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Serially Concatenated Coding for Broadcasting S-UMTS Applications

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Abstract – **Satellite-UMTS supports broadcast applications that involve transmission of the same encoded data over channels that may vary significantly. The same code must allow a user with a good channel to recover the information with low complexity, while a user with a bad channel should still be able to achieve an acceptable BER at the cost of increased complexity and/or decoding delay.**

To this end, we propose serially concatenated multilevel code structures that employ PSK modulation. The receiver has the flexibility to achieve turbo-code trellis-code or uncoded performance, depending on the decoding effort. Design considerations include the constituent encoder design and the use of a non-uniform constellation. Simulation results investigate the system's performance and highlight different parameters trade-offs.

I. INTRODUCTION

Broadcast applications involve transmission of information data encoded with the same channel code over channels that may significantly vary, depending on the end-user location. Even for point-to-point applications, mobility implies that the same user may come across different channels during a session, for example when a vehicle goes under a tunnel.

The Satellite-UMTS (S-UMTS) environment provides a direct application for a broadcasting coding scenario. S-UMTS supports 3G services that require a specific quality of service, which translates to Bit Error Rate (BER) upper-limits for the channel coding to meet, independently of the channel fading conditions. Moreover, in a satellite broadcast environment, feedback or retransmission of packets is either not generally possible or may involve excessive delay; the system has to typically rely solely on Forward Error Correction (FEC).

Coding may be designed so as to address the worst-case fading scenario, which leads to unnecessary receiver processing complexity for the vast majority of cases. Alternatively, coding may address an average fading scenario, which cannot provide a hard guaranty for the desired quality of service (QoS) level. Ideally, coding should allow a user with a good channel to recover the information with low complexity, while a user with a bad channel should still be able achieve an acceptable BER at the cost of increased complexity and/or some extra decoding delay. However, the transmitter can only employ a single channel code which is simultaneously broadcasted to all users; thus, the same channel code has to be able to provide the capability of soft trade-off in the processing complexity-achievable BER sense as a function of receiver decoding.

Finally, even if degradation due to severe fading cannot be completely avoided, it is desirable that the system performance follows an SNR-BER curve with smaller (smoother) slope than most codes naturally provide; in other words, a softer performance degradation. For example, for turbo codes the BER curve is characterized by a very abrupt waterfall area, where the BER drops orders of magnitude within tenths of a dB. It is undesirable for a user to observe such a huge performance difference (and concomitant QoS change) over so slight a channel variation.

With these considerations in mind, this paper proposes a serially concatenated coding scheme in the context of multilevel-coding. This technique was introduced in [1] as a method to realize coded modulation. Since then the field has received considerable attention and has enjoyed numerous applications (for a good review and bibliography list see, for example, [2]).

The performance improvement turbo codes can offer to multilevel coded systems has been previously investigated in the literature [2]. However, typically the performance of a generic turbo code is typically investigated, and turbo encoder design a the specific application is not considered.

This paper is organized as follows: Section II introduces and motivates the proposed structure. Section III addresses the design of each part of the concatenated code. Section IV presents simulation results, and finally Section V concludes the paper.

II. SYSTEM STRUCTURE

Fig. 1 depicts the proposed coding scheme structure, which employs a serially concatenated turbo code with 8-PSK modulation. For satellite applications, constant-

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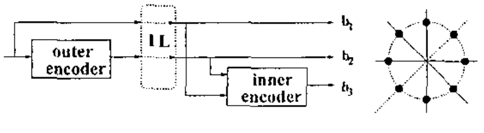

Fig. 1. Serially concatenated code structure employing 8-PSK.

amplitude modulation is desirable to avoid non-linear amplifier effects. However, the extension to different constellations is straightforward.

Another possible coding structure in the same context is presented in Fig. 2. In this case, each multilevel symbol consists of two 4-PSK symbols. Decoding only the convolutional code amounts to ignoring (punturing at the receiver) the second of the received symbols. The design of this code can be performed along the same lines of the code in Fig. 1; thus, it is not discussed here in a detail. For the rest of the paper, unless explicitly stated otherwise, we refer to the structure of Fig. 1.

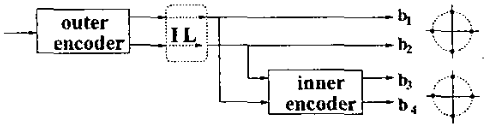

Fig. 2. Serially concatenated code structure employing 4-PSK.

