Hierarchical Kinematic Behaviors for Complex Articulated Figures

Ronan Boulic[1] and Ramon Mas[2]

(1) Computer Graphics Laboratory, Swiss Federal Institute of Technology, DI-LIG, CH1015 Lausanne, Switzerland

(2) Department of Mathematics and Computer Science, Balearic Islands University, 07071-Palma de Mallorca, Spain

Abstract

The hierarchical control decomposition provided by inverse kinematics is seldom used in the field of Computer Animation. In this chapter we first review the properties and limitations of inverse kinematics compared to other techniques dedicated to the motion control of complex articulated figures. Then, beyond the sole pseudo-inverse solution, we examine the homogeneous solution allowing the partial realization of a secondary *task* (or *optimization* or *behavior*). Case studies are presented in two distinct application areas of this technique : balance control and motion correction.

1 Introduction

In this chapter we explore the high potential of the *homogeneous solution* from inverse kinematic control. Its main interest is to allow partial fullfilment of a secondary task in the so-called *null space* without disturbing the achievement of the main task. We examine here the reasons why this property is of crucial interest for posture and animation design of complex articulated structures. The human articulated structure is considered as a good illustration of the degree of complexity we address here and it is used for most of the examples.

We first review the properties and limitations of inverse kinematics compared to other techniques when applied to the control of complex articulated figures. Then, we focus on secondary tasks, especially approaches for posture and animation design. We concentrate on two distinct application areas of this technique : balance control and motion correction.

2 Animating Complex Articulated Structures

Our purpose is to control and animate the posture of significantly complex articulated figures while still providing the end-user the necessary interactivity and specification flexibility. We first recall the order of complexity that we consider here and stress the specific requirements for the interactive manipulation of such entity. This will lead us to compare the inverse kinematics approach with other motion control techniques. We then develop a general expression of inverse kinematics with special emphasis on practical issues such as the calculation of jacobian and its inversion, the joint limits and the singularity management.

2.1 Key Characteristics of Complex Articulated Structures

We consider the animation of vertebrate animals including the human being. For our articulated figure a mechanical model of such an entity requires at least thirty *degrees of freedom* (referred as *dof*) for the limbs alone plus twelve *dof*s for a crude representation of the relative motion of the pelvis, the abdomen, the thorax, the neck and the head. So, even excluding the mobility of the clavicle and scapula, the simplest 3D model contains at least forty two *dof*s (chapter Calvert) (Badler et al. 1993) (Boulic et al. 1995). Then, a supplementary minimal set of forty *dof*s is necessary if simplified hands are to be considered too. Finally, after adding eight to twelve *dof*s approximating the clavicle and the scapula joints and some "aggregate vertebrae" for a smoother back bending (Monheit et al 1991), the total can easily raise to more than one hundred and twenty *dof*s depending on the resolution of the vertebrae model. The same evaluation is valid for most of the terrestrial vertebrate animals. The only significant variations are the following : first, the foot and hand models can be either more complex (a primate foot is similar to a hand) or simpler (horses have only one "finger") ; second, the spine can hold more (lizard) or less (frog) vertebrae and may include a tail.

From this statement we show in section 2.3 that inverse kinematics, although very simple, is still a pertinent tool to design postures and animation compared to other control techniques.

The second characteristic of these articulated structures directly results from its high dimension. These structures are extremely redundant, that is to say they have many more *dof* than are theoretically necessary to perform goal-oriented tasks in the cartesian space. This can be illustrated by the infinite number of possible postures to grasp an object. As a result it becomes very difficult to design and determine the "proper" posture to do so. Additional constraints or criteria have to be considered in order to guide the posture control. In such contexts, the hierarchical decomposition of inverse kinematics becomes a powerful feature as developed in section 2.4.3 and 3.

A fundamental aspect makes the articulated structure we consider here very specific compared to the traditional investigation field of Robotics. Apart from the dimension mentioned before, we want to stress that all bipedal animals have to face an intrinsic unstable balance problem whereas robots are normally rigidly fixed to their supporting area. Moreover, studies of mobile robots barely focus on such unstable structures. Some notable exception are the control studies of monopods, bipeds and quadrupeds in (Hodgins et al. 1991) and (Raibert et al. 1992). However, they only deal with the control of single support or flying phases, as in a running motion or hopping, due to the difficulty of managing double support phases, as in walking motion or standing.

2.2 Animation Design Criteria

The Animation of such a complex structure as the human being has a long history (Calvert 1991) (Badler et al. 1993) (Magnenat-Thalmann et al. 1990). Chapter *Calvert* is especially dedicated to the management of human animation involving multiple entities. We just recall here our requirements on motion control technique for such a complex entity:

- Maintain an interactive manipulation rate in order to allow direct user-intervention during motion control or any optimization process. In such a way, the animator can "shape" the motion or the posture concurrently with its evaluation.

- Retain techniques producing realistic, if not physically-based, output while involving the simplest possible control variables, i.e. joint angles, position and orientation of end effectors, mass distribution, and temporal derivatives of these variables.

- Provide input variables and operative concepts from which the animator has a very clear and intuitive understanding. Otherwise the high dimension of the system quickly overloads the animator capabilities. Some of these mental representations with low cognitive burden are the posture, position, displacement, mass and center of mass.

2.3 A Comparison of Inverse Kinematics with other control techniques

Physically realistic approaches, such as dynamic or optimal control are not yet suited for interactive design of human postures due to the delicate handling of their associated parametric space (torque, muscle activation) or the additional parameters added by the control approach (energy storage, management of the ground reaction force). They are hard to handle for an animator as stated in (Raibert et al. 1992). Regarding optimal control, the major limitation comes from the high dimension of the human figure preventing on-line interaction on current workstations (see chapter *Hégron*).

An interesting approach incorporates human strength into the solution of the motion control problem within a dynamic system paradigm (Lee et al. 1990). The belief is that strength is the foundation of a figure's posture (Lee 1993). A complete behavioral model is proposed to control deliberate movements (those not heavily influenced by dynamics). At each time step, it proceeds by selecting various path planning schemes (available torque, reduce moment, pull back, add joint) according to the current state of the system and some proposed external variables (force trajectory, comfort level, desired comfort level, perceived exertion). However, as it is related to biomechanical variables, this approach suffers from the arduous task of establishing the whole human strength model. There is no general model of human strength in the literature and partial results are often based on various simplifying assumptions one of which is the degree of consideration of adjacent joints' influence on a joint's strength (Lee 1993).

Conversely, pure inverse kinematic control has some limitations. It provides a local solution (see section 2.4.2) without simple means of knowing whether other solutions exist (Klein et al. 1983) except by using a global method which optimizes an integral cost criterion (Won et al. 1993). But such a global approach suffers from a lack of interactive capability. The local minima problem is the direct consequence of the local solution of inverse kinematics especially for highly redundant structures. However the optimization of a secondary criteria can be used to improve that aspect as already demonstrated in Robotics (Cleary et al. 1990)(see sections 2.4.4 and 3) while still being careful about unrealistic demands on mechanism performance induced by their naive use (Maciejewsky 1989). In fact the large range of possible secondary tasks greatly helps to ground inverse kinematics into more physically-based solutions.

Finally, a pertinent usage of the secondary task allows to take full advantage of inverse kinematics' intrinsic qualities : a low computational cost allowing interactive control, and a great flexibility of specification and manipulation of easy-to-understand control variables. In such a way, we can greatly extend the interactive postural design (Girard et al. 1985) (Boulic et al. 1993) (chapter Calvert) to posture control with higher realism. Its application to motion still requires negligible dynamics, i.e. a slow speed with minimal frictional and inertial effects, as already mentioned in (Badler et al. 1993).

2.4 An Explicit Task Hierarchy

Here we review the principle of direct and inverse kinematics and describe its general expression, then we discuss how to define the main task and evaluate the achievement potential of the secondary task. The jacobian construction and inversion is presented in detail prior to a discussion on two important aspects : joint limits and singularities.

2.4.1 Principle of Direct and Inverse Kinematics

As stated before, Inverse Kinematics is a technique mostly used to control or constrain strategic parts of the articulated figure, the so-called *end effectors*. Let us recall the technical justification of this technique. An end effector location depends on the current state of the joint parameters. The set of non-linear equations establishing the end effector location as a function of the joint state is called the direct geometric model in Robotics. Inverting it is possible as long as the dimension of both spaces is the same (Paul 1981). This problem has been solved for standard classes of robotic manipulators (i.e. with up to six dofs). However, as emphasized in section 2.1, we address classes of complex articulated structures which are highly redundant in accomplishing tasks. In such context the inversion of the geometric model is not possible. For these reasons, an alternate approach, the kinematic model, has become the privileged tool for positioning and animating such structures.

Let us now examine the principle of the direct kinematic model. It is based on the evaluation of instantaneous variations of the end effector(s) position and orientation, for each individual joint parameter, at the current state of the articulated system (direct kinematics). Figure 1 shows the resulting linerization of the direct geometric model obtained in this way (1D analogy shown for clarity).

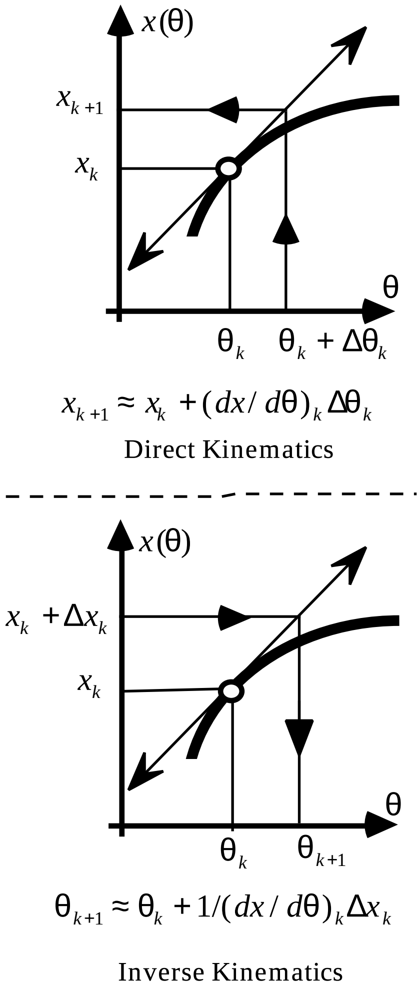


Figure 1: Linearization of the Direct Geometric Model.

