Deformable Surfaces using Physically-Based Particle Systems

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Abstract

This paper presents a simple method for creating and animating deformable surfaces using physically-based modeling. We represent a deformable surface as particle systems. Simple methods, such as, analytical solutions of the motion equations, discretization and time-step method, are used to solve the simulation equations. So, the surfaces have better stability and much faster speed of calculation. This method can be combined with the surfaces of arbitrary topology and the results are as realistic as that of continuous systems. The surfaces can be elastic, elastoplastic or plastic. This method can be widely and easily used in all kinds of deformable surfaces with good results.

Keywords: Deformable Surface, Physically Based, Particle Systems.

1. Introduction

The use of physically-based models for animating deformable surfaces is an important field in computer graphics. There are two successful approaches to this problem: particle systems and continuous systems.

Particle systems consist of a large number of single particle, all of which move under the influence of the forces, such as gravity and external forces, etc.. The particle systems [1, 2] were originally used to model fire, fireworks, waterfall, ocean, spray trees, grass and other nature phenomena. More recently, particle systems have been used to model deformable surfaces. An interesting work in this area is the description of cloth draping behavior by D. E. Breen [3, 4]. In this field, the interacting-particle methods have been used to develop a theoretical model of woven cloth and predicate the drape of woven cloth, such as cotton wool and etc.. T. L. Hilton and P. K. Egbert [8] also propose the use of vector fields for 3D particle systems. R. Szeliski and D. Tonnesen [10] define a model of surfaces with oriented particle systems which are a function of direction. The surface modeling can be used to split, join, or extend deformable surfaces without needing manual intervention and knowing the topology of the surfaces. The mass-spring systems are essentially particle systems with a fixed topology, each particle of which is connected with a finite number of neighboring particles by spring forces. The method is easy to be used, but the major problem of traditional particle systems is that the results are not as realistic as that of continuous systems. Another problem of particle-systems surface is that it is harder to achieve exact control over the shape of the surfaces[10]. Witkin and Heckbert [11] also proposed the use of particles systems to sample and control implicit surfaces.

The typical continuous systems of physically-based modeling is the elastically deformable models developed by Terzopoulos et al. This method has been successfully used in the animation of cloth [5] and other kinds of elastic surfaces [9]. The basic equation of this model can be written in Lagrange's form. For the convenience of discussion, we extract a part as following [6]:

$$\frac{\partial}{\partial t}\left(\mu \frac{\partial r}{\partial t}\right) + \gamma \frac{\partial r}{\partial t} + \frac{\delta \varepsilon(r)}{\delta r} = f(r,t) \qquad (1)$$

where, **r(a,t)** is the position of the particle **a** at time **t**. **m** is the mass density of the body at **a**. γ is the damping density, and **f(r,t)** represents the net externally applied forces. ε(r) is a functional which measures the net instantaneous potential energy of the elastic deformation of the body.

In order to solve these equations, discretization and the numerical solution of the equations, such as finite difference discretization techniques, are used. The discretization seems to change the continuous system into the mass-spring systems. In fact, by choosing certain types of non-linear springs, a mass-spring system can be made equivalent to a continuous model. But, the stability is one of the major problems of continuous systems, although adaptive time-step control can improve the stability [6]. The increased complexity is another problem, especially during the design and development of systems. As a result, the speed of the calculation is limited. All of these problems limit the application of this approach. For example, it is difficult to create and animate a complex character. On the other hand, there are also some problems to split, join or extend the deformable surfaces of continuous systems.

The present paper proposes the use of particle systems and a series of simple methods for creating and animating a deformable surface which has similar result as the continuous systems, but with a better stability and much faster speed of calculation. The shape of the surfaces is easily controlled.

2. The Equations and Solutions of the Deformable Surface

We assume that the deformable surface consists of many particles. When the particles move away from their rest positions in any direction under the external forces, the elastic forces will exert on the particles to pull them towards their original position. At the same time, the damping forces which are functions of the velocity also exert on the particles of the deformable surfaces.

