

A new print-based security strategy for the protection of valuable documents and products using moiré intensity profiles

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ABSTRACT

In response to the ever increasing need for new anticounterfeiting methods, a new integrated print-based authentication and security strategy is proposed for the protection of valuable documents and products, based on moiré intensity profiles. This strategy combines the advantages of microstructure artistic screening for the halftoning of black-and-white or color images, the application of specially designed mathematical transformations on the microstructure, and the possibility of using non-standard color separation for printing the microstructured image with non-standard custom inks. Using our moiré-based method, the overt (or covert) microstructured information from which the halftoned image is composed can be made clearly visible in the form of black-and-white or full-color moiré intensity profiles with varying displacements, magnification rates and orientations, generating an attractive, dynamic visual effect which is very difficult to counterfeit.

Keywords: print-based security features, document security, moiré intensity profiles, microstructure, artistic screening, geometric transformations, non-standard color separation.

1. INTRODUCTION

Counterfeiting of security documents such as banknotes, checks, certificates, travel documents etc. is becoming now more than ever a serious problem, due to the availability of high-quality and low-priced color photocopiers and desk-top publishing systems. The same is also true for valuable products such as CDs, DVDs, software packages, medical drugs, etc., that are often marketed in easy to counterfeit packages.

Various sophisticated means have already been introduced in the past for counterfeit prevention and for authentication of documents or valuable articles. Some of these means are clearly visible to the naked eye and are intended for the general public (overt features), while other means are hidden and only detectable by the competent authorities, or by automatic devices (covert features). Some of the already used anti-counterfeit and authentication means include the use of special paper, special inks, watermarks, micro-letters, security threads, holograms, etc. Nevertheless, there is still an urgent need to introduce further security elements, which do not considerably increase the cost of the produced documents or goods.

In response to this need, we present a novel print-based security and authentication strategy using moiré intensity profiles, which combines our know-how and experience in several highly specialized domains: Artistic screening, which enables the generation of high quality halftoned black-and-white or color images using halftone screens made of varying high-definition artistic microstructure-elements with any desired shapes and colors; authentication by means of moiré intensity profiles which clearly reveal this microstructure (be it overt or covert) and make it visible to the unaided eye; the use of mathematically transformed microstructures to further increase the security offered by the method; and the possibility of generating the microstructured image with non-standard inks by using our non-standard color separation method, in order to prevent reproduction of the document by standard CMYK inks. The different aspects of our global security strategy are presented in detail in the sections which follow. In Sec. 2 we describe the authentication of documents bearing specially designed microstructured images by means of moiré intensity profiles, which is the core of our security strategy. In Sec. 3 we explain how such microstructured images, either black-and-white or in full color, can be generated by our artistic screening

methods. In Sec. 4 we show how mathematically controlled geometric transformations can be applied to the microstructure as an additional level of security. In Sec. 5 we present the further advantages obtained by generating the microstructured images using our non-standard color separation method, and by printing them with non-standard inks. And finally, we present our main conclusions in Sec. 6.

2. AUTHENTICATION BY MEANS OF MOIRÉ INTENSITY PROFILES

Moiré effects have already been used in the past for the authentication of documents. For example, a well-known moiré-based method consists of printing on the original document some special repetitive patterns (screen traps) that generate a strong moiré effect when the document is counterfeited by means of halftone reproduction.¹ Similar methods are also applied to the prevention of digital photocopying or digital scanning of documents.² In all these cases, the presence of moiré patterns indicates that the document in question is counterfeit. Our method, on the contrary, takes advantage of the intentional generation of a moiré pattern whose existence, and whose precise shape, are used as a means of authenticating documents.

Our method³ is based on the moiré intensity profiles which are generated between two or more specially designed periodic dot-screens, at least one of which being a microstructured halftone image printed on the document itself. Each of these periodic dot-screens consists of a lattice of tiny dots, and is characterized by three parameters: its repetition frequency, its orientation, and its dot shapes. These periodic dot-screens are similar to dot-screens which are used in classical clustered-dot halftoning, but they have specially designed dot shapes, frequencies and orientations. When two properly designed black-and-white or color dot-screens are laid on top of each other, there appears in the superposition a highly visible repetitive moiré pattern of a predefined intensity profile shape and color (see Fig. 1), whose size, location and orientation are not fixed, but rather gradually vary as the screens are rotated or shifted on top of each other. As an example, this repetitive moiré pattern may consist of any predefined letters, digits or any other preferred symbols (such as the country emblem, the currency, etc.), either in black-and-white or in color. The theoretical explanation of this surprising phenomenon has been presented in detail, along with numerous illustrative figures, in Refs. 4, 5 and in Chapters 4 and 9 of Ref. 6. For the needs of our present application we only consider here the simplest case, in which the moiré intensity profile obtained in the superposition of periodic dot screens is a magnified, rotated and translated version of the cyclical convolution of the periods of the superposed screens (see two schematic examples in Fig. 2). As for the origin of the term “moiré intensity profiles”, see Ref. 6, Sec. 4.1.

