the conditions under which an equilibrium based on cooperation exists. As the problem is involved, we deliberately restrict ourselves to a static configuration. Note that our system model is an extension of the simple packet forwarding games presented in Section 1.1.

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[1]We consider the nodes as players in the packet forwarding game. As we have already mentioned in Section 1.1, we refer to the players with male pronouns.

3.2   Game-Theoretic Model of Packet Forwarding

3.2.1   Preliminaries

Let us consider an ad hoc network formed by a set of nodes $\mathcal{N}$. Each node has a given power range and two nodes are said to be neighbors if they reside within the power range of each other. We represent the neighbor relationship between the nodes with an undirected graph, which we call the *connectivity graph*. Each vertex of the connectivity graph corresponds to a node in the network, and two vertices are connected with an edge if the corresponding nodes are neighbors.

Communication between two non-neighboring nodes is based on multi-hop relaying. This means that packets from the source to the destination are forwarded by intermediate nodes. For a given source and destination, the intermediate nodes are those that form the shortest path[2] between the source and the destination in the connectivity graph. We call such a chain of nodes (including the source and the destination) a *route* and denote it by r. We call the topology of the network with a given set of communicating nodes a *scenario*.

We use a discrete model of time where time is divided into steps. We assume that both the connectivity graph and the set of existing routes remain unchanged during a time step, whereas changes may happen at the end of each time step. We assume that the duration of the time step is much longer than the time needed to relay a packet from the source to the destination. This means that a node is able to send several packets within one time step. This allows us to abstract away individual packets and to represent the data traffic in the network with *flows*. We assume CBR flows, which means that a source node sends the same amount of traffic in each time step. Note, however, that this amount may be different for every source node and every route.

3.2.2   Forwarding Game

We model the operation of the network as a game, which we call the *forwarding game*. The *players* of the forwarding game are the nodes, hence we use the two terms interchangeably in this chapter. In each time step t, each node i chooses a cooperation level $\kappa_i(t) \in [0, 1]$, where 0 and 1 represent full defection and full cooperation, respectively. Here, defection means that the node does not forward traffic for the benefit of other nodes, whereas cooperation means that he does. Thus, $\kappa_i(t)$ represents the fraction of the traffic routed through i in t that i actually forwards. Note that i has a single cooperation level $\kappa_i(t)$, which he applies to every route in which he is involved as a forwarder. We prefer to not require the nodes to be able to distinguish the flows that belong to different routes, because this would require identifying the source-destination pairs and applying a different cooperation level to each of them; this would probably increase the computation at the nodes significantly.

Let us assume that in time step t there exists a route r with source node src and ℓ forwarder nodes $f_1, f_2, \ldots, f_\ell$. Let us denote by $T_{src}(r)$ the constant amount of traffic that src wants to send on r in each time step. The throughput $\tau(r, t)$ experienced by the source src on r in t is defined as the fraction of the traffic sent by src on r in t that is delivered to the destination. Since we are studying cooperation in packet forwarding, we assume that the main reason for packet losses in the network is the non-cooperative behavior of the nodes. In other words, we assume that the network is not congested and that the number of packets dropped because of the limited capacity of the nodes and the links is negligible. Hence, $\tau(r, t)$

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[2]In other words, we abstract away the details of the routing protocol, and we model it as a function that returns the shortest path between the source and the destination. If there are multiple shortest paths, then one of them is selected at random.

can be computed as the product of $T_{src}(r)$ and the cooperation levels of all forwarder nodes:

$$\tau(r, t) = T_{src}(r) \cdot \prod_{k=1}^{\ell} \kappa_{f_k}(t) \tag{3.1}$$

In addition, we define the normalized throughput $\hat{\tau}(r, t)$ as follows:

$$\hat{\tau}(r, t) = \frac{\tau(r, t)}{T_{src}(r)} = \prod_{k=1}^{\ell} \kappa_{f_k}(t) \tag{3.2}$$

We will use the normalized throughput later as an input of the strategy function of src.

The benefit $b_{src}(r, t)$ of src on r in t depends on the experienced throughput $\tau(r, t)$. In general, $b_{src}(r, t) = \varphi_{src}(\tau(r, t))$, where the benefit function $\varphi_{src}(\cdot)$ is non-decreasing. In this work, we further assume that $\varphi_{src}(\cdot)$ is concave, derivable at $T_{src}(r)$, and $\varphi_{src}(0) = 0$. We place no other restrictions on $\varphi_{src}(\cdot)$. Note that the benefit function of different nodes may be different.

The cost $c_{f_j}(r, t)$ of the j-th forwarder node f_j on r in t is non-positive and represents the cost for node f_j to forward packets on route r during time step t. It is defined as follows:

$$c_{f_j}(r, t) = -T_{src}(r) \cdot C \cdot \hat{\tau}_j(r, t) \tag{3.3}$$

where C is the cost of forwarding one unit of traffic, and $\hat{\tau}_j(r, t)$ is the normalized throughput on r in t leaving node j. For simplicity, we assume that the nodes have the same, fixed transmission power, and therefore C is the same for every node in the network, and it is independent from r and t. $\hat{\tau}_j(r, t)$ is computed as the product of the cooperation levels of the forwarder nodes from f_1 up to and including f_j:

$$\hat{\tau}_j(r, t) = \prod_{k=1}^{j} \kappa_{f_k}(t) \tag{3.4}$$

In our model, the payoff of the destination is 0. In other words, we assume that only the source benefits if the traffic reaches the destination (information push). However, our model can be applied in the reverse case: all our results also hold when only the destination benefits from receiving traffic. An example of this case is a file download (information pull).

The *payoff* $u_i(t)$ of node i in time step t is then computed as

$$u_i(t) = \sum_{r \in \mathcal{SR}_i(t)} b_i(r, t) + \sum_{r' \in \mathcal{FR}_i(t)} c_i(r', t) \tag{3.5}$$

where $\mathcal{SR}_i(t)$ is the set of routes in t where i is the source, and $\mathcal{FR}_i(t)$ is the set of routes in t where i is an forwarder node.

3.2.3 Strategy Space

In every time step, each node i updates his cooperation level using a *strategy function* s_i. In general, i could choose a cooperation level to be used in time step t, based on the information he obtained in *all* preceding time steps. In order to make the analysis feasible, we assume that i uses only information that he obtained in the previous time step (i.e., he uses strategies of history-1 as introduced in Section 1.4.2).

More specifically, we assume that i chooses his cooperation level $\kappa_i(t)$ in time step t based on the normalized throughput he experienced in time step $t-1$ on the routes where he was a source:

$$\kappa_i(t) = s_i([\hat{\tau}(r, t-1)]_{r \in \mathcal{SR}_i(t-1)}) \qquad (3.6)$$

where $[\hat{\tau}(r, t-1)]_{r \in \mathcal{SR}_i(t-1)}$ represents the normalized throughput vector for node i in time step $t-1$, each element of which is the normalized throughput experienced by i on a route where he was source in $t-1$. The strategy of a node i is then defined by his strategy function s_i and his initial cooperation level $\kappa_i(0)$.

Note that s_i takes as input the normalized throughput and not the payoff received by i in the previous time step. The rationale is that i should react to the behavior of the rest of the network, which is represented by the normalized throughput in our model.

Our model requires that each source be able to observe the throughput in a given time step on each of his routes. We assume that this is made possible with high enough precision by using some higher level control protocol above the network layer.

3.3 Meta-Model

In this section, we introduce a meta-model in order to formalize the properties of the packet forwarding game defined in the previous section. In the meta-model, we focus on the evolution of the cooperation levels of the nodes; all other details of the model defined earlier (e.g., amounts of traffic, forwarding costs, and utilities) are abstracted away. Unlike in the model, in the meta-model and in the remainder of this chapter, we will assume that routes remain unchanged during the lifetime of the network. In addition, we assume for the moment that each node is the source of only one route.[3]

Let us consider a route r. The payoff received by the source on r depends on the cooperation levels of the forwarder nodes on r. We represent this dependency relationship between the nodes with a directed graph, which we call the *dependency graph*. Each vertex of the dependency graph corresponds to a network node. There is a directed edge from vertex i to vertex j, denoted by the ordered pair (i, j), if there exists a route where i is an forwarder node and j is the source. Intuitively, an edge (i, j) means that the behavior (cooperation level) of i has an effect on j. The concept of dependency graph is illustrated in Figure 3.1.

Now we define the automaton Θ that will model the unfolding of the forwarding game in the meta-model. The automaton is built on the dependency graph. We assign a machine M_i to every vertex i of the dependency graph and interpret the edges of the dependency graph as links that connect the machines assigned to the vertices. Each machine M_i thus has some input and some (possibly 0) output links.

The internal structure of the machine is illustrated in Figure 3.2. Each machine M_i consists of a multiplication[4] gate $\prod$ followed by a gate that implements the strategy function s_i of node i. The multiplication gate $\prod$ takes the values on the input links and passes their product to the strategy function gate.[5] Finally, the output of the strategy function gate is passed to each output link of M_i.

---

[3]We will relax this assumption in Section 3.5. We emphasize that all of our analytical results hold in the extended case as well.

[4]The multiplication comes from the fact that the experienced normalized throughput for the source (which is the input of the strategy function of the source) is the product of the cooperation levels of the forwarders on his route.

[5]Note that here s_i takes a single real number as input, instead of a vector of real numbers as we defined earlier, because we assume that each node is source of only one route.
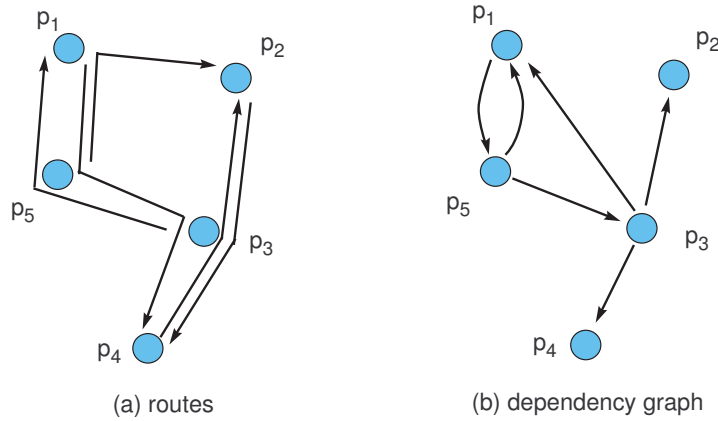
Figure 3.1: Representation of a network: (a) a graph showing 5 routes and (b) the corresponding dependency graph.
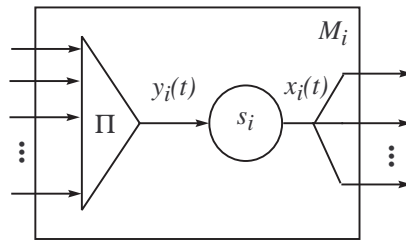
Figure 3.2: Internal structure of machine M_i.

The automaton Θ works in discrete steps. Initially, in step 0, each machine M_i outputs some initial value $x_i(0)$. Then, in step $t > 0$, each machine computes his output $x_i(t)$ by taking the values that appear on his input links in step $t - 1$.

Note that if $x_i(0) = \kappa_i(0)$ for all i, then in step t, each machine M_i will output the cooperation level of node i in time step t (i.e., $x_i(t) = \kappa_i(t)$), as we assumed that the set of routes (and hence the dependency graph) remains unchanged in every time step. Therefore, the evolution of the values (which, in fact, represent the state of the automaton) on the output links of the machines models the evolution of the cooperation levels of the nodes in the network.

In order to study the interaction of node i with the rest of the network, we extract the gate that implements the strategy function s_i from the automaton Θ. What remains is the automaton without s_i, which we denote by Θ_{-i}. Θ_{-i} has an input and an output link; if we connect these to the output and the input, respectively, of s_i (as illustrated in Figure 3.4), then we get back the original automaton Θ. In other words, the automaton in Figure 3.4 is another representation of the automaton in Figure 3.3, which captures the fact that from the viewpoint of node i, the rest of the network behaves like an automaton: The input of Θ_{-i} is the sequence $\overline{x}_i = x_i(0), x_i(1), \ldots$ of the cooperation levels of i, and his output is the sequence $\overline{y}_i = y_i(0), y_i(1), \ldots$ of the normalized throughput values for i.

By using the system of equations that describe the operation of Θ, one can easily express any element
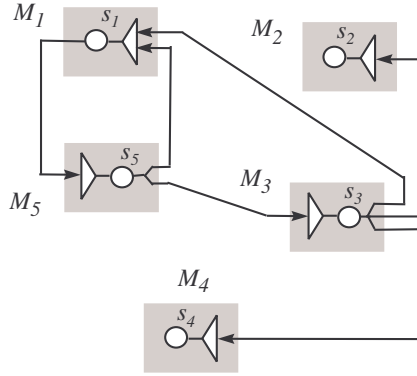
Figure 3.3: The automaton that corresponds to the dependency graph of Figure 3.1.
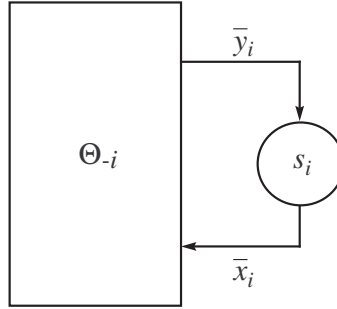
Figure 3.4: Model of interaction between node i and the rest of the network represented by the automaton Θ_{-i}.

$y_i(t)$ of sequence $\overline{y}_i$ as some function of the preceding elements $x_i(t-1), x_i(t-2), \ldots, x_i(0)$ of sequence $\overline{x}_i$ and the initial values $x_j(0)$ ($j \neq i$) of the machines within Θ_{-i}. We call such an expression of $y_i(t)$ the t-th *input/output formula* or the t-th *i/o formula* of Θ_{-i}, for short. It is important to note that the i/o formulae of Θ_{-i} may involve any strategy function s_j where $j \neq i$, but they never involve s_i. Considering again the automaton in Figure 3.3, and extracting, for instance, s_1, we can determine the first few i/o formulae of Θ_{-1} as follows:

$$
\begin{aligned}
y_1(0) &= x_3(0) \cdot x_5(0) \\
y_1(1) &= s_3(x_5(0)) \cdot s_5(x_1(0)) \\
y_1(2) &= s_3(s_5(x_1(0))) \cdot s_5(x_1(1)) \\
y_1(3) &= s_3(s_5(x_1(1))) \cdot s_E(x_1(2)) \\
\cdots & \qquad \cdots
\end{aligned}
$$

A *dependency loop* L of node i is a sequence $(i, v_1), (v_1, v_2), \ldots, (v_{\ell-1}, v_\ell), (v_\ell, i)$ of edges in the dependency graph. The length of a dependency loop L is defined as the number of edges in L, and it is denoted by $|L|$. The existence of dependency loops is important: if node i has no dependency loops,

then the cooperation level chosen by i in a given time step has no effect on the normalized throughput experienced by i in future time steps. In the example shown in Figure 3.1, nodes p_2 and p_4 have no dependency loops.

Every node i has two types of dependency loops; these types depend on the strategies played by the other nodes in the loop. If L is a dependency loop of i, and all other nodes $j \neq i$ in L play reactive strategies, then L is said to be a *reactive dependency loop* of i. If, on the contrary, there exists at least one node $j \neq i, j \in L$ that plays a non-reactive strategy, then L is called a *non-reactive dependency loop* of i.

3.4 Analytical Results

Our goal, in this section, is to study the *existence* of Nash equilibria of packet forwarding strategies. In the next section, we will investigate the *probability of fulfillment* of the conditions for possible Nash equilibria in randomly generated scenarios. The existence of a Nash equilibrium based on cooperation would mean that there are cases in which cooperation is "naturally" encouraged, i.e. without using incentive mechanisms. In the following, we use the model and the meta-model that we introduced earlier.

The goal of the nodes is to maximize the total payoff that they accumulate over time (i.e., we assume that they are long-sighted players). However, the end of the game is unpredictable. Thus, we apply the technique of discounting introduced in Section 1.4.3: We model the *finite* forwarding game with an unpredictable end as an *infinite* game where future payoffs are *discounted*. The discounted total payoff u_i of a node i is computed as the weighted sum of the payoffs $u_i(t)$ that i obtains in each time step t:

$$u_i = \sum_{t=0}^{\infty} [u_i(t) \cdot \delta^t] \tag{3.7}$$

where δ is the *discounting factor* with $0 < \delta < 1$, and hence the multiplicative factor decreases exponentially with t.

Let us repeat that $\mathcal{SR}_i(t)$ denotes the set of routes for which i is the source, and that $\mathcal{FR}_i(t)$ denotes the set of routes for which i is an forwarder node. As we assume that the routes remain static, meaning that $\mathcal{SR}_i(t)$ and $\mathcal{FR}_i(t)$ do not change over time, we will simply write $\mathcal{SR}_i$ and $\mathcal{FR}_i$ instead of $\mathcal{SR}_i(t)$ and $\mathcal{FR}_i(t)$. In addition, since we assume that each node is a source on exactly one route, $\mathcal{SR}_i$ is a singleton. We denote the single route in $\mathcal{SR}_i$ by r_i, and the amount of traffic sent by i on r_i in every time step by T_i. The cardinality of $\mathcal{FR}_i$ will be denoted by $|\mathcal{FR}_i|$. For any route $r \in \mathcal{FR}_i$, we denote the set of forwarder nodes on r upstream from node i (including node i) by $\mathcal{F}(r, i)$. Moreover, $\mathcal{F}(r)$ denotes the set of all forwarder nodes on route r, and $src(r)$ denotes the source of route r. Finally, the set of nodes that are forwarders on at least one route is denoted by $\mathcal{F}$ (i.e., $\mathcal{F} = \{i \in \mathcal{N} : \mathcal{FR}_i \neq \emptyset\}$).

Theorem 3.1. *If a node i is in $\mathcal{F}$, and he has no dependency loops, then his best strategy is All-D (i.e., to choose cooperation level 0 in every time step).*

Proof. Node i wants to maximize his total payoff u_i defined in (3.7). In our case, $u_i(t)$ can be written as:

$$
\begin{aligned}
u_i(t) &= b_i(r_i, t) + \sum_{r \in \mathcal{FR}_i} c_i(r, t) \\
&= \varphi_i(T_i \cdot y_i(t)) - \sum_{r \in \mathcal{FR}_i} T_{src(r)} \cdot c \cdot \prod_{k \in \mathcal{F}(r,i)} x_k(t)
\end{aligned}
$$

Given that i has no dependency loops, $y_i(t)$ is independent of all the previous cooperation levels $x_i(t')$ of node i, where $t' < t$. Thus, u_i is maximized if $x_i(t') = 0$ for all $t' \geq 0$. □

Theorem 3.2. *If a node i is in $\mathcal{F}$, and he has only non-reactive dependency loops, then his best strategy is All-D.*

Proof. The proof is similar to the proof of Theorem 3.1. Since all dependency loops of i are non-reactive, his experienced normalized throughput $\overline{y}_i$ is independent of his own behavior $\overline{x}_i$. This implies that his best strategy is full defection. □

From this theorem, we can easily derive the following corollary.

Corollary 3.3. *If every node j ($j \neq i$) plays All-D, then the best response of i to this is All-D. Hence, every node playing All-D is a Nash equilibrium.*

If the conditions of Theorems 3.1 and 3.2 do not hold, then we cannot determine the best strategy of a node i in general, because it very much depends on the particular scenario (dependency graph) in question and the strategies played by the other nodes.

Now, we will show that, under certain conditions, cooperative equilibria do exist in the network. In order to do so, we first prove the following lemma:

Lemma 3.4. *Let us assume that node i is in $\mathcal{F}$, and let us consider a route $r \in \mathcal{FR}_i$. In addition, let us assume that there exists a dependency loop L of i that contains the edge $(i, src(r))$. If all nodes in L (other than i) play the TFT strategy, then the following holds:*

$$y_i(t + \lambda) \leq \prod_{k \in \mathcal{F}(r,i)} x_k(t) \tag{3.8}$$

where $\lambda = |L| - 1$.

Proof. Let L be the following sequence of edges in the dependency graph: $(v_0, v_1), (v_1, v_2), \ldots, (v_\lambda, v_{\lambda+1})$, where $v_{\lambda+1} = v_0 = i$ and $v_1 = src(r)$. We know that each node is the source of a single route; let us denote by r_{v_j} ($0 < j \leq \lambda + 1$) the route, on which v_j is the source. It follows that $r_{v_1} = r$. In addition, we know the existence of edge (v_j, v_{j+1}) ($0 \leq j \leq \lambda$) in the dependency graph means that v_j is a forwarder on $r_{v_{j+1}}$. The following holds for every node v_j ($0 \leq j \leq \lambda$):

$$x_{v_j}(t) \geq \prod_{k \in \mathcal{F}(r_{v_{j+1}}, v_j)} x_k(t) \geq \prod_{k \in \mathcal{F}(r_{v_{j+1}})} x_k(t) = y_{v_{j+1}}(t) \tag{3.9}$$

Furthermore, since every node except for $v_0 = v_{\delta+1} = i$ plays TFT, we have the following for every $0 < j \leq \delta$:

$$x_{v_j}(t + 1) = y_{v_j}(t) \tag{3.10}$$

Using (3.9) and (3.10) in an alternating order, we get the following:

$$x_{v_0}(t) \geq \prod_{k \in \mathcal{F}(r_{v_1}, v_0)} x_k(t) \geq y_{v_1}(t) = x_{v_1}(t+1) \geq y_{v_2}(t+1) = x_{v_2}(t+2) \geq \ldots \geq y_{v_{\lambda+1}}(t+\lambda) \tag{3.11}$$

By substituting i for v_0 and $v_{\delta+1}$, and r for r_{v_1}, we get the statement of the lemma:

$$x_i(t) \geq \prod_{k \in \mathcal{F}(r,i)} x_k(t) \geq \ldots \geq y_i(t + \lambda) \tag{3.12}$$

□
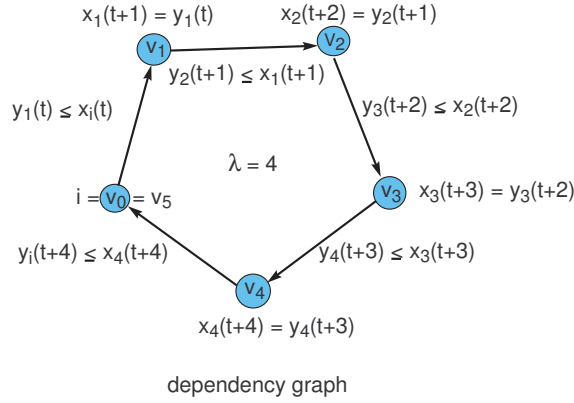
Figure 3.5: Example to illustrate the propagation of behavior as expressed formally in Lemma 3.4.

As an example, let us consider Figure 3.5, which illustrates a dependency loop of length 5 (i.e., $\lambda = 4$). According to Lemma 3.4, if nodes v_1, v_2, v_3, and v_4 play TFT, then the normalized throughput enjoyed by node i in time step $t + 4$ is upper bounded by his own cooperation level in time step t. Intuitively, this means that if node i does not cooperate, then this defection "propagates back" to him on the dependency loop. The delay of this effect is given by the length of the dependency loop.

Theorem 3.5. *Assuming that node i is in $\mathcal{F}$, the best strategy for i is full cooperation in each time step, if the following set of conditions holds:*

1. *for every $r \in \mathcal{FR}_i$, there exists a dependency loop $L_{i,src(r)}$ that contains the edge $(i, src(r))$;*

2. *for every $r \in \mathcal{FR}_i$,*

$$\frac{\varphi_i'(T_i) \cdot T_i \cdot \delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} > T_{src(r)} \cdot C \tag{3.13}$$

   where $\varphi_i'(T_i)$ is the value of the derivative[6] of $\varphi_i(\tau)$ at $\tau = T_i$, and $\lambda_{i,src(r)} = |L_{i,src(r)}| - 1$; and

3. *every node in $\mathcal{F}$ (other than i) plays the TFT strategy.*

The proof of Theorem 3.5 is provided in Section 3.7.

We have derived *necessary* conditions for spontaneous cooperation from Theorem 3.1 and 3.2. The fulfillment of the three conditions of Theorem 3.5 is *sufficient* for cooperation to be the best strategy for node i. We now discuss these three conditions one by one. *Condition 1* requires that node i has a dependency loop with all of the sources for which he forwards packets. *Condition 2* means that the maximum forwarding cost for node i on every route where i is a forwarder must be smaller than his possible future benefit averaged over the number of routes where i is a forwarder. Finally, *Condition 3* requires that all forwarding nodes in the network (other than node i) play TFT. This implies that all the dependency loops of node i are reactive. We note that the reactivity of the dependency loops can be based on other reactive strategies, different from TFT (for example Anti-TFT), but in that case the analysis becomes very complex. The analysis of the case when every node plays TFT is made possible

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[6]Recall the assumption that φ_i is derivable at T_i.

by the simplicity of the strategy function $s(y) = y$, which belongs to the TFT strategy. If all three conditions of Theorem 3.5 are satisfied, then node i has an incentive to cooperate, since otherwise his defective behavior will negatively affect his own payoff. However, as we will show in Section 3.6, *Condition 1* is already a very strong requirement that is rarely satisfied in randomly generated scenarios.

Both the All-C and TFT strategies result in full cooperation if the conditions of Theorem 3.5 hold. However, node i should not choose All-C, because All-C is a non-reactive strategy, and this might cause other nodes to change their strategies to All-D, as we will show in Section 3.6. Hence, we can derive the following corollary for cooperative Nash equilibria.

Corollary 3.6. *If the first two conditions of Theorem 3.5 hold for every node in $\mathcal{F}$, then all nodes playing TFT is a Nash equilibrium.*

In Section 3.6, we study *Condition 1* of Theorem 3.5, more specifically, the probability that it is satisfied for all nodes in randomly generated scenarios. Now, we briefly comment on *Condition 2*. As it can be seen, the following factors make *Condition 2* easier to satisfy:

- *Steep benefit functions:* The steeper the benefit function of node i is, the larger the value of its derivative is at $\tau = T_i$, which, in turn, makes the left side of (3.13) larger.

- *Short dependency loops:* In *Condition 2*, $|L_{i,src(r)}| = \lambda_{i,src(r)} + 1$ is the length of *any* dependency loop of node i that contains the edge $(i, src(r))$. Clearly, we are interested in the shortest of such loops, because the smaller $\lambda_{i,src(r)}$ is, the larger the value of $\delta^{\lambda_{i,src(r)}}$ is, which, in turn, makes the left side of (3.13) larger. It is similarly advantageous if δ is close to 1, which means, in general, that the probability that the game will continue is higher and thus possible future payoffs count more.

- *Small extent of involvement in forwarding:* The left side of (3.13) is increased if the cardinality of $\mathcal{FR}_i$ is decreased. In other words, if node i is a forwarder on a smaller number of routes, then *Condition 2* is easier to satisfy for i.