The so-called Jacobian of the system is the matrix gathering the first order variations. It is inverted (Whitney, 1969) (Liégeois, 1977) in order to obtain the joint variation realizing a desired variation of the end effector (inverse kinematics).

As shown on Figure 1, the linearization is valid only in the neighborhood of the current state of the articulated system and, as such, any desired variation has to comply with the hypothesis of small movements. We now explore in greater detail the mathematical expression of direct and inverse kinematics and then describe the construction of its components.

2.4.2 General Expression

The hierarchical decomposition resulting from the redundancy of the articulated system with respect to the task space was first introduced in (Liégeois, 1977). The first term of the solution is called the *pseudo-inverse solution* while the second term is called the *homogeneous solution* :

$$\Delta\theta = J^{+}\Delta x + (I - J^{+}J)\Delta z \qquad (1)$$

where

- $\Delta\theta$ is the unknown vector in the joint variation space, of dimension **n**.
- Δ**x** describes the so-called *main task* (or *behavior*) as a variation of one or more end effector(s) position and/or orientation in Cartesian space of dimension **m** .
- **J** is the Jacobian matrix of the linear transformation representing the first order approximation of the direct geometric model for the *main task* (figure 2).
- $\mathbf{J}^{+}$ is the unique pseudo-inverse of **J** providing the minimum norm solution, the so-called pseudo-inverse solution, which achieves the *main task* (figure 2).
- **I** is the identity matrix of the joint variation space (**n** x **n**)
- $(\mathbf{I}-\mathbf{J}^{+}\mathbf{J})$ is a projection operator on the *null space* of the linear transformation **J** .
- Δ**z** describes a *secondary task* (or *behavior*) in the joint variation space. Its projection on the null space constitute the homogeneous solution which is mapped by **J** into the null vector of the cartesian variation space, thus not affecting the realization of the main task (figure 2).

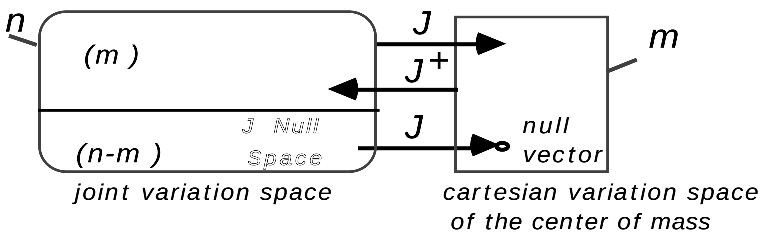
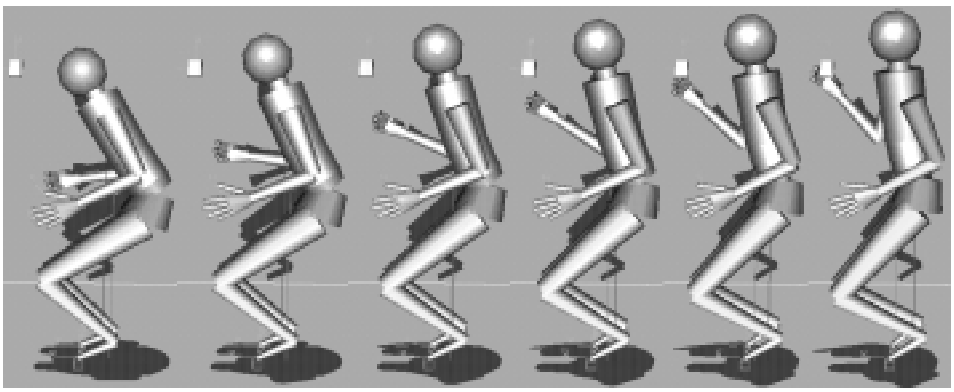
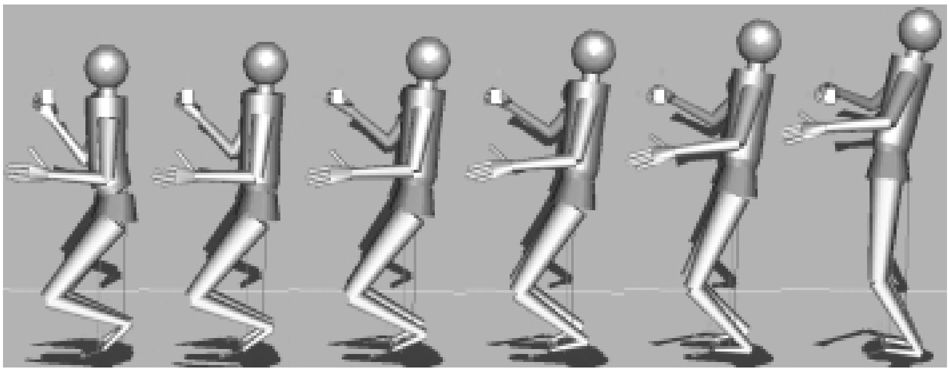


Figure 2 : Illustration of the joint variation space partitioning with Inverse Kinematics



a) Pseudo-inverse solution for a reaching task of the right hand (reach the white cube).



b) Homogeneous solution for the right hand (retain the white cube location)

Figure 3 : Inverse kinematics on complex articulated figure

2.4.3 Defining the Main Task

Inverse Kinematics has been successfully applied to the design of complex postures by interactively moving some end effector(s) attached to the articulated structure. It is possible to control only the end effector position or only its orientation (Girard et al. 1985) (Zeltzer et al. 1988) (Boulic et al. 1994d) (Chapter Calvert). Another approach considers the simultaneous position control of multiple end effectors attached at any places on the human tree structure (Badler et al. 1987).The resulting posture is a compromise depending on the relative weight which are also associated with each end effector. This method has been extended further to include combinations of position and orientation control under the generic name of goals (Phillips et al. 1990). In fact these goals are constraints analogous to mechanical joints as point on point, point on line, point on plane, as well as plane on plane, ball and socket, etc.(Dombre et al. 1985).

Now, before going further, we should explain the local character of the solution provided by this method (Klein et al. 1983). Figure 4 illustrates it in both the posture and cartesian spaces. Given a goal x_{final} to reach with an end effector, the final posture of the articulated figure (final1 or final2) depends on its initial configuration (respectively initial1 or initial2). The other postures belonging to the gray sub-space of figure 4 also complete the goal for the end effector. However, they cannot be evaluated directly ; they can only be estimated from the dimension of the null space. Again, it is possible to improve this aspect with the choice of a pertinent secondary behavior as introduced in the next section.

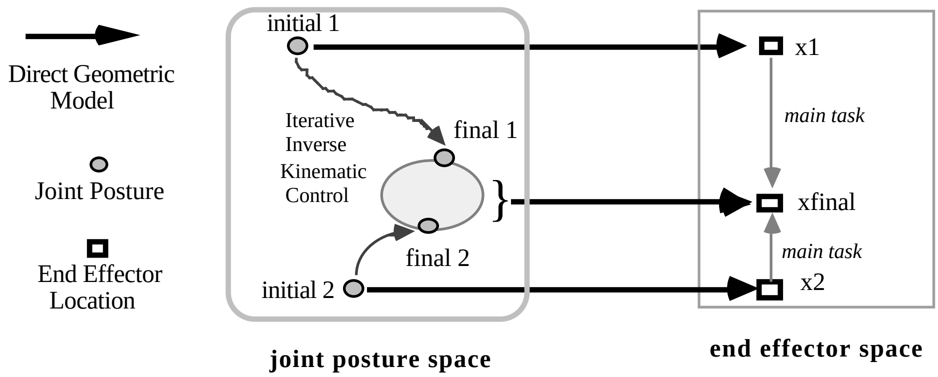


Figure 4 : Inverse kinematics provides a local solution depending on the initial posture

2.4.4 Evaluating The Potential of the Secondary Task

By definition the secondary task (or behavior) is partially realized by the projection on the null space (figure 2). This way the projected component does not modify the achievement of the main behavior because it is mapped into the null vector of the cartesian variation space by the linear transformation J. The secondary task usually expresses the minimization of a cost function and it is important to evaluate the potential of this optimization to succeed. First, let us assume that the main task belongs to the image space of **J** (i.e. all the target goals defined by the main task can be realized with the articulated structure). Then the null space's dimension is $n-m$.

From this information we can deduce to what extent the secondary behavior may be fulfilled, or rather, may not be fulfilled. To begin with, the joint space's dimension n must be greater than m in order to allow the null space to exist. Then it is easy to imagine the tradeoff between the dimension of the main task m and the remaining dimension to realize the secondary task $n-m$. The high redundancy of the human articulated structure is a very favorable context for the secondary tasks to be realized. Section 3 reviews the various approaches proposed for the secondary task and highlights the ones useful for computer animation.

2.4.5 Constructing the Jacobian Matrix

We have seen that the jacobian matrix can hold translation and/or rotational constraints in a very flexible manner according to the controlled dimensions.

We illustrate here how to build it for articulated systems with rotational joints. First we describe the translation jacobian followed by the jacobians for the rotation and the general case.

Figure 5 shows the construction of the translation jacobian in the 2D case from the end effectors' velocities due to the rotational joints θ.