It should be noted that the screen which is printed on the document itself (called the *basic screen*) needs not be of a constant halftone level. On the contrary, it may include dots of gradually varying sizes and shapes, and thus constitute a full, real halftoned image with varying tone levels (such as a portrait, a landscape, etc.), either in full gray levels or in color (see Fig. 3). Such a basic screen, made of halftone dots of varying sizes and microstructure shapes, can be generated using one of the techniques described in Sec. 3 below. The second screen (called the *master screen*) may typically comprise a dot-screen or a pinhole screen with the same frequency as the basic screen, whose dots consist of tiny openings on a dark, relatively opaque background. When an appropriately designed master screen is superposed on the corresponding basic screen, highly visible moiré intensity profiles become visible inside the image, showing a largely magnified version of the shapes and colors of the microstructured halftone dots from which the basic screen image is composed (see Fig. 4). The size and orientation of the moiré intensity profiles gradually vary as the master screen is rotated on top of the basic screen, and when the master screen is slowly shifted they scroll throughout the image. It should be mentioned that unlike in other moiré-based methods, where the shape of the moiré effects is determined by a fixed latent image that is physically located on the document, no latent images exist in our method, and all the spatial information which is made visible by the moiré intensity profiles is encoded in the specially designed forms of the individual dots from which the dot-screens are composed. As a consequence, unlike in the other methods where the moiré induced shape is fixed and delimited by the borders of the predefined latent image, in our method the moiré intensity profiles are not fixed, and they gradually vary (move, rotate, or become bigger or smaller) as the screens are shifted or rotated on top of each other, generating an attractive dynamic visual effect. Furthermore, unlike in the other moiré-based methods, where any slight shift of the revealing device causes a foreground / background color inversion (for example: Swiss flag / Red-Cross flag), all colors in our method remain stable and constant (in the example above: a Swiss flag remains a Swiss flag throughout). Another significant advantage of our method is its much larger angular tolerance: our moiré intensity profiles remain clearly visible within a range of more than

$\pm 10^\circ$, making them much more easy to observe than in other methods where the moiré effect is only visible when the revealer is superposed precisely at the specified angle.

The present method can be used in many different variants; a few variants of particular interest are mentioned here for the sake of illustration. In a first variant of our method, only the basic screen appears on the document itself, and the master screen is produced on a separate film that serves as a revealer. The master screen is superposed on the basic screen by the person or the apparatus that visually or optically validates the authenticity of the document, and the moiré effects in the superposition are observed by transmission, i.e. against a source of light. In another particularly attractive variant, the moiré intensity profile shapes can be visualized by superposing a basic screen and a master screen which are both located on two different areas of the same document. This way we obtain what is called a “self authenticating document”, i.e. a document which contains not only a hidden security feature, but also the decoder that is required to make it visible.⁷ Thus, when using this variant it is no longer necessary to carry a separate revealing device for authenticating such documents, and the man in the street can authenticate the document (a banknote, etc.) by simply folding it over itself and holding it against any source of light, such as an open window, the sky, an artificial light source, etc. In a third variant of our method, the master screen is a sheet of microlenses. This variant applies equally well to both transparent support, where the moiré is observed by transmission, and to opaque support, where the moiré is observed by reflection.

The variants mentioned above may be basically considered as overt security features. However, covert versions of our method can also be designed, in which the information carried by the basic screen is masked using any of a variety of techniques, so that even an inspection under a microscope or a strong magnifying glass would not reveal this hidden information. This can be done, for example, by introducing perturbations into the microstructure used to generate the basic screen, by means of mathematical, statistical or logical Boolean operations. If the inserted perturbation is non-periodic, or if it is periodic but with a different period or orientation than the basic screen, this masking effect will not hamper the appearance of the moiré intensity profiles when the master screen is superposed, because of the averaging effect of the cyclical convolution. However, this masking effect will prevent the visualization of the information carried by the dot shapes of the basic screen without using the master screen, for example by a mere inspection of the document under a microscope (see Fig. 5). It should be noted that since masked basic screens are generated by a computer program, they can be made so complex that even professionals in the graphic arts cannot re-engineer them without having the original computing programs specially developed for creating them.

As it is well known in the security printing community, microprinting remains one of the most effective security elements in security documents, because of the difficulty in reproducing it by photocopiers or desk-top publishing systems. However, this robust security feature is not very useful for the man in the street, because its verification requires a strong magnifying glass or a microscope.⁸ Using our moiré-based method this problem can be eliminated by choosing the second variant described above, where the periodic microstructure incorporated into the image printed on the document is revealed by a master screen which is located on a different area of the same document, and which serves as an integrated “virtual microscope”.

The fact that moiré effects generated between superposed dot-screens are very sensitive to any microscopic variations in the screened layers makes any document protected using the present method very difficult to counterfeit, and serves as a means to distinguish easily between a real document and a counterfeited one. Any attempt to counterfeit a document produced using our method by photocopying, by means of a desk-top publishing system, by a photographic process, or by any other counterfeiting method, be it digital or analog, will inevitably corrupt (even if slightly) the size or the shape of the tiny screen dots of the basic or master screens comprised in the document (for example, due to resampling, dot-gain, ink-propagation, etc.). But since moiré effects between superposed dot-screens are very sensitive to any microscopic variations in the screens, this makes any document protected by the present method very difficult to counterfeit, and serves as a means to distinguish between a real document and a counterfeited one (compare Figs. 4(a) and 4(b)). Furthermore, unlike previously known moiré-based anticounterfeiting methods, which are mainly effective against counterfeiting by digital equipment (digital scanners or photocopiers), the present method is equally effective in the cases of analog or digital equipment.