The outer encoder is rate $1/2$ systematic and encodes the constellation-labeling Most Significant Bits (MSB) b_1 and b_2. The inner encoder is rate $2/3$ systematic, with the parity output encoding the Least Significant Bit (LSB) b_3. Thus, three levels of decoding are possible that achieve uncoded performance (decode only the MSB bit b_1), convolutional code performance (decode the two MSB bits b_1 and b_2) and turbo code performance (decode all three bits), respectively. The spectral efficiency is1 bit/sec/Hz for all levels of decoding.

A serially concatenated structure is proposed because for both the outer encoder of a serially concatenated code, and the simple convolutional code, the performance is determined by the output sequence characteristics, i.e., in both cases the design criteria maximize the output free distance. Thus, the same encoder may be optimized for this dual role without conflict. This is not the case for parallel concatenated turbo codes where the constituent encoder mapping from input to output sequences greatly affects the turbo code performance.

The interleaver in Fig. 1 performs symbol interleaving. Density evolution chart comparisons in [3] indicate that symbol-interleaved serially concatenated convolutional codes (SCCC) can convergence at a lower SNR than bit-interleaved SCCC. The same interleaver can play the role of a channel interleaver for the convolutional code.

The structure of Fig. 1 allows graceful incorporation with existing standards. Indeed, by using the standard's convolutional code as the outer encoder, the existing receivers would still be able to operate.

Finally, uncoded performance may be useful for such applications as fast tuning to different broadcasted satellite programs and zapping [4].

III. DESIGN CONSIDERATIONS

The design of the inner and outer constituent encoder needs to optimize the performance of both the serially concatenated code and the incorporated convolutional code.

A. Symbol Interleaving

The use of a symbol interleaver implies that the constituent encoders should be optimized for symbol-wise distance properties. An analytical upper bound to the bit error probability in [5] derived design guidelines for the SCCC constituent encoders. Repeating the analysis for symbol-wise input along the lines of [5], the following guidelines [6] for the symbol-interleaved case can be derived:

- the outer constituent encoder should be optimized for output symbol-wise Hamming distance, or in other words, effective code length [7].
- the inner constituent encoder should have infinite symbol-wise impulse response[1] $d_{s1}^H = \infty$, and be optimized for symbol effective distance (Hamming d_{s2}^H or Euclidean d_{s2}^E depending on the application).

B. Convolutional Code

The convolutional code decoder employs only the two MSB bits b_1 and b_2, without any information about the LSB bit b_3. In fact, because of the large interleaver, the distribution of b_3 can be uncorrelated from b_1 and b_2 with uniform distribution.

Thus, the convolutional code employs a 4-point modulation, where each transmitted value will be represented

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[1] We use the following notation for distance d_*^*: the superscript refers to the output distance, Hamming (H) or Euclidean (E). The number in the subscript denotes the input weight, whether bit-wise (b) or symbol-wise (s). We always imply squared Euclidean distance. For example, d_{s2}^E stands for the output squared Euclidean distance when the symbol-wise input weight is two.

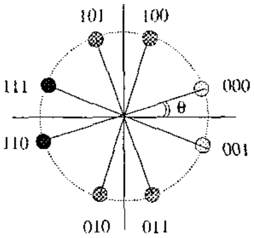

Fig. 3. Non-uniform 8-PSK constellation, described by the angle θ, $0 \leq \theta \leq \pi/8$.

by one of two points with equal probability. The uncertainty involved, as well as the reduced minimum inter-constellation distance, incur a performance degradation, as compared to employing the regular 4-PSK constellation.

The performance loss can be reduced by using a non-uniformly spaced 8-PSK constellation, where points corresponding to the same two MSB bits are clustered closer together, as Fig. 3 illustrates. The angle θ allows to describe all the non-uniform constellations between 4-PSK for $\theta = 0$, and uniformly-spaced 8-PSK for $\theta = \pi/8$.

To assess the expected convolutional code performance when employing the two MSB bits of the non-uniform 8-PSK constellation, Fig. 4 plots the cut-off rate for different angles θ. Note that even a small reduction of θ from $\pi/8$ to $3\pi/32$ can make a difference, as the simulation results in Section IV will also demonstrate.