For each joint θ_i, the instantaneous velocity v_i on the end effector E due to a unit variation of θ_i is given by :

$$v_i = \omega_i \times O_iE \qquad (2)$$

where x is for the cross product of the unit instantaneous rotation vector ω_i of joint θ_i with the lever arm vector O_iE. All the vectors have to be expressed in a common frame. In this 2D case all the rotational vectors are perpendicular to the plan. However formula (2) is still valid in the 3D case where the rotational vectors are freely oriented. So, finally the translation jacobian is given by :

$$J_T = \left[v_0 \vdots v_1 \vdots v_2 \vdots v_3 \right] \qquad (3)$$

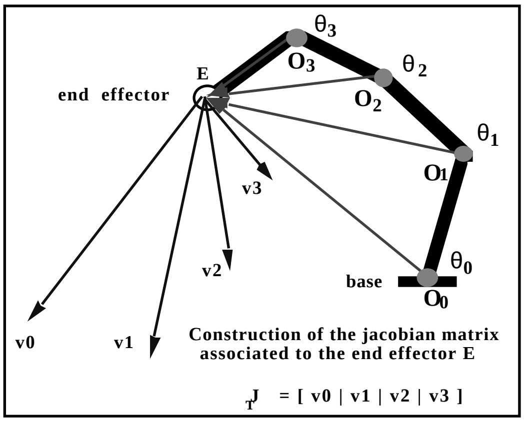


Figure 5 : building a translation Jacobian

Building the rotational jacobian is even simpler. The instantaneous rotations on the end effector due to a unit variation of the joints θ_i are simply the instantaneous rotation vectors ω_i themselves. The reason for this comes from the rigid solid hypothesis for the so-called *augmented body* including all the body segments after joint θ_i (on the end effector side of the articulated structure). This hypothesis is intrinsic to the kinematic model and states that the instantaneous rotation is the same for any part of the solid, i.e. at location O_i as at location E. So the rotational jacobian just gathers the set of ω_i vectors expressed in a common frame :

$$J_R = [\omega_0 \vdots \omega_1 \vdots \omega_2 \vdots \omega_3] \qquad (4) \qquad \text{and} \qquad J = \begin{bmatrix} J_T \\ \hline J_R \end{bmatrix} \qquad (5)$$

The complete jacobian for one end effector integrates the translation and rotational jacobians just by combining them as a column matrix. As mentioned before, some dimension may be omitted to reflect the behavior of a mechanical joint. In the multiple end effectors case the associated jacobian can be built by piling the partial jacobians representing the goals associated with each end effector.

2.4.6 Inverting the jacobian matrix

The first iterative algorithm proposed to compute the pseudo-inverse of a rectangular matrix is given in (Gréville 1960). Although efficient, this approach cannot be adapted to overcome the singularities (or rather the singularity neighborhood as developed in 2.4.8). For this reason, an approach based on the Singular Value Decomposition is to be preferred (Press et al. 1992). A specific nonlinear programming approach with variable metric, which avoids the computation of the pseudo-inverse, has been chosen in (Phillips et al. 1990).

2.4.7 Handling Joint Limits

Only the approach described in (Phillips et al. 1990) and (Badler et al. 1993) explicitly takes into account the joint limits as additional inequality constraints for their nonlinear programming problem. In the classical and more general case of equation (1) the joint limits do not explicitly appear. So, when a joint reaches such a limit, various approaches can be followed :

- Eliminating this variable is a bad solution as it remains locked there.

- A simple approach is to evaluate the solution and to truncate the joints values which are beyond their joint limits. However, such a blind truncation of both main and secondary behaviors introduces a bias into the solution which can lead to a local minimum different from the expected main behavior.

- Using the secondary task to avoid the joint limits has been proposed in the literature (Liégeois, 77) but there is no guarantee of permanent avoidance as it is only partially realized on the null space. Moreover such optimization may not be desirable as discussed in section 3.2.3 .

- An alternate method is to remove all variables, which are at their limit range from the Jacobian matrix, but only for the computation of the projection operator onto the null space (Boulic et al. 1994b). In this way, the secondary behavior has no component along these dimensions. The joint value truncation alters only the main component of the joint variations for the angles at their limit value. Then the remaining error in the main goal is further reduced in the next iteration, thus exponentially converging to its realization without local minimum.

Furthermore, the secondary behavior should not be evaluated whenever the number of joints reaching their limit value is equal to the dimension of the null space. It is easy to understand that, for each degree of freedom reaching its limit, the dimension of the null space is also decreased until there is no "space" for a secondary task.

2.4.8 Managing Singularities

Another limitation of Inverse kinematic control appears whenever the configuration becomes singular. This situation is due to an alignment of the segments constituting the articulated figure leading to a loss of mobility of the effector(s) in that direction. The Jacobian has a loss of rank or, even worse, a very small singular value along that Cartesian dimension. Then, inverting the matrix leads to a very large singular value for the pseudo-inverse. Finally, the resulting pseudo-inverse solution has its norm growing accordingly to infinity when a movement is required along the singular cartesian direction. Such problem appears quite frequently for a human model. For example, it should be mentioned that the usual standing position is nearly singular regarding displacements along the vertical dimension.

An elegant solution to this problem has been proposed in (Maciejewsky, 1990) which limits the norm of the solution provided by inverse kinematics. The computation of the so-called damped least square solution is related to the Singular Value Decomposition (Press et al. 1992) by preventing the singular values of the jacobian to be zero with a so-called damping factor. Then, the singular values of the inverted damped jacobian are prevented to reach the important values inducing the former instabilities of the joint variation solution.

2.5 Controlling the Center of Mass Position

The center of mass is one of these fundamental concepts which are intuitive and well mastered by an animator. It is especially useful to evaluating the state of balance of a static posture from the relative position of the center of mass and the support area. If its vertical projection lies in the support area then the posture can be considered well balanced. Otherwise, if we want to design complex balanced postures (e.g. for dance postures), it is necessary to provide a simple means for its position control in order to move it over the support area.

Inverse kinematics has been first used for that purpose by envisioning the center of mass as an end effector attached to the lower torso because its average human location is slightly frontal to the base of the spine (Phillips et al. 1991). Unlike the true center of mass which local coordinates are changing in the lower torso coordinate system after each posture update, their end effector remains fixed until the constraint satisfaction process is solved. Then the center of mass is recomputed and usually it does not coincide with the end effector. Therefore, the constraint satisfaction process is repeated until the center of mass position is close enough to the end effector. In this approach, the only articulated chain influencing the center of mass position is of the dominant leg ; i.e. the one bearing most of the body weight (hip, knee, ankle). The authors can achieve balance behaviors for a standing human model (Phillips et al. 1991) (Badler et al. 1993).

However it is too complex to generalize this method for any arbitrary articulated figure and even for a human structure with a different support or attach (hanging by the hands, sitting etc.). Furthermore, the influence of the joint variables is not related to the mass distribution in the body but to the geometrical lever arm (figure 5). Such a geometric approach clearly explains the observed distortions between the positions of end effector and the real center of mass.

Several other approaches have also considered the control of the center of mass. The system presented in (Girard, 1987) dedicated to bipedal walking, is mostly kinematic but also includes some dynamic rules so as to maintain the center of mass within the support polygon. A similar approach has defined kinematic and dynamic rules so as to partially control the balance via the position of the center of mass, and to minimize the effort developed by the muscles (Maiocchi, 1991). The approach is applied to optimize various motions such as walking, sitting on a chair and climbing stairs. Despite the fact that the author can successfully manage the balance in the case of sitting (and especially, getting up from a seat), the control of the center of mass is actuated only with the bending of the torso, which limits its range of application.

Recently, a general approach has been introduced in (Boulic et al. 1994b&c) which overcomes the theoretical weakness previously mentioned. The key point of this approach is to evaluate the **kinetic** influence of the joints based on the fraction of the total body mass they support, i.e., their *augmented body*. Here, the augmented body concept is used for both the imaginary rigid solid it represents and the equivalent mass it holds. Figure 6 shows the center of mass of all the augmented bodies of an arbitrary articulated chain and also outlines the augmented body associated with one joint. It also highlights related variables for the following demonstration of direct kinetics principle.

The basic principle of direct kinetics is to relate instantaneous joint rotations to the corresponding instantaneous translation of the total center of mass. According to the hypothesis of an instantaneous rigid body, the instantaneous velocity $V_{G_{ai}}$ on the center of mass G_{ai} from the augmented body associated with joint θ_i is given by :

$$V_{G_{ai}} = \omega_i \times O_i G_{ai} \qquad (6)$$

where the instantaneous rotation ω_i is of unit magnitude and O_i as center of rotation (Figure 6). For direct kinetics we need to evaluate the corresponding velocity V_{G_i} of the center of mass of the whole body. It is given by applying the principle of the conservation of the momentum to the augmented body of mass m_{ai} and velocity $V_{G_{ai}}$ and to the whole body of mass m and velocity V_{G_i} . :

$$mV_{G_i} = m_{ai} V_{G_{ai}} \qquad (7)$$

So

$$V_{G_i} = (\frac{m_{ai}}{m}) V_{G_{ai}} \qquad (8)$$

V_{G_i} constitutes one column of the direct kinetic jacobian matrix J_G (an alternate demonstration of direct kinetics is developed in (Boulic and Mas, 1994). Conversely, *inverse kinetics* provides the instantaneous joint rotation realizing a desired instantaneous translation of the total center of mass. The kinetic jacobian is inverted in exactly the same way as the kinematic jacobian. Inverse kinetic control is equivalent to expression (1).

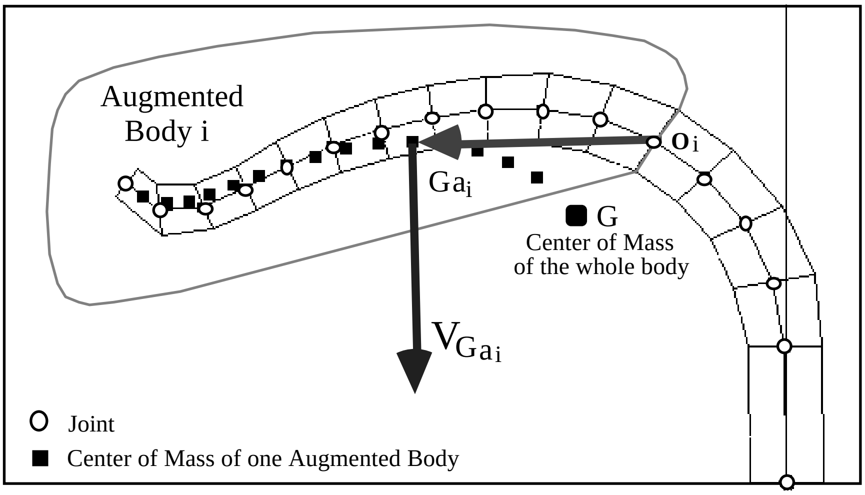


Figure 6 : total and partial (augmented bodies) centers of mass
for a simple articulated chain

3 Beyond the Pseudo-Inverse Solution

In Computer Animation very few applications of inverse kinematics go beyond the pure inverse kinematic control. We first review how the secondary task has been treated in Robotics and then we present a selection of secondary tasks tailored for Computer Animation.