It should be noted that the dot-screens which appear on the document itself may be printed on the document like any screened (halftoned) image, within the standard printing process, and therefore no additional cost is incurred in the document production.

Finally, a similar method can be also used for the protection of valuable products such as CDs, DVDs, software packages, medical drugs, etc. In such cases the basic screen may be located on the product itself or on its external wrapping or packaging, and the master screen can be located on a separate film, or attached to the same product or to its packaging.

3. GENERATION OF MICROSTRUCTURED BASIC-SCREEN IMAGES BY ARTISTIC SCREENING

As we have seen in the previous section, the basic screen which is printed on the document may typically consist of a halftoned black-and-white or color image with microstructured halftone dots. Such images can be produced by means of several artistic screening concepts and tools that we have specially developed for security printing.

Artistic screening is a computer-based image reproduction technique which generates halftone screens incorporating freely created artistic screen-elements, and which uses them for the halftoning of any given grayscale images.^{9,10} In the classical clustered-dot halftoning technique, the original continuous-tone image is converted into a screen of small circular or elliptic dots whose sizes depend on the tone levels within the original image. However, artistic screening provides a new additional layer of information which is incorporated in the shapes of the screen dots, and which only becomes visible by close inspection of the printed image using a strong magnifying glass. For example, Fig. 3 shows a painting by Vermeer that has been halftoned with a screen whose microstructure elements (screen dots) have, in all intensity levels being used, the shape of the letter D in a nice calligraphic font. Note that the shapes of the screen dots which make up the halftoned image may be also varied, either depending on the intensity levels of the original image (for example: fish in the dark areas, which gradually turn into birds in the bright areas), or even within a constant intensity level (for example: fish-shaped dots in the bottom of the image, which gradually turn into bird-shaped dots in the top of the image, independently of the tone levels of the image).

Classical clustered-dot halftoning techniques are based on ordered dither threshold arrays which determine the growing behaviour of the screen dots, i.e. the order in which the corresponding binary screen-element pixels are to be turned on (=inked) as the gray level becomes darker. Consequently, the screen dot shapes generated for darker gray levels necessarily encompass the screen dot shapes which are generated for lighter gray levels; in other words, a pixel which has been turned on at a certain gray level will remain turned on for all the subsequent darker gray levels. However, this inherent property of the dither screening method may be too restrictive for the design of true freely-created artistic screens; indeed, the evolution of the screen dot shape as the gray level increases may require that a pixel which has already turned on at a lower gray level be temporarily turned off at some higher gray levels. Therefore, our artistic screening method is not based on dither threshold arrays, but on a completely different paradigm. A set of fixed predefined dot contours determines the precise screen dot shape at some chosen intensity levels. The evolution of the screen dot contours is then defined over the entire intensity range by interpolating over this set of predefined fixed dot contours. Once the evolving shape of the halftone dot boundary is defined exactly for every discrete intensity level, the screen elements associated with each intensity level are rasterized by filling their associated screen dot contours. The resulting precalculated screen elements are then used for the halftoning of the given image. A generalization of this technique may also allow to gradually vary the dot shapes across the image, in addition to the dot size and shape variations according to the intensity levels of the image.

This artistic screening technique is basically useful for the halftoning of monochromatic images or bi-color images (foreground color and background color). However, as already mentioned in Sec. 2, our moiré-based authentication method is not only limited to such cases; on the contrary, it may largely benefit from the use of different colors in the dot-screens being used.

One way of using colored dot-screens in our moiré-based method is similar to the standard multichromatic printing technique, where several (usually three or four) dot-screens of different colors (usually: cyan, magenta, yellow and black) are superposed at different orientations in order to generate a full-color image by halftoning. If one of these colored dot-screens is generated by artistic screening and used as a basic screen according to our method, the moiré intensity profile that will be generated between this basic screen and a black-and-white master screen will closely approximate the color of this basic screen. Furthermore, if several of the different colored dot-screens are generated by artistic screening and used as basic

screens according to our method, each of them will generate with a black-and-white master screen a moiré intensity profile approximating the color of the basic screen in question.

Another possible way of using colored dot-screens is by generating a basic screen whose individual screen elements are composed of sub-elements of different colors. A schematic example of such a basic screen is illustrated in Fig. 6, in which each of the screen dots of the basic screen has a triangular shape, and is sub-divided into sub-elements of different colors, as indicated by the different hachures in the figure. Such multichromatic screens need not necessarily be constant, and by gradually varying the relative area occupied by each color within the screen dot they can be used to generate real halftoned color images with varying tone levels and colors. When a black-and-white master screen is superposed on such a multichromatic basic screen, a similar multichromatic moiré effect is obtained, where not only the shape of the moiré profiles is determined by the screen elements of the basic screen but also their colors. For example, in the case of the basic screen shown in Fig. 6, the moiré profiles obtained will be triangular, and each of them will be sub-divided into colored zones as shown in the figure. An important advantage of this method as an anticounterfeiting means is gained from the extreme difficulty in printing perfectly juxtaposed sub-elements of the screen dots, due to the high precision it requires between the different colors in multi-pass color printing. Only the best high-performance security printing equipment which is used for printing security documents such as banknotes is capable of giving the required precision in the alignment of the different colors. Registration errors which are unavoidable when counterfeiting the document on lower-performance equipment will cause small shifts between the different colored sub-elements of the basic screen elements; such registration errors will be largely magnified by the moiré effect, and they will significantly corrupt the form and the color of the moiré profiles obtained by the master screen.