We can obtain the motion equations of the particles on the surface from a general form of Lagrange's equation for a system of particles.

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i \qquad \boldsymbol{i} = 1,2,...n \qquad (2)$$

where, **n** is the degree of freedom for a system, $q_1, \ldots, q_n$ are the generalized coordinates, **T** is the kinetic energy of the system, Q_i is the generalized forces corresponding to q_i.

Because a particle has three degrees of freedom (**n**=3), we can obtain three equations of motion for each particle.

$$T = \frac{1}{2} m\dot{q}_i^2 \qquad \boldsymbol{i} = 1,2,3$$

$$\delta(W.D.) = -K(q_i - q_{i0})\delta q_i - c\dot{q}_i\delta q_i + (F_i + P_i)\delta q_i \qquad \boldsymbol{i} = 1,2,3$$

Where, $\delta(W.D.)$ is the virtual work. W.D means Work Done. δq_i is the virtual displacement which is an assumed infinitesimal displacement of a particle. **m** is the mass of a particle, **c** is the damping parameter, **K** is the stiffness parameter.
The result is:

$$m\ddot{q}_i + c\dot{q}_i + K(q_i - q_{i0}) = F_i + P_i \qquad \boldsymbol{i} = 1,2,3 \qquad (3)$$

or $$m\ddot{q}_i + c\dot{q}_i + Kq_i = F_i + P_i + Kq_{i0} \qquad \boldsymbol{i} = 1,2,3 \qquad (4)$$

The first term on the left of equation (3) is the inertial force of the particle. The second term is the damping force. The third is the elastic force due to the particle moving away from its original position q_0, which is important for the integrity of the surface. When the external forces exert on it, the surface will be deformed until the elastic forces balance the external forces. The first term F_i on the right of equation (3) is the sum of external forces which exert on the particle. The second term P_i is the sum of internal forces among the particles. Of course, the equations (3) and (4) are also expressed by the vectors.

We also can obtain the equation (3) from (1) if suitable $\delta\varepsilon(r)$ and discretization are used.

If **m**, **c** and **K** are supposed to be constant and are not equal to zero, the equation (4) is a linear and non-homogeneous difference equation of second order with constant coefficients. If forces on the right of equation are constant, which means the direction and magnitude of the forces are constant, the analytical solutions may be obtained. We shall discuss the case with variable forces and variable coefficients later. For the time being, the problem become easy. Using the roots r_1 and r_2 of the auxiliary equation, we may find two independent solutions of corresponding homogeneous equations. The general solutions of it are:

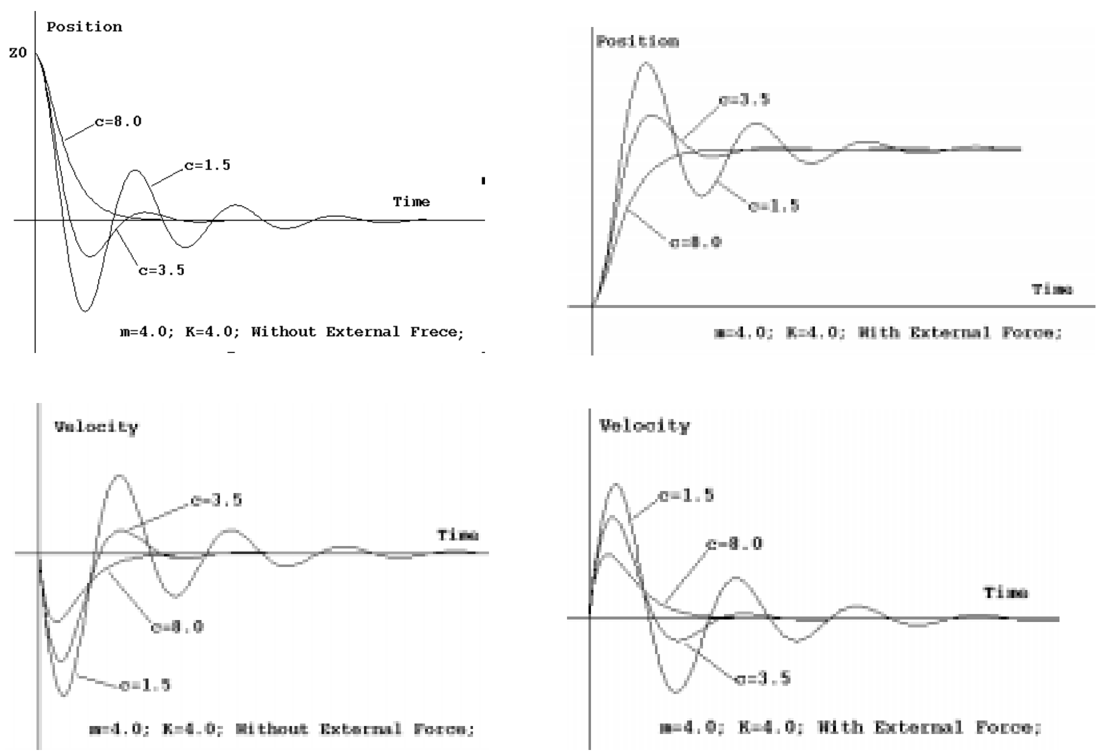
1. $r_1 \neq r_2$; $\qquad q_i = C_1 e^{r_1 t} + C_2 e^{r_2 t} \qquad \boldsymbol{i} = 1,2,3$

2. $r_1 = r_2 = r$; $\qquad q_i = (C_1 + C_2 t)e^{rt} \qquad \boldsymbol{i} = 1,2,3$

3. $r_1 = a + bi$, $r_2 = a - bi$; $\qquad q_i = e^{at}(C_1 \cos bt + C_2 \sin bt) \qquad \boldsymbol{i} = 1,2,3$

It is easy to find the particular solution of the given non-homogeneous equation(4), while the general solution of the non-homogeneous equation(4) is the sum of the particular solution and the general solution of corresponding homogeneous equations. If the surface is isotropic all of the particles have same mass **m**, same damping parameter **c** and same stiffness parameter K. The problem is to find the initial value and the particular solution for forces**.**

How about the features of this motion equation? Fig. 1 shows the features of the motion equation under different conditions. From the figures we can know several features of the particles on surfaces. Firstly, all the functions of position, velocity and acceleration of the particle have continuous second derivative on $(0, +\infty)$. Secondly, if the external forces do not exert on the particle, the particle has high ability to go back to its rest position Z_0 (original position). Then, if the external forces exert on the particle, the particle can smoothly move to the position which depend on the forces and at which the elastic forces balance the external forces. Finally, the time and behavior of movement depend on the damping parameter **c**, stiffness parameter **K** and the mass **m** of the particle. On other words, the features of movement depend on the properties of the surfaces. As a result, the particle systems with such features are suitable to create and animate deformable surfaces.



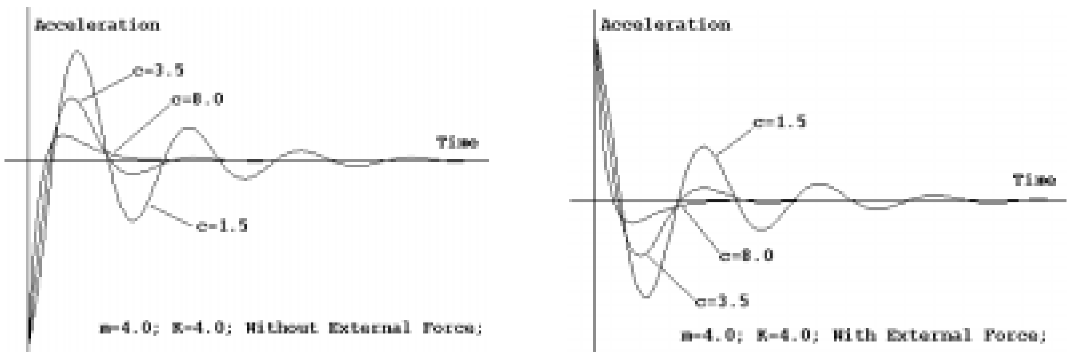


Fig. 1. The features of the motion equation

3. Using Discretization and Time-step Method

Because of using the analytical solution, discretization and time-step method, our approach is flexible for creating and animating the surface. In this section, we shall discuss several problems we often meet when the method is applied.