3.1 The Robotics Viewpoint

Various secondary tasks have been proposed to enhance the control of redundant manipulators. Their purpose is to optimize a cost function while carrying out a task in cartesian space. Two great classes of secondary tasks can be found depending on the space of interest : the configuration space (internal criteria) or the cartesian space (external criteria). The following section present them in greater detail. The next section highlights the kinetic limitation of the secondary task.

3.1.1 Optimization criteria

The literature is rich of cost functions designed to improve various and sometimes conflicting aspects of the manipulator state, either internal or external. We now briefly explain the chief advantages for the most important of them.

Joint limits avoidance :

It was the first secondary task to be proposed in (Liégeois, 1977). In order to achieve a joint limit avoidance behavior the cost function to minimize is the squared norm of the difference between the current posture _ and the mid-range posture $_M$. The mid-range posture is the one for which each joint holds the middle value of its range. The gradient vector is simply twice the difference vector :

$$\Delta Z = -2(\theta - \theta_M) \qquad (9)$$

Singularities avoidance :

Singular configurations of the manipulator are linked to a loss of rank in the jacobian matrix and their proximity is linked to the vanishing of the jacobian singular values to zero. So, any measure reflecting the current state of the singular values is a measure of the proximity to a singular configuration. The singular values should be globally maximized in order to balance the potential end effector movements in all the directions of the cartesian space (including rotation). For these reasons the cost functions aiming at the singularity avoidance are sometimes called dexterity or manipulability. The dexterity index (Klein, 1984) is the smallest singular value while the manipulability index (Yoshikawa, 1985) is the product of all the singular values. This latter is also equal to :

$$\sqrt{\left|JJ^T\right|} \quad (10)$$

Another approach minimizes the condition number which is defined as the ratio of the higher singular value to the lower one (Cleary et al. 1990). Finally, let us recall that the damped least square method successfully overcomes the inherent instabilities occurring at the singularity neighborhood (Maciejewski, 1990) with a modified pseudo-inverse (Cf. section 2.4.8). However, it does not improve the manipulability which remains very small along some cartesian dimensions in such context.

Obstacle avoidance :

Obstacle avoidance is the main concern of the secondary tasks expressed in the cartesian space. In (Maciejewski et al. 1985) the manipulator point closest to the obstacles is given an instantaneous repulsive velocity. We can call this point a secondary end effector with its specific jacobian J_S and pseudo-inverse J_S^+. The main task clearly has some influence on the secondary effector motion. It is necessary to evaluate it and subtract it from the desired repulsive velocity in order to compensates it in the final solution. Expression (11) summarizes the successive transformations realizing that behavior (Maciejewski, 1989) :

$$\Delta\theta = J^+\Delta x + \left[J_s(I - J^+J)\right]^+(\Delta x_s - J_s J^+\Delta x) \quad (11)$$

In (Espiau et al. 1985) and (Boulic, 1986) the collision avoidance is based on the simulation of multiple proximeter sensors providing an approximation of the short range distance to the obstacles (from one to twenty centimeters with narrow and wide fields). Their small size allows installing a great number of them (from twelve to thirty two) on the end effector and the segments of the manipulator. Furthermore, their fast measurement processing guarantees the real-time control. The distance measurements are exploited to synthesize a repulsive kinematic torsor on each segment of the manipulator (translation and rotation velocities). The repulsive torsor associated to the end effector is integrated in the main task thus adapting to changing environments. The other repulsive torsors are mapped onto the joint variation space with the transpose jacobian associated with each segment. Expression (12) recalls the final solution of this approach (k is indexing the segments and k_{max} is the end effector):

$$\Delta\theta = J^+(\Delta x + \Delta x_{k_{max}}) + (I - J^+J)\left(\sum_{k=k_{min}}^{k_{max}-1} J_k^T \Delta x_k\right) \quad (12)$$

Other interesting approaches worth mentioning are :

- manipulator workspace investigation (Kumar et al. 1981)
- improvement of the dynamic response (Salisbury et al. 1985)
- minimization of the joint torque (Suh et al. 1987)
- improvement of the kinetic energy distribution (Cleary et al. 1990)
- multiple criteria integration with normalization and prioritization (Cleary et al. 1990)
- global optimization for various performance indexes (Won et al. 1993)

3.1.2 Kinetic Limitations of the secondary task

Despite the pleasant theoretical partitioning of task provided by inverse kinematics, its solution can be physically inaccurate when applied to real redundant manipulators. Imposing an arbitrary secondary task for their control can induce significant error tracking for the main task and generate unrealistic joint torques (Klein et al., 1987).

Manipulators have a mass distribution and therefore a kinetic energy whenever moving. Consequently, the kinetic behavior of the homogeneous solution should be controlled and limited in order to obtain realistic torque requirements. This is especially true when redundancy is solved at the level of acceleration as has been demonstrated in (Maciejewski, 1989).

3.2 The Animation Viewpoint

Again let us recall that the articulated systems we deal with are, at least kinematically, much more complex than the one simulated in Robotics. This may explain the lukewarm interest of Computer Animation researchers in Robotics secondary tasks. For example, the joint limits avoidance and singularity avoidance are of little interest for the human figure because the natural resting postures mostly adopted by humans while standing are close to some joint limits (knee and elbow extension, hip and shoulder adduction) and nearly singular for vertical displacements of any part of the body.

Specific secondary tasks have to be proposed bringing solutions for such problems as :

- interactive posture manipulation with goal-oriented constraints
- automatic enhancing of posture realism with respect to balance, natural postures.
- automatic correction of pre-recorded joint motions with goal-oriented constraint

We review now the significant propositions made in these directions.

3.2.1 Interactive Posture Manipulation

The hierarchical nature of inverse kinematics solution always guarantees the realization of the main task. For example, it is possible to move and/or orient some end effector interactively, then to decide that it remains fixed (<=> null vector for the main task) and work on adjusting the posture within the null space of the main task. Thus, the posture adjustment does not affect the goal oriented constraints of the main task.

This can be achieved in two ways :

- specifying a cost function attracting the current posture to an interactively defined posture in the same way we were avoiding the joint limits with an attraction to the mid-range posture (Liégeois, 1977). This is used in (Girard et al. 1985) with an additional weighting factor for each joint (also interactively defined).

- using the cartesian space secondary tasks with attracting velocities instead of repulsive velocities. The user can easily pick any part of the articulated chain as a secondary end effector and interactively attract it to some desired location.

3.2.2 Global Balance Optimization

The global balance optimization was introduced in (Boulic et al. 1994b). It focuses on the minimization of torques due to the gravitation force around the center of support (also called support torques) . These torques tend to rotate the whole body around the center of support under the influence of gravity. The cost function C_S to minimize expresses that every independent support torque T_i , should be geared to zero rather than their algebraic sum alone:

$$C_S = \sum_{i=1}^{n} \|T_i\|^2 \qquad (13)$$

The relation between cost function C_S and the joints has been established in order to express its gradient vector in term of joint variations. Figure 7 illustrates the different elements entering this relationship for the augmented body associated with joint i.

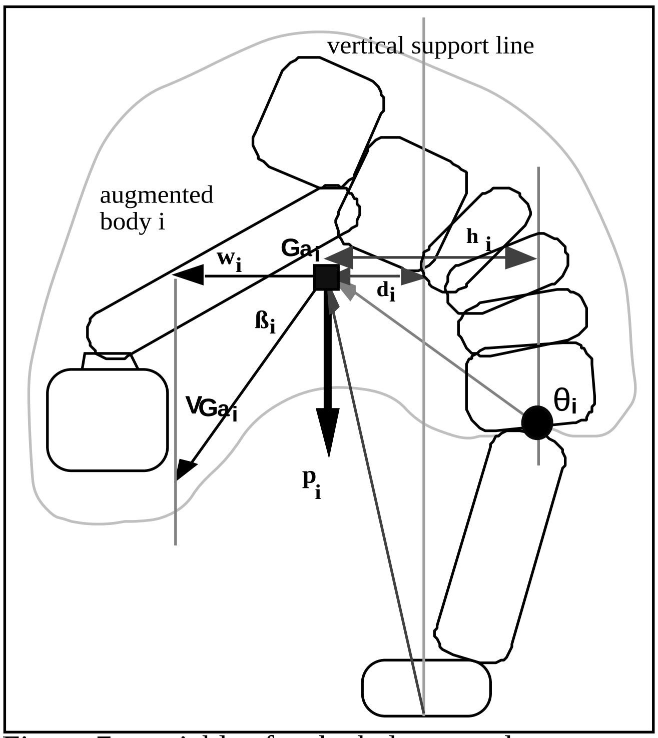


Figure 7 : variables for the balance and rest posture secondary tasks

First, its support moment comes from the action of the weight p_i and is directly proportional to the distance d_i between G_{ai} and the vertical support line.

Minimizing M_i is equivalent to minimize d_i .

Now, the influence of a joint variation on d_i can be deduced from its influence V_{Gai} on point G_{ai} projected on the axis supporting distance d_i (referred as W_i).