In practice, such multichromatic basic screens can be generated and used for the halftoning of any given color images by means of our multicolor artistic dithering method.¹¹ This method is a generalization of standard single-color ordered dithering; it converts the color intensities of the given image into a multicolor non-overlapping surface coverage. Multicolor dithering can be used to generate high-quality color images (portraits, landscapes, etc.) whose screen dots are made of multichromatic artistic shapes such as symbols, letters, color ornaments, etc. Moreover, this method is particularly useful for printing images made of a set of non-standard inks, as will be explained in Sec. 5 below.

4. ADDITIONAL SECURITY DUE TO GEOMETRIC TRANSFORMATIONS

In order to add further protection and to make counterfeiting even more difficult, mathematically controlled geometric transformations can be applied to the basic/master screens and their microstructure elements. Such geometrically transformed screens are more difficult to generate and extremely hard to reverse engineer; furthermore, thanks to their varying frequencies they can be also used as screen traps against digital photocopying or reproduction. Also, as we will see in Sec. 5 below, when printed with non-standard inks they cannot be faithfully reproduced by standard color separation and reproduction techniques. Hence, geometrically transformed screens can offer higher security against counterfeiting, practically at the same cost.

The problem is, however, that in the general case moiré intensity profiles which result from the superposition of non-periodic structures such as curvilinear gratings or geometrically transformed dot-screens are extremely distorted, and they do not preserve the shapes of the original screen elements. However, thanks to the mathematical theory that we have recently developed in Ref. 12 and in Chapter 10 of Ref. 6, it becomes possible by using certain mathematical rules to synthesize non-periodic, geometrically transformed screens which in spite of being non-periodic in themselves, still generate, when they are superposed on top of one another, periodic moiré intensity profiles with clearly visible and undistorted elements throughout the superposed area, just like in the fundamental case presented in Sec. 2. An example of such geometrically transformed screens is shown in Fig. 7. If we use the geometrically transformed screen of Fig. 7(a) to halftone a given image (which is *not* geometrically transformed in itself), we obtain a halftoned image (basic screen) with a varying-frequency halftone screen (see Fig. 8). When printed on the security document, this image can obviously be used as a screen trap against digital photocopying or reproduction. However, in addition, when the corresponding master screen (Fig. 7(b)) is superposed onto this image, clearly visible and undistorted moiré intensity profiles appear in the superposition (see Fig. 9), just as in the periodic case discussed in Sec. 2.

It should be stressed that this remarkable property is not true for all non-linear geometric transformations, and it only works with special mathematically designed transformations. Since such transformed screens are generated by a computer program, and the mathematical formulas being used for their generation can be kept secret, it becomes extremely difficult even for professionals in the graphic arts to re-engineer these special screens without having the original programming tools and mathematical formulas which were used for creating them. Thanks to the magnification property inherent to the moiré effect, any small deviations in such re-designed dot screens will unavoidably cause significant distortions in the resulting moiré intensity profiles.

5. ADDITIONAL SECURITY BY NON-STANDARD COLOR SEPARATION FOR CUSTOM INKS

Printing a valuable document using non-standard inks rather than the standard CMYK (cyan, magenta, yellow and black) inks provides an additional level of security. By choosing non-standard custom inks that are completely outside the gamut of colors which can be reproduced by CMYK inks, one can guarantee that the colors of a document which has been counterfeited by a standard desk-top publishing system or by any standard press will considerably differ from the colors of an authentic document.¹³ Furthermore, various special-purpose inks exist for the particular needs of security document printing, such as optically variable inks (OVI) whose colors vary depending on the viewing angle,¹⁴ or various metallic or other special-purpose inks, whose optical effects are practically impossible to simulate by standard CMYK printing. Many modern banknotes and security documents use, indeed, such non-standard inks as an additional security feature. However, non-standard inks are usually used either as individual solid inks or in single-ink halftone gradations, and they are rarely used in halftone combinations of several inks. One of the reasons is that color separation for non-standard inks is a non-trivial task, particularly if a realistic color image such as a human portrait is to be reproduced using such inks. But even for artistic images with non-realistic colors the use of non-standard inks is a non-trivial task for the designer, since it is extremely difficult to predict the precise colors that will be obtained by multicolor halftoning using non-standard inks.

As already mentioned in Sec. 3, our multicolor artistic dithering method¹¹ is particularly useful for printing images made of non-standard inks. This is due to its inherent use of tetrahedral color separation, which works equally well with standard or non-standard inks. Given a desired set of standard or non-standard primary colors (inks) to be used, each of these colors and each of their permitted combinations (e.g. their pairwise superpositions) is represented as a point in a given 3D color space such as RGB or CIE-XYZ. The 3D volume covered by this color point set, called the printable gamut, is then segmented into exhaustive and disjoint tetrahedra. The vertices of each tetrahedron correspond to four neighboring colors from this color point set. (It should be noted that a point set in a 3D space may have many different tetrahedrizations; a few considerations on the choice of the best tetrahedrizations for our needs have been given in Ref. 15).