3.1 The external and internal forces

In our method, the important forces are the forces which control the shape evolution of the surfaces. The force **F** in the equation (3) is the external force which exert on the particle. For the surfaces, the force field should be defined or calculated, such as gravity field, external force field etc.. External forces are directly controlled by the user.

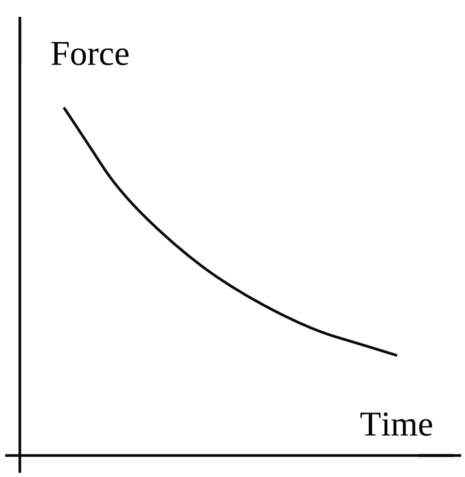
The force **P** is the sum of the internal forces which exert among the particles. The interparticle forces **P** can be determined by the property of the surface. They may have a lot of forms depending on the application. In some situations, we don't need to consider the forces among the particles at all, namely $\mathbf{P} = 0$. For example, the application of the skin in section 4. In the example of deformable head in section 4, we only calculate the radial forces approximately according to the change of the volume and do not consider the forces between the particles. In this situation, the elastic forces balance the external forces, and the elastic forces of the particles guarantee the integrity of the surface. In some other situation, such as the membrane shrinking around a jack, the forces between the particles should be considered in order to obtain more realistic results. For this example, the forces among the particles are supposed to be the elastic forces. In fact, we have a lot of methods to define the forces among the particles. Using discretization, the elastic forces with variable stiffness can be obtained. Forces can be also defined to be pull or push forces proportional to the distance between two particles. Also, the forces with very large stiffness can be defined so that the distance between the particles is almost fixed, and so on.

We can get all kinds of effect of woven cloth by defining the internal forces between the particles and using the suitable topology of the surfaces. For example, we use the quadrilateral polygons and define that one direction is warp and the other direction is weft, and the forces on the warp and the forces on the weft are defined to be different. For instance, the forces on the warp are a little larger than those on the weft because the warp is usually wider than the weft. We can get the effect of the plain weave. If we change the topology of surface and the internal forces between the particles, we can get another kind of woven cloth.

The external forces **F** and the internal forces **P** control the movement and shape of the surface. The forces only have influence on the particular solution of the given non-homogeneous equation(4) and do not have influence on the general solution of it. So, the shape of surfaces can be easily controlled by controlling the forces.

3.2 The variable forces and variable coefficients

In the last section, the forces were supposed constant on the right of equation(3). Generally, the forces are not constant, but functions of the position and time. If the forces are not constant, we also can find the analytical solution in some situations. In fact, this is difficult and often useless because discretization and time-step method have been used to solve the problems. Consider, for example, the variable force in Fig. 2, discretization can be used as shown in Fig. 3. In this case, the force is constant during each interval of the time. And, the discretization guarantees the analytical solutions of the motion equations (4) and thus changes the problem from a variable force to finding particular solutions.