The resulting term of the gradient vector is proportional to :

$$\Delta Z_{S_i} = -2 \; m_{ai} \; d_i \; \|W_i\| \qquad (14)$$

The global minimum of this cost function corresponds to the configuration sub-space where the centers of gravity of all the augmented bodies lie on the vertical line of support (see figure 7).

3.2.3 Attraction to a Natural Rest Posture

The approach presented in (Boulic et al. 1994b) considers the rest posture as a useful concept for the posture optimization of complex mechanisms presenting an active behavior as animals and human models. They assume it to be the global minimum among standing postures regarding muscular cost according to some biomechanical and physiological studies (Jouffreoy et al. 1990) (Kuo et al. 1993). For this reason, they propose a cost function converging to the rest posture based on kinetic information.

Although an attraction gradient vector like the one of expression (9) clearly leads to the rest posture _r , it does not convey kinetic information and therefore is not a valid rest performance index. For this reason, a second factor representing such effort scales this cost function. This is the torque exerted by the augmented body weight with respect to the origin O_i of the joint rotation axis. Figure 7 illustrates the quantity h_i directly influencing the torque for joint i. Finally, the gradient term retained for the effort minimization is proportional to :

$$\Delta Z_{E_i} = -2 \; m_{ai} \; h_i \; (\theta_i - \theta_{ri}) \qquad (15)$$

Some gradient terms can be locally null whenever their torque vanishes due to the vertical alignment of the G_{ai} and O_i. By construction, all terms vanish only for the rest posture.

3.2.4 Cascaded Control

Kinetic and Kinematic control schemes share a common space, i.e. the joint variation space, thus allowing their integration into more sophisticated architecture as developed in (Boulic et al. 1994b). Extending expression (1) to integrate the center of mass control as a secondary behavior of a classic kinematic control is straightforward and bears some similarity with expression (11) from (Maciejewski et al. 1985).

Such a secondary task is equivalent to ensure the algebraic sum of support moment to be null, *i.e.* to maintain the total center of mass on the vertical line of support. This sub-optimal approach has been identified in (Phillips et al. 1990) but not treated kinetically and globally. In discrete form, the cascaded architecture with an additional level of secondary task is:

$$\Delta\theta = J_e^+ \Delta x_e + (I - J_e^+ J_e)(J_G^+ \Delta x_G + (I - J_G^+ J_G)\Delta z_o) \qquad (16)$$

Where J_e is the Jacobian of the kinematic transformation describing the effector control (for example a reach behavior), J_G is the Jacobian of the kinetic transformation describing the center of gravity control (balance behavior). Here the optimization behavior Δz_o is integrated through a second partitioning level of the joint variation space. It could also directly share the kinematic null space with the kinetic control.

Moreover, the underlying hierarchy of expression (16) can be inverted to favor the center of mass control over the reach behavior :

$$\Delta\theta = J_G^+ \Delta x_G + (I - J_G^+ J_G)(J_e^+ \Delta x_e + (I - J_e^+ J_e)\Delta z_o) \quad (17)$$

3.2.5 Reference Motion Deformation

One nice feature provided by inverse kinematics is the goal-oriented motion specification in cartesian space. Keyframing of the goal parameters is also possible as was demonstrated in (Phillips et al. 1991).

However, unlike Robotics where high level specification of complex tasks is better made in the cartesian space, in Computer Animation the space truly expressing the character of an articulated figure motion is the joint space. It is well known that the motion developped using inverse kinematics is poor in that respect, resembling puppet motion. So it is rarely used to animate characters directly. Rather, it is a valuable tool to define realistic postures for an interpolation tool (Girard et al. 1985) (Chapter Clavert). There are several other important motion sources expressed in the joint space :

- simulation of physically-based motions (Chapter Hégron)
- functional models based on dynamics and keyframing (Bruderlin et al. 1989) or biomechanical studies of specific motion pattern as walking (Boulic et al. 1990)

Furthermore, strong requirements on the motion realism have recently stimulated the development of performance animation systems (Chapter Maiocchi). One challenging task in this domain is to elaborate robust and versatile tools converting raw measurement data into joint trajectories of the performer model. It is easy to understand this interest as motion patterns expressed in joint space can be played back by a reasonable range of virtual human models without noticeable dynamic discrepancies (see (Raibert et al. 1992) on scaling and Dynamics). The second reason comes from the large set of joint motion manipulating tools allowing refinement of the motion (filtering, editing, layering etc.)

However, even the best of converters cannot exactly translate the motion character accurately and precisely in the joint space because all the articulated models of the human being (or other animal) heavily rely on simplifying assumptions. One example among others is the knee joint which has six degrees of freedom biomechanically, while only two rotations are usually retained in computer animation models (Boulic et al. 1994a). Moreover, the parameters of the performer skeleton are never very accurate due to the difficulty in measuring such characteristics. So, from these statements we already know that the motion reproduced on the performer model can be significantly different from its real counterpart. Finally, the computer animation character itself can be radically different from the performer.

All these problems generally have the cascading effect losing the initial cartesian constraints achieved by the performer (e.g. the foot may enter the ground etc.). So, in such a context the reusability of a joint space motion is quite poor. There is a need for tools editing highly coordinated motion while retaining the coordination information at the joint level. This analysis has led to the proposal of combining direct and inverse kinematic control schemes (Boulic et al. 1992). It's purpose is to deform coordinated joint motion by distributing on all joints the motion deformation resulting from cartesian constraints. The secondary task plays the key role in that approach. So, given an original joint motion to edit, let us now review the concepts guaranteeing that objective :

- Tracking a reference joint motion as secondary task :

The basic idea is to consider the original joint motion as a reference motion which tracking is enforced through inverse kinematics secondary task. The simplest tracking approach is realized with an attraction to each successive reference postures (figure 8) and the associated gradient vector is similar to expression (9). The main task simultaneously ensures the realization of desired cartesian constraints over some specified end effector(s).

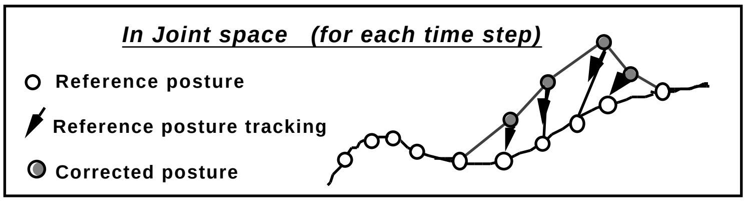


Figure 8 : Reference configuration tracking as a secondary task

- Half-space as a convenient class of Cartesian constraints :

The user is more interested in local reshaping of the motion both in time and space. For this reason half-space constraints (planar, cylindrical, spherical) are preferred as they divide the cartesian space into an "allowed zone" and a "forbidden zone".

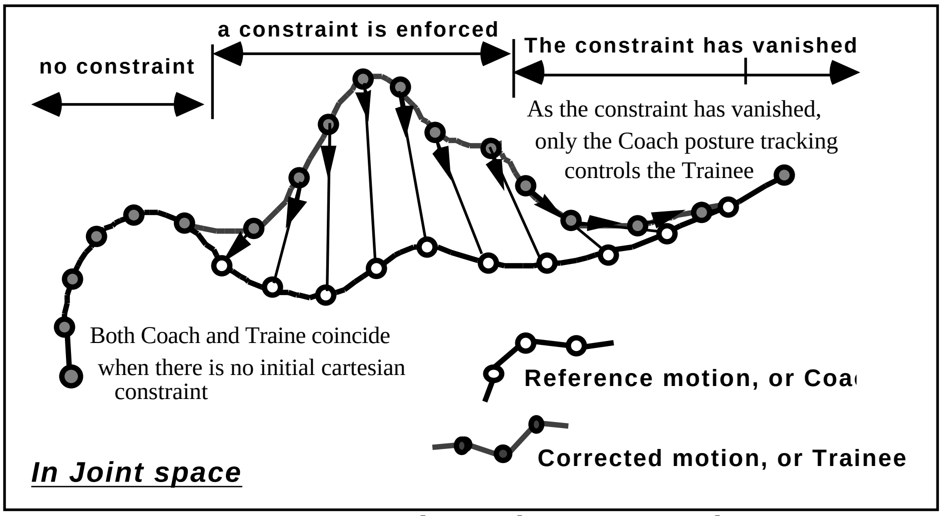


Figure 9 : The *coach-trainee* metaphor

- The Coach-Trainee Metaphor :

In order to ensure a continuity requirement on the corrected motion, the most important concept of this approach is to duplicate the part of the articulated structure which requires motion correction (e.g. a leg, an arm, more). The original articulated structure is simply replicating the reference motion, also called the *coach* motion. It serves as a guide for the duplicated articulated structure supporting the corrected motion, also called the *trainee* motion. This *coach-trainee* metaphor is suggested by sport training as the adaptation of the coach reference movement into a trainee movement. This movement is the closest possible with respect to a different context of body structure and/or Cartesian constraints (figure 9).

- <u>The transition function :</u>

Even with the coach-trainee concept, the solution presents a first order discontinuity at the interface of the half-space constraint. For this reason the boundary of the half-space constraints is given a thickness and a smooth switching is performed on the resulting *transition zone* (using a *transition function* f). Then, the formulation of the combined direct and inverse kinematic control scheme is given by the following equation (one dimensional constraint case) :

$$\Delta\theta = J^{+}(f\Delta x + (1-f)J\Delta z) + (I - J^{+}J)\Delta z \quad (17)$$

with $0 \leq f \leq 1$

f = 1	provides	$\Delta\theta = J^{+}\Delta x + (I - J^{+}J)\Delta z$	(equation (1))
f = 0	provides	Δz	(secondary task only)

When multiple constraints are defined, each constrained dimension is associated with one transition function and the previous scalar term f in (17) becomes a diagonal matrix F containing the transition fucntion. The transition function f has two parameters. First, trainee goal, the relative position of coach with respect to the transition zone. Second, trainee deviation, the relative position of the trainee with respect to its goal. Details can be found in (Boulic et al. 1992).