Once the tetrahedrization of the printable gamut has been done, color separation of each input value from the given color image is obtained by finding in which tetrahedron within the 3D color space the given color value is located, and by calculating the barycentric coordinates d_1, d_2, d_3, d_4 that express this input color value as a linear combination of the tetrahedron's vertices. These barycentric coefficients give the relative amounts of the primary (or secondary) colors C_1, C_2, C_3, C_4 from our color point set that are required to reproduce the given input color. A detailed description of our multicolor dithering algorithm can be found in Ref. 11. It should be noted that this method ensures by construction that the contributing primary or secondary colors be printed side by side, since it converts a barycentric combination of color values into a multicolor non-overlapping surface coverage. This method is therefore ideal for high-end printing equipment that benefits from high registration accuracy, and that is capable of printing with non-standard inks, thus making the printed document very difficult to counterfeit, yet easily authenticated by means of our moiré-based method, as explained above.

A further significant advantage of using non-standard inks in conjunction with our moiré-based method is obtained when using a basic screen with varying frequencies, as described in Sec. 4 above. Due to the high frequencies incorporated in some areas of the variable-frequency basic screen, if the basic screen is printed on the document using a non-standard ink color (such as deep blue), it will not be possible to faithfully reproduce it using standard color printing methods. Such methods require a superposition of two or more screens (the screens used for the standard cyan, magenta, yellow and black inks), but the use of such a screen superposition to represent the original variable-frequency screen would unavoidably result in highly objectionable artifacts or moirés between the superposed variable-frequency screens. This provides an additional protection against counterfeiting at the same price.

6. CONCLUSIONS

We propose an integrated print-based authentication and security strategy for valuable documents and products, using moiré intensity profiles. This strategy combines our know-how and experience in several highly specialized domains: The incorporation of microtext or any other artistic microstructure into a black-and-white or color halftone image (a portrait, a landscape, etc.) that is printed on the document; first-line or second-line authentication by means of moiré intensity profiles which clearly reveal this microstructure (be it overt or covert) and make it visible to the unaided eye; the use of mathematically transformed microstructures to further increase the security offered by the method; and the possibility of using our non-standard color separation method for printing the microstructured image with non-standard inks, in order to prevent reproduction of the document by standard CMYK inks.

Of course, each of these techniques can be used as an independent security feature on its own, but the combination of all or at least some of them together in the same security element makes our method extremely robust and difficult to counterfeit. Different variants of our method have been tested in collaboration with leading security printers such as Orell Füssli, and the results we obtained confirmed, indeed, our expectations.

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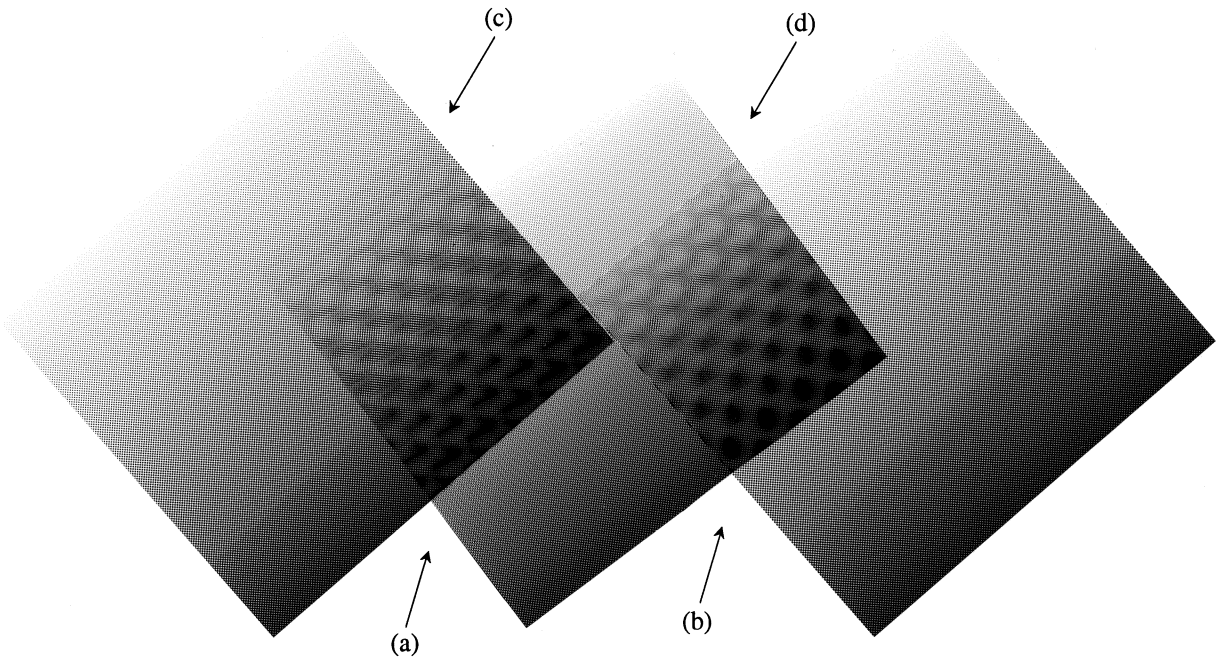


Figure 1: The superposition of dot-screens with specially designed dot-shapes and frequencies may yield spectacular moiré intensity profiles with any desired shapes. Note in particular the black “1”-shaped moiré in (a) and the white “1”-shaped moiré in (d); these moiré intensity profiles are schematically explained in Fig. 2.