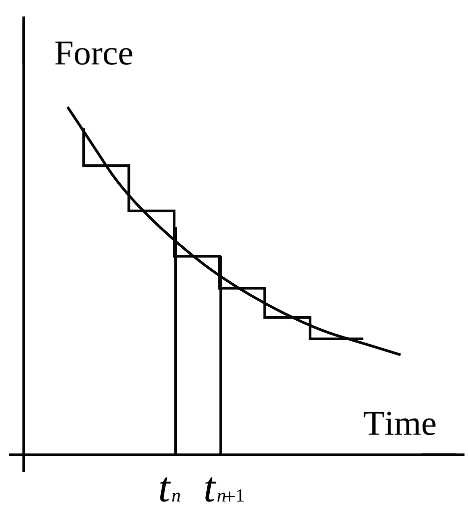


Fig. 2. Variable force

Fig. 3. Variable force (discretization)

Similarly, we can also use the discretization to extend the motion equation (3) to an equation with variable coefficients. First, the discretization of variable coefficients is performed with the coefficients constant during time interval. Then, motion equations may be solved as equations with constant coefficients. For a new time interval, the coefficients are changed using new discrete values, and the analytical solutions can be determined again. By repeating the process step by step, the problem with variable coefficients is transformed into the problem with constant coefficients.

3.3 The elastoplastic model in graphics

Because the method is flexible, we can make the deformable surface to be elastic or elastoplastic, even plastic as required.

For the sake of clearness, we briefly discuss the idea of elastoplastic by means of a stress-strain diagram. A stress-strain diagram correspond to the relationship between load and deformation, which usually depends on the material [7]. Different materials have different stress-strain diagrams. Fig. 4 shows an typical engineering stress-strain diagram in tension. The diagram shown in Fig. 4 defines two ranges of material behavior, the elastic and the plastic (or inelastic) ranges. In general, the elastic range is the part of a linear relation between the stress and the strain, which is represented by segment OB in Fig. 4. If the load is removed within the range, the material will regain its original dimensions and is said to behave elastically. Beyond the range the material dose not regain its original dimensions and the permanent deformation appears. This is the plastic or inelastic range which the segment BF represents in Fig. 4. the plastic range shows a nonlinear relation between the stress and the strain. The dash line which parallels the segment OA in Fig. 4 represents the behavior of the material in unload after the material are loaded up to G, which means the material is elastoplastic.

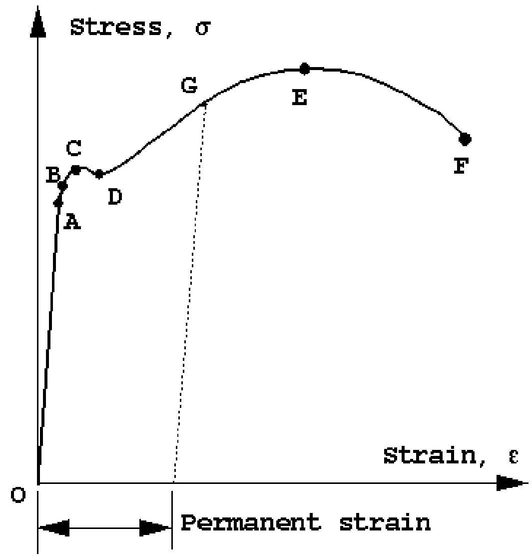


Fig. 4. Behavior of material

The permanent strain should be determined by the stress. But, we do not exactly calculate the stress and strain on the surface and do not consider the state of stress and strain for any point. This means that we just use the idea of elastoplastic of mechanics in the graphics rather than the strict method of elastoplastic theory. From Equation (3), we know, when the forces are removed the particles will go back to original positions q_0. For each step the deformation of the surface is calculated. If the deformation is large enough over the elastic limit, the permanent deformation is calculated by all kinds of the approximate method. Then, the permanent deformation is used instead of the original position q_0. So, when the forces are removed the permanent deformation appears on the surface.

4. Simulation Examples

In this section, we describe a few examples that can be animated in near-real-time on SGI Indigo-2.