The first two theorems state that if the behavior of node i has no effect on his experienced normalized throughput, then defection is the best choice for i. In addition, Corollary 3.3 says that if every node always defects, then this is a Nash equilibrium. Theorem 3.5 leads to Corollary 3.6, which shows the existence of a cooperative equilibrium (each node playing TFT) under certain conditions.
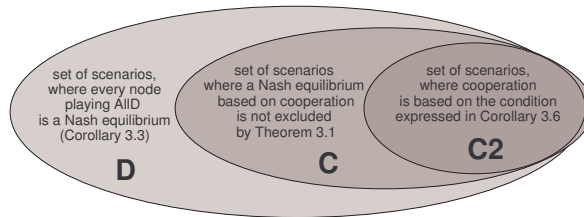
Figure 3.6: Classification of scenarios defined by our analytical results.

Figure 3.6 shows a classification of scenarios from the cooperation perspective. In the figure, set D denotes the set of all possible scenarios; indeed, we know from Corollary 3.3 that all nodes playing All-D is a Nash equilibrium in any possible scenario. Set $C2$ contains those scenarios where the conditions of Corollary 3.6 hold. Hence, all nodes playing TFT is a Nash equilibrium in every scenario in $C2$. Finally,

set C contains those scenarios, where the condition of Theorem 3.1 does not hold for any of the nodes in $\mathcal{F}$, or in other words, where every node in $\mathcal{F}$ has at least one dependency loop. Determining the Nash equilibria in the scenarios that belong to set $C \setminus C2$ is still an open research problem. In Section 3.6, we will describe our simulation results that quantify the size of the above sets.

3.5 Extended Model

In this section, we show that the assumption that each node is a source on only one route can be relaxed. For this, we slightly modify our initial model illustrated in Figure 3.2, and we define another machine representation for the node. This new representation is illustrated in Figure 3.7. The main novelty is that the outputs of the multiplication gates that correspond to the various routes for which the node is the source are aggregated into a single value using an aggregation function $a(\cdot)$.
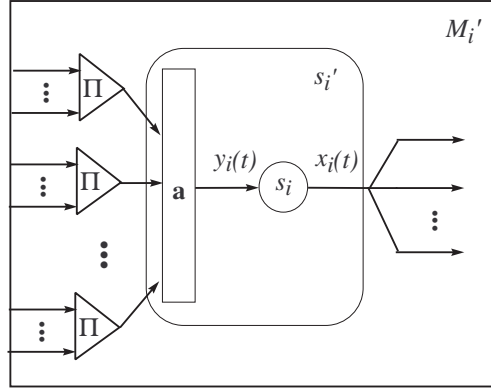
Figure 3.7: Modified machine for a node (M_i'). s_i' represents the modified strategy function for node i.

Theorems 3.1 and 3.2 still hold for an arbitrary $a(\cdot)$, because the existence of dependency loops does not depend on the internal structure of the machine representing the nodes. Moreover, assuming that $a(\cdot)$ has the property that $a(y_1, y_2, \ldots y_{|\mathcal{SR}_i|}) \leq y_k$ for all $1 \leq k \leq |\mathcal{SR}_i|$, we can prove a theorem similar to Theorem 3.5, where *Condition 1* and *Condition 3* are unchanged, and *Condition 2* has a slightly different form:

Theorem 3.7. *Assuming that node i is in $\mathcal{F}$, the best strategy for i is full cooperation in each time step, if the following set of conditions holds:*

1. *for every $r \in \mathcal{FR}_i$, there exists a dependency loop $L_{i,src(r)}$ that contains the edge $(i, src(r))$;*

2. *for every $r \in \mathcal{FR}_i$,*
$$\frac{\delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} \cdot \sum_{q \in \mathcal{SR}_i} \varphi_i'(T_i(q)) \cdot T_i(q) > T_{src(r)} \cdot C$$

   where $T_i(q)$ denotes the amount of traffic that node i sends as a source on route q, while φ_i' and $\lambda_{i,src(r)}$ have the same meaning as in Theorem 3.5; and

3. *every node in $\mathcal{F}$ (other than i) plays the TFT strategy.*

The proof of Theorem 3.7 is analogous to that of Theorem 3.5 and therefore we omit it.

An example for such a function $a(\cdot)$ is the minimum function (which we will use in our simulations). This choice of $a(\cdot)$ represents a pessimistic perception for the node: he considers the minimum normalized throughput he receives on the different routes as an aggregate view of all his routes. Similarly to Corollary 3.6, if the (modified) conditions of Theorem 3.5 hold for every node that is a forwarder on at least one route, then all nodes playing TFT is a Nash equilibrium.

3.6 Simulation Results

We have run a set of simulations to determine the probability that the conditions of our theorems and their corollaries hold. In particular, our goal is to estimate the probability that the first condition of Theorem 3.5 holds for every node in randomly generated scenarios.[7] In addition, we also estimate the probability that the condition of Theorem 3.1 does not hold for any of the nodes in randomly generated scenarios. These probabilities quantify the size of sets $C2$ and C, respectively.

In our simulations, we randomly place nodes on a toroid area.[8] Then, for each node, we randomly choose a number of destinations and we determine a route to these destinations using a shortest path algorithm. If several shortest paths existed to a given destination, then we randomly choose a single one. From the routes, we build up the dependency graph of the network. The simulation parameters are summarized in Table 3.1.

Parameter	Value
Number of nodes	100, 150, 200
Distribution of the nodes	random uniform
Area type	Torus
Area size	1500x1500m, 1850x1850m, 2150x2150m
Radio range	200 m
Number of destinations per node	1-10
Route selection	shortest path

Table 3.1: Parameter values for the simulation

Note that we increase the network size and the simulation area in parallel in order to keep the node density at a constant level. All the presented results are the mean values of 1000 simulation runs.

In the first set of simulations, we investigate the probability that the first condition of Theorem 3.5 holds for every node (the size of the set $C2$ in Figure 3.6). Among the 1000 scenarios that we generated randomly, we observed that there was not a single scenario in which the first condition of Theorem 3.5 was satisfied for all nodes. Thus, we conclude that the probability of a Nash equilibrium based on TFT as defined in Corollary 3.6 is very small.

In the second set of simulations, we investigate the proportion of random scenarios, where cooperation of all nodes is not excluded by Theorem 3.1. Figure 3.8 shows the proportion of scenarios, where

---

[7]The second condition of Theorem 3.5 is a numerical one. Whether it is fulfilled or not very much depends on the actual utility functions and parameter values (e.g., amount of traffic and discounting factor) used. Since, by appropriately setting these parameters, the second condition of Theorem 3.5 can always be satisfied, in our analysis, we make the optimistic assumption that this condition holds for every node in $\mathcal{F}$.

[8]We use this area type to avoid border effects. In a realistic scenario, the toroid area can be considered as an inner part of a large network.

each node in $\mathcal{F}$ has at least one dependency loop (the scenarios in set C in Figure 3.6) as a function of the number of routes originating at each node. We can observe that for an increasing number of routes originating at each node, the proportion of scenarios, where each node has at least one dependency loop, increases as well. Intuitively, as more routes are introduced in the network, more edges are added to the dependency graph. Hence, the probability that a dependency loop exists for each node increases. Furthermore, we can observe that the proportion of scenarios in which each node has at least one dependency loop decreases, as the network size increases. This is due to the following reason: the probability that there exists at least one node for which the condition of Theorem 3.1 holds increases as the number of nodes increases.

Figure 3.8 shows that the proportion of scenarios, where cooperation of all nodes is not excluded by Theorem 3.1 (set C) becomes significant (with respect to set D) only for cases in which each node is a source of a large number of routes. This implies that the necessary condition expressed by Theorem 3.1 is a strong requirement for cooperation in realistic settings (i.e., for a reasonably low number of routes per node).
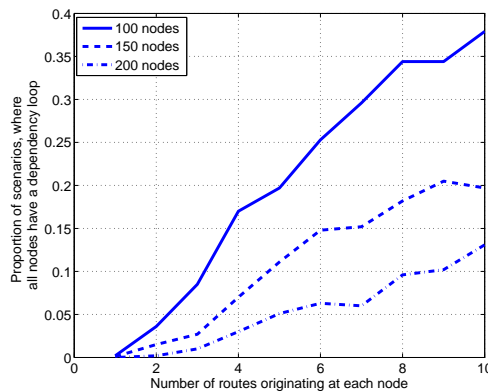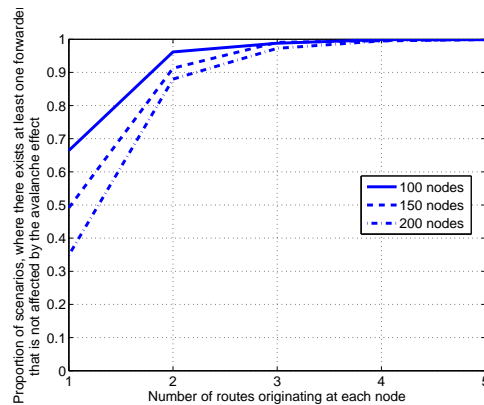

Figure 3.8: Proportion of scenarios, where each node that is a forwarder has at least one dependency loop.

Now let us consider the case, in which the nodes for which Theorem 3.1 holds begin to play All-D. This non-cooperative behavior can lead to an "avalanche effect" if the nodes iteratively optimize their strategies: nodes that defect can cause the defection of other nodes. We examine this avalanche effect in a simulation setting as follows.

Let us assume that each node is a source on one route. First, we identify the nodes in the set of forwarders $\mathcal{F}$ that have All-D as the best strategy due to Theorem 3.1. We denote the set of these defectors by $\mathcal{Z}_0$. Then, we search for sources that are dependent on the nodes in $\mathcal{Z}_0$. We denote the set of these sources by $\mathcal{Z}_0^+$. Since the normalized throughput of the nodes in $\mathcal{Z}_0^+$ is less than or equal to the cooperation level of any of their forwarders (including the nodes in $\mathcal{Z}_0$), their best strategy becomes All-D, as well, due to Theorem 3.2. Therefore, we extend the set $\mathcal{Z}_0$ of defectors, and obtain $\mathcal{Z}_1 = \mathcal{Z}_0 \cup \mathcal{Z}_0^+$. We extend the set $\mathcal{Z}_k$ of defectors iteratively in this way until no new sources are affected (i.e., $\mathcal{Z}_k \cup \mathcal{Z}_k^+ = \mathcal{Z}_k$). The remaining set $\mathcal{F} \setminus \mathcal{Z}_k$ of nodes is not affected by the behavior of the nodes in $\mathcal{Z}_k$ (and hence the nodes in $\mathcal{Z}_0$); this means that they are potential cooperators. Similarly, we can investigate the avalanche effect when the nodes are sources of several routes. In that case, we take the pessimistic assumption that the defection of a forwarder causes the defection of his sources. Then, we can iterate the search for the nodes that are affected by defection in the same way as above.

In Figure 3.9, we present the proportion of scenarios, where there exists a subset of nodes that are not affected by the defective behavior of the initial All-D players. We can see that this proportion converges rapidly to 1 as the number of routes originating at each node increases. The intuitive explanation is that increasing the number of routes per source (i.e., adding edges to the dependency graph) decreases the probability that Theorem 3.1 holds for a given node. Thus, as the number of routes per sources increases the number of forwarders that begin to play All-D decreases and so does the number of nodes affected by the avalanche effect.

Figure 3.9: Proportion of scenarios, where at least one node is not affected by the defective behavior of the initial nodes.

Additionally, we present in Figure 3.10 the proportion of forwarder nodes that are not affected by the avalanche effect. The results show that if we increase the number of routes originating at each node, the average number of unaffected nodes increases rapidly. For a higher number of routes per node, this increase slows down, but we can observe that the majority of the nodes are not affected by the defective behavior of the initial All-D players.
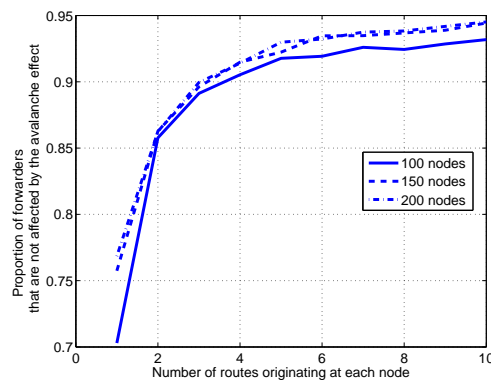
Figure 3.10: Average proportion of forwarder nodes that are not affected by the avalanche effect.

3.7 Proof of Theorem 3.5

Proof. In this proof we will express the maximum possible value of the total payoff u_i for node i in general. Then we will show that the maximum corresponds to the case in which node i fully cooperates. First, we introduce the linear function $\psi(\tau) = \varphi_i'(T_i) \cdot \tau + \varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i$. Function $\psi(\cdot)$ is the tangent of function φ_i at $\tau = T_i$. Note that due to the fact that φ_i is non-decreasing and concave, we have that $\psi(\tau) \geq \varphi_i(\tau)$ for all τ; in addition, we have equality at $\tau = T_i$ (i.e., $\psi(T_i) = \varphi_i(T_i)$).

By definition, the total payoff u_i of node i is the following:

$$
\begin{aligned}
u_i &= \sum_{t=0}^{\infty} \left[b_i(r_i, t) + \sum_{r \in \mathcal{FR}_i} c_i(r, t) \right] \delta^t \\
&= \sum_{t=0}^{\infty} \left[\varphi_i(T_i \cdot y_i(t)) - \sum_{r \in \mathcal{FR}_i} T_{src(r)} \cdot C \cdot \prod_{k \in \mathcal{F}(r,i)} x_k(t) \right] \delta^t
\end{aligned}
\tag{3.14}
$$

Because of *Condition 1* and *Condition 3*, we can use Lemma 3.4 to obtain the following inequality for every $r \in \mathcal{FR}_i$:

$$
\prod_{k \in \mathcal{F}(r,i)} x_k(t) \geq y_i(t + \lambda_{i,src(r)})
\tag{3.15}
$$

which leads to the following upper bound on u_i:

$$
u_i \leq \sum_{t=0}^{\infty} \left[\varphi_i(T_i \cdot y_i(t)) - \sum_{r \in \mathcal{FR}_i} T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \delta^t
\tag{3.16}
$$

Since the first term of the right side of (3.16), $\varphi_i(T_i \cdot y_i(t))$, is independent of r, the following holds:

$$
\varphi_i(T_i \cdot y_i(t)) = \sum_{r \in \mathcal{FR}_i} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|}
\tag{3.17}
$$

By substituting the right side of (3.17) into (3.16), we get the following:

$$
\begin{aligned}
u_i &\leq \sum_{t=0}^{\infty} \left[\sum_{r \in \mathcal{FR}_i} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} - \sum_{r \in \mathcal{FR}_i} T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \delta^t \\
&= \sum_{r \in \mathcal{FR}_i} \left[\sum_{t=0}^{\infty} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t - \sum_{t=0}^{\infty} T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \cdot \delta^t \right]
\end{aligned}
\tag{3.18}
$$

Let us consider the first term of (3.18). We will now split up the summation that goes from $t = 0$ to ∞ into two summations such that one goes from $t = 0$ to $\lambda_{i,src(r)} - 1$, and the other goes from $t = \lambda_{i,src(r)}$

to ∞. We shift the index in the second sum such that the summation goes from $t = 0$ to ∞ again:

$$
\begin{aligned}
&\sum_{t=0}^{\infty} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t \\
&= \sum_{t=0}^{\lambda_{i,src(r)}-1} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t + \sum_{t=\lambda_{i,src(r)}}^{\infty} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t \\
&= \sum_{t=0}^{\lambda_{i,src(r)}-1} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t + \sum_{t=0}^{\infty} \frac{\varphi_i(T_i \cdot y_i(t + \lambda_{i,src(r)}))}{|\mathcal{FR}_i|} \cdot \delta^{t+\lambda_{i,src(r)}}
\end{aligned}
\tag{3.19}
$$

By writing (3.19) back into (3.18), we get the following:

$$
\begin{aligned}
u_i \;\leq\; \sum_{r \in \mathcal{FR}_i} \Bigg[&\sum_{t=0}^{\lambda_{i,src(r)}-1} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t \\
&+ \sum_{t=0}^{\infty} \left[\frac{\varphi_i(T_i \cdot y_i(t + \lambda_{i,src(r)}))}{|\mathcal{FR}_i|} \cdot \delta^{\lambda_{i,src(r)}} - T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \cdot \delta^t \Bigg]
\end{aligned}
\tag{3.20}
$$

Let us consider the first term of (3.20). Since the utility function φ_i is non-decreasing and $y_i(t) \leq 1$, we get the following:

$$
\begin{aligned}
\sum_{t=0}^{\lambda_{i,src(r)}-1} \frac{\varphi_i(T_i \cdot y_i(t))}{|\mathcal{FR}_i|} \cdot \delta^t &\leq \sum_{t=0}^{\lambda_{i,src(r)}-1} \frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} \cdot \delta^t \\
&= \frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} \cdot \frac{1 - \delta^{\lambda_{i,src(r)}}}{1 - \delta}
\end{aligned}
\tag{3.21}
$$

Now let us consider the second term of (3.20). By using the fact that $\psi(\tau) \geq \varphi_i(\tau)$ for all τ, we get the following:

$$\sum_{t=0}^{\infty} \left[\frac{\varphi_i(T_i \cdot y_i(t + \lambda_{i,src(r)}))}{|\mathcal{FR}_i|} \cdot \delta^{\lambda_{i,src(r)}} - T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \cdot \delta^t$$

$$\leq \sum_{t=0}^{\infty} \left[\frac{\psi(T_i \cdot y_i(t + \lambda_{i,src(r)}))}{|\mathcal{FR}_i|} \cdot \delta^{\lambda_{i,src(r)}} - T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \cdot \delta^t$$

$$= \sum_{t=0}^{\infty} \left[\frac{\varphi_i'(T_i) \cdot T_i \cdot y_i(t + \lambda_{i,src(r)}) + \varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i}{|\mathcal{FR}_i|} \cdot \delta^{\lambda_{i,src(r)}} - T_{src(r)} \cdot C \cdot y_i(t + \lambda_{i,src(r)}) \right] \cdot \delta^t$$

$$= \sum_{t=0}^{\infty} \left[\frac{\varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i}{|\mathcal{FR}_i|} \cdot \delta^{\lambda_{i,src(r)}} + \left(\frac{\varphi_i'(T_i) \cdot T_i \cdot \delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} - T_{src(r)} \cdot C \right) y_i(t + \lambda_{i,src(r)}) \right] \cdot \delta^t$$

$$= \frac{\varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i}{|\mathcal{FR}_i|} \cdot \frac{\delta^{\lambda_{i,src(r)}}}{1 - \delta} + \sum_{t=0}^{\infty} \left(\frac{\varphi_i'(T_i) \cdot T_i \cdot \delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} - T_{src(r)} \cdot C \right) y_i(t + \lambda_{i,src(r)}) \cdot \delta^t \quad (3.22)$$

$$\leq \frac{\varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i}{|\mathcal{FR}_i|} \cdot \frac{\delta^{\lambda_{i,src(r)}}}{1 - \delta} + \sum_{t=0}^{\infty} \left(\frac{\varphi_i'(T_i) \cdot T_i \cdot \delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} - T_{src(r)} \cdot C \right) \cdot \delta^t \quad (3.23)$$

$$= \frac{\varphi_i(T_i) - \varphi_i'(T_i) \cdot T_i}{|\mathcal{FR}_i|} \cdot \frac{\delta^{\lambda_{i,src(r)}}}{1 - \delta} + \left(\frac{\varphi_i'(T_i) \cdot T_i \cdot \delta^{\lambda_{i,src(r)}}}{|\mathcal{FR}_i|} - T_{src(r)} \cdot C \right) \cdot \frac{1}{1 - \delta}$$

$$= \frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} \cdot \frac{\delta^{\lambda_{i,src(r)}}}{1 - \delta} - \frac{T_{src(r)} \cdot C}{1 - \delta} \quad (3.24)$$

where in the transition from (3.22) to (3.23), we used *Condition 2* and the fact that $y_i(t + \lambda_{i,src(r)}) \leq 1$. By using (3.21) and (3.24) in (3.20), we get the following:

$$u_i \leq \sum_{r \in \mathcal{FR}_i} \left[\frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} \cdot \frac{1 - \delta^{\lambda_{i,src(r)}}}{1 - \delta} + \frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} \cdot \frac{\delta^{\lambda_{i,src(r)}}}{1 - \delta} - \frac{T_{src(r)} \cdot C}{1 - \delta} \right]$$

$$= \frac{1}{1 - \delta} \cdot \sum_{r \in \mathcal{FR}_i} \left[\frac{\varphi_i(T_i)}{|\mathcal{FR}_i|} - T_{src(r)} \cdot C \right]$$

$$= \frac{1}{1 - \delta} \cdot \left(\varphi_i(T_i) - C \cdot \sum_{r \in \mathcal{FR}_i} T_{src(r)} \right) \quad (3.25)$$

Now let us consider what payoff is achieved by node i if he fully cooperates in every time step. In this case, since all the other nodes play TFT, every node will always fully cooperate, and hence, every node will experience a normalized throughput equal to 1 in each time step. This can easily be derived from the i/o formulae describing the behavior of the nodes, which take a simple form due to the simplicity of the strategy function of the TFT strategy. As a consequence, we have that $y_i(t) = 1$ for every t, and $x_k(t) = 1$ for every k and for every t. In this case expression 3.14 becomes:

$$u_i = \sum_{t=0}^{\infty} \left[\varphi_i(T_i \cdot y_i(t)) - \sum_{r \in \mathcal{FR}_i} T_{src(r)} \cdot C \cdot \prod_{k \in \mathcal{F}(r,i)} x_k(t) \right] \delta^t$$

$$= \frac{1}{1 - \delta} \cdot \left(\varphi_i(T_i) - C \cdot \sum_{r \in \mathcal{FR}_i} T_{src(r)} \right)$$

This means that by fully cooperating, the payoff of node i reaches the upper bound expressed in (3.25); in other words, there is no better strategy for node i than full cooperation.   □

3.8 Related Work

3.8.1 Incentive Mechanisms in Ad Hoc Networks

The operation of ad hoc networks relies on the contribution of nodes. Several researchers have realized that this cooperation is not obvious and have proposed solutions to give nodes incentive to contribute. There are basically two approaches to encourage nodes: (i) by denying service to misbehaving nodes by means of a reputation mechanism or (ii) by remunerating honest nodes, using for example a micro-payment scheme. We provide an overview of these approaches below.

Marti *et al.* [MGLB00] consider an ad hoc network where some misbehaving nodes agree to forward packets but then fail to do so. They propose a mechanism, called *watchdog*, in charge of identifying the misbehaving nodes, and another mechanism, called *pathrater*, that deflects the traffic around them. The drawback of their solution is that misbehaving nodes are not punished, and thus there is no incentive for the nodes to cooperate. To overcome this problem, Buchegger and Le Boudec [BB02] as well as Michiardi and Molva [MM02] define protocols that are based on a reputation system. In both approaches, the nodes observe the behavior of each other and store this knowledge locally. Additionally, they distribute this information in reputation reports. According to their observations, the nodes are able to behave selectively (e.g., nodes may deny forwarding packets for misbehaving nodes). However, such a scheme requires a reliable authentication scheme, otherwise it is vulnerable to the Sybil attack [Dou02]. Note that authentication is an open issue in ad hoc networks and the Sybil attack is proven to be always possible if a central authority is not present in the network. Luo *et al.* [LKZ$^+$04] use a ticket certification service to identify misbehaving nodes. their scheme relies on a collaboration of nodes to decide about their neighbors. Mahajan *et al.* [MRWZ05] propose using anonymous probes for neighbor testing to encourage nodes to cooperate in an ad hoc network.

Other researchers proposed schemes that employ a virtual currency system to encourage cooperation. Zhong, Yang and Chen [ZYC03] present a solution, where an off-line central authority collects *receipts* from the nodes that relay packets and remunerates them based on these receipts. They rely on public key cryptography to process each packet. Thus, their solution might be too complex in an ad hoc network. Another solution, presented by Buttyan and Hubaux [BH00, BH03], is based on a virtual currency, called a *nuglet*: If a node wants to send his own packets, he has to pay for it, whereas if the node forwards a packet for the benefit of another node, he is rewarded. However, some mechanisms of this solution (e.g., the generation of nuglets) still need to be investigated. Many researchers have proposed auction- and pricing-based schemes to encourage routing and packet forwarding in ad hoc networks [AE03, CN04, ISCM05, MQ05a, YL05, WL06]. Zhong *et al.* [ZLLY05] present a comprehensive game-theoretic analysis of strategic behavior in routing and packet forwarding in ad hoc networks. They propose a VCG-based mechanism extended with cryptographic techniques to encourage truthful behavior.

3.8.2 Cooperation without Incentive Mechanisms

The proposals that we have just described were based on heuristics. There was a need for a formal description of the cooperation problem in ad hoc networks.

In [SNCR03], the authors propose a game theoretic model that considers cooperation from the energy efficiency point of view. They consider a maximal battery level and an expected lifetime for each node,

and they group the nodes into *energy classes* according to this information. They derive the energy class for a connection as the minimum of the energy classes of the participants. The energy class is a novel idea that allows the authors to express the heterogeneity of devices. They define time steps as a unit of operation for the nodes, as we also do in our framework. However, in contrast to our approach, they do not take into account the topology of the network and the existing communication flows. Instead, they assume that a single communication session with random participants is generated in each time step. Based on the random session generation, they show that cooperation emerges within the network, because, by the nature of the random participation in the sessions, nodes have a symmetric interaction pattern. However, in reality, the interactions between nodes are likely to be asymmetric; this is practically true in the extreme case of a static network. In this chapter, we have shown that spontaneous cooperation exists only if the interaction between the nodes is balanced and we have also shown that this property does not hold in general. Our conclusion justifies the need for incentive mechanisms, that should reestablish the balance between the utilities of nodes, for example by remunerating nodes that contribute more.

The authors of [SNCR03] provide a framework that relies on two mechanisms: the first communicates energy class information while the second enables the relays of a session to communicate their decision to the source (accept or refuse relaying). These mechanisms are needed to optimize the nodes' contribution with respect to energy conditions. From the security point of view, however, these mechanisms are vulnerable. This is an important issue, since the whole analysis is about selfish nodes that want to maximize their utility, even if it means disobeying the network protocols. Cheating can be done as follows. First, a high energy node could use his own identity when sending his own packets and pretend to be a low energy node when asked to forward packets. By doing this, he could decrease his load in terms of packet forwarding. This kind of selfish behavior could be detected using an appropriate authentication scheme, combined with a cheating detection mechanism. Second, in [SNCR03] it is assumed that once nodes agree to relay packets in a session, they do so. But there is no guarantee that a node really complies to its promise. Thus, an additional mechanism should be applied to punish nodes whenever it is necessary. Although far from perfect, our model relies on the *real* behavior of the nodes (and not on their declared behavior), and does not require any form of authentication.