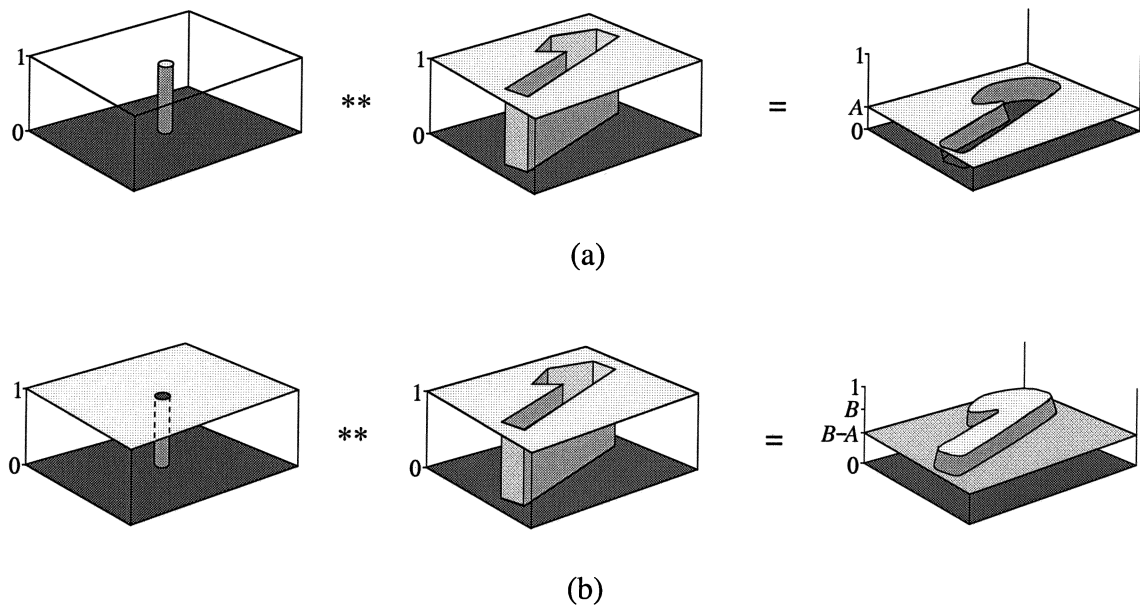


Figure 2: Schematic explanation of the “1”-shaped moiré intensity profiles of Fig. 1: (a) The cyclical convolution of tiny white dots or openings from the first screen with dots of any chosen shape from the other screen gives dots of essentially the same chosen shape. (b) The cyclical convolution of tiny black dots from the first screen with dots of any chosen shape from the other screen gives dots of essentially the same chosen shape, but in inverse-video.

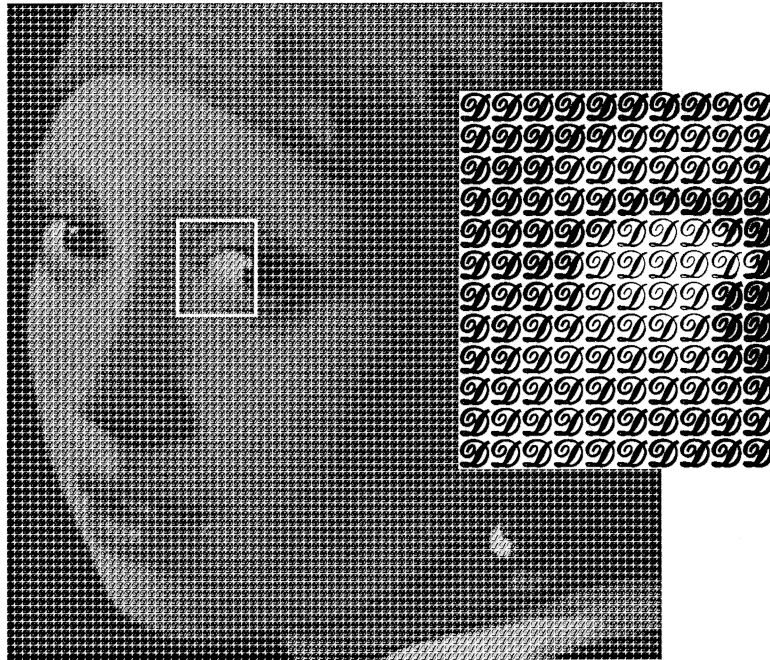
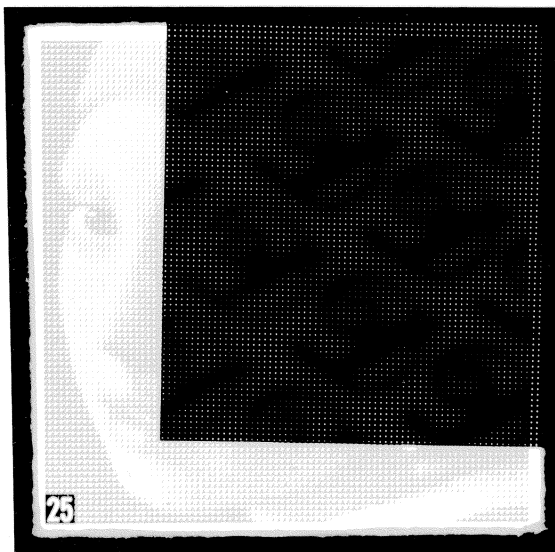
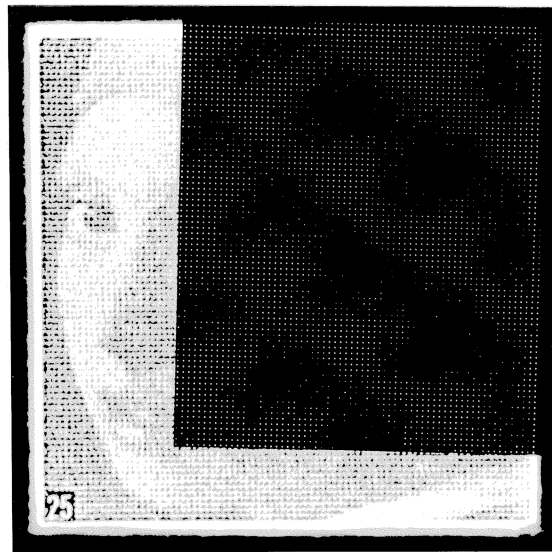