4 Case Studies

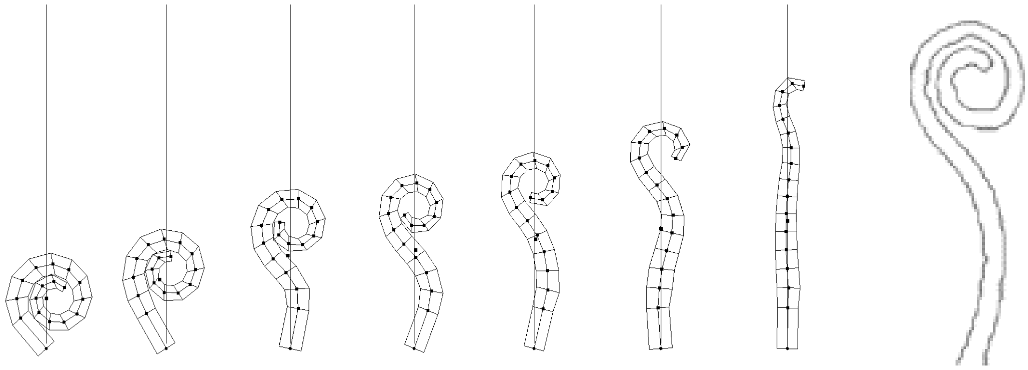
The following case-studies illustrate most of the techniques presented in section 3.2 . Regarding the methods dedicated to automatic posture optimization (from section 3.2.2 to 3.2.4) let us recall that only the model mass distribution information is required in addition to standard inverse kinematics information. This can be derived as a first approximation from a volume distribution or identified roughly from photographs or X-ray multiple views. Conversely, it is very difficult to find data on more complex quantities such as strengths and general joint modeling as required by other approaches. Whenever possible the simulation results are compared with real postures obtained from images.

4.1 Balance Control

First some 2D examples clearly demonstrate the efficacy of the balance control as a secondary task for chain structures (at least fifteen *dof*s). Then a complex 3D human model (tree-structure with at least forty dofs) confirms these approaches as a valid alternative to complex physically-based simulations. The 2D drawing convention reflects the mass distribution of the articulated structure in the following way: a segment S_i is defined between joint i and joint i+1. It has a length L_i and a mass m_i. The idea is to have the segment surface proportional to the segment mass. So its width l_i is proportional to (m_i / L_i). The segment width is displayed only at the end-effector side of the segment in order to draw a continuous envelop reflecting a continous mass distribution.

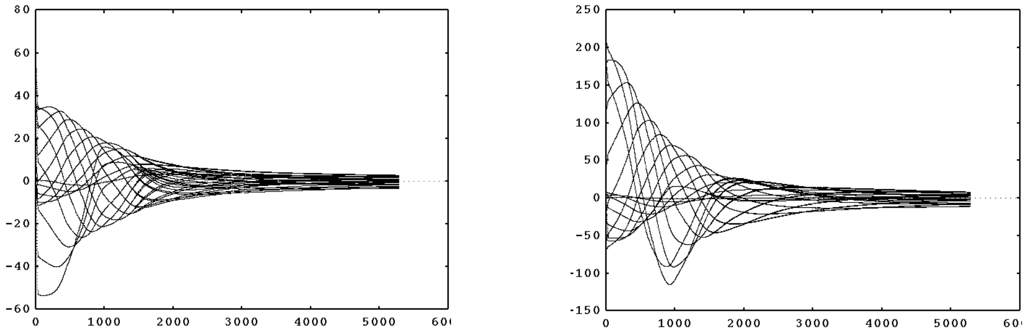
4.1.1 Global Balance Optimization in joint space (2D)

A simple fern "fiddle head" simulation highlights the interest of minimizing the support moments.



a b

Figure 10 : a) Unrolling of a fern with a minimization of the support moments
b) Outline of a real fern "fiddle head", polypodium aspidium (Blossfeldt, 1929)



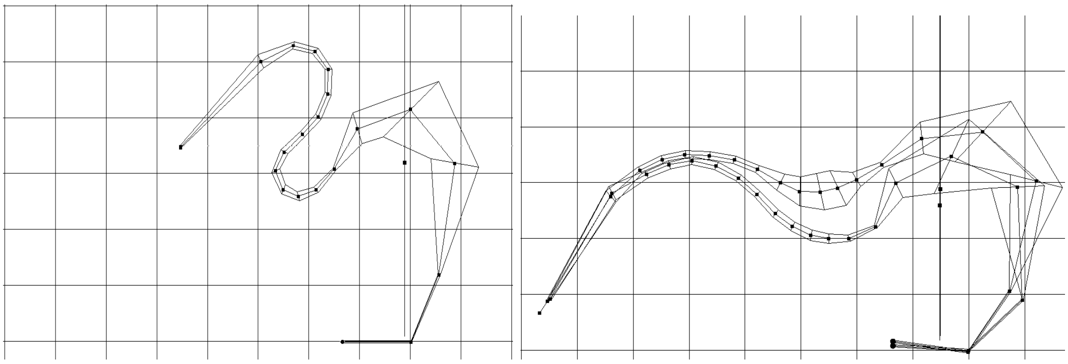
(a) (b)

Figure 11 : evolution of support torques (a) and joint torques (b) for the fern simulation

Although there is no explicit specification to unroll the fern "fiddle head", this motion implicitly derives from the support moment minimization (Figure 10). Here, the center of gravity is moving upward from the combined opening and closing variations of the augmented bodies in order to align their center of gravity on the vertical line of support. One can also notice the slight swaying along the vertical line of support. The torques display these characteristics (Figure 11) while globally decreasing to zero (Boulic et al. 1994b).

4.1.2 Comparison of Cascaded Control with other approaches

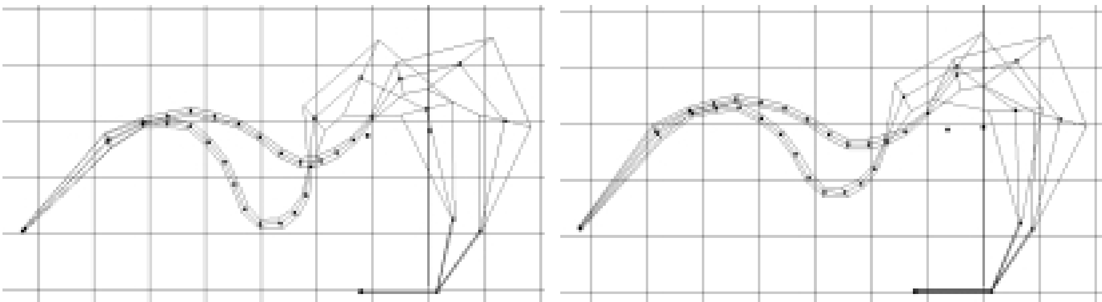
The bird example is of great interest due to some radiographs (see p 79 in (Mc Lelland 1989)) which have been used to identify the initial rest posture of the bird and to validate the posture predicted with cascaded control. Three of the body segments are rigidly connected in this simulation. Their purpose is to adjust the mass distribution so that the center of mass projection lies just in front of the "palm" joint in the rest posture .



a b

Figure 12 : a) rest posture (from a radiograph); Comparison of final drinking postures :
b) real drinking posture (thin neck) and cascaded control (thick neck base)

The cascaded control of inverse kinematics with inverse kinetics as a secondary task is used to ensure a reaching and orienting task of the beak as main behavior (for drinking) while maintaining the balance of the subject as secondary behavior.



a b

Figure 13 : a) Cascaded Control (black) and inverse kinematics alone (gray).
b) Cascaded Control (black) and inverse kinematics with attraction to rest.

Figure 12 shows the initial (real posture) and final stages (real and simulated). The simulated posture has a thicker neck ; this is just to model the mass distribution of the water drunk. As can be seen in figures 13a and 13b the cascaded control is clearly superior regarding the balance control. However, the real drinking posture bears some similarities with the one resulting from the attraction to the rest posture (12a) as also suggested in the Neurobehavioral Morphology literature (Zweers et al. 1994). A complementary simulation would be to integrate the attraction to the rest posture as the secondary task of inverse kinetics, itself being the secondary task of the reach behavior.

4.1.3 Human Posture Optimization

As one could notice in Figure 3, the human posture space is highly redundant for simple reach tasks. We illustrate here how to take advantage of the redundancy to optimize the balance of the human model while carrying complex reach tasks with one or more end effector(s).

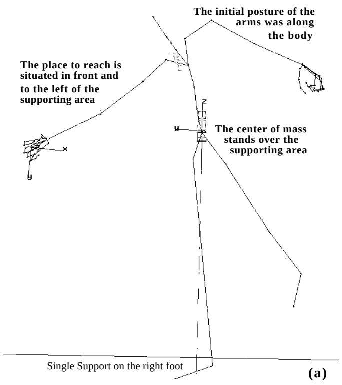


Figure 14 : side view   Figure 15 : front view

The general tree structured human model is rooted at the right foot. The projection of the center of gravity should be in the so-called support polygon made by the foot between the ankle and toe joints. It should always remain in that space to maintain the balance of the whole body.

When the toe joint is locked the so-called vertical line of support is passing through the center of support located a few centimeters in front of the ankle joint.

In the present simulations, the cascaded control is used with priority given to reach behaviors (figure 14, 15, 16).

In the first example, the balance is difficult to achieve because the reach target lies far in front (Figure 14) and on the left side of the body while the support is on the right foot (figure 15). In the second example, three end effectors are attracted by one another to simulate the action of lacing a shoe (Fig. 16).