4.1 The skin with wrinkles

This method is suitable to create and animate the skin with the wrinkles because the surface developed by the method has the high ability to go back to original position. In fact, the real skin appears smooth under the pressure of the blood and the muscle, and most of wrinkles on the skin always appear at same place. Fig. 5 show a piece of skin of a forehead. At first, the skin with wrinkles is made, which is easy to be done. The user can design various wrinkles on the skin. Then, put it on the frontal bone by the forces, and let it smooth as (a). when the forces are released (the eyebrow move upward), the wrinkles appear on the forehead. (b) - (d) show the different wrinkles. Finally, the forces exert on the skin again (the eyebrow move downward), the skin become smooth again.

4.2 The head

Fig. 6 illustrate another application of the method using our Marylin's polygonal head. Every vertex of the polygon is used as a particle with mechanical properties attached. Therefore, the geometrical surfaces are changed into the elastic surfaces of physically based modeling. In fact, this method can be combined with surfaces of arbitrary topology. At the beginning, no forces are exerted on the head as (a). When the forces exert on the top of the head, the head are deformed as (b) - (d). When the forces are removed, the deformed head recovers to its original shape as (e) and the top of head can vibrates several times around the original position, which can be controlled by the damping parameter **c**.

4.3 The membrane shrinking around a jack

A shrink wrap effect is shown in Fig. 7. Fig.7(a) shows the model of the jack which consist of six balls on the cross rods. Fig.7(b) shows the membrane surround the jack before shrinking. When the external forces exert on it, the membrane will shrink until the elastic forces balance the external forces. Fig.7(c) shows the shrinking result under small forces. Fig.7(d) shows the shrinking result under large forces.

4.4 The elastoplastic surface

Fig. 8 show a simple test of the elastoplastic surface on shell. The slider can control the permanent deformation on the shell. Fig.8(a) shows the shell which the force is exerting on. The other images show the different permanent deformation on the shell when the force is removed, from 100% elastic surface (plasticity = 0) to 100% plastic surface (plasticity = 100%).

5. Discussion

In this paper, we have shown that deformable surfaces based on particle systems may be as realistic as that using continuous systems. These particle-based surfaces have also many advantages as stated above. From a mathematical point-of-view, we have used a series of simple methods in this paper, such as analytical solution, discretization and time-step method. All the methods can approximately change the mathematical problems to solve differential equations which don't have analytical solution into elementary mathematics. Therefore, we can make use of a lot of methods of elementary mathematics to define the internal forces, to calculate the position and so on. We also use approximate methods to calculate the stress and strain on the elastoplastic model in graphics. Of course, we can calculate the exact value of the stress and strain field using elastoplastic theory, but this is not necessary. The methods we use are simple, but they satisfy the requirements for good visual results.

The method proposed has many advantages:

- *Good stability and convenience*: Because the analytical solutions are used, there is no problem of convergence. The deformable surface is very stable even under very large deformations. The calculation is very easy to implement, and the visual result is realistic.
- *Efficiency*: Because the calculation is more simple and more effective, the result can be obtained quickly. Using the method, interactive animation can be obtained. For example, for Marilyn's head deformation, 60 frames were obtained on SGI Indigo-2 in 20 seconds (3 frame / second). For the example of membrane shrinking around a jack, 60 frames were produced in 25 seconds to 40 seconds (1.5 - 2.4 frame / second). Time is dependent on the condition of the surface, such as forces, stiffness and so on.
- *Wide application*: The method may be easily applied to any kind of deformable surface without topology limitation, from small deformations to very large deformations, from a movement with small damping, even no damping, to a movement with very large damping, etc. So, the method can be used to create and animate the skin with or without wrinkles, cloth, paper, rubber, plastic and other surfaces etc..

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Computational Geometry Journal and the *CADDM* journal. She is the President of the Computer Graphics Society (CGS).

Color Figure Captions

Fig. 5. Skin of a forehead with wrinkles
Fig.6. Head with elastic surfaces
Fig.7. Membrane shrinking around a jack
Fig. 8. Test of elastoplastic surface