A major contribution of [SNCR03] is the investigation of both the existence and emergence of cooperation in wireless ad hoc networks; in this chapter, we focus only on the existence of cooperative equilibria. Another important result of [SNCR03] is the proof that the emerging cooperative Nash equilibrium is Pareto-efficient (thus it is a *desired* outcome of the packet forwarding game).

3.9   Summary

In this chapter, we presented a game-theoretic model in order to investigate the conditions for cooperation in wireless ad hoc networks, in the absence of incentive mechanisms. Because of the complexity of the problem, we restricted ourselves to a static network scenario. We then derived conditions for cooperation from the topology of the network and the existing communication routes. We introduced the concept of dependency graph, based on which we were able to prove several theorems. As one of the results, we proved that cooperation solely based on the self-interest of the nodes can *in theory* exist. However, our simulation results showed that *in practice*, the conditions of such cooperation are virtually never satisfied. We concluded that with a very high probability, there exist some nodes that have All-D as their best strategy and therefore, these nodes need an incentive to cooperate. In this chapter, we also showed that the behavior of these defectors affects only a fraction of the nodes in the network; hence, local subsets of cooperating nodes were not excluded. It is important to notice that our approach does

not require a node to keep track of the behavior of other nodes. Thus, our solution does not require any node authentication.

Our results apply for example to mesh networks, where the mesh access points are operated by home users. In such networks, the communications flows are likely to be static and unbalanced, because most of the communication is from a wired access point. Our results show that an incentive mechanism (e.g., one of the mechanisms proposed in the literature) is most probably needed if the users want to ensure the correct functioning of the network.

Publications: [FBH03], [FHB06]

Chapter 4

Packet Forwarding in Wireless Ad Hoc Networks – the Dynamic Case

4.1 Introduction

Ad hoc networks have the potential to increase the flexibility of wireless communication systems. They, however, also require novel operating principles. In particular, due to the absence of fixed infrastructure, most of the functions (routing, mobility management, in some cases even security) must rely on the cooperation between the nodes.

The most fundamental of these functions is packet forwarding. Cooperation is straightforward if all the nodes are under the control of a single authority, as is usually the case in military networks or for rescue operations: in these cases, the interest of the mission by far exceeds the vested interest of each participant.

However, if each node is his own authority, the situation changes dramatically: The most reasonable assumption is then to consider that each node will try to maximize the benefit he gets by using the network, even if this means adopting a selfish behavior. This selfishness can mean not participating in the unfolding of mechanisms of common interest, notably to spare resources, including battery energy.

Over the last few years, several researchers have proposed incentive techniques to encourage nodes to collaborate, be it by circumventing misbehaving nodes [MGLB00], by making use of a reputation system [BB02, MM02], or by relating the right to benefit from the network to the contribution to the common interest of a node provided thus far [BH03]. These proposals have been based on heuristics, and are therefore rather difficult to compare with each other.

Very recently, Srinivasan *et al.* [SNCR03] have proposed a formal framework, based on game theory, to study cooperation without incentives. They have identified the conditions under which cooperation is a Nash-equilibrium. In order to do this, their system model is quite simple: For each route to be set up, they randomly select several nodes to be part of it; as a result, their approach does not take the topology of the network into account.

Our own approach has essentially the same goal as this seminal work; however, we believe that the network topology is important, and we therefore include it in our model. In Chapter 3, we have already studied the *static* case, meaning that we have assumed that nodes do not move. We have identified the network topologies under which cooperation can be an equilibrium, and we have shown that the likelihood for these topologies to exist is extremely small.

In this chapter, we pursue exactly the same ambition, but we now consider that the nodes can move.

As a consequence, we have to adopt a different model. Due to the complexity of the problem, we deliberately devote a substantial part of the chapter to a simplified scenario (all nodes are located - and shuffled - on a ring). In this way, we are able to formulate and to prove several theorems. Then, by means of simulations, we study the more general (and more realistic) case where the nodes move on a plane; thus, we can easily assess to what extent the situation differs from the ring scenario.

Our main contribution is to show that cooperative Nash-equilibria are much more likely to happen with mobile than with static nodes. In addition, we quantify how much "generosity" the nodes should grant in order to make these equilibria feasible.

4.2   Modeling Packet Forwarding as a Game

4.2.1   System Model

We assume that there exists a set of nodes $\mathcal{N}$ who want to communicate via multiple hops. We denote the number of nodes $|\mathcal{N}| = N$. Each node uses an omnidirectional antenna with the same radio range. Hence, there is a bidirectional communication link between two nodes if they reside within the radio range of each other.

We assume that the packets originating from a source node are sent via ℓ nodes that are expected to forward the packet. We call a *route* the communication path defined by the source, the forwarders and the destination. We assume that each node is the source of one route. We also assume that the routes last for the duration of the game. The study of routing behavior is out of the scope of this work, so we assume a shortest routing protocol that establishes a route between a given source and a given destination.

We assume an end-to-end mechanism that enables a source to detect the loss of a packet (e.g., at the transport layer), hence, we do not require an additional acknowledgement from the forwarders to the source. This means that the source can observe the fact that a packet is lost, but he cannot tell where, when and how it happened. We further assume that each packet loss is due to the defective behavior of the nodes and not due to link errors or congestion.

We introduce the following notation to identify our investigation scenarios:

$$\textit{Scenario-}\ell$$

where *Scenario* stands for the given scenario and ℓ (or $\bar{\ell}$) stands for the constant (or average) number of forwarders of each route.

4.2.2   Game-Theoretic Model

In this section, we present a game-theoretic framework to investigate the conditions of cooperation in packet forwarding. We model packet forwarding as an infinite-horizon game using the technique called *limit of means* [OR94]. Similarly to the model in Section 3.3, we assume that each node as a player interacts with the rest of the network. Hence, we do not require the nodes to use a reliable authentication mechanism to identify each other. This assumption greatly reduces the complexity of the assumed underlying mechanisms.

In our model, we assume that the source benefits from the arrival of a packet at the destination. But the model can be adapted to the case in which the destination benefits if a packet successfully arrives.

We split up the time in discrete steps. At the end of each time step (denoted by t), each node evaluates the results of his interaction with the network in the following way. Each node maintains two variables, which are the basis for his strategy function. The *benefit* $b_i(t)$ represents the number of packets *until step*

t that were originated at node i and were successfully received by his corresponding destinations. The *cost* $c_i(t)$ represents the number of packets *until step t* that node i forwarded for other nodes. We define the *interaction ratio* $\rho_i(t)$ at step t as the ratio of these two values (i.e., $\rho_i(t) = \frac{b_i(t)}{c_i(t)}$). If $c_i(t) = 0$, we set $\rho_i(t) = \Omega$, where Ω is an arbitrarily large number with $\Omega < \infty$.

Each node decides for each packet whether to forward it or not, using his own strategy. The strategy of node i is defined in the following way:

- The initial move $m_i(0)$ of node i (for the value $\rho_i(0)$) is to forward (F) or drop (D) the packets he is asked to forward

- For each subsequent packet:

    - If $\rho_i(t) \geq \kappa_i$, then play F,

    - otherwise play D.

The value κ_i represents the *cooperation level* of node i.

In Table 4.1, we show that specific values of κ_i correspond to strategies presented in Section 1.4 (shown in Table 1.5). In particular, *TFT* is the strategy that imposes the benefit for node i to be equal to his cost in the network (taking into account the average number of forwarding nodes ($\bar{\ell}$) on his routes).

Strategy	Initial move	κ_i
All-D (always defect)	D	∞
All-C (always cooperate)	F	0
TFT (Tit-for-Tat)	F	$\frac{1}{\bar{\ell}}$

Table 4.1: Cooperation levels for three highlighted strategies.

In principle, a node could decide to update his behavior after each packet processing; however, this would be too fine grained. Therefore, we assume that a node reconsiders his decision only at the end of each time step. This means that we evaluate simultaneously all packets that the node sends and forwards in each step.

We define two constants for each node: (i) B_i stands for the benefit from a single packet for node i, if the packet reaches the destination and (ii) C_i is the forwarding cost at node i for a single packet. For the sake of simplicity, we assume that $B_i = B$, $\forall i$ (i.e., each node enjoys the same benefit if a packet successfully goes through) and $C_i = C, \forall i$ (i.e., each node suffers the same cost for each packet it forwards).

In this chapter, we assume that the overall payoff of the node is linearly dependent of the benefit of the node. The aim of each node is to maximize his *total average payoff* $\bar{u}_i$.

$$\bar{u}_i = \liminf_{t \to \infty} \bar{u}_i(t) \tag{4.1}$$

where $\bar{u}_i(t)$ is the *average payoff*:

$$\bar{u}_i(t) = \frac{B \cdot b_i(t) - C \cdot c_i(t)}{t} \tag{4.2}$$

Node i could maximize his total average payoff by decreasing his cost. However, this might cause the defection of other nodes that might be forwarders for node i.

In our model, each source sends a small amount of information at each step that corresponds to a unit of information. For better understanding, we refer to this unit of information as a *packet*.[1]

## 4.3   Analysis of Nash-equilibria with a Single Forwarder

To illustrate our approach, we begin with the analysis of a simple and deliberately unrealistic scenario.

4.3.1   Investigation Scenario

We assume that the nodes are organized in a ring (an example network with four nodes is shown in Figure 4.1). Each node is the source of one route. We also assume that each route has one forwarder (i.e., $\ell = 1$), which is the next node in clockwise direction from the source (according to our notation introduced in Section 4.2.1, this scenario is *Ring-1*). In general, we assume that $B > C$ (meaning that the node has a "natural" incentive to send packets).

In order to mimic mobility, at the end of each step, we randomly shuffle the nodes on the ring. We assume that the time for a topology change is much higher than the time to send a packet from the source to the destination. Thus, the network is considered to be static during the sending of a single packet.
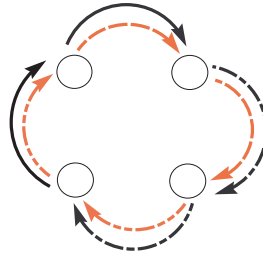
Figure 4.1: A ring network with four nodes and four routes. Each node is a source of one route and each route has one forwarder.

4.3.2   Equilibrium of TFT Strategies

In the following, we show that nodes playing the TFT strategy constitute a scenario with stable cooperation. Let us first assume that each node plays TFT except node i. If node i drops a packet, then he decreases the payoff of a source node src whose packet does not arrive. Because each node except node i plays TFT, this means that each node $j \neq src$ will have $\rho_j = 1$. Since node src forwards in this step, his interaction ratio will drop below the strategy constant ($\rho_{src} \leq \kappa_{src} = 1$). In the next step node src drops a packet for another node src' and his interaction ratio will be again equal to one ($\rho_{src} = \kappa_{src} = 1$). But then the interaction ratio of node src' decreases below one. Because every node applies the TFT strategy, this packet dropping behavior propagates through the network until it gets back to node i.

Lemma 4.1. *In Ring-1, if any node i defects once (and otherwise always cooperates) and all other nodes $j \neq i$ play TFT ($\kappa_j = 1$), then the defection affects node i in expectedly $N - 1$ steps (meaning his payoff is reduced because of his own defection).*

---

[1]Note that our concept of *packet* is general in the sense that it does not correspond to any specific protocol packet, but it contains a given number of protocol packets. The number of protocol packets is limited by the fact that the time to send a *packet* must be much shorter than the time for topology change.

Proof. Let us consider the expected number of steps (denoted by t') after which this dropping affects node i (meaning that another node drops his packet).

Without loss of generality, let us assume that node i drops a packet in step 0 and in step t' another node drops the packet of node i for the first time. As all the other nodes play TFT, this event exists with very high probability. In other words, in all steps $t < t'$, other nodes forwarded for node i, but in step t' it is not the case.

Recall that N stands for the number of nodes in the network. Given the scenario, there exists a node j at each step that defects. The probability q_1 that the defecting node is a forwarder for node i in any step is given by:

$$q_1 = \frac{1}{N-1} \tag{4.3}$$

Thus, the probability q_2 that the defecting node is a forwarder for node i in step t' for the first time (i.e., no defecting node was a forwarder for node i in the previous steps ($0 < t \leq t' - 1$) corresponds to a geometric distribution with respect to t':

$$q_2 = (1 - q_1)^{t'-1} \cdot q_1 \tag{4.4}$$

The expected value of the geometric distribution is (substituting the given q_1 value):

$$E[t'] = \frac{1}{q_1} = N - 1 \tag{4.5}$$

This means that the defection comes back to node i expectedly in $N - 1$ steps. $\square$

Our aim is to identify the number of defections node i can do without decreasing his payoff. If this number is equal to 0, it means that node i is better off cooperating in each time step. Let us denote by $x(t)$ the number of packets dropped by node i until step t. We denote by $y(t)$ the number of packets that were generated at node i and were dropped by other nodes until step t.

We refer to nodes who play non-reactive strategies (e.g., All-C or All-D) as *sinks*. These nodes do not propagate defections. In our approach, we want to define the strategy that results in the highest payoff for node i, thus we assume that his output is *a priori* independent of his input. Thus, we consider node i as a sink.

Because we assume that every node except node i plays TFT, all the defections in the network are consequences of the defections of node i (recall from Section 4.2.1 that we assume that there exist no packet losses due to link errors or congestion). Let us denote the set of defecting nodes by $\mathcal{D}(t)$. Then, the number of propagating defections in the network (in this case $|\mathcal{D}(t)|$) is given by:

$$|\mathcal{D}(t)| = x(t) - y(t) \tag{4.6}$$

Since there are N nodes in the network, we can state that:

$$|\mathcal{D}(t)| \leq N - 1 \tag{4.7}$$

This means that the number of propagating defections is upper bounded by the number of nodes on the ring excluding node i.

Lemma 4.2. *In Ring-1, if node i defects a finite number of times (and otherwise always cooperates) and all other nodes $j \neq i$ play TFT ($\kappa_j = 1$), then $\liminf_{t \to \infty} E[|\mathcal{D}(t)|] = 0$.*

Proof. Let us assume that the node defects K times, where $K < \infty$, and cooperates in all the other steps. For the sake of simplicity, we assume that the node defects in the first K steps and consider the expected value of $|\mathcal{D}(t)|$ in the subsequent steps.

For any step $t > K$, we can write the expected number of propagating defections:

$$
\begin{aligned}
E[|\mathcal{D}(t)|] &= (|\mathcal{D}(t-1)| - 1) \cdot \frac{|\mathcal{D}(t-1)|}{N-1} + |\mathcal{D}(t-1)| \cdot \left(1 - \frac{|\mathcal{D}(t-1)|}{N-1}\right) \\
&= |\mathcal{D}(t-1)| - \frac{|\mathcal{D}(t-1)|}{N-1} \\
&= |\mathcal{D}(t-1)| \cdot \left(1 - \frac{1}{N-1}\right)
\end{aligned}
\tag{4.8}
$$

Because $1 - \frac{1}{N-1} < 1$, (4.8) shows us that the expected number of defecting nodes is decreasing (and it is never increasing after $t > K$). Hence we obtain the condition of the theorem. $\square$

Now let us formulate a theorem for cooperation for a single node.

Theorem 4.3. *In Ring-1, if every node $j \neq i$ plays TFT, then the best strategy for node i is a strategy that results in full cooperation (meaning a strategy with $\kappa_i \leq 1$).*

Proof. We assume that each node wants to maximize his total average payoff per step over an infinite game as expressed in (4.1). Let us first compute the average payoff until step t:

$$
\begin{aligned}
\bar{u}_i(t) &= \frac{B \cdot b_i(t) - C \cdot c_i(t)}{t} \\
&= \frac{B \cdot (t - y(t)) - C \cdot (t - x(t))}{t} \\
&= \frac{(B - C) \cdot t - (B \cdot y(t) - C \cdot x(t))}{t} \\
&= (B - C) - \frac{(B \cdot y(t) - C \cdot x(t))}{t}
\end{aligned}
\tag{4.9}
$$

Let us assume that node i cooperates at each step; then $y(t) = x(t) = 0$ for any step t (because the other nodes play TFT). In this case, the average payoff for node i is given by:

$$
\bar{u}_i(t) = B - C
\tag{4.10}
$$

To be superior to the always cooperating strategy, an alternative strategy should be such that the second term in (4.9) is positive (taking also the minus sign into account):

$$
\begin{aligned}
-\frac{(B \cdot y(t) - C \cdot x(t))}{t} &> 0 \\
-B \cdot y(t) + C \cdot x(t) &> 0 \\
-B \cdot (x(t) - |\mathcal{D}(t)|) + C \cdot x(t) &> 0 \\
(C - B) \cdot x(t) + B \cdot |\mathcal{D}(t)| &> 0 \\
B \cdot |\mathcal{D}(t)| &> (B - C) \cdot x(t) \\
\frac{B \cdot |\mathcal{D}(t)|}{B - C} &> x(t)
\end{aligned}
\tag{4.11}
$$

We know that the expected number of propagating defections $|\mathcal{D}(t)|$ is finite. Furthermore, according to Lemma 4.2, if the node defects finite number of times, then the expected number of propagating defections goes to zero with the number of steps. Using this statement in (4.11), we obtain the following:

$$\liminf_{t \to \infty} E[|\mathcal{D}(t)|] = 0 \Rightarrow \liminf_{t \to \infty} x(t) = 0 \tag{4.12}$$

Thus, if the node wants to maximize his total average payoff, his best strategy is to cooperate in every time step. $\square$

Since TFT results in full cooperation in this specific scenario, we can state the following corollary.

Corollary 4.4. *In Ring-1, if every node plays TFT, it is a Nash-equilibrium.*

4.3.3 Other Nash-Equilibria

Now we focus on the following question: If some of the nodes choose the strategy All-C instead of TFT, does it undermine the Nash-equilibrium among the nodes?

Theorem 4.5. *In Ring-1, if there exists one node j who plays a non-reactive startegy (i.e., he is a sink for defections) and every node $i \neq j$ plays TFT, it is a Nash-equilibrium, if $\frac{B}{2} > C$.*

Idea of the proof: In this proof we will show, that if any node i defects in the network, then it propagates through the network, until it is absorbed by either node i himself or by the sink node j. This means that half of the defections are sinked by node i. Thus, it is worth to defect, if the cost spared by defection is greater than the decrease of the benefit.

Proof. Remember that in every step we shuffle the nodes in the network. Assume that node i defects once and this defection is propagating among the nodes till it is sinked by either node i or j (the two nodes that do not play TFT). Let us define the following events for a given step t:

- E_i: The node that is affected by the original defection of node i in step t is a forwarder for node i.

- E_j: The node that is affected by the original defection of node i in step t is a forwarder for node j.

Because of random shuffling, we can write:

$$Pr\{E_i\} = \frac{1}{N-1}$$
$$Pr\{E_j\} = \frac{1}{N-1}$$

The probability that either event happens can be expressed as follows. In the derivation, we use the fact that it is impossible that the node that defects forwards for both node i and j ($Pr\{E_i \cdot E_j\} = 0$).

$$Pr\{E_i + E_j\} = Pr\{E_i\} + Pr\{E_j\} - Pr\{E_i \cdot E_j\} = Pr\{E_i\} + Pr\{E_j\} = \frac{2}{N-1} \tag{4.13}$$

Because of the symmetric nature of the scenario, the defection is sinked by half of the time by either nodes (i and j). Thus, we can state that

$$y(t) = \frac{x(t)}{2} \tag{4.14}$$

meaning that half of the defection of node i causes a benefit loss for him. The other half of the loss goes to node j.

Now let us investigate the average payoff of node i. Using (4.14) in (4.9), we obtain:

$$\bar{u}_i(t) = \frac{B \cdot b_i(t) - C \cdot c_i(t)}{t} \tag{4.15}$$

$$= (B - C) - \frac{(B \cdot y(t) - C \cdot x(t))}{t} \tag{4.16}$$

$$= (B - C) - B \cdot \frac{x(t)}{2t} + C \cdot \frac{x(t)}{t} \tag{4.17}$$

$$= (B - C) - \frac{x(t)}{t} \cdot (\frac{B}{2} - C) \tag{4.18}$$

The suggested behavior of node i is determined by the second term in (4.18). Since $x(t)$ and t are non-negative numbers, we investigate the sign of the expression $\frac{B}{2} - C$. There can be three cases:

- If $\frac{B}{2} > C$, then (4.15) is maximized for $x(t) = 0$ (no defection at all).

- If $\frac{B}{2} = C$, then both cooperation and defection result in the same payoff. A rational node can choose both cooperation and defection (however, one would assume defection in this case).

- If $\frac{B}{2} < C$, then (4.15) is maximized for $x(t) = t$ (defection in all steps).

We can conclude that an always cooperating strategy (TFT or All-C) is the best strategy for node i for $\frac{B}{2} > C$ even, if there is node j that plays All-C strategy. $\qquad\square$

Assume now that there exists a group of players $\Gamma \subset \mathcal{N}$ who play a non-reactive strategy. We can generalize Theorem 4.5 as follows.

Theorem 4.6. *In Ring-1, if there exists a set of players Γ who play non-reactive strategies (where $i \notin \Gamma$) and the other nodes except node i play TFT, then node i should always cooperate if $\frac{B}{|\Gamma|} > C$.*

The proof of the theorem is the same as before substituting Γ everywhere for 2. Intuitively, we say, that if the benefit for one step is greater than the decrease in cost, then the node should rather cooperate.

We finally mention the "worst case" situation in the following theorem:

Theorem 4.7. *In Ring-1, if every node plays All-D, then it is a Nash-equilibrium.*

The proof is trivial: In this case a node does not receive any benefit, no matter what strategy he plays. Hence, it is beneficial for he to defect as well.

4.4   Cooperation with Multiple Forwarders

Now we extend our analysis for routes with several forwarders, still considering the nodes placed on a ring. We assume that each node is a source of one route. Each route has exactly ℓ forwarder nodes (i.e., we denote the scenario by *Ring-ℓ*). We also assume that $B > \ell \cdot C$.

As an example, let us assume that the number of forwarders on each route is two. Thus, each node i has two forwarder nodes that are the two nodes clockwise from node i (an example is presented in Figure 4.2).

As in the previous case, at the end of each step, we randomly shuffle the nodes on the ring. We assume that the number of forwarders (ℓ) is a known parameter for each node.
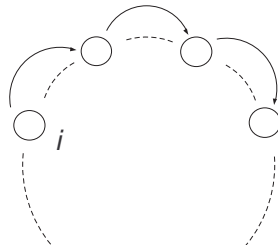
Figure 4.2: Ring of nodes with an example route from node i ($\ell = 2$ in this case)

Theorem 4.8. *In Ring-ℓ, if every node except node i plays TFT, then the best strategy for node i is a strategy that results in full cooperation (play TFT as well, or an even more generous strategy ($\kappa_i \leq \frac{1}{\ell}$)).*

Proof. We prove the theorem for $\ell = 2$ which corresponds to the example of Figure 4.2; as we will see, the proof can be extended to any value of ℓ. As before, let us denote the number of defecting nodes at step t by $\mathcal{D}(t)$.
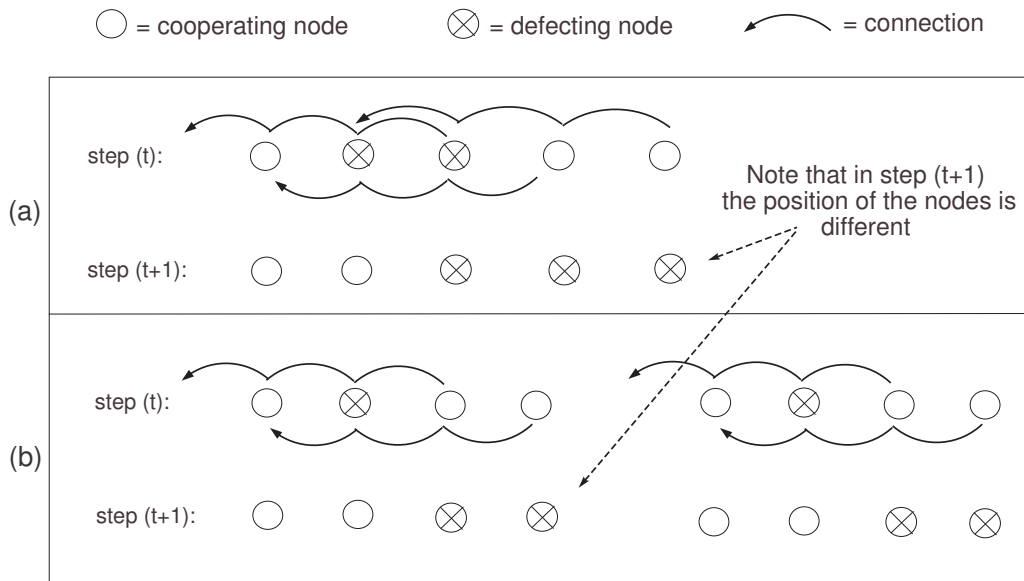
Figure 4.3: Propagation of defections on the ring for $\ell = 2$, $\mathcal{D}(t) = 2$: In (a) the two defecting nodes are neighbors on the ring in step t; in this case, they cause the defection of at least two other nodes (note that if node i is among the nodes whose packets were dropped, then he might sink the defection). In (b) the defecting nodes are not neighbors; thus, they cause the defection of four other nodes.

Let us assume that node i defects in an arbitrary step T. It causes the defection of the nodes whose packets are dropped (in this case two nodes, because $\ell = 2$ and the topology is a ring). In the next step, these two nodes cause the defection of other nodes, and so on. We will show that the number of defecting nodes is non-decreasing: If we have $\mathcal{D}(t)$ defecting nodes in step t (where $t > T$), then the number of

defecting nodes in step $t + 1$ is $\mathcal{D}(t + 1) \geq \mathcal{D}(t)$. An example for $\mathcal{D}(t) = 2, \ell = 2$ is presented in Figure 4.3. If the defecting nodes in step t happen to become neighbors on the ring (Figure 4.3a), then they drop the packets for three nodes. Since node i is a sink for defection, if node i is among the three nodes, then $\mathcal{D}(t + 1) = 2$, otherwise, $\mathcal{D}(t + 1) = 3$. If they are not neighbors (Figure 4.3b), then $\mathcal{D}(t + 1) = 3$ or $\mathcal{D}(t + 1) = 4$ depending on whether node i is among the source nodes or not.

One can see that this propagation of defection continues until it reaches all nodes in the network. During this procedure, node i's payoff decreases as more and more nodes defect in the network. Assume that from T' every node defects in the network. Then, the average payoff $u_i(t)$ of node i becomes zero for $t \geq T'$.

Before node i defects in step T, none of his packets are dropped ($y(t) = 0$). During the propagation of his defection (in the steps $T < t < T'$), y' packets are dropped that originate from node i. After step T', all nodes in the network, except node i, defect.