We suggest the readers to try themselves

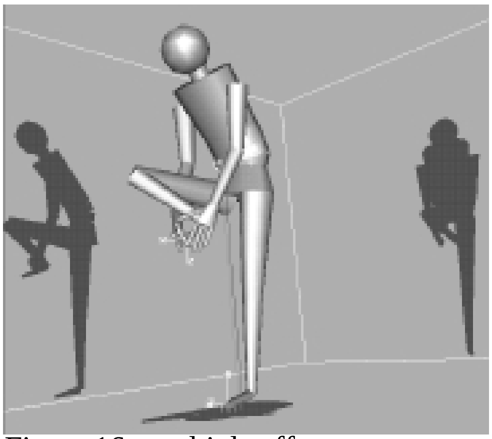


Figure 16 : multiple effector case

4.2 Walking Motion Correction

The Coach-Trainee method for motion correction is illustrated on a walking motion. Unlike other walking motion generators (Girard et al. 85) (Ko et al. 1993) (Bruderlin et al. 1989) the one proposed in (Boulic et al. 1990) is established in the joint space. It relies on normalized trajectories coming from multiple biomechanical studies in order to retain the natural dynamics of the motion. As such it is intrinsically associated with a statistical average of the human articulated structure. So it may happen that the walking motion associated to a specific instance of human skeleton faces some cartesian discrepancies during the support phase (i.e. foot entering the floor). This problem is in fact very common and happens each time one wants to reuse a motion expressed in joint coordinates on a different articulated structure than the one used for performing the motion (or simulating or designing etc.). We recall now two examples from (Boulic et al. 1992a) with a single end effector (figure 17) and two end effectors (figure 19).

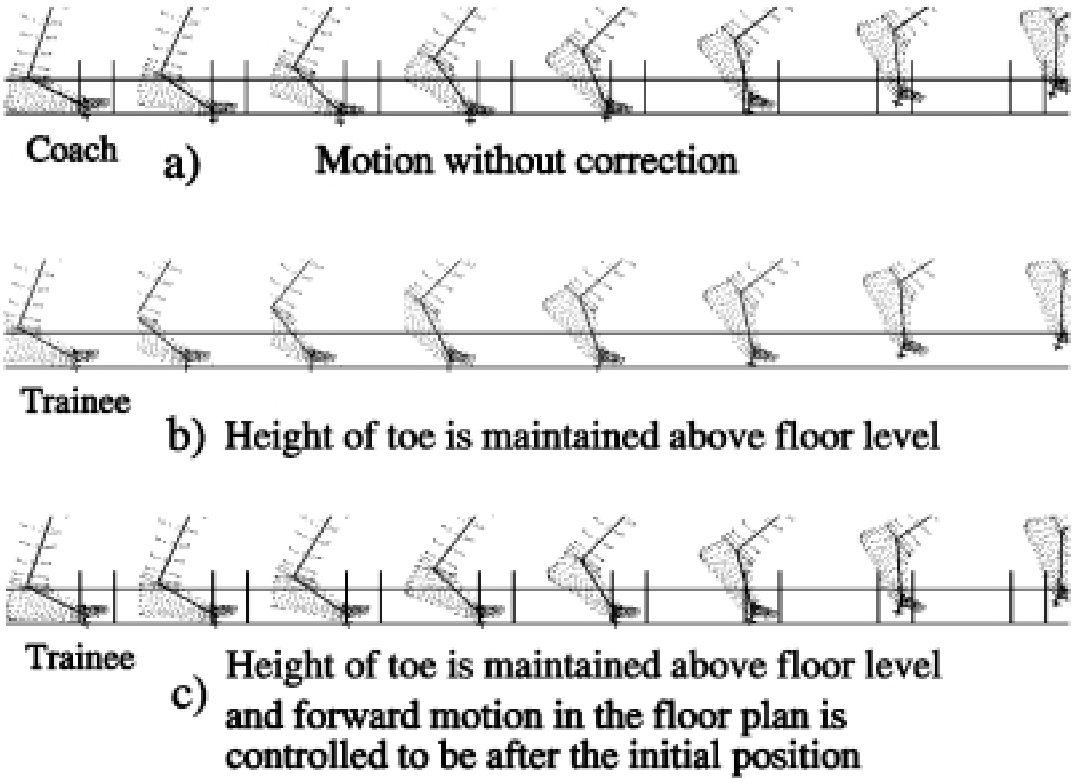


Figure 17 : single end effector with one and two constrained dimension(s) (planar halfspace)

Figure 17a highlights the type of correction which is handled by the method. First the vertical dimension only is corrected resulting in a backward sliding of the foot (17b), then an additional vertical planar halfspace is added to prevent the backward sliding without restraining the forward motion (17c).

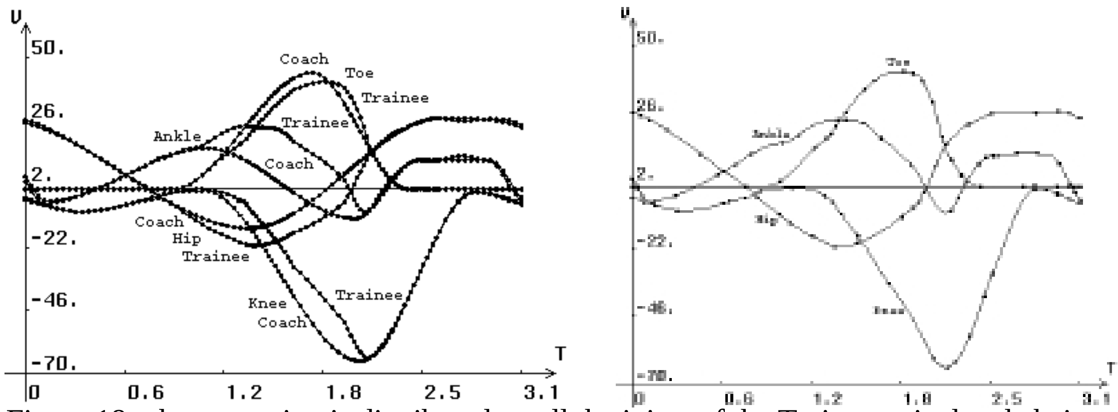


Figure 18 : the correction is distributed on all the joints of the Trainee articulated chain

The correction is distributed on the chain including the hip, knee, ankle and toe flexion-extension joints (figure 18). As can be noticed on these curves, it is preferable to perform small corrections in order to retain the natural dynamics of the initial motion. If larger corrections are required, an incremental methodology is preferred (Boulic et al. 1992b). The curves on the left show the raw sampled data of the coach and the trainee motions ; it is important to reduce the number of samples whithout affecting the shape of curves (Boulic et al. 1992b). Then only it becomes easy to edit and adjust them with standard curve design tools ; the curves on the right show the slight adjustment of the trainee motion after data reduction (Boulic et al. 1994d).

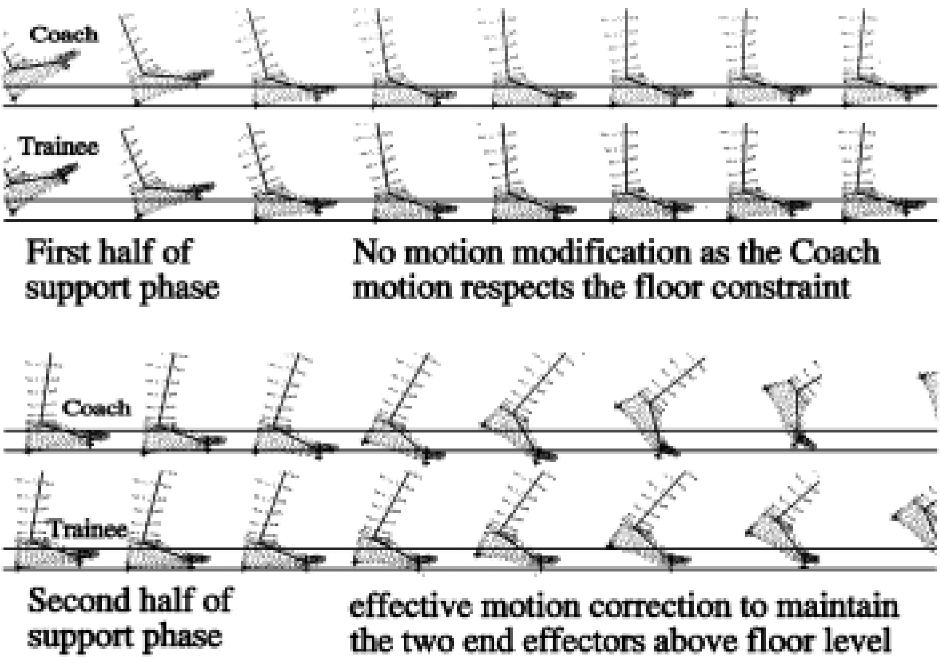


Figure 19 : two end effectors with one constrained dimension (planar halfspace)

Figure 19 shows an example with two end effectors corrected only along the vertical dimension. First we can see that the motion is unchanged as the end effectors do not violate the forbidden zone (19a), then we can evaluate the motion deformation resulting from an effective correction on both effectors simulatneously (19b).

5 Future Directions

Within the present context of complex articulated structures, other secondary tasks have to be proposed in the following directions :

• Strength Optimization : once normalized data on human strength are available for the whole body, it is important to build a comfort performance index based on the human strength in order to obtain more realistic postures (important concepts are introduced in (Lee, 1993) in a dynamic context). Associated with the balance control, such secondary task would greatly enhances Ergonomics studies for workplace evaluation.

• Multiple Support inverse kinetics : human and animals rely most of the time on multiple support to ensure an equibrilium state. Until now the motion is rooted on the dominant support, i.e. the one bearing most of the body weight (Badler et al. 1993) (Boulic et al. 1994). It is highly desirable to be able to control the center of mass position by taking into account how the body mass is distributed on the different supporting sites.

• Enhancing the motion correction : first order correction of the motion with the Coach-Trainee approach show interesting results but the resulting motion still has to be improved with traditional tools. In fact the correction provides useful hints on how the correction should be distributed on the articulated chain and the designer can locally adjust the smoothness of the joint curves. Furthermore, the Coach-Trainee approach can be applied to the inverse kinetics context where the center of mass would be a special case of end effector and the motion would be corrected whenever the posture becomes out of balance.