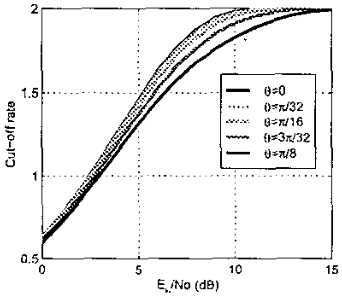

Fig. 4. Cut-off rate for different angles θ in Fig. 3, when employing only the two MSB bits of the constellation.

C. Serially Concatenated Code

Convolutional code performance is usually compared against cut-off rate, while concatenated codes are typically compared against capacity. Using a non-uniform constellation affects adversely the capacity of the 8-PSK constellation, as Fig. 5 indicates. However, the loss is not significant for a small reduction in the angle θ.

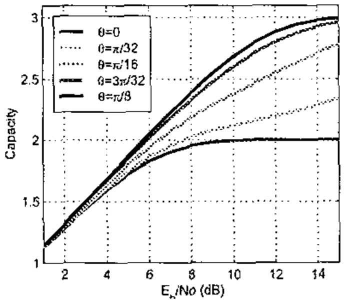

Fig. 5. Capacity for different angles θ in Fig. 3, when using the information of all three bits of the constellation labeling.

C.1 Inner encoder

The inner encoder is designed to have infinite symbol-wise impulse response, and maximize the squared Euclidean distances d_{s2}^E, d_{s3}^E, etc, taking into account the non-uniform constellation shape.

Using a fully systematic inner encoder fully systematic makes possible the separate convolutional code decoding. Moreover, as is observed in the literature [8], systematic bits contribute to an early turbo code convergence.

C.2 Outer encoder

According to the asymptotic code design criteria, the outer encoder should be maximized for output distance. The maximum output Hamming distance a code can achieve increases with the code memory. Thus increasing the outer code number of memory elements and free distance offers a good asymptotic performance for the serially concatenated code, and, moreover, improves the convolutional code performance.

However, the larger the number of memory elements and corresponding free distance of the outer code, the higher the SNR that the serial concatenated turbo code converges. This effect has been observed in the literature [9], and is discussed in [6].

In other words, a small free distance outer code leads to a wider SNR gap between the concatenated and convolutional code performance. In this paper we opt for such an outer encoder, for the following reason: because our system has the capability of shifting between the two curves the gap between them does not affect the overall system performance. Also, smaller number of memory elements leads to reduced decoder complexity.

IV. SIMULATION RESULTS

This section presents simulation results for the multilevel concatenated code over AWGN channel.

A. 8-PSK serially concatenated code

We consider the structure in Fig. 1 with the constellation labeling depicted in Fig. 3.

For the simulations, we use a feedforward outer encoder with $m = 3$ memory elements and generator polynomials $\{01,\ 017\}$ in octal notation. This code achieves effective code length four and output Hamming distance four, which is the upper limit for a systematic code of this complexity.

The inner encoder is symbol-wise recursive systematic with $m = 3$ memory elements. Both the constituent encoders were identified through exhaustive search. The SCCC employs a semi-random symbol-interleaver of length 1000 symbols, with spread parameters $(10, 0, 1)$ as described in [10]. All the following simulation results encode the information with the full serial concatenated code, but employ different decoders.

Fig. 6 depicts the BER performance achieved when decoding only the outer encoder for different angles θ in Fig. 3.

TABLE I

CODES OPTIMIZED FOR SQUARED EUCLIDEAN EFFECTIVE DISTANCE, RATE 2/3 SYSTEMATIC, M=3 MEMORY ELEMENTS

enc	θ	polynomials	ds_2
C1	$\pi/8$	$\{013,\ 01,\ 07,\ 07,\ 00\}$	4.58579
C2	$3\pi/32$	$\{015,\ 02,\ 07,\ 02,\ 03\}$	4.33706
C3	$\pi/16$	$\{015,\ 01,\ 03,\ 06,\ 03\}$	4.15224

Fig. 7 depicts the serial concatenated code performance, when employing non-uniform 8-PSK constellation with different θ. The solid line corresponds to six decoder iterations and the dotted line to twelve decoder iterations. For each θ a different inner encoder was identified through exhaustive search. Table I summarizes the inner encoders generator polynomials and distance characteristics. The encoder polynomials are described according to the notation in [10].