Figure 3: “Girl with a Pearl Earring” by Vermeer, halftoned with a dot screen whose microstructure elements (screen dots) consist of the letter D in a nice calligraphic font (magnified approx. 4 times). The inset shows a largely magnified detail from the eye.



(a)

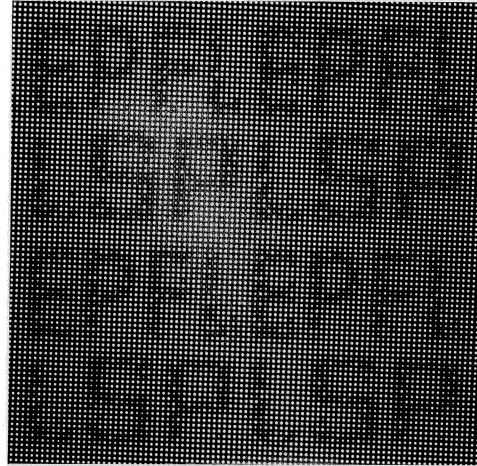


(b)

Figure 4: (a) Moiré intensity profiles with the shape of the calligraphic letter D which appear when a corresponding master screen is superposed on top of the halftoned image (basic screen) of Fig. 3. (b) The result obtained when the same master screen is laid on a photocopy of the basic screen of Fig. 3. Both (a) and (b) are magnified approx. 3 times.



(a)



(b)

Figure 5: A covert variant of our method: (a) A largely magnified view of an unintelligible masked basic screen, as it is observed under the microscope. (b) By superposing a corresponding master screen on top of this masked basic screen, moiré intensity profiles in the form of “EPFL/LSP” become clearly visible.

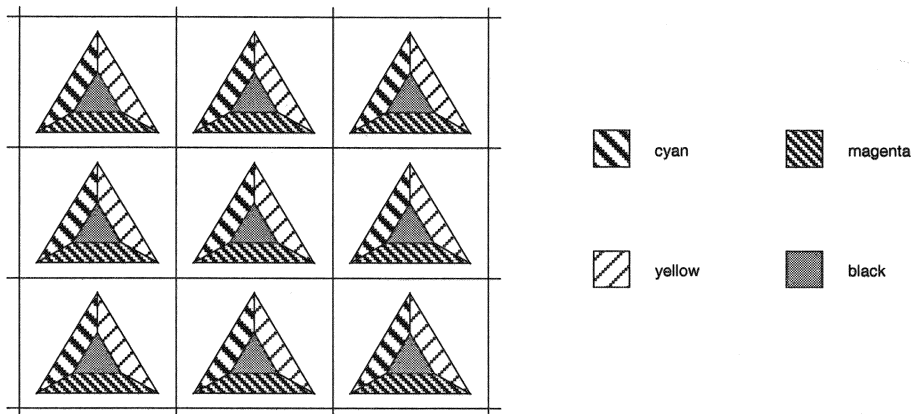
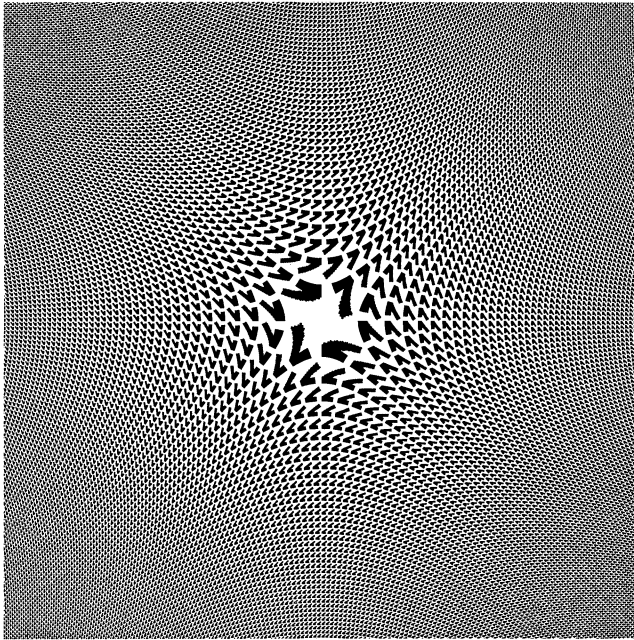
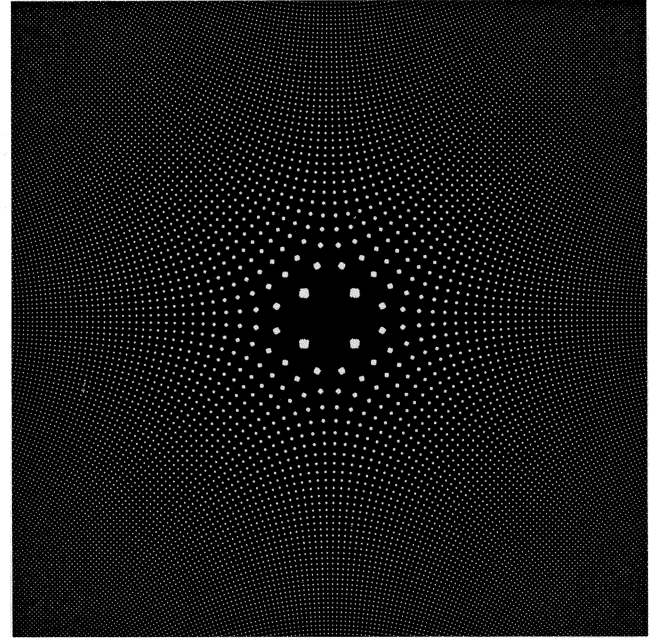