6 Conclusion

The various examples shown along this chapter clearly demonstrate the interest of inverse kinematics for the posture and motion control of complex articulated structures. The main condition is to associate the pseudo-inverse solution with a homogeneous solution driven by a pertinent secondary task.

Owing to the hierarchical nature of inverse kinematics control we can ensure the realization of the main task while optimizing the secondary task.

Thus we feel that inverse kinematics can be physically grounded while still providing a low cost computational solution for complex articulated structures (forty *dof*s and more). Moreover, using intuitive concepts and variables allow the designer to overcome the great complexity of the posture design of such articulated structure.

7 Acknowledgments

The research was partly supported by the Swiss National Research Foundation and OFES. The authors are grateful to Srikanth Bandi and Serge Rezzonico for improving the style, and to Roberto Maiocchi for fruitful discussions.

8 References

Badler N., Manoocherhi K. and Walters G. (1987) "Articulated Figure Positioning by Multiple Constraints", IEEE CGA, 7(6), pp.28-38

Badler N., Phillips C. and Webber B. (1993) "Simulating Humans, Computer Graphics Animation and Control", Chapter 4 "Behavioral Control", Oxford University Press 1993.

Blossfeld K. (1929) "Art Forms in Nature: Examples from the Plant World Photographed Direct from Nature", New-York E. Weyhe, ISBN 0-486-24990-5, republished by Dover NY

Boulic R. (1986) "Conception Assistee par Ordinateur de Boucle de Commande Referencee Capteurs en Robotique et en Teleoperation", These de Docteur-Ingenieur, IRISA-Universite de Rennes I

Boulic R., Thalmann D and Magnenat-Thalmann N. (1990) "A global human walking model with real time kinematic personification" The Visual Computer, 6(6)

Boulic R. and Thalmann D. (1992a) "Combined Direct and Inverse Kinematic Control for Articulated Figure Motion Editing", Computer Graphics Forum ,2(4), pp.189-202.
-> also reprinted in "Recent Techniques in Human Modeling, Animation, and Rendering", Course notes n°80, SIGGRAPH 93, Anaheim, California

Boulic R. and Thalmann D. (1992b) "TRACK, a Kinematic Goal-Oriented Animation System for Coordinated Editing of Joint-Space Based Motions", Third Eurographics Workshop on Animation and Simulation, Cambrige England, September 5&6th 1992.

Boulic R., Huang Z., Magnenat-Thalmann and Thalmann D. (1994) "Goal Oriented Design and Correction of Articulated Figure Motion with the TRACK System",*Journal of Computer and Graphics,* Vol 18 n°4, pp 443-452, Pergamon Press, October 1994

Boulic R., Balmer D., Huang Z. and Thalmann D. (1994a) "A Stereoscopic Restitution Environment for 3D anaysis of Gait", third Syposium of 3D Analysis of Human Motion, Stockholm, July 1994

Boulic R. and Mas R. (1994b) "Inverse Kinetics for Center of Mass Position Control and Posture Optimization", Technical Report 94/68, Computer Sciences Department, EPFL, DI-LIG, Switzerland

Boulic R., Mas R. and Thalmann D. (1994c) "Inverse Kinetics for Center of Mass Position Control and Posture Optimization", Race Workshop on "Combined real and synthetic image processing for broadcast and video production (Monalisa Project)", Hamburg, 23-24 November 1994. Y. Parker & S. Wilbur (Eds.), Workshop in Computing Series, Springer-Verlag, ISBN 3-540-19947-0

Boulic R., Huang Z., Magnenat-Thalmann and Thalmann, D. (1994d) "Goal Oriented Design and Correction of Articulated Figure Motion with the TRACK System",*Journal of Computer and Graphics,* Pergamon Press, 18 (4), pp. 443-452

Boulic R., Capin T., Kalra P., Lintermann B., Moccozet L., Molet T., Huang Z., Magnenat-Thalmann N., Saar K., Schmitt A., Shen J. and Thalmann D. (1995) "A system for the Parallel Integrated Motion of Multiple Deformable Human Characters with Collision Detection", EUROGRAPHICS' 95, Maastricht.

Calvert T. (1991) "Composition of Realistic Animation Sequences for Multiple Human Figures" in "Making Them Move: Mechanics, Control, and Animation of Articulated Figures", Badler, Barsky & Zeltzer Editor, Morgan Kaufmann, pp.35-50

Cleary K. and Tesar D. (1990) "incorporating multiple criteria in the operation of redundant manipulators", IEEE Conf. RA Vol 1, pp. 618-624

Dombre E., Fournier A., Quaro C. and Thevenon J.B. (1985) "Design of a CAD-CAM system for robotics on a microcomputer", proc. of 3rd Int. Symposium on Robotic Research Gouvieux, France

Espiau B. and Boulic R. (1985) "Collision Avoidance for Redundant Robots with Proximity Sensors", 3rd Int. Symposium on Robotics Research, Gouvieux, France

Gréville T.N.E (1960) "Some application of the pseudo-inverse of a matrix", SIAM review 2(1)

Girard M. and Maciejewski A.A. (1985) " Computational Modeling for the Computer Animation of Legged Figures". Computer Graphics 19 (3), pp.263-270

Girard M.(1987) "Interactive Design of 3D Computer-Animated Legged Animal Motion" , IEEE CGA , 7(6)

Hodgins J.K. and Raibert M. (1991) "Adjusting Step Length for Rough Terrain Locomotion", IEEE Trans. on Robotics and Automation, 7(3), pp 289-298

Jouffroy F.K., Niemitz C. and Stack M.H. (1990) "Nonhuman Primates as a Model to Study the Effect of Gravity on Human and Nonhuman Locomotor Systems", in "Gravity, Posture and Locomotion in Primates", edited by Jouffroy, Stack and Niemitz, "Il Sedicesimo" Firenze, Italy

Klein C.A. and Huang C.H. (1983) "Review of Pseudo-Inverse Control for use with Kinematically Redundant Manipulators", IEEE Trans. on SMC 13(3)

Klein C.A. (1984) "Use of redundancy in the design of robotic systems, Proc. of second Int. Symposium of Robotic Research, H. Hanafusa and H. Inoue, Eds. Cambridge, MA:MIT Press, 1985, pp.207-214

Klein C.A. and Chirco A.I. (1987) "Dynamic simulation of a kinematically redundant manipulator system", Journal of Robotic Systems, 4(1), pp.5-23

Ko H. and Badler N.I. (1993) "Intermittent Non-Rhythmic Human Stepping and Locomotion", First Pacific Conference on Computer Graphics and Applications, Seoul

Kuo A.D. and Zajac F.E. (1993) "Human Standing Posture : Multi-Joint Movement Strategies Based on Biomechanical Constraints", in "Progress in Brain Research", Vol 97, Allum, Allum-Mecklemburg,Harris&Probst editors, Elsevier Science Publishers

Lee P., Wei S., Zhao J. and Badler N.I. (1990) "Strenght Guided Motion", Computer Graphics 24 (4), pp253-262

Lee P.L.Y. (1993) "Modeling Articulated Figure Motion with Physically- and Physiologically-based Constraints", PhD Dissertation in Mechanical Engineering and Applied Mechanics, University of Pennsylvania

Liégeois A. (1977) "Automatic Supervisory Control of the Configuration and Behavior of Multibody Mechanisms" IEEE Trans SMC, 7(12), pp 868-871

Maciejewski A.A. and Klein C.A. (1985) "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environment", Int. Journal of Robotic Research, 4(3), pp.109-117

Maciejewski A.A. (1989) "Kinetic Limitations on the Use of Redundancy in Robotics Manipulators", Proc. of IEEE Conf. on Robotics and Automation, pp 113-118

Maciejewski A.A. (1990) "Dealing with Ill-Conditioned Equations of Motion for Articulated Figures", IEEE CGA 10,3, pp 63-71

Mac Lelland J. (1989) "Form and Function in Birds" vol 4, King & Mac Lelland Editors, Academic press Publisher, London, pp.79.

Magnenat-Thalmann N. and Thalmann D. (1990) "Synthetic Actors in 3D Computer Generated Films", Springer-Verlag, New-York

Maiocchi R. (1991) "A Knowledge based approach to the Synthesis of human motion", IFIP TC5/WG5.10 "Modeling in Computer Graphics", Tokyo.

Monheit G. and Badler N. (1991) "A Kinematic Model of the Human Spine and Torso" IEEE CGA 11(2), pp.29-38

Paul R. (1981) "Robot manipulators mathematics programming and control", MIT Press

Philips C.B., Zhao J. and Badler N.I., (1990) "Interactive Real-Time Articulated Figure Manipulation Using Multiple Kinematic Constraints", Computer Graphics 24 (2),pp 245-250

Phillips C.B.and Badler N. (1991) " Interactive Behaviors for Bipedal Articulated Figures" Computer Graphics 25 (4), pp 359-362

Press W.H., Teukolski S.A., Wetterling W.T. and Flannery B.P. (1992) "Numerical Recipees in C", Cambridge Press, second edition

Raibert M.H. and Hodgins J.K. (1992) "Animation of Dynamic Legged Locomotion", Computer Graphics 25 (4), pp 349-358

Whitney D.E. (1969) "Resolved motion rate control of maniplators and human prostheses" IEEE Trans. Man-Machine Systems MMS-10, pp.47-53

Won J.H., Choi B.W. and Chung M.J. (1993) "A unified approach to the inverse kinematic solution for a redundant manipulator", Robotica Vol 11, pp.159-165

Yoshikawa T. (1985) "Manipulability of robotics mechanism", Int. Journal of Robotic Research, 4(2)

Zeltzer D. and Sims K. (1988) "A Figure Editor and Gait Controller to Task Level Animation", SIGGRAPH 88 Tutorial Notes on Synthetic Actors : The impact of A.I. and Robotics on Animation

Zweers G., Bout R. and Heidweiller J. (1994) "Motor Organization of the Avian Head-Neck System", Davies M.N.O. and Green P.R. (Eds.) "Perception and Motor Control in Birds", Springer Verlag Berlin Heidelberg, pp.201-221