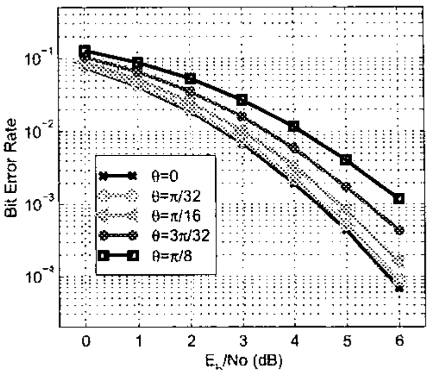

Fig. 6. BER performance achieved when decoding only the outer encoder for different angles θ in Fig.

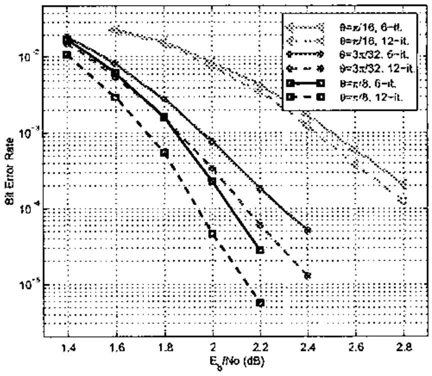

Fig. 7. BER performance achieved by the serially concatenated code for different angles θ, with an interleaver of length 1000.

Fig. 8 plots the performance of all three levels of decoding for $\theta = 3\pi/32$. For the serially concatenated code we plot the BER achieved with two, four, six and twelve decoder iterations. The uncoded performance is included. The straight dotted lines indicate an example performance-complexity trade-off curve that a specific application may select.

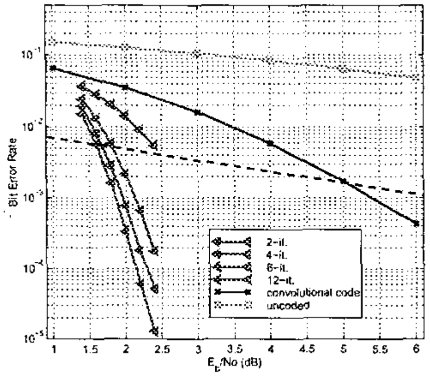

Fig. 8. BER performance of all three decoding levels for $\theta = 3\pi/8$, and interleaver length 1000.

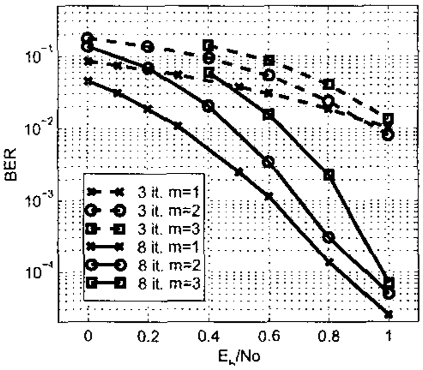

Fig. 9. BER performance of the system in Fig. 2, with inner encoders of different free distance and interleaver of length 1000.

B. 4-PSK serially concatenated code

TABLE II

CODES OF RATE 1/2 THAT ACHIEVE MAXIMUM OUTPUT HAMMING DISTANCE FOR THEIR NUMBER OF MEMORY ELEMENTS

m	polynomials	d^H_{free}
1	{02, 03}	3
2	{05, 07}	5
3	{015, 017}	6

Fig. 9 demonstrates the performance (and the effect of the outer encoder free distance) to the system of Fig. 2. The inner encoder has generator polynomials {013, 01, 07, 07, 00} (notation as described in [10]) and achieves $d_{s2} = 4.586$. The employed outer convolutional encoders, presented in Table II achieve the upper free distance bound for their number of memory elements. The behavior of the code performance as a function of the free distance is analyzed in [6]. The dotted and solid lines correspond to three and eight decoder iterations respectively. The interleaver length is 1000 symbols, and the spectral efficiency 1/2 bits/sec/Hz.

V. CONCLUSIONS

This paper proposed serially concatenated multilevel coding schemes for broadcasting applications, that allow the encoder to transmit at the same information rate, while independently each decoder decides its level of decoding as a function of its own channel conditions. System design parameters were discussed, and the performance was evaluated through simulation results.

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