In step $t > T'$, the number of packets that were originating at node i and are dropped by other nodes is:

$$y(t) = 0 + y' + (t - T') = t - (T' - y') \tag{4.19}$$

We know that $x(t) \leq \ell \cdot t$. Then, we can express the average payoff for $t > T'$ as follows:

$$
\begin{aligned}
\bar{u}_i(t) &= \frac{B \cdot b_i(t) - C \cdot c_i(t)}{t} \\
&= \frac{B \cdot (t - y(t)) - C \cdot (\ell \cdot t - x(t))}{t} \\
&= \frac{(B - \ell \cdot C) \cdot k - (B \cdot y(t) - C \cdot x(t))}{t} \\
&= (B - \ell \cdot C) - \frac{(B \cdot y(t) - C \cdot x(t))}{t} \\
&= (B - \ell \cdot C) - B \cdot \frac{t - (T' - a)}{t} + C \cdot \frac{x(t)}{t} \\
&= (B - \ell \cdot C) + C \cdot \frac{x(t)}{t} - B \cdot \left(1 - \frac{(T' - y')}{t}\right) \\
&= -\ell \cdot C + C \cdot \frac{x(t)}{t} + B \cdot \frac{(T' - y')}{t} \tag{4.20} \\
&\leq B \cdot \frac{(T' - y')}{t}
\end{aligned}
$$

We can write the total average payoff $\bar{u}_i$ as:

$$\bar{u}_i = \liminf_{t \to \infty} \bar{u}_i(t) = 0 \tag{4.21}$$

If node i cooperates then $T' = t$, $y' = 0$ and $x(t) = 0$. This results in $\bar{u}_i(t) = B - \ell \cdot C$ from (4.20). Hence, the best strategy for the node is to cooperate in all step. The theorem can be proved for any value of ℓ in a similar way. $\qquad\square$

From this theorem we can conclude on the following corollary.

Corollary 4.9. *In Ring-ℓ, if every node plays TFT, it results in a Nash-equilibrium.*

If some of the nodes play All-C, then they might be a sink for the defective behavior. We can now formulate a theorem for sinks in the multi-hop forwarding case.

Theorem 4.10. *In Ring-ℓ, if a set of nodes Γ play All-C and the rest of the nodes except node i play TFT, then the best strategy for node i ($i \notin \Gamma$) is to always cooperate if $|\Gamma| < \ell$.*

Proof. Because of the multi-hop scenario, node i is a forwarder in ℓ routes. If he defects, he causes the defection of ℓ nodes who are sources on these routes. We distinguish two cases:

1. If $|\Gamma| < \ell - 1$ or at any step some of the ℓ sinks do not forward for node i. In this case, they cause the defection of more than one node in the first step. This implies that in the subsequent steps, the defective behavior will reach the whole network. Let us consider step T when all nodes are defecting except node i and the nodes who are in Γ. Because the number of forwarders ℓ is greater than the number of All-C players $|\Gamma|$, there is at least one forwarder node on the route originating from node i that drops the packet. Thus, after step T, no packet of node i reaches the destination. Hence, the proof for Theorem 4.8 applies for this case. Again, the best strategy for node i is to cooperate in every step.

2. If $|\Gamma| = \ell - 1$ and the $|\Gamma|$ sinks are within the ℓ nodes who defect in each and every step. Clearly, because of the random shuffling at each step, the probability of this sequence of events is extremely small. In this case, only one node will change to defection in the next step. In the subsequent steps, the defective behavior of this one node propagates until it is sunk by node i himself. Hence the proof in Theorem 4.3 applies (we are back to the case where one defection propagates in the network, because the other defections are constantly sunk by the All-C players). Thus, in this case, defection is not beneficial for the node.

We can thus conclude that the best strategy for node i is to always cooperate. $\square$

Corollary 4.11. *In Ring-ℓ, if a set of nodes Γ play All-C in the network, where $|\Gamma| < \ell$ and the other nodes play TFT, then it is a Nash-equilibrium.*

The corollary expresses that the cooperative equilibrium is resistant to the phenomenon of *drift* [Hof01], provided that the number of sinks is below a threshold given by the number of forwarders on each route.

Note that the above mentioned analysis does not apply for sinks playing All-D. If at least one such node exists in the network, it might cause the whole network to defect, independently of the behavior of node i. In this case, the best strategy for node i is to defect in every step.

4.5 Simulation Results

In this section, we present simulation results where we vary the route length: Instead of having a constant number of forwarders for a route, we choose the number of forwarders between two values. We first investigate the ring network and then a more realistic network scenario.

Our analysis presented in Sections 4.3 and 4.4 relies on the fact that each route has the same number of forwarders. This enables the TFT strategy to constitute a Nash-equilibrium. The idea is that each node contributes as much as he receives from the network. If the number of forwarders varies, this balance can be undermined. In order to tolerate this possible difference of interaction at each node, we introduce a new strategy.

Inspired by [Axe84], we call *Generous Tit-For-Tat (GTFT)* a strategy that overestimates the required cost of packet forwarding in the network. Thus, a node playing this strategy is *generous*, because he is

willing to contribute more to the network than to benefit from it. If a node i plays the GTFT strategy, he uses the following strategy constant:

$$\kappa_i = \frac{1}{\bar{\ell} + g_i} \tag{4.22}$$

where $\bar{\ell}$ stands for the average number of forwarders for all the routes of the network during his whole lifetime and g_i stands for the *generosity* of the node. For the sake of simplicity, we choose $g_i = g$, $\forall i$. Note that we get the usual TFT strategy if $g = 0$.

4.5.1   Simulations on a Ring Network

We performed simulations on a ring network with the parameters provided in Table 4.2. We performed each simulation as follows. First, we place nodes with uniform probability on the ring. Then, we generate a route for every node with the given average number of forwarders. Then, we let every node send a packet on the route for which he is the source. At the end of the step, we release the routes. We repeated this procedure for the number of steps.

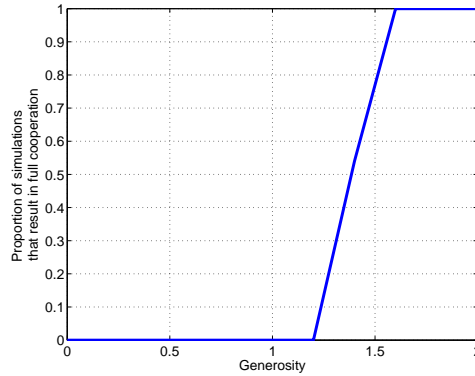| Parameter	Value
Number of nodes	100
Avg. number of forwarder nodes	2
Distribution of the number of forwarders	uniform
Mobility	random shuffling
Number of simulations	200

Table 4.2: Parameter values for the simulation on the ring

We performed simulations for the average number of forwarders equal to two (we denote the scenario by *Ring-$\bar{2}$*). We observe that the network always converges to one of the two extreme states: either all nodes cooperate or all nodes defect. Figure 4.4 shows the proportion of simulations that result in full cooperation as a function of the generosity. We can observe that the generosity must be reasonably high (compared to the average number of forwarders) to have full cooperation in the network. If the generosity is above a given threshold (in the example this threshold is equal to 1.6), all simulations result in full cooperation.

4.5.2   Simulations on a Realistic Network

We simulated a realistic network with the parameters provided in Table 4.3. We performed each simulation as follows. First, we place nodes with uniform probability in the simulation area. Then, we generate a route for every node with the given average number of forwarders (we denote the scenario by *Real-$\bar{\ell}$*). Then, we let every node send a packet on the route for which he is the source. We repeat this procedure for the number of steps. In order to improve our simulation scenario further, we introduce a more realistic route generation model. Instead of generating a route for each node at each step, we generate a new route only if the old one breaks because of mobility.

We used the random waypoint model with the parameters presented in Table 4.4. In our first simulation, we set the duration of a step to 1024 seconds. In this case, the expected time a node travels to a destination (given the average speed and pause time presented in Table 4.4) is much shorter than the duration of one step. Thus, with this setting, we approximate the random shuffling of nodes (between two steps the network topology changes completely).

Figure 4.4: The proportion of simulations on the ring that end with full cooperation between the nodes (in *Ring-$\bar{2}$*)

| Parameter	Value
Number of nodes	100
Number of steps per simulation	500
Duration of one step	variable (1-1024 s)
Area type	Toroid plane
Area size	1500 m x 1500 m
Avg. number of forwarder nodes	2
Distribution of the number of forwarders	uniform
Number of simulations	200
Radio range	250 m

Table 4.3: Parameter values for the simulation on the realistic network

Figure 4.5 shows the proportion of simulations that result in full cooperation as a function of the generosity. We can see that the realistic route generation introduces an additional difference of the required cost of packet forwarding for the nodes, thus a much higher generosity is needed to ensure full cooperation. The reason for this is that generosity is required to cope with the worst case situation, where a node is a forwarder in a number of routes that is higher than the average number of forwarders on a route ($\bar{\ell}$). Regarding the worst case, some nodes might forward on more routes in the realistic scenario than on the ring. Hence, more generosity is required to ensure full cooperation.

In the same model, we investigate the effect of mobility on cooperation. We increase the step duration exponentially (2 to the power of x seconds, where $x = 0, 1, \dots$), and we observe the required generosity

| Parameter	Value
Mobility model	Random waypoint
Speed	1-19 m/s
Distribution of speed	uniform
Average pause time	10 s

Table 4.4: Parameter values for the Random Waypoint mobility model
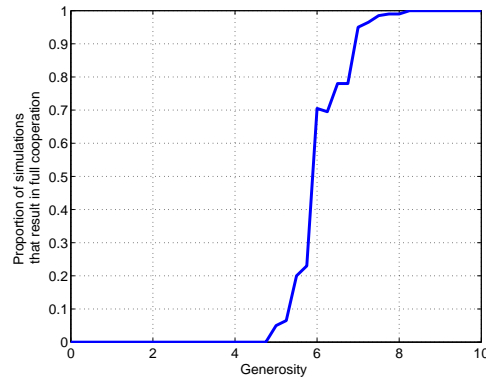
Figure 4.5: The proportion of simulations on a realistic scenario that end with full cooperation between the nodes (with *Plane-$\bar{\ell}$*); step duration is 1024 seconds

level that ensures that 95 % of the simulations result in full cooperation (we call this value the *generosity threshold*). Figure 4.6 presents the generosity threshold as a function of the duration of a step (which represents the effect of mobility). We see that if the length of one step is small (meaning that mobility is small), then a higher generosity threshold is required. The higher the mobility, the lower the generosity threshold. This result is fully consistent with our previous results in Chapter 3: The absence of mobility is a major hurdle for "spontaneous" cooperation.

As explained before, generosity is needed for nodes that are forwarders in a high number of routes compared to the average number of forwarders in a route. This situation represents the worst case for a node. If the duration of the step is small, then this worst case situation is valid for several steps and the node has to be more generous to cope with the cumulative effect of the situation. If mobility increases (meaning that the topology of the network changes more between the steps), then the duration of a worst case situation is shorter and less generosity is required to cope with its cumulative effect. For a detailed investigation of the effect of mobility on the duration of paths, the reader is referred to [SBKH03].
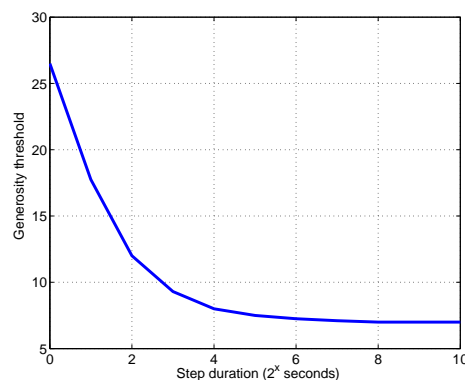
Figure 4.6: Generosity threshold ensuring full cooperation as a function of the duration of one step (i.e., the effect of mobility)

4.6 Summary

In this chapter, we studied the effect of mobility on the level of cooperation in selfish packet forwarding. We adopted a game-theoretic approach, in which a node considers that he plays against the rest of the network. In this model, the node does not need to distinguish between the behavior of the other nodes, which has the benefit of avoiding the intricacies of node authentication or the burden of complex schemes based on reputation. With this new model, we stated and proved several theorems, expressing the conditions for the existence of cooperation; we quantified the tolerance to the phenomenon of drift, as well as the level of generosity required, in the case the routes have varying lengths. We then considered a more realistic model, where the nodes are on the plane and move according to the random waypoint mobility pattern. We showed that the generosity required to reach cooperation is much higher in this case, and we quantified the relationship between mobility and cooperation. We concluded that cooperation is easier to reach when mobility is higher.

Our results extend our previous results in Chapter 3. In this chapter, we showed that mobility increases cooperation in mobile ad hoc networks, such as personal wireless networks formed by pedestrians in a city area. We have seen that some degree of generosity is nevertheless needed to bootstrap the network operation.

Publication: [FHB03, FBH04]

Part III

Non-Cooperative Behavior of Network Operators

Chapter 5

Packet Forwarding in Multi-Domain Sensor Networks

5.1 Introduction

Multi-hop wireless networks provide both new networking environments and extensions of existing network infrastructures. Sensor networks, in particular, emerge as a new paradigm of a large scale wireless network for data gathering purposes. Sensor networks have the potential to extend the current solutions and to open the possibility for more precise environmental monitoring.

In the literature of sensor networks, it is generally assumed that the sensors are under the control of a single operator. In real deployments of sensor networks, it is reasonable to assume, however, that different sensor networks are going to be deployed independently of each other in the same area. Typical examples of future co-located deployments will be found in freight transport (e.g., vehicle, container, and material tracking sensors co-located with control sensors in the warehouse, airport, harbor, or train station), in environmental monitoring (e.g., forest fire, earthquake and flood detection sensors), in intelligent buildings (e.g., material tracking, environmental control, and building state monitoring networks), and in animal monitoring (e.g., where each subset of a herd belongs to a different owner).[1] Even if the sensors perform different tasks, the communication interface between them is likely to be standardized, making them able to cooperate with each other.

In this chapter, we consider sensor networks that are deployed in the same area, but are controlled by different operators. In such a situation, sensors may reduce transmission energy if their packets are forwarded by sensors that belong to another operator. There is the risk, however, that the sensors belonging to another operator drop the packets. The reason can be denial of service attack, lack of agreement on the common goal, etc.

Our goal is to determine the best strategy for the operators who control the sensor networks; for this purpose, we make use of game theory. We do not rely on any cooperation enforcement mechanism, but rather we want to see whether cooperation can exist based solely on the self-interest of the operators. Our simulation results show that cooperation of co-located sensor networks extends their lifetime, thus operators are better off if they cooperate with each other.

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[1]For a survey on sensor network applications see [ASSC02].

5.2 Game-Theoretic Model

In this section, we present our system model and the non-cooperative game that enables us to investigate selfish packet forwarding in co-located sensor networks.

5.2.1 System Model

We assume a set of small, battery-powered devices called *sensors*. For simplicity, we assume that each battery has the same maximum energy it can store, which we denote by **B**. We assume that two sensors are able to communicate with each other if they reside within each other's transmission range, even if they belong to different sensor networks; in other words, inter-operability is ensured by the device manufacturers. We assume an ideal channel without packet losses; in other words, we assume that each packet loss is due to the strategic behavior of the sensors.

We also assume that the sensors perform a given task and that they periodically report their measurements to one or several base station nodes in the network. We refer to the base stations as *sinks*. We further assume that the measurement data can be included in a single message that we call a *packet*. We assume that packets have the same size. Hence, we express the *transmission cost* C^{tr} for a single packet as a function of the transmission distance, in particular we assume $C^{tr} = C^{tr}_{unit} \cdot d^\alpha$, where C^{tr}_{unit} is a constant that includes antennae characteristics, d is the distance of the transmission and $2 \leq \alpha \leq 5$ is the *path loss exponent* [Rap02]. Without loss of generality, we assume that $C^{tr}_{unit} = 1$. We assume that a fixed energy for computation is included in the transmission cost. We introduce the *unit* of energy as the transmission cost to the distance of one meter. We also assume that the energy consumed by receiving and processing a packet is fixed and we denote the *reception cost* by C^{rx}.

We assume that two sensor networks, each controlled by a different *operator*, are co-located on the same area. We investigate two scenarios:

- *Separate sinks:* The sinks belong to different operators (e.g., each sensor network has its own sinks).

- *Common sinks:* The sinks are common resources used by all operators.

We call the set of networking elements controlled by operator i, a *domain* $\mathcal{D}_i$. In the case of the common sink scenario only the sensors belong to $\mathcal{D}_i$, whereas in the case of the separate sink scenario the sinks controlled by i are also part of $\mathcal{D}_i$. We define the a domain to be *inactive* as the time when the battery of the first sensor in the domain is depleted.

We assume that there exists an energy efficient routing algorithm that enables sensors to send packets to the sinks via several hops. The design of energy efficient routing algorithms is a focus of on-going research efforts. Throughout this chapter, we assume that the routing protocol establishes a minimum energy path from each sensor to the sink, as it is presented by Ye *et al.* in [YCLZ01]. Note that we assume that routing is performed properly; we postpone the investigation of selfish behavior in routing as a separate problem to our future work. In our model, two routes are established for each sensor: one route in the own network (non-cooperative routing) and one route in the common network of all operators (cooperative routing). If a sensor runs out of battery, then its domain is excluded from the game and routes are recalculated. We also assume that the communication from the sinks to the sensors is performed via a single-hop, (i.e., the sinks have sufficient energy to reach their sensors directly).

5.2.2 Game

Game theory [FT91] provides an appropriate tool to model strategic decision situations. In our system, the operators have to decide, whether they help each other to increase the lifetime of their network or they ignore the possible help from other sensor networks and rely on their own network to achieve their goal. We model this cooperative packet forwarding situation, as a multi-stage game $\mathbf{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$, where $\mathcal{N}$ denotes the set of *players*, $\mathcal{S}$ the set of strategies and $\mathcal{U}$ is the set of payoff functions. We assume that if a domain becomes inactive (as defined in Section 5.2.1), it is excluded from the game. The game ends when both domains become inactive. Note that in the last phase of the game there is only one domain.

We assume that the time is divided into time units called *time steps*. Once per time step t the sensors of each domain send measurement packets towards the sinks. The length of a time step is defined by the frequency of the packet sending and it has no effect on the game. We assume that the time step is long enough to deliver all packets to the sink. Correspondingly, we assume that the sensors wake up almost synchronously to report to the sinks. We will consider the effect of asynchronous wake-up in our future work.

In each time step, each of the players i has to define two actions for his domain (a) whether his sensors and sinks should forward the packets of sensors in domain $\mathcal{D}_j$, where $j \neq i$, or not (in case they are asked to forward), and (b) whether to request the sensors and sinks belonging to other domains to forward the packets of sensors in $\mathcal{D}_i$ or to send the packets only within $\mathcal{D}_i$.[2] We refer to the decision of any player i in time step t as a *move* $m_i(t)$. Note that the players apply the same move for each packet of each other domain in a given time step t. We use the following short notation for the possible moves of the players:

- *DD (don't ask/drop)*: *do not ask* others to forward and *drop* all packets from others if asked for help

- *DF (don't ask/forward)*: *do not ask* others to forward and *forward* all packets from others if asked for help

- *AD (ask/drop)*: *ask* others to forward and *drop* all packets from others if asked for help

- *AF (ask/forward)*: *ask* others to forward and *forward* all packets from others if asked for help

We further assume that each player has to perform a move exactly once in each time step. We denote the vector of the moves of all players in time step t by $\bar{m}(t)$. Note that the decision does not affect the reception. We assume that if a sensor is active, it will always receive packets.

We assume that the operation of a sensor domain i is successful in time step t if $\rho_i(t)$, the proportion of his sensors from which it has received measurement data is above a given success threshold ξ_i. In this case, player i obtains a fixed *benefit* ($b_i(t) = B$). If the measurement is not successful (meaning that $\rho_i(t) < \xi_i$), then the player receives no benefit in time step t ($b_i(t) = 0$).

Player i also has a *cost* in time step t denoted by $c_i(t)$ that represents the total transmission and reception cost of all sensors that belong to i for all packets (both for own packets and packets for the opponents). In general, we can assume that $B >> c_i(t)$ in any time step t, meaning that the potential benefit received from successful information sending is higher than the value of the total cost (i.e., it is worth to send packets towards the sinks). We assign a *payoff* $u_i(t) = g_i(t) - c_i(t)$ to each player i for each time step of the game.

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[2]Note that in the separate sink scenario, the decision of player i applies also to the sinks in $\mathcal{D}_i$.

A *strategy* $s_i \in \mathcal{S}$ is a function that defines the move of player i for a time step $t + 1$ given the success of player i in the previous time step. In order to reduce the complexity of the sensors, it is reasonable to assume that there is a pre-programmed packet forwarding strategy stored at each sensor. Each sink informs his own sensors about the success of gathering the last measurement, as an input of this strategy[3] [IK03], hence the feedback can be included in a single bit. Our solution is beneficial, because it minimizes the reception energy of the feedback.

In our analysis, we define the *total payoff* as $u_i = \sum_{t=0}^{T} u_i(t)$, where T denotes the lifetime of the domain controlled by player i. The goal of the players is to maximize their total payoff in the game. Intuitively, this goal means to report measurement successfully as many times as possible, while minimizing their energy consumption (maximizing their lifetime).

We assume that players are rational and that rationality is a common knowledge (meaning that they know that the others are rational as well). We also assume that the game is of complete information, i.e., each player knows each element in **G** and thus he is able to analyze it and act according to the analysis.

5.3   Simulation Results

In this section, we present our simulation results in which we have identified the best packet transmission strategies in randomly generated scenarios. We also quantify the difference between equilibrium strategies.

We assume two operators that deploy their sensor network in the same area, in such a way that they are initially connected. In our simulations, we investigate both the separate sink and the common sink scenario: In the separate sink scenario, we put one sink per domain in two different positions; and in the common sink scenario, we put a single sink in the middle of the simulation area.

Table 5.1 presents our simulation parameters.[4]

Parameter	*Value*
Number of sensors per domain	10–50 (25)
Distribution of the sensors	uniformly random
Area size	40x20m
Battery of sensors (**B**)	100000 units
Benefit constant (B)	100000
Reception energy (C^{rx})	100 units
Path loss exponent	2–5 (4)
Success requirement (ξ_i)	1.0
Positions of the sinks (separate sinks)	[10,10] and [30,10]
Positions of the sink (common sink)	[20,10]
Route selection	minimum energy path

Table 5.1: Parameter values of the simulations

For a given set of parameters, we performed 100 simulation runs, each corresponding to a different topology of the sensors. For each simulation run, we performed an exhaustive search on the available strategy space to identify possible Nash equilibria. For each player, we determined the Nash equilibrium

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[3]In the common sink scenario, the common sinks inform the sensors in each domain.
[4]We present the default values of variables in parenthesis.

(or several Nash equilibria) that result(s) in the highest payoff. We observed that in each of the selected Nash equilibria, the game stabilizes in the following pair (or pairs) of moves:

- **Defective equilibrium:** The players end up in playing the moves DD-DD.

- **Cooperative equilibrium:** The players end up in playing the moves AF-AF.

- **Other equilibria:** The equilibrium is different from the ones above.

Defective equilibria always exist in the network, which is not always true for cooperative and other equilibria. Consequently, if several types of equilibria exist in the network (typically both defective and cooperative equilibria), we define the efficiency Φ as the ratio of the payoff achieved by defection with respect to the payoff achieved by cooperation:

$$\Phi = \sum_{i \in \mathcal{N}} \frac{u_i^{defective}}{u_i^{cooperative}} \tag{5.1}$$

Figure 5.1 presents the number of different equilibria in the separate sink scenario as a function of the network size. We can observe that the number of cooperative equilibria is much higher than the number of other types of equilibria. Furthermore, as we increase the number of sensors in the domains, the number of simulation runs with cooperation as the best equilibrium decreases.
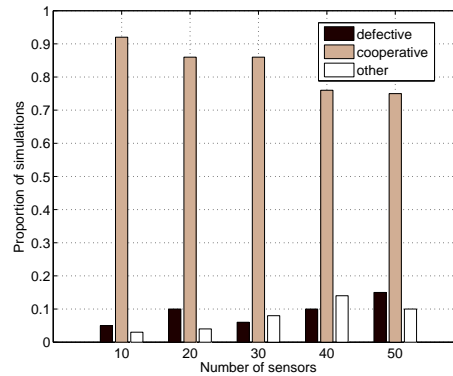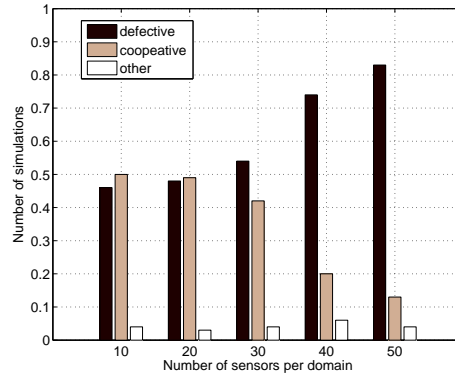

Figure 5.1: The effect of network size on cooperation in the separate sink scenario.

It is important to emphasize that in the separate sink scenario, the players control their sinks as well. Thus, the dominance of cooperation might be the result of the presence of the sink of the other domain (which enables shorter routes) and not the result of cooperation between the sensor networks. To investigate the effect of cooperation in the sensor networks, we present results in the common sink scenario. Our results show that this effect decreases as network size increases.

Figure 5.2 presents the number of different equilibria in the common sink scenario as a function of the network size. We can see that the number of defective equilibria is approximately the same than the number of cooperative equilibria. As we increase the network size, however, the number of defective equilibria increases and the number of cooperative equilibria decreases. The reason is that with the increasing density of sensors, the reception power dominates the energy consumption.

Figure 5.3 presents Φ as a function of the network size. We see that in the separate sink scenario, Φ is much less than in the common sink scenario, thus choosing cooperation is more beneficial with respect to defection. In both scenarios, Φ increases with the network size.

Figure 5.2: The effect of network size on cooperation in the common sink scenario.
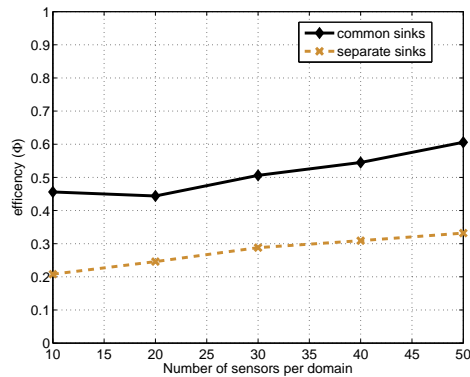
Figure 5.3: Ratio of the payoff achieved by defection with respect to the payoff achieved by cooperation as a function of the network size.

Figure 5.4 presents the number of different equilibria in the common sink scenario as a function of α. The figure shows that as the path loss exponent increases (which represents a more hostile environment) the number of defective equilibria drops significantly; at the same time the number of cooperative equilibria increases significantly. This shows that the more hostile the environment is, the more beneficial the cooperation is.

Figure 5.5 presents Φ as a function of the path loss exponent. As no cooperative equilibria exist for $\alpha = 2$, we can present the results only for $\alpha > 2$. We can observe that the more hostile the environment is, the better cooperation is with respect to defection.

5.4   Summary

In this chapter, we presented a game-theoretic model to study cooperation in multi-domain sensor networks. Multi-domain sensor networks present a likely scenario in civilian applications, where various authorities deploy their sensor networks to perform different tasks. Nonetheless, sensor technology and protocols might be standardized enough to enable the sensors to cooperate.

In our study, based on the assumption that the sensors have limited computation and energy resources,
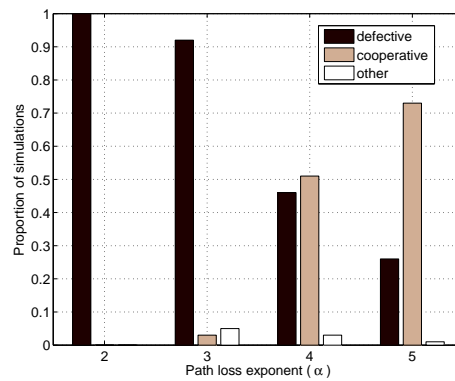
Figure 5.4: The effect of α on cooperation in the common sink scenario.
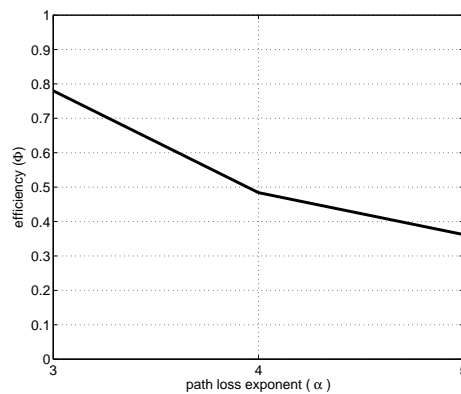
Figure 5.5: Ratio of the payoff achieved by defection with respect to the payoff achieved by cooperation Φ as a function of α.

we investigated cooperation in the absence of incentive mechanisms. Our results showed that saving energy by cooperation provides a "natural incentive" for the operators. The advantage of cooperation is twofold: (a) the operators can largely benefit by providing service of their sinks for other's sensor networks and (b) if sinks are common resources, then cooperative packet forwarding is beneficial for sparse networks or if the environment is hostile. These results show viable methods for improving the performance of sensor networks using cooperation.

Publication: [FBH05]

Chapter 6

Wireless Operators in a Shared Spectrum

6.1 Introduction

Cellular networks are notoriously difficult to design and operate; in particular, defining the optimal location of the base stations and fine tuning their configuration parameters is very challenging. For this reason, government agencies (such as the FCC in the US) have sold or rented, for example by auction, each operator a frequency band for its exclusive usage in a given country or region. Only a small part of the whole spectrum is allocated as a *shared spectrum*, in which networks function in the same (unlicensed) frequency band.

With the progress of technology and the fast growing demand for ubiquitous high-speed wireless services, it is clear that the pressure towards more flexibility of the usage of the spectrum will only increase. Therefore, the government agencies are likely to adapt the current regulations in order to increase the proportion of the unlicensed spectrum as discussed in [Ben04, FF03].

The evolution towards unlicensed frequency bands can lead to a better usage of the spectrum. Yet, it would also create a novel situation, in which the base stations of different operators would interfere with each other. An operator may be tempted to let its base stations transmit at the maximum authorized level. But by doing this, it would maximize interference not only to its own base stations, but also to the base stations of the other operators, and to all mobile devices in the power range of its base stations; in addition, it would face the danger that the other operators retaliate by behaving in the same way.

In our work, we assume that mobile users can *freely roam* across the base stations located in their neighborhood, attaching to the one offering the most favorable signal quality (i.e., the base station with the strongest pilot signal) and bandwidth, irrespectively of the operator to which the base station belongs.[1] From the interference perspective, this operating principle is much more efficient than the current practice, because it enables mobile devices to find the "closest" base station in the area and hence mobile devices and base stations can significantly decrease their transmission power. This free roaming could be beneficial for both operators and users, because the former could serve an increased set of users, and the latter could enjoy various services across several operators.

We also assume that each operator wants to cover the largest possible area by increasing the transmission range of its base stations. At the same time, it wants to minimize interference. These two contradictory goals correspond to the willingness to maximize the number of users who can attach to its base stations. We model this situation as a game between operators in terms of power control of the base

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[1]The users might have other attachment preferences based on subscription type, past experience, etc. We will consider the extension of user attachment behavior in our future work.

stations. We believe our contribution to be one of the first steps towards a deeper understanding of the trade-offs of operating cellular networks in shared spectrum.

Note that the general problem of power control of base stations is hard to solve (i.e., NP-complete); it is characterized by the following dimensions: (i) the size of the base station sets, (ii) the geographic locations of the base stations and (iii) their possible radio ranges.

6.2 Model

6.2.1 System Model

We make the following assumptions with respect to the communication network. We assume two wireless communication networks, each operated by an *operator* and we call the operators A and B. Operator $o \in \{A, B\}$ controls a set of *base stations* denoted by $\mathcal{B}_o$. We denote the union of all base stations by $\mathcal{B}$. There exist no base stations that are located at the same place and belong to different operators.[2] We also assume several *users* equipped with *mobile devices* to access the communication network. The networks reside in a given *service area*, where the operators want to provide wireless access for the users. We restrict ourselves to two operators in order to provide an insight in the basic principles of cooperation in a multi-operator context. Note that the problem is hard to solve for a general network topology as presented in Sections 6.5.

We assume that the radios of the base stations and the mobile devices are compatible, meaning that any user is able to access the network via any of the base stations. Base stations and mobiles operate on the same unlicensed band of the frequency spectrum. Each of these devices might perform power control to optimize its transmission power and reduce interference. This optimization can be realized in three ways: the power control of the pilot signal of the base stations, of the downlink (base station to mobile) and of the uplink (mobile to base station). In this chapter, we focus on the first technique and we postpone the investigation of the other two techniques to our future work. To mitigate interference, the shared frequency band is usually split up into channels (i.e., separated frequency sub-bands), but the pilot signal is typically emitted on a single shared channel for all the base stations, which results in mutual interference of the pilot signals.

According to the *physical model* of signal propagation [Rap02], the pilot signal of a base station $i \in \mathcal{B}$ can be received by a user device v if its *signal-to-interference-plus-noise ratio (SINR)* exceeds a reception threshold $SINR_{min}$:

$$SINR_{iv} = \frac{P_i \cdot d_{iv}^{-\alpha}}{N_0 + \sum_{j \in \mathcal{B}, j \neq i} P_j \cdot d_{jv}^{-\alpha}} \geq SINR_{min} \tag{6.1}$$

where P_i is the transmission power of base station i and N_0 is the Gaussian thermal noise. We normalize the effect of the antenna characteristics and correspondingly we assume that the channel gain depends only on the distance d_{iv} of the transmitter and the receiver and the path loss exponent $2 \leq \alpha \leq 5$. The path loss exponent characterizes the radio signal propagation properties of the environment. Hence, (6.1) corresponds to the Friis free space radio signal propagation equation (see [Rap02] Equation (4.1)). It captures how the reception power depends on the most important factors, namely on the transmission power and the distance between transmitter and receiver. Note that we consider the local average of the received pilot signal as described in [Rap02]. In reality, on a small time scale, the pilot power signals

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[2]To reduce operating costs, operators of current cellular networks often share the same site. However, if users can freely roam, then this site sharing does not make sense anymore.

have a time-varying property due to fading. In our future work, we will consider a more realistic radio signal modeling that incorporates fading and more realistic path loss models. We assume that (6.1) holds for every point in the service area for at least one base station (i.e., there is full coverage) and that the user device v attaches to the base station i with the best SINR.

We abstract away the mobiles and assume that their expected position is uniformly distributed over the service area. Note that this also means a balanced load on the base stations (i.e., no users have to switch base stations due to the lack of available capacities). We leave the topic of other user distributions for future work.

Let us assume that the pilot signals propagate in an open area, meaning $\alpha = 2$. Then the distribution of user devices defines a Multiplicatively Weighted Voronoi power diagram (MW power diagram) [OBSC00], which determines the set of points in the service area (potential places of user devices) that are attached to a given base station. In the MW power diagram, a point belongs to a base station if it is "closer" to it than to any other base station, where the distance is defined as follows:

Definition 6.1. *The* multiplicatively weighted power distance *between the points v and i is defined as:*

$$d_{mpw}(v, i; w_i) = \frac{d_{iv}^2}{w_i} \tag{6.2}$$

where d_{iv} is the Euclidian distance between the points v and i and w_i is a weight assigned to point i.

We can define the *Voronoi region $V(i)$* around a base station $i \in \mathcal{B}$ as the set of points v that are "closer" to point i than to any other point j. Hence, we can write $V(i)$ as:

$$V(i) = \{v | d_{mpw}(v, i; w_i) \leq d_{mpw}(v, j; w_j), \forall i \neq j\} \tag{6.3}$$

We can write the *Voronoi diagram $V(\mathcal{B})$* of all base stations $\mathcal{B}$ as:

$$V(\mathcal{B}) = \bigcup V(i) \tag{6.4}$$

where $i \in \mathcal{B}$.

Due to the complex shape of the Voronoi diagram with multiplicatively weighted distances, it is difficult to derive analytical solutions for the pilot power control problem. Hence, we apply a *radio range model* that is widely used in the literature. We will show in Section 6.3.4 that the principles derived from the range model hold for the physical model as well.

Let us derive from (6.1) the *radio range (R_i)* of the pilot signal of the base station i as the Euclidian distance within which the users are able to attach to this base station if there is no interference from other devices:

$$R_i = \sqrt[\alpha]{\frac{P_i}{SINR_{min} \cdot N_0}} \tag{6.5}$$

According to the radio range model, we can define the Additively Weighted Voronoi power diagram (AW power diagram) [OBSC00]. In the AW power diagram, the distance is defined as follows:

Definition 6.2. *The* additively weighted power distance *between the points v and i is defined as:*

$$d_{apw}(v, i; w_i) = d_{iv}^2 - w_i \tag{6.6}$$

where d_{iv} is the Euclidian distance between the points v and i and w_i is a weight assigned to point i.

In this chapter, we substitute $w_i = R_i^2$ and hence we obtain a *Voronoi diagram in the Laguerre geometry* [OBSC00]. This model corresponds to a Voronoi diagram, where the distance is defined as a tangential Euclidean distance to circles centered at the base stations' locations and radii corresponding to their radio ranges.

We assume that the base stations are placed on the vertices of a two-dimensional lattice in an alternating way such that any base station that belongs to operator A has four neighboring base stations that belong to operator B (a small part of the network is shown in Figure 6.1). Let us call d the smallest Euclidian distance between base stations. In Section 6.4 and 6.5, we will extend our model to general network topologies.
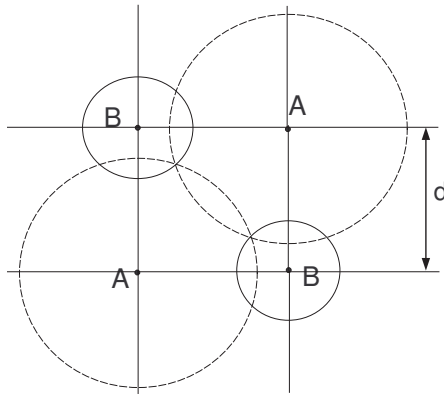
Figure 6.1: Base stations on the vertices of a two-dimensional lattice. Here A is the operator with a larger radio range.

To further specify our model, we assume that:

- **A1:** Operators want to provide wireless access service everywhere. Thus, no place remains uncovered in the service area.

- **A2:** Operators can estimate or measure their coverage including the action of other operators.

- **A3:** Each base station belonging to the same operator has the same radio range. We show in Section 6.5 that relaxing this assumption makes the power control problem NP-complete.

- **A4:** There exists a limitation P_{MAX} on the transmission power of any base station, which is defined by the regulator of the wireless spectrum. Then, the maximum radio range R_{MAX} can be derived from (6.5) by substituting $P_i = P_{MAX}$. Furthermore, if the radio ranges of all base stations $i \in \mathcal{B}$ are equal, we denote the minimum radio range for which **A1** holds by $R_{MIN} = \frac{\sqrt{2}}{2}d$.

- **A5:** The users can freely roam between any of the base stations (i.e., the operators do not forbid roaming between their networks).

- **A6:** Users are uniformly distributed over the area and hence, the expected load is the same on every base station.

- **A7:** The base stations and the mobile devices have omnidirectional antennae. The investigation of the effect of directional antennas is part of our future work.

These assumptions ensure an open spectrum environment, in which users enjoy ubiquitous wireless connectivity. In particular, we make Assumption **A3**, as well as the assumption that the base stations are placed on the vertices of a grid, to make the model tractable. This special scenario is reasonable for a small number of base stations, such as for a small city network. We will show stability points for this special model. We study this special model, because we want to provide some quantitative insights into the power control problem. The general problem is very involved: We show that if operators can set an arbitrary radio range for their base stations (i.e., **A3** does not hold), then the power control problem is hard to solve.

6.2.2 Power Control Game

We model the power control problem with two operators as a two-player, nonzero-sum game. We refer to the two operators as *players* A and B, respectively. Due to **A3**, we designate the radio ranges of the pilot signal of the players by R_A and R_B. The *strategy* of the players defines their best radio range. In this work, we consider only pure strategies, because mixed strategies might not be feasible for pilot power adjustment due to their incurred overhead. The goal of the players is to maximize the area they cover with their pilot signal as expressed by their *payoff* function. To express the payoff of the players formally, let us introduce the following concepts.

Assume that the two players choose a different radio range. Let us call the player with the larger radio range *heavy* and the player with the smaller range *light*. Let us denote the radio range of the heavy player by R_H and the one of the light player by R_L (recall from **A3** that a player has the same range for all of its base stations). In this section, we assume that A is the heavy player and B is the light player (i.e., $R_A = R_H$ and $R_B = R_L$); note, however, that it can be the opposite due to the symmetric situation. Since the placement of the base stations is symmetric and the players apply the same radio range to all of their base stations, we can analyze the game considering two neighboring base stations, as shown in Figure 6.2. Consequently, we designate both the players and their respective BSs by the same letter.

We define the *useful coverage area (O_i)* for any base station i as its Voronoi region $V(i)$ in the radio range model (i.e., in the Voronoi diagram in the Laguerre geometry). We define the *interference area (Y_i)* for a base station i as the covered area where the signal of i is not the strongest (i.e., the difference between total covered area and the useful coverage area).

$$Y_i = R_i^2 \cdot \pi - O_i \tag{6.7}$$

Note that the useful coverage area of a player depends on the radio range of the other player. Accordingly, we can distinguish two cases as follows.

If both players have a non-empty useful coverage area, as presented in Figure 6.2a, then the following condition holds:

$$R_H < \sqrt{R_L^2 + d^2} \tag{C1}$$

We can express the useful coverage area of the heavy player by calculating the area of the octagon. As shown in Figure 6.2, this area can be calculated based on the distance d of the two base stations, the distances $\overline{AM}$ and $\overline{BM}$ between the base stations and the middle point M as well as the ranges R_H and R_L. Thus, we can write the useful coverage area as follows:

$$O_H = \frac{d^4 + 2d^2(R_H^2 - R_L^2) - (R_H^2 - R_L^2)^2}{d^2} \tag{6.8}$$
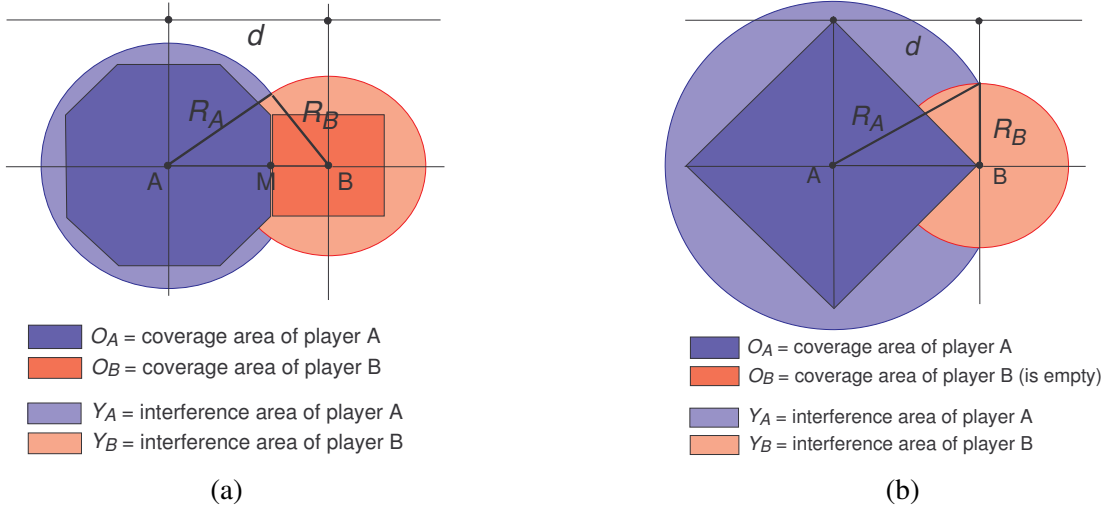
Figure 6.2: Coverage and interference area of a base station, illustrated with two base stations: (a) both base stations have a coverage area; (b) the base stations of the light player are overwhelmed by the base stations of the heavy player and thus the light player has no coverage area at all.

The useful coverage area of the light player is as follows:

$$O_L = \frac{(d^2 - R_H^2 + R_L^2)^2}{d^2} \tag{6.9}$$

If Condition **C1** does not hold, then the light player is overwhelmed by the heavy player, meaning that the pilot signal of the heavy player is the strongest everywhere (as presented in Figure 6.2b). If the heavy player overwhelms the light player, the useful coverage area functions are as follows:

$$O_H = (\sqrt{2}d)^2 = 2d^2 \tag{6.10}$$

$$O_L = 0 \tag{6.11}$$

In addition to **C1**, we can derive a condition for the radio ranges of the two players from **A1** from the geometry presented in Figure 6.1 as follows:

$$R_H^2 \geq (\frac{\sqrt{2}}{2}d - R_L)^2 + (\frac{\sqrt{2}}{2}d)^2 = d^2 - \sqrt{2}dR_L + R_L^2 \tag{C2}$$

In the limit case, in which the equality holds in (**C2**), they just cover the service area (as shown in Figure 6.1).

From (**C2**) and **A4**, we can derive the definition interval for R_H:

$$\sqrt{d^2 - \sqrt{2}dR_L + R_L^2} \leq R_H \leq R_{MAX} \tag{6.12}$$

Similarly, from (**C2**) we get the bounds on R_L knowing that it is positive and smaller than R_H:

$$\max\{0, \frac{\sqrt{2}}{2}(d - \sqrt{-d^2 + 2R_H^2})\} \leq R_L \leq R_H \tag{6.13}$$

The upper bound comes from the fact that $R_H \leq \frac{\sqrt{2}}{2}(d + \sqrt{-d^2 + 2R_H^2})$ for all values of R_H. Note that the expressions in (6.12) and (6.13) always take real values.

We assume that the goal of the players is to maximize their payoff, in other words to maximize their useful coverage area while minimizing their interference area (i.e., the area, which is in their radio range, but they do not cover eventually). We define the payoff per base station for player i playing R_i given that the other player j plays R_j at its base stations as follows:[3]

$$u_i(R_i, R_j) = O_i - \gamma_i \cdot Y_i = (1 + \gamma_i) \cdot O_i - \gamma_i \cdot R_i^2 \cdot \pi \qquad (6.14)$$

where $\gamma_i \geq 0$ is a *cooperation parameter* that defines how much player i cares about the size of his interference area. Note that the cooperation parameter provides a general method to model both internal considerations of the operator such as cooperativeness, as well as external cooperation enforcement mechanisms such as an agreement between operators or a *power price* induced by the regulator of the spectrum.

Let us graphically present the payoffs of the players based on expression (6.14). Figure 6.3a presents an example for the payoff of the heavy player for a fixed value of R_L and Figure 6.3b presents the payoff of the light player for a fixed value of R_H. In the next section, we derive stability points in the game using these payoff functions.
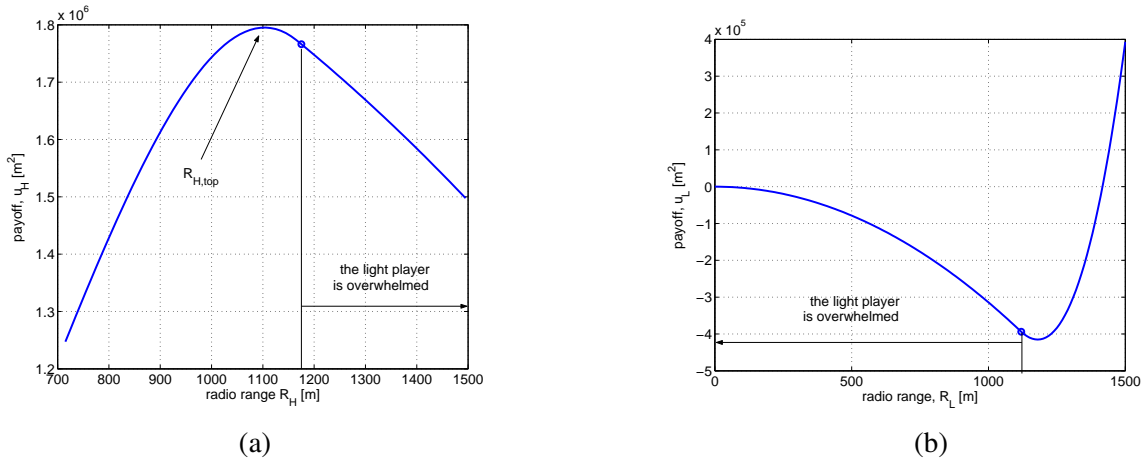
Figure 6.3: Payoff function (a) of the heavy player for $d = 1km$, $\gamma_H = 0.1$, $R_L = 0.6km$ and $R_{MAX} = 1.5km$ (defined by the regulator); and (b) of the light player for $d = 1km$, $\gamma_L = 0.1$ and $R_H = 1.5km$.

6.3 Single-Stage Game

In this section, we consider a *single-stage game*, where both players simultaneously choose their radio range once and for all. This corresponds to the case in which the base stations are not able to perform power control during the operation of the network, thus the radio power has to be set manually at the installation of the base stations. We use this basic scenario to study the basic equilibria of the power

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[3]Note that due to the specific scenario, the payoff of player i can be calculated by multiplying u_i with the number of its base stations. In this scenario, we refer to the payoff per base station as the payoff of the player.

control game. We extend our investigation to more complex scenarios in the following sections. We make use of the concept of Nash equilibrium and best response introduced in Section 1.2.3.

6.3.1   Best Response Function

We derive the best response function for the heavy player from the payoff functions presented in Figures 6.3a. For the heavy player, its payoff is a concave function with a maximum point $R_{H,top}$ as shown in Figure 6.3a. We can derive $R_{H,top}$ by maximizing (6.14) with the coverage area defined in (6.8).

$$R_{H,top} = \frac{\sqrt{2(1 + \gamma_H)(d2 + R_L^2) - d2\gamma_H \pi}}{\sqrt{2}\sqrt{1 + \gamma_H}} \tag{6.15}$$

We can identify different best response strategies for this case, corresponding to the definition interval of the payoff function, as follows.

1. If $R_{H,top} < \sqrt{d^2 - \sqrt{2}dR_L + R_L^2}$, then the best response of the heavy player is the lower bound in (6.12), because the payoff function is strictly decreasing:

$$br_H(R_L) = \sqrt{d^2 - \sqrt{2}dR_L + R_L^2} \tag{6.16}$$

2. If $\sqrt{d^2 - \sqrt{2}dR_L + R_L^2} < R_{H,top} < R_{MAX}$, then the best response is (this corresponds to Figure 6.3a).

$$br_H(R_L) = R_{H,top} \tag{6.17}$$

3. Finally, if $R_{H,top} > R_{MAX}$, then the best response of the heavy player is:

$$br_H(R_L) = R_{MAX} \tag{6.18}$$

For the light player, the best response strategy should be one of the bounds as defined in (6.13) as shown in Figure 6.3b. If the upper bound in (6.13) applied, then the light player would have reason to become a heavy player (i.e., apply a radio range larger than the upper bound in (6.13)). Hence, it is enough to compare only the lower bound with the best response solutions derived in (6.16), (6.17) and (6.18). The lower bound has two cases as expressed in (6.13).

Let us now define the *critical range* of player j as the range R_j for which the payoff of player i, whether the heavy or the light player, is equal. We denote the critical radio range by R_j^*. If player j plays a radio range larger than the critical range of player i, then player i should be the light player.

We can now derive the best response function of the players by substituting (6.16) through (6.18) in the payoff function and compare them with the payoff playing $R_i = 0$. The result is shown in Figure 6.4. We can notice that the critical range (identified by the vertical lines) decreases as the cooperation parameter γ_i increases.

Table 6.1 presents the critical ranges for player i as a function of γ_i, and we show the numerical values of the limits in Table 6.2. If $\gamma_i < \frac{d^2}{\pi \cdot R_{MAX}^2 - d^2}$, then the critical range is larger than the maximum range R_{MAX} and hence, the best response is never $R_i = 0$. Note that $\gamma_{LIM1} = \frac{d^2}{\pi \cdot R_{MAX}^2 - d^2} \leq \frac{1}{\pi - 1}$ for any $R_{MAX} > d$. If $\gamma_i \geq \gamma_{LIM3}$, then the critical range $R_j^* \leq R_{MIN}$ and the player necessarily plays R_{MIN}.
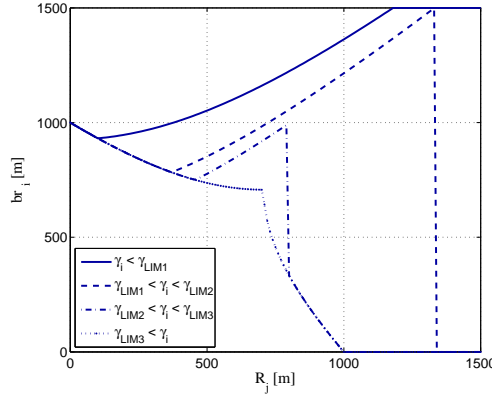
Figure 6.4: Best response function of player i for various γ_i values for $d = 1km$ and $R_{MAX} = 1.5km$

Value of γ_i	Critical range: R_j^*
$\gamma_i < \gamma_{LIM1}$	$R_j^* > R_{MAX}$, no critical range exists
$\gamma_{LIM1} \leq \gamma_i < \gamma_{LIM2}$	$d < R_j^* < R_{MAX}$
$\gamma_{LIM2} \leq \gamma_i < \gamma_{LIM3}$	$R_{MIN} < R_j^* < d$
$\gamma_i \geq \gamma_{LIM3}$	$R_j^* \leq R_{MIN}$, no critical range exists

Table 6.1: Critical radio ranges depending on the value of γ_i

6.3.2 Nash Equilibria in the Single Stage Game

Let us designate the stability points in the game as follows. NE_{MIN} denotes a pure-strategy Nash equilibrium in which the players play the radio ranges (R_{MIN}, R_{MIN}). Similarly, we define the Nash equilibrium NE_{MAX} for the joint action (R_{MAX}, R_{MAX}). We write $NE_{MIN,i,j}$ if the players just cover the service area, but they have a different radio range (i.e., their radio ranges define the limit case in (**C2**) as shown in Figure 6.1). In the subscript, i refers to the player with the larger radio range (i.e., i is the heavy player).

We can identify pure-strategy Nash equilibria in the single stage game based on Definition 1.4 by searching for the intersections of the possible best response functions shown in Figure 6.4 using the corresponding equations (6.13), (6.16), (6.17) and (6.18).

1. The radio ranges of the players just cover the service area. They play one of the minimum Nash equilibria (meaning NE_{MIN} or $NE_{MIN,i,j}$). This case holds for $\gamma_A > \gamma_{LIM2}$ or $\gamma_B > \gamma_{LIM2}$.

2. There is a unique Nash equilibrium NE_{MAX} if
   $\gamma_A < \gamma_{LIM1}$ and $\gamma_B < \gamma_{LIM1}$;

3. No Nash equilibrium exists if
   $\gamma_{LIM1} < \gamma_A < \gamma_{LIM2}$ and $\gamma_B < \gamma_{LIM1}$ or vice versa.

Table 6.3 shows the types of different Nash equilibria as a function of the cooperation values of the players.

γ_{LIM1}	$\frac{d^2}{\pi \cdot R_{MAX}^2 - d^2} < \frac{1}{\pi - 1} \approx 0.46$
γ_{LIM2}	$\frac{2(-4 + 2\pi - \sqrt{2}\pi)}{8 - 8\pi + \pi^2} \approx 0.59$
γ_{LIM3}	$\frac{2}{\pi - 2} \approx 1.75$

Table 6.2: Numerical values of the limits of γ_i
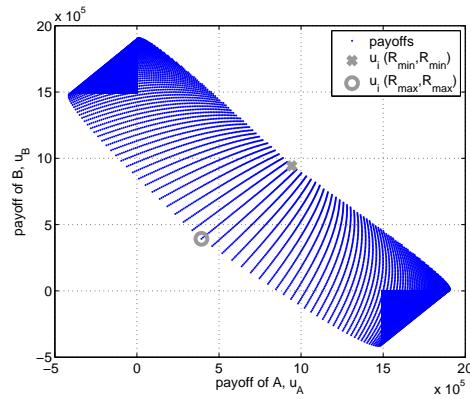
	$\gamma_B < \gamma_{LIM1}$	$\gamma_{LIM1} < \gamma_B < \gamma_{LIM2}$	$\gamma_{LIM2} \le \gamma_B < \gamma_{LIM3}$	$\gamma_B \ge \gamma_{LIM3}$
$\gamma_A < \gamma_{LIM1}$	NE_{MAX}	*no NE*	$NE_{MIN,A,B}$	$NE_{MIN,A,B}$
$\gamma_{LIM1} < \gamma_A < \gamma_{LIM2}$	*no NE*	*no NE*	$NE_{MIN,A,B}$	$NE_{MIN,A,B}$
$\gamma_{LIM2} \le \gamma_A < \gamma_{LIM3}$	$NE_{MIN,B,A}$	$NE_{MIN,B,A}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$
$\gamma_A \ge \gamma_{LIM3}$	$NE_{MIN,B,A}$	$NE_{MIN,B,A}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$ NE_{MIN}

Table 6.3: Nash equilibria (NE) in the single stage game as a function of the cooperation values.

6.3.3 Equilibrium Selection

From Table 6.3, we can observe that there exist a variety of Nash equilibria depending on the parameters (i.e., cooperation, maximum radio range) in the power control game. In order to assess the success of the players in these Nash equilibria, we use the concept of *Pareto-superiority* as defined in Definition 1.7.

In order to assess the feasible region (all possible values of the payoffs) of the radio ranges, we show the payoffs for each possible values of R_A and R_B for $\gamma_A = \gamma_B = 0.1$ in Figure 6.5.[4] Furthermore, let us distinguish the $NE_{MIN,i,j0}$ state in which the heavy player plays $R_i = d$ and the light player plays $R_j = 0$.

Figure 6.5: The payoffs for the possible values of R_A and R_B if $d = 1km$ and $\gamma_A = \gamma_B = 0.1$.

The following theorem shows that, depending on the parameter values, the minimum Nash equilibria can be socially optimal.

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[4]The parallel lines are the result of the precision of the simulations.

	$\gamma_B < \gamma_{LIM1}$	$\gamma_{LIM1} < \gamma_B < \gamma_{LIM2}$	$\gamma_{LIM2} \leq \gamma_B < \gamma_{LIM3}$	$\gamma_B \geq \gamma_{LIM3}$
$\gamma_A < \gamma_{LIM1}$	NE_{MAX}	*no NE*	$NE_{MIN,A,B}$	$NE_{MIN,A,B0}$
$\gamma_{LIM1} < \gamma_A < \gamma_{LIM2}$	*no NE*	*no NE*	$NE_{MIN,A,B}$	$NE_{MIN,A,B0}$
$\gamma_{LIM2} \leq \gamma_A < \gamma_{LIM3}$	$NE_{MIN,B,A}$	$NE_{MIN,B,A}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$	$NE_{MIN,A,B0}$
$\gamma_A \geq \gamma_{LIM3}$	$NE_{MIN,B,A0}$	$NE_{MIN,B,A0}$	$NE_{MIN,B,A0}$	$NE_{MIN,A,B}$ $NE_{MIN,B,A}$ NE_{MIN}

Table 6.4: The best (Pareto-superior) Nash equilibria in the single stage game as a function of the cooperation values.

Theorem 6.1. *If several $NE_{MIN,i,j}$ Nash-equilibria exist in the grid scenario, then*

1. *If $\gamma_A > \gamma_{LIM3}$ and $\gamma_B < \gamma_{LIM3}$, then only $NE_{MIN,B,A0}$ is Pareto-superior.*

2. *If $\gamma_A < \gamma_{LIM3}$ and $\gamma_B > \gamma_{LIM3}$, then only $NE_{MIN,A,B0}$ is Pareto-superior.*

3. *Otherwise none of them is Pareto-superior to the others.*

Proof. Suppose that in any state of the game player i increases its radio range. It is easy to see that the payoff of player increases if $\gamma_i < \frac{2}{\pi-2}$. Similarly, the payoff of the other player increases if its range increases and we have $\gamma_j < \frac{2}{\pi-2}$. Hence, if both $\gamma_i < \frac{2}{\pi-2}$ and $\gamma_j < \frac{2}{\pi-2}$, then the increase of the payoff of one of the players results in the decrease of the payoff of the other player. Hence, any $NE_{MIN,i,j}$ state is Pareto-optimal. However, if player j has $\gamma_j > \frac{2}{\pi-2}$, then decreasing its range increases its payoff and hence the only Pareto-optimal state is $NE_{MIN,i,j0}$. $\square$

Based on Theorem 6.1, we can identify the most beneficial Nash equilibria from Table 6.3. We express this modified solution in Table 6.4.

Table 6.4 shows that if the operators are cooperative, then they should play the minimum radio range with which they are able to cover the service area. Furthermore in a fair solution, they should both play R_{MIN}. However, if one of the players does not cooperate and the other does, then the non-cooperative player can increase its radio range to force the cooperative player out of the game. If none of the players cooperate, then they will end up in both playing the maximum radio range R_{MAX}.

6.3.4 Discussion

Our model based on the Voronoi diagram in the Laguerre geometry results in useful coverage areas with straight separation lines. We adopted this model, because if we applied the physical radio model based on (6.1), it would be difficult to derive a closed-form expression for the coverage and hence for the payoff of the players. We now use a numerical method to compare the radio range model to the physical model and to show that the principles derived in our model hold for the physical model as well.

We compare the useful coverage areas in both models as follows. We transform the continuous area into a discrete area by substituting it with a grid of interval ϵ as shown in Figure 6.6. In our numerical study we use a grid of 100x100 points.

For a given set of radio ranges, we determine the number of points that belong to the base station in the middle of the considered area in each of the radio models. This results in an empirical value of the useful coverage area. We substitute this useful coverage area value into (6.14) to obtain the payoff of the
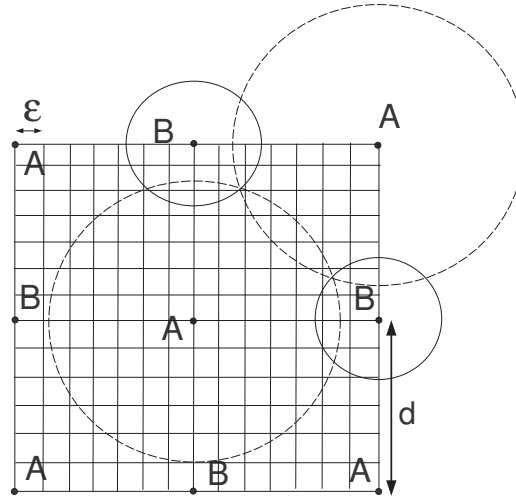
Figure 6.6: Discrete area model with points in ϵ distance.

players in both cases and then we calculate the best responses from the payoff function. Figure 6.7 shows an example of the best response of player i who controls the base station in the middle of Figure 6.6 for each of the radio models.
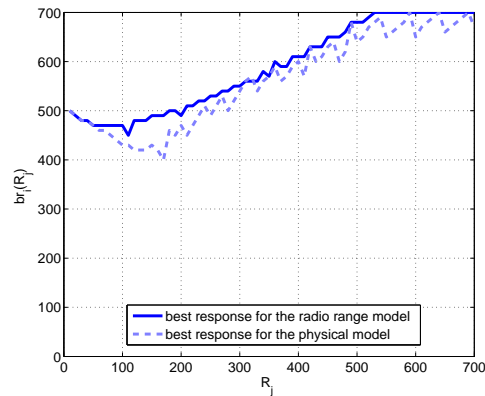
Figure 6.7: Best response function of player i for $\gamma_i = \gamma_j = 0.1$, $d = 500m$ and $R_{MAX} = 700m$.

We can observe that the best response functions are very similar for the two models. We performed our numerical analysis for various values of γ_i and γ_j and it resulted in the same conclusion. Hence, the conclusions about the Nash equilibria in the radio range model hold for the physical model as well. However, the derivation of the precise values of γ_i and γ_j requires an extensive set of numerical calculations. This complexity justifies our study based on the radio range model.

6.4 Repeated Game

In the previous section, we assumed that the radio range of the base stations has to be set in advance and no power adjustment is possible. In this section, we consider the possibility of an iterative power control in a *repeated game*. We assume that the operators do not know the end of the game, hence we study the problem in an infinite repeated game model with discounting [Axe84, FT91]. We will show that cooperation (i.e., both players playing R_{MIN}) can be enforced in the cases in which no cooperative equilibrium exists.

In the repeated game, we assume that the game is split up into steps denoted by t. In each step, player $i \in \{A, B\}$ adjusts the radio range of his base stations according to his strategy s_i.

Furthermore, let us define the discounted total payoff in T time steps as:

$$u_i(t) = \sum_{t=0}^{T} u_i(t) \cdot \delta^t \tag{6.19}$$

where $0 < \delta < 1$ is the *discounting factor*, which expresses the value of future payoffs for the players. Here, we interpret he discounting factor as a value related to the probability that the game ends in the subsequent time step. Based on this interpretation, we assume that the discounting factor is the same for both players.

We now prove a theorem to show that non-cooperation based on R_{MAX} is a Nash equilibrium.

Theorem 6.2. *Both players playing R_{MAX} in all time steps is a Nash equilibrium if $\gamma_i < \gamma_{LIM1}$ and $\gamma_j < \gamma_{LIM1}$ holds.*

Proof. Let us assume that player i plays R_{MAX} all the time. Since the decision of the other player does not affect player i's radio range, we can analyze the game by time steps. In any time step, player i necessarily becomes the heavy player (or they are of equal weight). If $\gamma_i < \gamma_{LIM1}$ and $\gamma_j < \gamma_{LIM1}$, then the best strategy of the other player is $R_j = R_i = R_{MAX}$ in every time step. $\qquad\square$

In this case, the players are in a socially non-optimal equilibrium. We have seen in Theorem 6.1 that for high γ_A and γ_B values, cooperation does not need to be enforced. We will now prove conditions that enable the players to enforce cooperation for other cooperation values. We prove in this section that they can do better, by applying a strategy called *Punisher*.

Definition 6.3. *If player i plays the* Punisher *strategy, he plays R_{MIN} in the first time step. For any further time steps, he plays:*

- *R_{MIN} in time step t if player j played R_{MIN} in time step $t-1$, or*

- *R_{MAX} for the next Υ_i^p time steps, if player j played anything else.*

The parameter Υ_i^p, called the *punishment interval*, defines the number of time steps for which player i punishes the other player. Note that the Punisher strategy is similar to the *Tit-For-Tat (TFT)* strategy introduced in Section 1.4.2.[5] The major difference is that it retaliates any defection by playing R_{MAX} instead of copying the same behavior. Furthermore, the Punisher strategy is different from the *Trigger* strategy defined in [MCWG95], because the Punisher strategy imposes a punishment that is comparable to the amount of misbehavior and thus it is able to recover from erroneous defections.

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[5]TFT defines the choice of a given player in the next time step, whereas the Punisher strategy defines the punishment interval as a set of subsequent time steps.

Assume now that player j plays the Punisher strategy. If player i cooperates as well, they both play R_{MIN}. In this case they both have the *cooperative payoff* $u_i^{coop} = u_H(R_{MIN}, R_{MIN})$.[6] If player i defects, then he obtains a *cheating gain* $u_i^{cheat} = u_H(br_i(R_{MIN}), R_{MIN})$. Substituting γ_i for γ_H, we can obtain the best response value $br_i(R_{MIN})$ from (6.15):

$$br_i(R_{MIN}) = \frac{d^2(3 - (\pi - 3)\gamma_i))}{2(1 + \gamma_i)} \tag{6.20}$$

If we substitute $R_j = R_L = R_{MIN}$ into (6.14), we get a cheating gain $u_i^{cheat} = u_H(br_i(R_{MIN}), R_{MIN})$:

$$u_i^{cheat} = \frac{d^2(8 + (16 - 6\pi)\gamma_j + (8 - 6\pi + \pi^2)\gamma_j^2)}{4(1 + \gamma_j)} \tag{6.21}$$

After the defection, player j retaliates by playing R_{MAX} and player i plays his best response to this radio range. In this case player i has the *defection payoff* $u_i^{def} = u_H(br_i(R_{MAX}), R_{MAX})$. If $br_i(R_{MAX}) = R_{MAX}$, then i's payoff for each time step in the next Υ_i^p time steps is the defection payoff $u_i^{def,max}$:

$$u_i^{def,max} = (1 + \gamma_i)d^2 - \gamma_i R_{MAX}^2 \pi \tag{6.22}$$

If $br_i(R_{MAX}) = 0$, then i's payoff for each time step in the next Υ_i^p time steps is the defection payoff $u_i^{def,0}$:

$$u_i^{def,0} = 0 \tag{6.23}$$

Otherwise, if player i played R_{MIN}, he obtained a cooperation payoff u_i^{coop} for all the Υ_i^p time steps:

$$u_i^{coop} = \frac{d^2(2 - (\pi - 2)\gamma_i)}{2} \tag{6.24}$$

Cooperation can be enforced using the Punisher strategy as proven in the following theorem.

Theorem 6.3. *A Nash equilibrium NE_{MIN} based on R_{MIN} is enforceable with the* Punisher *strategy (i.e., player j is able to punish the defection of player i) if*

1. $\gamma_i < \frac{2}{\pi - 2}$ *and*

2.

$$\frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) < 1 \tag{6.25}$$

where $\delta \leq 1$ and $\gamma_i, \gamma_j \neq 0$.

If the above condition holds, the punishment interval is defined by:

$$\Upsilon_j^p \geq \log_\delta \left(1 - \frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) \right) - 1 \tag{6.26}$$

---

[6]Note that for $R_A = R_B$, $u_H(R_A, R_B) = u_L(R_A, R_B)$. Hence we can apply any of the two payoff functions.

Proof. Let us assume that player i deviates in time step t_0. Let us assume that he applies the best option, hence he plays $br_i(R_{MIN})$. The Punisher strategy played by player j reduces the discounted total payoff of player i for the time interval from t_0 to $t_0 + \Upsilon_j^p$ if:

$$u_i^{cheat} + u_i^{def} \cdot \sum_{t=1}^{\Upsilon_j^p} \delta^t \leq u_i^{coop} \cdot \sum_{t=0}^{\Upsilon_j^p} \delta^t \tag{6.27}$$

If $\delta = 1$, we can write (6.27) as follows:

$$u_i^{cheat} + u_i^{def} \cdot \Upsilon_j^p \leq u_i^{coop} \cdot (\Upsilon_j^p + 1) \tag{6.28}$$

Hence, we obtain the following bound on the punishment interval:

$$\Upsilon_j^p \geq \frac{u_i^{cheat} - u_i^{coop}}{u_i^{coop} - u_i^{def}} \tag{6.29}$$

Note that if $\delta = 1$, then cooperation is always enforceable.

If $\delta < 1$, we can transform the sums in (6.27) to the same intervals:

$$u_i^{cheat} - u_i^{def} + u_i^{def} \cdot \sum_{t=0}^{\Upsilon_j^p} \delta^t \leq u_i^{coop} \cdot \sum_{t=0}^{\Upsilon_j^p} \delta^t \tag{6.30}$$

Since the sums are geometric sequences, we can write that:

$$u_i^{cheat} - u_i^{def} \leq (u_i^{coop} - u_i^{def}) \cdot \frac{1 - \delta^{\Upsilon_j^p + 1}}{1 - \delta} \tag{6.31}$$

If $\gamma_i < \frac{2}{\pi - 2}$, then $u_i^{coop} - u_i^{def} > 0$. Furthermore, we have $1 - \delta > 0$, and we can rewrite the inequality:

$$\frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) \leq 1 - \delta^{\Upsilon_j^p + 1} \tag{6.32}$$

Reordering the inequality gives us:

$$\delta^{\Upsilon_j^p + 1} \leq 1 - \frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) \tag{6.33}$$

This gives the condition on Υ_j^p, because the left side of (6.33) is strictly positive. Thus the inequality cannot be fulfilled if the right side of (6.33) is non-positive, meaning that:

$$\frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) \leq 1 \tag{6.34}$$

If the condition in (6.34) holds, we can take the logarithm of both sides in (6.33). Since $\delta < 1$, the logarithm function is strictly decreasing and hence the direction of the inequality changes.

$$\Upsilon_j^p \geq \log_\delta \left(1 - \frac{u_i^{cheat} - u_i^{def}}{u_i^{coop} - u_i^{def}} \cdot (1 - \delta) \right) - 1 \tag{6.35}$$

Due to the symmetric situation, the same arguments apply for the opposite case that defines the punishment interval for player j. $\qquad \square$

Note that for $\delta = 1$, cooperation can always be enforced using the Punisher strategy. This principle is expressed in general by the Nash folk theorem 1.4.4.

The typical value of Υ_j^p is small (for $d = 1km$, $\gamma_i = 0.1$, $R_{MAX} = 1.5km$ and $\delta = 0.1$, the value is $\Upsilon_j^p = \lceil 1.23 \rceil = 2$). For higher values of γ_i, R_{MAX} and δ, the punishment interval is one time step (i.e., there is an immediate punishment). Figure 6.8 illustrates the average per time step payoff of a player for both cooperation and defection. One can observe that cooperation is more beneficial, because defection is quickly retaliated by the other player.
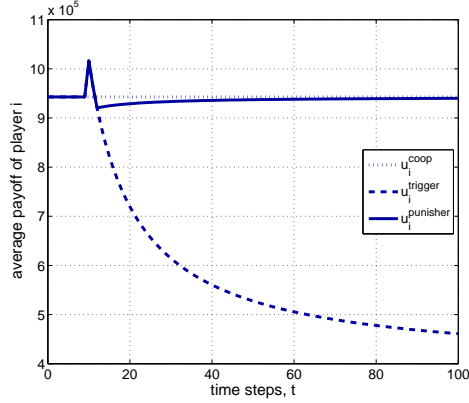
Figure 6.8: Average payoff of player i for $d = 1km$, $\gamma_i = 0.1$, $R_{MAX} = 1.5km$ and $\delta = 0.1$ if player j applies a punishment. One-time defection is quickly retaliated and hence cooperation is the best choice. The Trigger strategy stabilizes in infinite punishment, and the Punisher strategy returns to the cooperative state.

Based on Theorem 6.3, we state the following result.

Corollary 6.4. *If both players play the* Punisher *strategy and the conditions of Theorem 6.3 hold, then it results in a Nash equilibrium.*

6.5 NP-Completeness of the General Problem

In this section, we analyze the power control problem for general network topologies and for general values of radio ranges in the single stage game.

The goal of player i is to allocate the radio ranges such that their overall payoff $u_i = \sum_{k=1}^{|\mathcal{B}_i|} u_k$ is maximized, where $|\mathcal{B}_i|$ is the number of bases stations that belong to player i and the payoff per base station u_k is as follows (derived from (6.14)).

$$u_i = \sum_{k=1}^{|\mathcal{B}_i|} \left[(1 + \gamma_i) \cdot O_k - \gamma \cdot R_k^2 \cdot \pi \right] \tag{6.36}$$

where O_k is the useful coverage area and R_k is the radio range of base station k.

We can now formulate the following theorem.

Theorem 6.5. *Finding the maximum payoff of player i for general values of radio ranges is NP-complete.*

Proof. To prove the theorem, let us consider the special case of finding the optimal radio range allocation in the presence of a single operator. In this case, operator i has the payoff:

$$u_i = \sum_{k=1}^{|\mathcal{B}_i|} \left[(1 + \gamma_i) \cdot O_k - \gamma \cdot R_k^2 \cdot \pi \right] \tag{6.37}$$

Let us denote the whole service area by $O_i = \sum_{k=1}^{|\mathcal{B}_i|} O_k$. Since the γ_i values are the same for all base stations, we can reformulate the payoff as:

$$u_i = (1 + \gamma_i) \cdot O_i - \gamma_i \cdot \pi \sum_{k=1}^{|\mathcal{B}_i|} R_k^2 \tag{6.38}$$

Under the assumption that $\alpha = 2$, the power is proportional to the square of the radio range. Chamaret *et al.* [CJK$^+$97] as well as Värbrand and Yuan [VY03] have proven that finding the minimum power allocation in the network of a cellular operator while maintaining the total coverage is NP-complete. Hence, the minimum value of u_i cannot be determined in polynomial time. Because the problem is NP-complete for the special case of one operator, we conclude that it is NP-complete in the general game as well. □

Since finding the maximum payoff for an operator is NP-complete in general, it is impossible to calculate the best responses for a given player in polynomial time. Thus, we can state the following result.

Corollary 6.6. *Finding Nash equilibria in the power control game for general values of radio ranges is NP-complete.*

6.6   Related Work

Game theory is used to study the power control of user devices in wireless networks, notably in cellular systems as studied in [ABSA02, GM01, HBH06, JH98, MW01, MCPS05, XSC03] and [ZZHJ04]. Game theory is also used to study cooperation in wireless ad hoc networks, for example in [CGKO03, MQ05a] and [SNCR03], in particular for cooperative power control [MK05]. A general framework for resource allocation in wireless network is addressed in [DCS03].

Recently, the coexistence of multiple Internet Service Providers (ISPs) was studied by Shakkottai and Srikant in [SS05]. They consider both transit and customer prices for the ISPs. They show that if the number of ISPs competing for the same customers is large, then it can lead to price wars. In addition to this work in wired networks, the coexistence of wireless operators in a non-shared spectrum is addressed in two contributions. In [HHLM04], Halldórsson *et al.* study channel assignment strategies for Wi-Fi operators. They use the maximum graph coloring problem to identify Nash equilibria and they also provide a bound on the price of anarchy of these equilibria. In another paper [ZdV05], Zemlianov and de Veciana consider the scenario, in which users are able to choose between a cellular network and a Wi-Fi network. They show that congestion sensitive strategies are better than proximity-based strategies. None of these works considers the power control of the base stations.

Our work addresses the problem of pilot signal power control in shared spectrum networks. Haykin provides a comprehensive overview [Hay05] of the current tendencies and research challenges in shared spectrum communications in general. One of the challenges, namely opportunistic spectrum access, is addressed in the paper of Wang *et al.* [WLX03].

6.7   Summary

In this chapter, we investigated the problem of co-existing wireless operators in a shared spectrum. This scenario becomes more and more important with the expansion of wireless networks in a shared spectrum. Although we focused on a cellular network model, our study can be adapted to investigate spectrum sharing in networks using the WiFi or WiMax technologies as well. Our study highlighted a crucial problem in the coexistence of networks, specifically that operators have to compete for the common set of users. As a result of this competition, the network performance might change in an undesired way.

In our study, we assumed that the operators apply power control at the base stations to mitigate interference, while providing a permanent service to the users. Our contribution in this chapter is threefold. First, we showed that Nash equilibria exist if the operators set the power of their base stations at the beginning of the operation of the network. We identified different equilibrium situations that depend on the cooperativeness of the operators. Second, we proved a condition for which a socially optimal Nash equilibrium exists and that it can be enforced using punishments. Third, we showed that the solution of the power control problem is NP-complete for a general topology of base stations.

In general, our results showed which operation points are beneficial for the players and how these should be achieved. We demonstrated that reactive behavior based on repeated games could be applied to enforce a desirably low transmission power in future wireless scenarios where operators compete for users.

Publication: [FH06b]

Chapter 7

Border Games in Cellular Networks

7.1 Introduction

Today's cellular networks operate on separate frequency bands to avoid interference between them. The operators of these networks obtain an exclusive right to use a given frequency band in their respective country. However, the division based on frequency bands does not apply across national borders. The operators have to resolve their conflicts across the borders themselves. One of the issues is when mobile users of one operator attach to the network of the operator of the other country while still being in their own country. This problem is referred to as *accidental roaming* [INT06, Lee06]. Often, the operators make make mutual agreements to resolve these conflicts, but these agreements are difficult to enforce, because they require the mutual cooperation of the operators. The proper understanding of this special case is important for the future studies of the coexistence of cellular networks, because there exist many examples where cities reside close to a national border. One can find examples for the border scenario on each of the continents such as Geneva, Basel or Aachen in Europe; San Diego and Detroit in the USA; or Hongkong and Singapore in Asia.

In this chapter, we consider the problem of strategic behavior of operators on the border of their cellular networks. We consider the operators of 3G cellular networks, such as the *Universal Mobile Telecommunication System (UMTS)* for example, that are based on the *Code Division Multiple Access (CDMA)* technology [HT02, Rap02, Sch05]. Note however, that the problem we highlight in this chapter applies to any CDMA network. In these networks, the base stations emit pilot signals to help users to assess the available channel quality and to attach to the base station with the best offered quality. According to the current definition in the UMTS standard, the pilot power for the base stations is determined at the network dimensioning phase and remains fixed afterward. However, as the number of users changes, the operators may adjust the network parameters. This *slow adaptation* of the pilot signal power is part of the network re-dimensioning process and hence it exists on a large time scale. On the other hand, the technology enables the base stations to quickly adapt their pilot signals to the actual usage. This *fast adaptation* technique is commonly referred to as *cell breathing* [HT02, Rap02, Sch05].

In this work, we assume that the operators control the power of the pilot signal of their base stations to attract more users over time. Several methods (e.g., cell-breathing [Rap02, Sch05]) have been proposed to implement fast adaptation in CDMA networks. We survey them in Section 7.6. In our work, however, we focus on the slow adaptation problem. We study how the network operators can fine-tune their pilot power in the presence of other operators given a certain user distribution. We investigate whether this situation leads to a game and we study the properties of the equilibria of power control strategies.
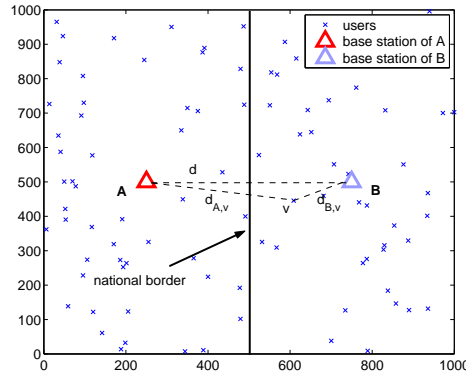
Figure 7.1: Network scenario with two base stations.

7.2 Model

7.2.1 System Model

We consider a scenario with two cellular network operators A and B. We assume that their networks are separated by a national *border*. The operators operate their network based on the principles of the CDMA method. We assume that the two operators acquired the same frequency band for their networks in their respective country. This means that their networks interfere along the border. We assume that each operator controls a set of *base stations (BS)* $\mathcal{B}_i$, where $i \in \{A, B\}$. We refer to the set of all base stations as $\mathcal{B} = \bigcup_i \mathcal{B}_i$. We also assume a set of *users* $\mathcal{M}$ equipped with *wireless devices* who access the communication network. For the sake of convenience, we assimilate the operators with their base stations and the users with their devices. In order to get an insight, we study the case in which each operator has one BS and we refer to the BSs by the letters of their operators (i.e., base station A and B). This single-cell model is often considered in the literature [Jak94, LZJH04]. The network scenario is shown in Figure 7.1.

We assume that the radios of the base stations and the mobile devices are compatible, meaning that any user is able to access the network via any of the base stations. We further assume that the antennas of the BSs and wireless devices are omnidirectional. Note that the results derived in this chapter are still valid if the operators use sectorized antennas that point towards the national border. Sectorized antennas have more impact in the general scenario, where the operators have several base stations each. The study of this general scenario is the main focus of our ongoing work.

Throughout this chapter, we assume that the users are not associated with any of the operators (i.e., they are roaming users) and thus they attach to the base station with the best signal quality.

In CDMA networks, power control is used to mitigate the near-far effect [Rap02], to optimize the transmission power of the devices and to reduce interference. In this chapter, we focus on the *downlink (or forward link) power control of the pilot signals* emitted by the base stations. The pilot signal helps the wireless devices to perform the following tasks:

- detection of the available base stations,

- synchronization with them and

- estimation of the channel quality and handover decision based on this estimation.

In particular, we focus on the problem of how the network operators can determine the pilot signal power that will potentially attract the highest number of users. We leave the study of the competitive fast adaptation problem as a future work.

In the remainder of this chapter, we present the physical model of CDMA. As mentioned earlier, the pilot signal is used to attract users. If several users attach to a given base station, their transmissions are performed on different *channels*. In CDMA-based cellular networks, unlike GSM networks, channels are not separated in different frequencies, but use different codes. Hence each transmission uses the same frequency band. In theory, the codes from one base station are orthogonal, meaning that the transmissions to different receivers do not interfere with each other. In practice, there exists some interference between concurrent transmissions from a given base station because of multipath propagation. This interference is called the *own-cell interference*. In addition, there is an interference caused by the transmissions of other base stations, called the *other-cell interference*.

Let us consider the scenario shown in Figure 7.1. According to the *physical model* of signal propagation in a CDMA system [TV05], we can write the *signal-to-interference-plus-noise ratio (SINR)* of the pilot signal of base station $i \in \mathcal{B}$ to user $v \in \mathcal{M}$ as:

$$SINR_{iv}^{pilot} = \frac{G_p^{pilot} \cdot P_i \cdot d_{iv}^{-\alpha}}{N_0 \cdot \mathbb{W} + I_{own}^{pilot} + I_{other}^{pilot}} \tag{7.1}$$

where $G_p^{pilot} = \frac{\mathbb{W}}{\mathbb{R}^{pilot}}$ is the *processing gain* for the pilot signal, $\mathbb{W}$ is the available bandwidth, $\mathbb{R}^{pilot}$ is the data rate of the pilot signal, P_i is the power of the transmitted pilot signal of BS i, d_{iv} is the distance between BS i and user v, α is the path loss exponent, N_0 is the noise spectral density, and I_{own}^{pilot} as well as I_{other}^{pilot} are the own-cell and the other-cell interferences that affect the pilot signal of BS i.

Let us first express the own-cell interference I_{own}^{pilot}:

$$I_{own}^{pilot} = \zeta \cdot d_{iv}^{-\alpha} \left(\sum_{w \in \mathcal{M}_i} T_{iw} \right) \tag{7.2}$$

where ζ is the *orthogonality factor* (also called the *own-cell interference factor*) that expresses the non-orthogonality between the different transmissions from BS i. Furthermore, $\mathcal{M}_i$ is the set of users at BS i and T_{iw} is the traffic power assigned to user $w \in \mathcal{M}_i$ by BS i.

Similarly, we can write the interference I_{other}^{pilot}:

$$I_{other}^{pilot} = \eta \cdot \sum_{j \neq i} d_{jv}^{-\alpha} \left(P_j + \sum_{w \in \mathcal{M}_j} T_{jw} \right) \tag{7.3}$$

where η is the *other-to-own-cell interference factor*, d_{jv} is the distance between BS j and user v. Furthermore P_j is the pilot signal power of BS j, whereas $\mathcal{M}_j$ is the set of users at BS j and T_{jw} is the traffic power assigned to user $w \in \mathcal{M}_j$ by BS j.

Similarly to (7.1), we can express the SINR for the traffic signal T_{iv}:

$$SINR_{iv}^{tr} = \frac{G_p^{tr} \cdot T_{iv} \cdot d_{iv}^{-\alpha}}{N_0 \cdot \mathbb{W} + I_{own}^{tr} + I_{other}^{tr}} \tag{7.4}$$

where $G_p^{tr} = \frac{\mathbb{W}}{\mathbb{R}^{tr}}$ is the *processing gain* for the traffic signal, $\mathbb{W}$ is the available bandwidth, $\mathbb{R}^{tr}$ is the data rate of the specific traffic signal, and I_{own}^{tr} as well as I_{other}^{tr} are the own-cell and the other-cell interferences that affect the traffic signal of BS i to user v.

Let us write the own-cell interference I_{own}^{tr} for the traffic signal as:

$$I_{own}^{tr} = \zeta \cdot d_{iv}^{-\alpha}(P_i + \sum_{w \neq v, w \in \mathcal{M}_i} T_{iw}) \tag{7.5}$$

and the interference from other BSs j as:

$$I_{other}^{tr} = I_{other}^{pilot} = \eta \cdot \sum_{j \neq i} d_{jv}^{-\alpha}(P_j + \sum_{w \in \mathcal{M}_j} T_{jw}) \tag{7.6}$$

Furthermore, we can express the *carrier-to-interference ratio (CIR)* as a function of SINR:

$$CIR_{iu}^{pilot} = \frac{SINR_{iv}^{pilot}}{G_p^{pilot}} \tag{7.7}$$

Similarly, we can write the CIR of the traffic signal:

$$CIR_{iv}^{tr} = \frac{SINR_{iv}^{tr}}{G_p^{tr}} \tag{7.8}$$

where G_p^{tr} is the processing gain for the traffic signal from BS i to user v.

In UMTS systems [HT02], the processing gain for the pilot signal is $G_p^{pilot} = 256 \approx 14.3$ dB and the available bandwidth (for the spread signal) is $\mathbb{W} = 3.84$ MHz. The processing gain of the traffic signal G_p^{tr} depends on the bitrate of the application running on the user device. In this work, we refer to different types of communication as the *traffic type*, namely audio ($\mathbb{R}^{tr} = 12.2$ kbps), video ($\mathbb{R}^{tr} = 144$ kbps) and data ($\mathbb{R}^{tr} = 384$ kbps) flows.[1] Accordingly, we distinguish different requirements for different traffic types as presented in [HT02]. We summarize these parameters in Table 7.1.

traffic type	required SINR	processing gain	required CIR
pilot	$\approx$ -6 dB	14.3 dB	-20 dB
audio, $\mathbb{R}^{tr} = 12.2$ kbps	5 dB	25 dB	-20 dB
video, $\mathbb{R}^{tr} = 144$ kbps	1.5 dB	14.3 dB	-12.8 dB
data, $\mathbb{R}^{tr} = 384$ kbps	1 dB	10 dB	-9 dB

Table 7.1: UMTS parameters [HT02].

In wireless networks, the authorities impose a transmission power limit to the devices. In UMTS networks, the base stations must emit their signal below 43 dBm = 20 W [HT02]. This limit is called the *downlink power budget*. In addition, this power budget must be split between the control channel signals, such as the pilot signal, and the traffic channel transmissions. The actual utilization of the power budget is called the *load* of the base station. As the load increases, the *bit-error-rate (BER)* at the user devices increases exponentially [HT02]. Hence, the BS load is typically kept such that the BER does not exceed a certain threshold, for example 10^{-3}. Here, we assume that the BS load is kept below 10 W.

In order to determine the average usage of the two networks, we developed a numerical simulator in MATLAB. We summarize the parameters of our simulation in Table 7.2. In each simulation run, we distribute the users according to the uniform distribution[2] and calculate the number of users that attach to each of the BSs based on the physical model developed in this section (i.e., using Equations (7.1)–(7.8) and the requirements shown in Table 7.1). We repeat this experiment several times for each power setting and we obtain the average number of users at each BS.

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[1] For simplicity, we consider only constant bitrate traffic.

[2] Note that we use a random uniform user distribution in our study, but our qualitative results hold for any user distribution.

Parameter	Value
simulation area size	1 km^2
BS positions	(250 m, 500 m) and
	(750 m, 500 m)
default distance between BSs, d	500 m
user distribution	random uniform
number of simulations	500
default path loss exponent, α	4
BS max power	43 dBm = 20 W
BS max load	40 dBm = 10 W
BS standard power, P^s	33 dBm = 2 W
BS min power	20 dBm = 0.1 W
power control step size, P_{step}	0.1 W
orthogonality factor, ζ	0.4
other-to-own-cell interference factor, η	0.4
user traffic types:	audio, $\mathbb{R}^{tr} = 12.2$ kbps
	video, $\mathbb{R}^{tr} = 144$ kbps
	data, $\mathbb{R}^{tr} = 384$ kbps
required CIR (audio, video, data):	-20 dB, -12.8 dB, -9 dB
expected incomes (θ_{audio}, θ_{video}, θ_{data}):	10, 20, 50 CHF/month

Table 7.2: Simulation parameters (based on [HT02]).

7.2.2 Power Control Game

We model competitive power control using game theory [FH06a, FT91, Gib92, OR94]. We define a two-player non-cooperative *power control game* **G** with the operators as *players*. In this game, the *strategies* of the operators determine the pilot transmission power of their base stations. Formally, we can write the strategy of operator i as the pilot signal power value of his BS:

$$s_i = P_i \tag{7.9}$$

where 0 W $< P_i <$ 10 W is the pilot signal power of BS i. According to the UMTS standard, the BSs transmit their pilot signal with approximately 33 dBm = 2 W. We denote this standard pilot power by P^s. We call the set of strategies of all players a *strategy profile* $s = \{s_1, s_2\}$.[3] In our game, the players have the same strategy set S.

The operators define their strategies in order to maximize their *expected payoff* u_i:

$$u_i = \sum_{v \in \mathcal{M}_i} \theta_v \tag{7.10}$$

where θ_v is the *expected income* obtained by serving user v of a certain traffic type. Suppose that each user has the same traffic type, for example audio. Then the expected payoff obtained at BS i is:

$$u_i = |\mathcal{M}_i| \cdot \theta_{audio} \tag{7.11}$$

---

[3]Note that one can easily extend the definitions in the power control game to several BSs and operators.

We further assume that the income[4] per user increases according to the data rate of the given service, thus $\theta_{audio} < \theta_{video} < \theta_{data}$. We obtain the expected income by performing several simulation runs with various pilot power settings as described in the previous section. This results in an *expected payoff matrix* for the two players. We apply the classic game-theoretic concepts on this payoff matrix. We express the payoffs of the players in *Swiss francs (CHF)* to emphasize the monetary advantage.

In order to get an insight into the strategic behavior of the operators, we apply the game-theoretic concepts of best response, Nash equilibrium and Pareto-optimality introduced in Sections 1.2.3 and 1.2.6.

We present our results using a symmetric scenario of the base stations and assuming that the users are uniformly distributed in the simulation area. Note that the result qualitatively hold for and base station placement and any user distribution. Naturally, in these cases, the Nash equilibrium strategies and payoffs are going to by asymmetric.

7.3   Is There a Power Control Game?

In this section, we study the behavior of the operators in a single-stage game. We first assume that one of the operators does not play and show that the other operator has an incentive to be strategic.[5] Second, we consider the case in which both operators have the possibility to adjust their pilot power and show that they are better off by doing so. We obtain our simulation results using the simulation environment described in Section 7.2.1.

7.3.1   Only Operator A is Strategic

First, we consider the case where only operator A is strategic and adjusts the pilot power of his BS to attract more users, whereas operator B operates his BS according to the standard pilot power of $P^s = 2$ W. To quantify the advantage of the strategic player, we define the concept of *normalized payoff difference* Δ_i.

Definition 7.1. *The* normalized payoff difference Δ_i *is the normalized difference between the maximum payoff of player i and his payoff using the standard power P^s assuming that the other player j uses P^s.*

$$\Delta_i = \frac{\max_{s_i}\left(u_i(s_i, P^s)\right) - u_i(P^s, P^s)}{u_i(P^s, P^s)} \tag{7.12}$$

Suppose that there are on average 10 users of the data traffic type in the simulation area. We show the payoffs of players A and B as a function of the pilot signal power P_A as well as the sum of their payoffs in Figure 7.2. Figure 7.2a shows these payoffs for $\alpha = 2$, whereas Figure 7.2b presents the same results for $\alpha = 4$. We observe that in both cases the operators are able to serve all users in the area using certain power values. If all users are served, then the game is a zero-sum game. In the zero-sum game, if player A adjusts his pilot power and obtains the increase of Δ_A, he causes the decrease of Δ_A in the payoff of the non-strategic player B. Furthermore, the payoff function of operator A has a unique maximum point. It is interesting to observe that the maximum payoff point requires a higher pilot power than $P^s = 2$ W. Because the two operators serve all the users in this case, the normalized payoff difference Δ_A of player A means the decrease of Δ_B in the payoff of the non-strategic player B. Hence, we conclude that operator A should be strategic and adjust his pilot signal. Note that we get qualitatively the same result for different user traffic types.

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[4]Note that the income is defined by the total amount of downloaded data, which can vary according to the length of communication sessions. If we change these income values, our results only change quantitatively, but not qualitatively.

[5]Due to symmetry, we only show the results for player A.
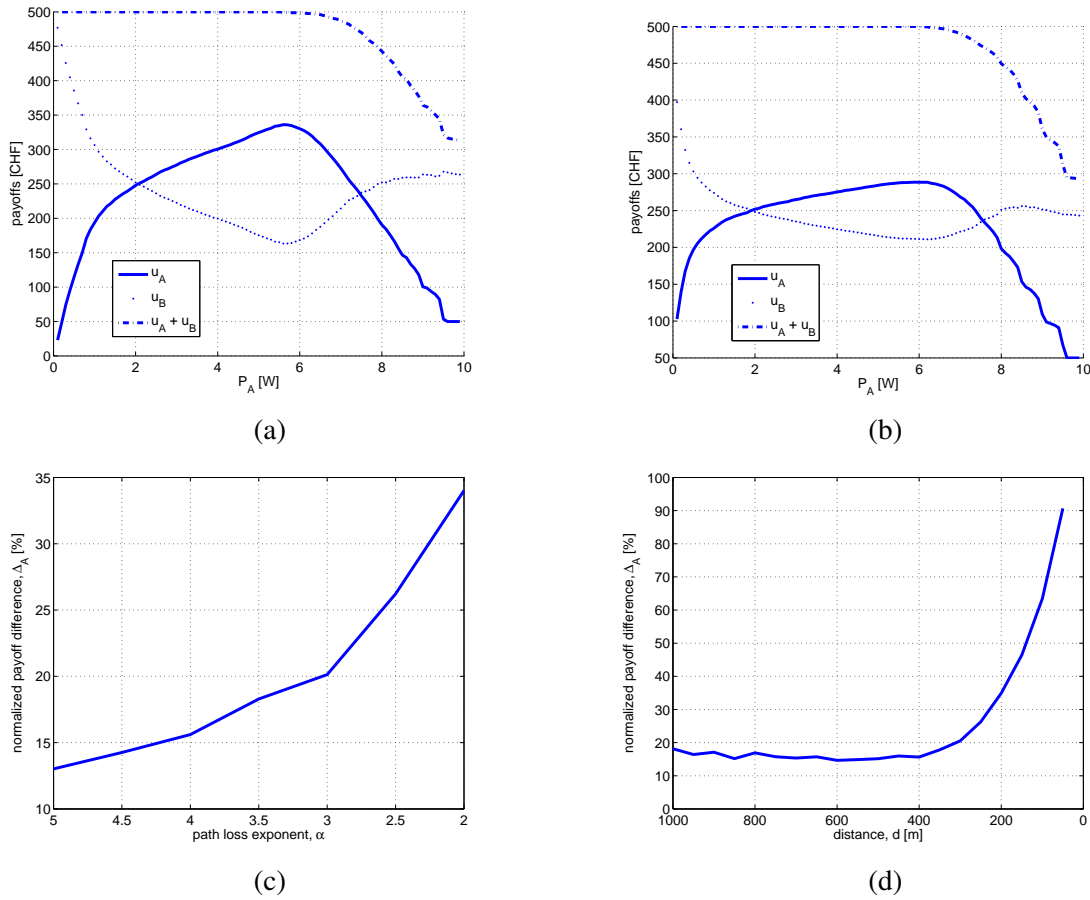
Figure 7.2: Payoffs of the players as a function of the pilot power of player A for 10 data users: (a) for $\alpha = 2$ and (b) for $\alpha = 4$. We also show the normalized payoff difference Δ_A as a function of (c) the path loss exponent α and (d) the distance d between the two BSs.

Figures 7.2a and Figure 7.2b show that the value of the normalized payoff difference Δ_A depends on the parameter α. We show this dependency in Figure 7.2c. One can observe that Δ_A increase as α decreases. The reason is that by low α values the pilot signals propagate easier giving a higher benefit to A if he uses higher pilot power. The value of Δ_A also depends on the distance d between the two BSs as shown in Figure 7.2d. As the distance decreases, Δ_A increases exponentially. The reason for this increase is the same as discussed before. In the remainder of the chapter, we choose the conservative default values $\alpha = 4$ and $d = 500$ m for the simulations. We will show that even with these conservative values, the players have an incentive to fine-tune their pilot powers.

7.3.2 Both Operators are Strategic

In the second set of simulations, we assume that both operators adjust their pilot power. We still consider 10 data users in the simulation area. We provide the payoff of player A as a function of his pilot power P_A in Figure 7.3a. We obtain different payoff curves as the pilot power of the other BS P_B increases.
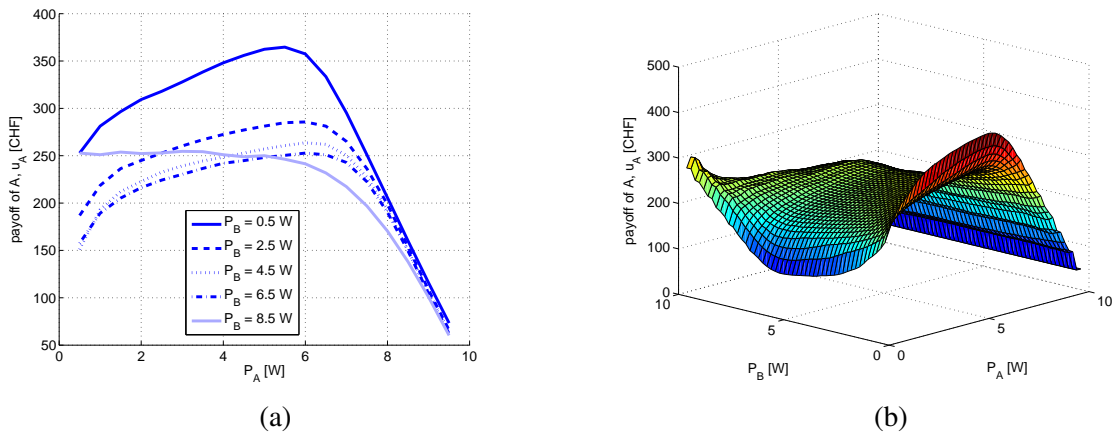
Figure 7.3: Payoff of player A as a function of his pilot power if there exist 10 data users.
Both operators are strategic, hence we present this payoff for various values of P_B in (a).
We show the complete payoff surface in (b).

We can observe that each of the payoff functions has a unique maximum point for P_A. Moreover, this maximum point depends on the pilot power of the other BS, P_B. For low values of P_B, the maximum payoff value decreases as P_B increases. In Figure 7.3b, we show the payoff surface for operator A as a function of the pilot power values of the two BSs.
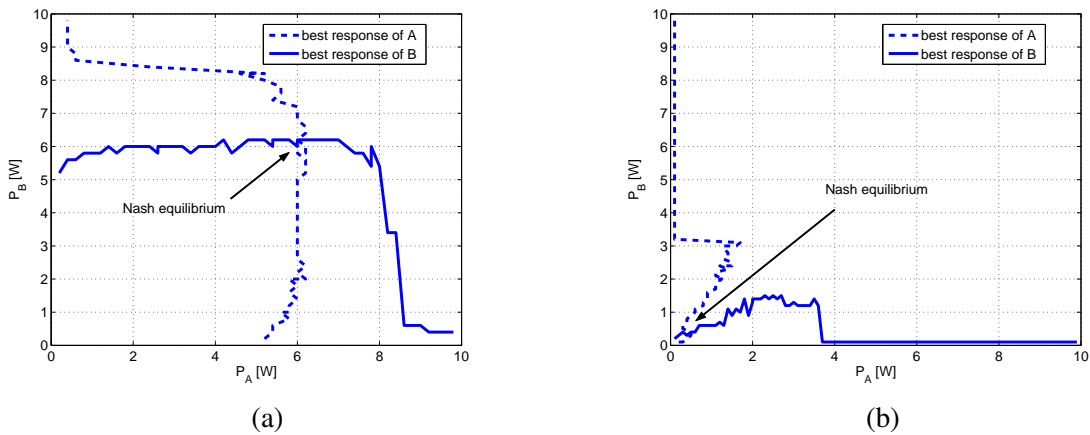
Figure 7.4: Best response functions for the two players with (a) 10 data users, (b) 100 data users.

Using the two payoff surfaces, we derive the best response functions (i.e., the set of maximum payoff points) for the operators as shown in Figure 7.4 for various user densities. Based on the concept of best responses introduced in Section 7.2.2, we can identify the Nash equilibria in the power control game as shown in Figures 7.4a for 10 data users and Figures 7.4b for 100 data users. We see that there exists a unique Nash equilibrium point defined as the crossing point of the two best response functions. Note that for 10 data users the Nash equilibrium strategy profile defines $P_A = P_B = 6$ W, which are higher than
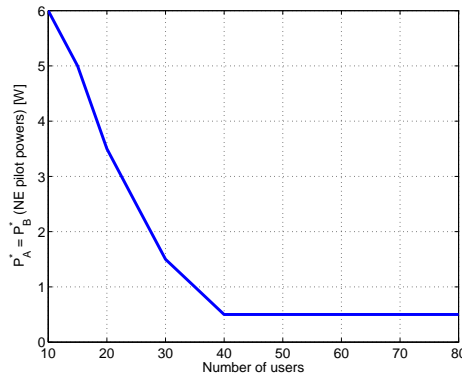
Figure 7.5: Nash equilibrium pilot power values as a function of the user density.

the standard pilot powers. For 100 data users the Nash equilibrium strategy profile defines $P_A = P_B = 0.5$ W. The reason is that the capacities of BSs saturate by using a relatively small power and hence there is no reason for them to go above these pilot power values.

Next, we study the pilot power values in the Nash equilibrium as a function of the number of users. We show the results in Figure 7.5. Due to symmetry in the user distributions, the Nash equilibrium pilot power is the same for both players. We observe that the Nash equilibrium pilot powers decreases as the number of users increases. For high user densities, the Nash equilibrium pilot powers stabilize at the value of 0.5 W.

In the following set of experiments, we study the *efficiency* of the system in a Nash equilibrium with respect to the case in which the players both use the standard power P^s. To this end, we investigate the *payoff region*, i.e. the payoff values for various pilot power levels. We identify the payoffs corresponding to the Nash equilibrium, the standard pilot power setting using P^s and the payoffs that correspond to Pareto-optimal strategy profiles. In particular, we can define the *Pareto frontier* as the set of Pareto-optimal payoff points. In our case, the Pareto-optimal payoff points characterize the system-efficient solutions.

Figure 7.6a shows the achieved payoffs as a function of the pilot power values P_A and P_B for 10 data users. We observe that in this case the Pareto frontier defines a straight line, because in a Pareto-optimal strategy profile each user in the system is attached to one of the BSs. Furthermore, the standard pilot powers and the Nash equilibrium strategy profile result in the same payoffs for the players and in addition they both lie on the Pareto frontier. This means that the players achieve a desirable state from the system point of view. Recall, however, that in this case the Nash equilibrium strategy profile requires higher pilot powers than the standard setting.

We present the payoffs for 100 data users in Figure 7.6b. In this case the Pareto-optimal points do not form a straight line anymore, because some users cannot be served. Another observation is that the Nash equilibrium is still close to Pareto-optimality, but the standard solution becomes very inefficient.

Following the previous experiment, we formally express the efficiency of the standard and the Nash equilibrium solutions compared to the best Pareto-optimal point (i.e., the Pareto-optimal strategy profile in which the sum of the payoffs for the two players is maximized). To this end, let us define the following two concepts:

Definition 7.2. *The* price of anarchy *[KP99] is the ratio between the total payoff achieved by the two*

(a)   (b)
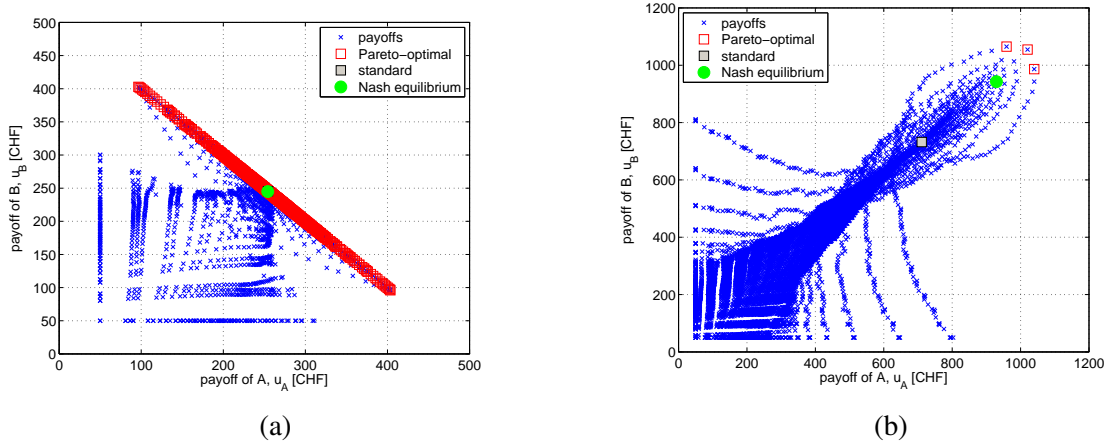
Figure 7.6: The payoff region with all possible payoffs for (a) 10 data users and (b) 100 data users. We highlight the Nash equilibrium, the payoff of the standard powers and all Pareto-optimal points.

*players in the best Pareto-optimal point and in the Nash equilibrium.*

Definition 7.3. *The* price of conformance *is the ratio between the total payoff achieved by the two players in the best Pareto-optimal point and when using the standard pilot powers P^s (i.e., being non-strategic).*

We perform a set of experiments to measure these values for increasing user densities. Figure 7.7 presents the price of anarchy and the price of conformance as a function of the user density assuming they have data traffic. We see that both prices increase as the number of users increases. As we have seen in Figure 7.6a, both the standard payoff point and the Nash equilibrium achieves Pareto-optimality if there is a small number of users. Hence, the two prices are very close to one. As the user density increases, we observe that both prices increase and then stabilize around a constant value. Note, however, that the price of anarchy stabilizes close to one, whereas the price of conformance stabilizes around 1.4. This shows that for a high number of users, the players can achieve a higher payoff if both of them are strategic.

7.4   Convergence to a Nash Equilibrium

We have seen in the previous section that the expected payoff function for a certain player is continuous and has a unique maximum point. In this section, we propose a distributed algorithm to achieve the Nash equilibrium in a given scenario.

The algorithm is similar to the better-response dynamics [FM01], i.e., where each player tries to improve his payoff in each step. They continue to increase their pilot powers, until they pass over the maximum payoff point and then they change to pilot power decrease. The players stop the optimization if they reach their maximum payoff again. Since the two players might change their pilot powers in the same optimization step, they define the payoff curve for the other player (i.e., as presented in Figure 7.3a). Thus, the payoff of player i that stopped optimizing might change due to the strategic behavior of the other player j. If this is the case, player i continue the optimization procedure. Due to the above properties, the convergence algorithm might oscillate around the maximum payoff points. To resolve this potential instability, we include a probability with which the players update their pilot powers in
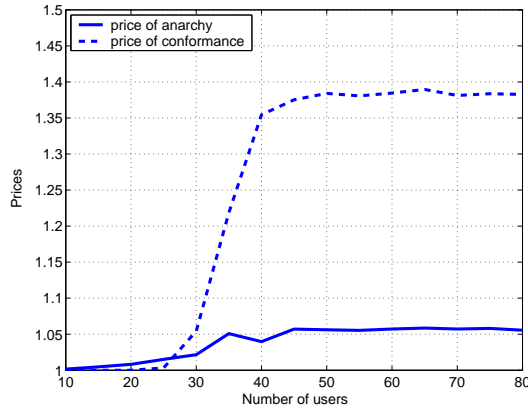
Figure 7.7: The price of anarchy and the price of conformance as a function of the user density.

each step and denote it by q. This probability ensures that sequential moves appear in the distributed optimization. We provide the pseudo-code for this procedure as shown in Algorithm 7.1.

Algorithm 7.1 Distributed convergence algorithm to achieve the NE

1: **for all** player i **do**
2:     set pilot power $P_i = 0.1W$
3:     set the direction of optimization $dir_i = +1$
4: **end for**
5: set power control step size $P_{step} = 0.1W$
6: **while** $\exists$ BS that optimizes **do**
7:     **for** all player i **do**
8:         **if** u_i changed compared to previous step **then**
9:             CONTINUE the optimization for player i
10:         **end if**
11:         update P_i with a probability $0 < q < 1$
12:         $P_i = P_i + dir_i \cdot P_{step}$
13:         **if** u_i decreased **then**
14:             {the optimization passed the maximum payoff value}
15:             $dir_i = -dir_i$
16:         **end if**
17:         **if** reached the previously passed maximum payoff point again **then**
18:             STOP optimization
19:   **end if**
20:     **end for**
21: **end while**

Figure 7.8 shows the evolution of the pilot power values applying Algorithm 7.1. We observe that the pilot power values follow the linear increase defined in the algorithm. After reaching the Nash equilibrium pilot power values, the algorithm stabilizes after certain steps.

Figure 7.9 shows the evolution of the payoffs during the convergence process. We see that the algorithm deviates from the Nash equilibrium payoffs while the pilot powers increase. As soon as the pilot powers reach the Nash equilibrium strategies, the payoffs remain close to the Nash equilibrium payoffs as well.

Figure 7.10 shows the convergence path to reach a Nash equilibrium. Because the Nash equilibrium
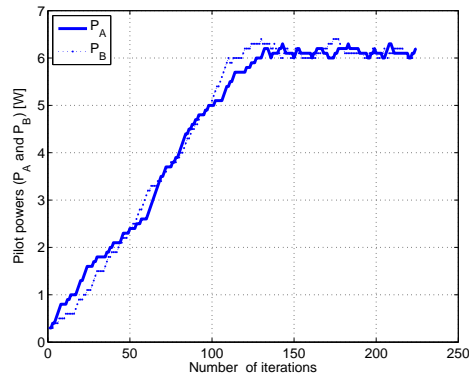
Figure 7.8: The evolution of the pilot power values using Algorithm 7.1.
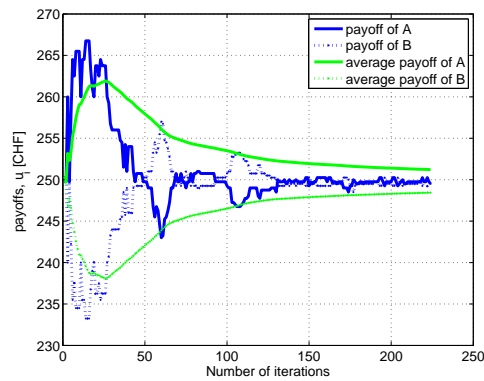

Figure 7.9: The evolution of the payoffs using Algorithm 7.1.

payoffs are almost equivalent to the starting payoffs, one might ask why the players start the optimization at all. We can observe from Figure 7.10, however, that the players reach higher payoffs during the optimization procedure. Since for low user densities, the game is a zero-sum game, the increase of u_i means the decrease of u_j. Player j adjusts his pilot powers as soon as he becomes aware of this loss. Hence, the payoffs equalize again.

7.5   Power Control Game with Power Cost

We have seen that the operators are able to serve all users in the area if the user density is low. We observe, however, that the Nash equilibrium pilot powers are higher than the standard value. Recall that the payoff function defined in (7.10) does not include the possible cost due to the operation with high pilot power. Let us now extend the expected payoff function defined in (7.10) to capture this important aspect of the power control game. We introduce two cost values for each player. The first cost denoted by C_i^{op} shows the *operating cost* of a BS i. This includes the aging of devices and hence the maintenance costs. The other cost, C_i^{subj}, expresses the *subjective cost* of player i. This covers every other aspect such
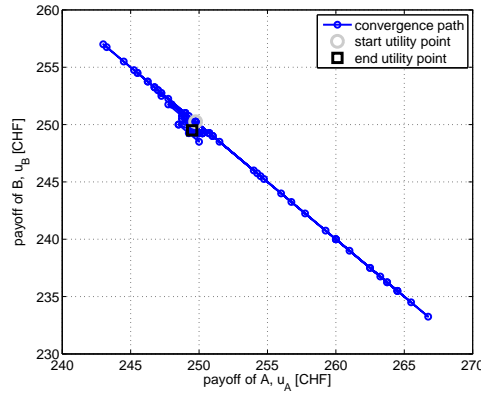
Figure 7.10: Convergence path of an example simulation run using Algorithm 7.1.

as the risk of lawsuits or potential bad reputation due to high emission power. Without loss of generality, we assume that these cost are an increasing function of the downlink transmission power of the base stations.

According to the above description, we can extend the notion of expected payoff as:

$$u_i = (\sum_{v \in \mathcal{M}_i} \theta_v) - C_i^{op} - C_i^{subj} \tag{7.13}$$

We define a non-cooperative power control game with the new expected payoff function introduced in (7.13) and denote it by $\hat{\mathbf{G}}$. We assume that the players are able to calculate the Nash equilibrium of the original game G with no power cost. Hence we define the strategy in the extended game $\hat{\mathbf{G}}$ as the choice between the standard and the Nash equilibrium strategies. Formally, we can write the strategies in $\hat{\mathbf{G}}$ as:

$$s_i = \{P_i^*, P^s\} \tag{7.14}$$

Let us call U the expected payoff that the players obtain by serving half of the total number of users. As we have seen in Section 7.3.2, if they play the Nash equilibrium strategy profile by low user densities, then it requires a higher pilot power from each operator. Without loss of generality, we denote by C^* the additional cost imposed by the Nash equilibrium compared to the standard pilot power setting P^s. The cost C^* includes both the operating and the subjective costs. Recall that we defined the normalized payoff difference Δ_A in Section 7.3.1. Due to symmetry $\Delta_A = \Delta_B$ and we denote it by Δ. In the extended game $\hat{\mathbf{G}}$, we assume that the normalized payoff difference is higher than the corresponding cost of using higher pilot power, thus $\Delta > C^*$.

We present the payoff matrix of the game $\hat{\mathbf{G}}$ in Table 7.3. In each payoff pair, the first payoff belongs to player A, whereas the second to player B.

To emphasize the structure of the payoff matrix, let us substitute the values $U = 3$, $\Delta = 2$ and $C^* = 1$. Substituting these values in Table 7.3, we obtain Table 7.4. From the payoff matrix, one can realize that the game $\hat{\mathbf{G}}$ is equivalent to the well-known Prisoner's Dilemma [FT91, Gib92, OR94]. Analogously, the strategy P^s corresponds to cooperation, whereas the strategy P_i^* corresponds to defection. This means that in the Nash equilibrium, each player uses high power and the resulting payoffs are lower than if both had complied.

Player B

		P^s	P_B^*
Player A	P^s	U,U	$U - \Delta, U + \Delta - C^*$
	P_A^*	$U + \Delta - C^*, U - \Delta$	$U - C^*, U - C^*$

Table 7.3: Payoff matrix of the game $\hat{\mathbf{G}}$.

Player B

		P^s	P_B^*
Player A	P^s	3,3	1,4
	P_A^*	4,1	2,2

Table 7.4: The extended power control game $\hat{\mathbf{G}}$ corresponds to the Prisoner's Dilemma.

7.6   Related Work

Power control has been extensively studied in the context of cellular networking. Baccelli *et al.* [BBT03] consider downlink power allocation and admission control in CDMA networks relying on stochastic geometry. Hanly and Tse [HT99] as well as Catrein *et al.* [CIM04] consider power control and capacity in CDMA networks. There is a very little literature about pilot power optimization, though [KCL99, VY03].

Game theory is used to study the power control of user devices in wireless networks, notably in cellular systems as studied in [ABSA02, AFB+05, GM01, HBH06, JH98, MW01, MCPS05, LMS02, XSC03] and [ZZHJ04]. A general framework for resource allocation in wireless network is addressed in [DCS03].

Recently, the coexistence of multiple Internet Service Providers (ISPs) was studied by Shakkottai and Srikant in [SS05]. They consider both transit and customer prices for the ISPs. They show that if the number of ISPs competing for the same customers is large, then it can lead to price wars. In another paper [SAK06], Shakkottai *et al.* consider the problem of non-cooperative multi-homing in WLANs. Zemlianov and de Veciana study a scenario in [ZdV05], in which users are able to choose between a cellular network and a Wi-Fi network. They show that congestion sensitive strategies are better than proximity-based strategies. Félegyházi and Hubaux [FH06b] consider the competition between different operators in terms of pilot power control of their base stations. They show that in the pilot power control game a socially desirable Nash equilibrium exists and that it can be enforced by punishments.

7.7   Summary

In this chapter, we studied the problem of competitive pilot power control in two CDMA networks that reside on two sides of a national border. We were motivated by many real-life complaints from cellular network users. This problem is especially significant in cities such as Geneva, where the French network attracts users on the Swiss territory.

We investigated whether the operators of these networks have an incentive to adjust their pilot signal powers. To get an insight into the problem, we considered the single-cell case with two base stations. Initially, we assumed that only one operator can adjust the pilot signal power of his base station. We showed that he has an incentive to behave strategically and quantified the effect of various parameters on the increase of his payoff. We further showed that when the user density is low and if both operators

behave strategically, then their payoffs are similar whether they adjust their pilot powers or not. We recognized that the two solutions require different pilot powers. If the user density is high, then the Nash equilibrium is more efficient than using the standard pilot powers, which suggests that the operators again have an incentive to be strategic. Finally, we extended the payoff function to include the cost of using high pilot powers. The game with power costs corresponds to the well-known Prisoner's Dilemma: The players still have an incentive to adjust their pilot powers, but their strategic behavior leads to a sub-optimal Nash equilibrium.

Border games represent an interesting and rarely advertised problem in cellular networks. Several real-life examples show that this problem is a real burden to users who live near national borders. From our private communication with cellular operators, our impression is that they do not pay enough attention to the problem, mostly for financial reasons or because they indeed benefit from the accidental roaming of the users. In this chapter, we uncovered the fundamental mechanisms that drive the operators towards this behavior. The goal of our work is twofold: we want to raise the awareness of the users and to help the operators to counter this problem by showing them the appropriate game-theoretic tools.

Publication: [FCDH07]

Conclusion

Wireless networks provide services that have become crucial to our everyday life. The recent evolution of wireless networks points towards decentralized wireless access: On the users' side, devices have become more sophisticated and programmable than before; on the operators' side, wireless infrastructure devices (such as access points) have become affordable and easily manageable. This ease of programming for both users and operators opens the door to selfish behavior: (i) the users can selfishly modify the original programs of their devices in order to exploit the available network services and (ii) prospective wireless operators can easily deploy their networks and compete with existing large operators.

In this thesis, our objective is to assess the effect of selfish behavior on the efficiency of wireless networks. We studied this effect in a wide range of wireless networks and for a set of important problems. In our study, we relied on the framework of non-cooperative game theory. Our analysis characterized the vulnerability of wireless networks to selfish behavior and enabled us to suggest game-theoretic techniques to counteract.

In the first part of the thesis, we introduced game theory with examples tailored to wireless network engineers. This comprehensive summary is important, because, at the time of this writing, virtually all of the available textbooks about game theory are written for the practitioners of other disciplines, e.g. for economists or political scientists. Our game theory tutorial enables wireless engineers to get a flavor of incentive design and hence educates them to take this important aspect into account at the design of wireless communication protocols.

In the second part of the thesis, we made several contributions studying the efficiency of wireless networks with selfish users. First, we formalized the channel allocation problem in competitive wireless networks using a non-cooperative, single-stage game. Based on this model, we showed that a Nash equilibrium channel allocation achieves load balancing over the set of available channels. Another contribution was the detailed study of the efficiency and fairness properties of the Nash equilibria relying on the widely-known concept of the price-of-anarchy and max-min fairness, respectively. We pursued our analysis to identify a subset of Nash equilibria that are also resistant against a coalition of selfish users. Finally, we designed two convergence algorithms to achieve the load-balancing Nash equilibria and proved their convergence theoretically or by simulations. The results show that efficient channel allocation can be reached even if the devices are selfish. This is particulary encouraging because the channel allocation model can be easily extended to model upcoming mesh and cognitive radio networks.

Second, we studied whether incentives in packet forwarding are needed to encourage participants in an ad hoc networks. Unlike existing work, we provided a formal model for static networks that takes the topology into account. We derived a cooperative solution from this model, but our results showed that the likelihood that spontaneous cooperation exists in general is very small. Thus, we concluded that external incentives are required to maintain cooperative packet forwarding in ad hoc networks. Extending our previous results, we studied the effect of mobility on cooperation. We concluded that mobility promotes cooperation, although some generosity of the participants is always required to bootstrap the network

operation.

The recent evolution of social networks on the Internet are driven by the users. We believe that a similar evolution might happen in the wireless domain. In fact, social community networks based on WiFi are already extending to provide wireless access. Our results show that selfish users degrade the performance of such social wireless networks, but this undesired effect can be countered by appropriate behavior of the other users in repeated interactions. Our results apply to user operated networks such as community mesh networks or mobile personal area networks.

In the third part of the thesis, we contributed to ongoing research in wireless sensor networks and shared spectrum communication. In this part, we focused on wireless networks run by non-cooperative operators. First, we proposed a game-theoretic model to investigate the potential of cooperation in a joint packet forwarding and power control problem. Our results showed that the benefits of energy saving encourages the sensor network operators to cooperate. The advantage of cooperation is twofold: (a) the authorities can largely benefit by providing service of their sinks for other's sensor networks and (b) if sinks are common resources, then cooperative packet forwarding is beneficial for sparse networks or in hostile environments. These results show viable methods to improve the performance of sensor networks using cooperation.

Second, we envisioned a scenario where users can freely roam across cellular networks operated in a shared spectrum and considered the competitive power control of these networks. We provided a formal model of a single-stage game for this scenario and identified the Nash equilibria depending on the sensitivity of networks operators to interference. In a repeated game, we showed that a socially desirable Nash equilibrium (i.e., the one that produces the least interference) can be enforced and we presented a strategy to achieve it. Although we focus on cellular networks in our model, our results show guidelines for the coexistence of other wireless networks, such as WiFi networks.

Finally, we turned our attention to existing 3G cellular networks and highlighted an important short-coming, namely the existence of cross-border interference. We were motivated by many real-life complaints of the cellular network users. We developed a game-theoretic model of pilot power control and showed that the operators have an incentive to behave strategically at their borders, i.e., to adjust the transmit power of their pilot signals. We studied the existence and properties of Nash equilibria in an empirical model and showed their efficiency for different user densities. We provided a distributed convergence algorithm to achieve the identified Nash equilibria and characterized its convergence properties. We also extended the payoff function to include the cost of using high pilot powers. The game with power costs corresponds to the well-known Prisoner's Dilemma: The players still have an incentive to adjust their pilot powers, but their strategic behavior leads to a sub-optimal Nash equilibrium. This implies that cellular operators should carefully agree about their pilot power control on national borders.

Competition between wireless operators increases with the emergence of new wireless networks. We studied the coexistence of wireless operators in existing and prospective wireless networks. Our results demonstrate that the strategic behavior of wireless operators can result in a in undesired operation of their networks (e.g., by using high transmission powers). But, we have shown appropriate coordination techniques that can mitigate the effect of selfish behavior. We have shown that by using distributed coordination relying on game theory, the wireless operators can avoid the undesired network operation. These results give guidelines on how to design incentive-aware protocols in future wireless networks with heterogeneous technologies and operators.

Future Research Directions

In this thesis, our main focus was to study incentives for cooperative behavior in wireless networks. In existing works, various incentive mechanisms were proposed to promote cooperation, for example in packet forwarding in ad hoc networks, but their need was never formally justified. Based on the second part of the thesis, in the future we will focus on designing incentives in prospective wireless networking technologies, notably in cognitive radio. We believe that incentives for cooperation in cognitive radio networks are essential, because cognitive radio devices are strategic by definition.

Pervasive wireless networking is a vision shared by many researchers. To achieve this goal, wireless networks have to coexist such that users can use their services regardless of time and location. Based on the third part of the thesis, we will further study the coexistence of wireless networks, notably in unlicensed spectrum. We will pay particular attention to the competition between unlicensed network operators providing best-effort services (e.g., WiFi networks) and licensed network operators with guaranteed quality-of-service (e.g., cellular networks). We believe that competition between various network operators will further increase as more and more networking services based on pervasive data access are available.

We are also interested in incentive problems in the area of security and privacy. Existing examples show that current security protocols to protect systems and the privacy of users do not work properly. Most of the time, the basic security protocols are correct, but there are inappropriate or there are not any incentives to apply them correctly. Hence, a promising research area is the design of incentive schemes that facilitate the appropriate deployment of security and privacy protocols.

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Index

MÁRK FÉLEGYHÁZI

Research and teaching assistant

Laboratory for computer Communications and Applications (LCA)

School of Computer and Communication Sciences (IC)

EPFL (Ecole Polytechnique Fédérale de Lausanne), Switzerland

Station 14

CH-1015 Lausanne, Switzerland

e-mail: mark.felegyhazi@epfl.ch

url: http://people.epfl.ch/mark.felegyhazi

phone: +41 21 6936486

PERSONAL

Born in Budapest, Hungary on May 10, 1978. Citizen of Hungary.

Languages: Hungarian - native, English - fluent, German - fluent, French - intermediate

RESEARCH

My research focuses on selfish behavior in wireless networks. During my PhD work at EPFL, I have analyzed the incentives for cooperation in various wireless networks using game theory and have shown which of them require external incentives to enforce cooperative behavior. My previous work at Ericsson Research included Mobile IP protocol testbed implementation and the design of ad hoc network protocols for the Bluetooth short-range wireless technology.

EDUCATION

PhD. student in communication systems, *Sep. 2002 – present*

EPFL, SWITZERLAND

    thesis title: *Non-cooperative Behavior in Wireless Networks*

    advisor: Prof. Jean-Pierre Hubaux

    expected graduation: April 2007

Doctoral school in communication systems, *Oct. 2001 – Jul. 2002*

EPFL, SWITZERLAND

Diploma (Dipl. Ing.) in electrical engineering, *Sep. 1996 – Jun. 2001*

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS (BME), HUNGARY

    thesis title: *Development and Evaluation of a Dynamic Bluetooth Scatternet Formation Procedure*

    advisors: Dr. György Miklós and Dr. András Valkó (Ericsson Research, Hungary)

    major in telecommunication, minor in mobile communication

    Hungarian and German certificate (from UNIVERSITÄT KARLSRUHE (TH), GERMANY)

PROFESSIONAL EXPERIENCE

Associate member, *2006 – present*

LABORATORY OF CRYPTOGRAPHY AND SYSTEM SECURITY (CRYSYS LAB.)

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS (BME), HUNGARY

Research and teaching assistant, *Sep. 2002 – present*

SCHOOL OF COMPUTER AND COMMUNICATION SCIENCES (IC)

EPFL, SWITZERLAND

Trainee, *1999 – 2001*

ERICSSON RESEARCH, HUNGARY

TEACHING

Teaching assistant

Mobile Networks, EPFL – 2005, 2007
Self-Organized Mobile Networks, EPFL – 2003, 2004, 2005, 2006
Informatics I-II - C++ programming, EPFL – 2004, 2005
Object Oriented Programming - JAVA, EPFL – 2002

Supervised projects

Julien Freudiger, *Anonimity in Vehicular Networks*, course, 2007
Christophe Waber, *Channel Assignment in Mesh Networks*, master, 2007
Diego Dufour, *Border Games in Cellular Networks*, semester, 2006
Sivan Altinakar, *Cellular Operators in a Shared Spectrum*, master, 2006
Shirin Saeedi, *Multi-radio Channel Allocation in Competitive Wireless Networks*, course, 2005
Megha Goel, *A Routing Architecture for Wireless Messaging Ad Hoc Networks*, internship, 2004
Martin Klauser, *Connection Establishment Models in Wireless Ad Hoc Networks*, semester, 2003

PROFESSIONAL ACTIVITIES

Reviewer for scientific journals

ACM/Kluwer Mobile Networks and Applications (MONET), IEEE Transactions on Wireless
Communications (TWC), IEEE Transactions on Mobile Communications (TMC),
Wiley Transactions on Communications, IEEE/ACM Transactions on Networking (ToN)

Reviewer for scientific conferences, workshops

ACM Mobicom, ACM Mobihoc, IEEE Infocom, IEEE WiOpt, ACM WiSe, IEEE DySPAN, ESAS

AWARDS

Selected poster at Mobicom 2005, in the Student Research Competition (best 7 posters)
EPFL doctoral school fellowship, Oct. 2001 – Jul. 2002
3rd price at the Students' Scientific Conference, Budapest, Oct. 2000
2nd price at the Students' Scientific Conference, Budapest, Oct. 1999
Fellowship of Universität Karlsruhe (TH), Sep. 1998 – Feb. 1999
Fellowship of the Budapest University of Technology and Economics (BME), 1996 – 2001

REFERENCES

Prof. Jean-Pierre Hubaux, Full Professor,
EPFL, Switzerland
Room BC 207, +41 21 6932627, jean-pierre.hubaux@epfl.ch

Prof. Levente Buttyán, Associate Professor,
BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS (BME), Hungary
Room IE 419, +36 1 4631803, buttyan@hit.bme.hu

Prof. Peter Marbach, Associate Professor,
UNIVERSITY OF TORONTO, Canada
Room BA 5232, +1 416 9781680, marbach@cs.toronto.edu

Prof. Srdjan Čapkun, Assistant Professor,
ETHZ, Switzerland
IFW C 41.2, +41 44 632 7190, capkuns@inf.ethz.ch

Dr. András Valkó, Manager,
TRAFFIC LAB, ERICSSON RESEARCH, Hungary
+36 1 437 71 00 (central number), andras.valko@ericsson.com

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15. Gy. Miklós and M. Félegyházi, **Traffic Dependent Bluetooth Scatternet Optimization Procedure**, US Patent Nr.: 09/666529, Jul. 2000.

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