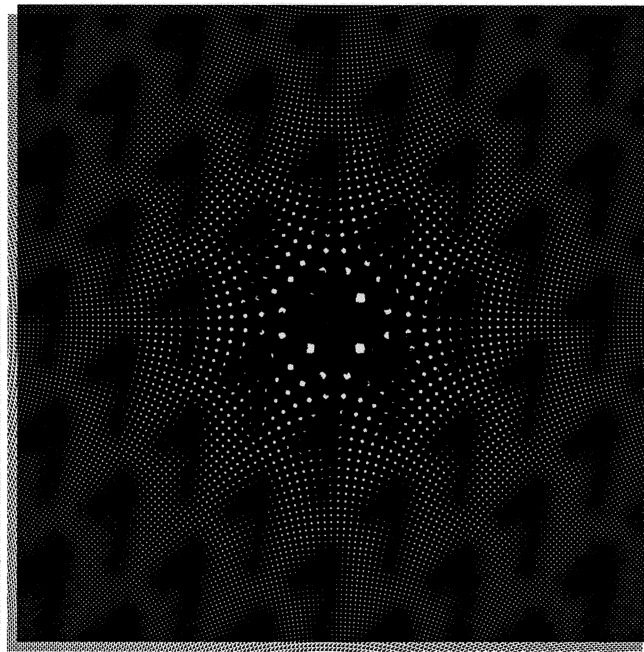
Figure 6: A schematic example of a multicolor basic screen, in which each screen dot is sub-divided into sub-elements of different colors. Each type of hachure represents a different color (for example: cyan, magenta, yellow and black).



(a)



(b)



(c)

Figure 7: (a) A basic screen consisting of distorted “1”s. (b) The corresponding master screen. Both screens have been distorted by the same special non-linear geometric transformation $\mathbf{g}(x,y) = (2xy, x^2 - y^2)$. As shown in (c), the moiré intensity profiles obtained when screen (b) is laterally shifted on top of screen (a) consist of a purely periodic and undistorted version of screen (a), although both of the superposed screens (a) and (b) are distorted.

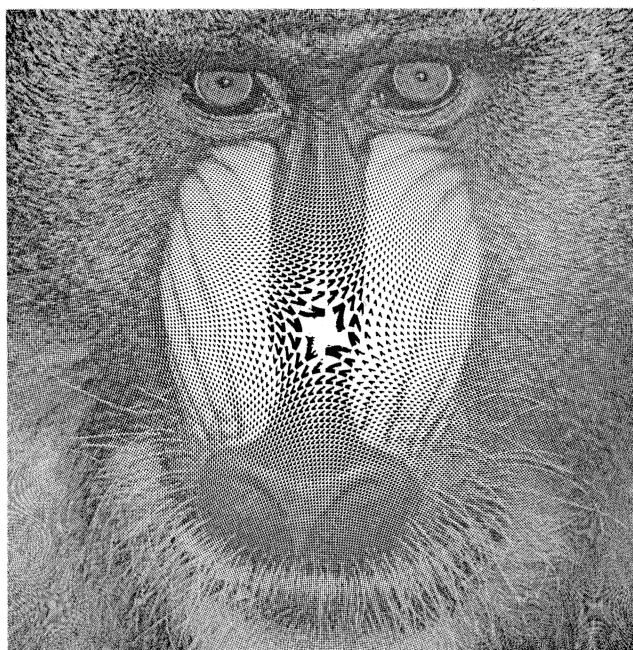


Figure 8: A real image halftoned with the geometrically transformed dot-screen of Fig. 7(a). The original color figure is reproduced here in black-and-white, magnified approx. 4 times. (Note that the scaling factor is not the same as in Fig. 7).

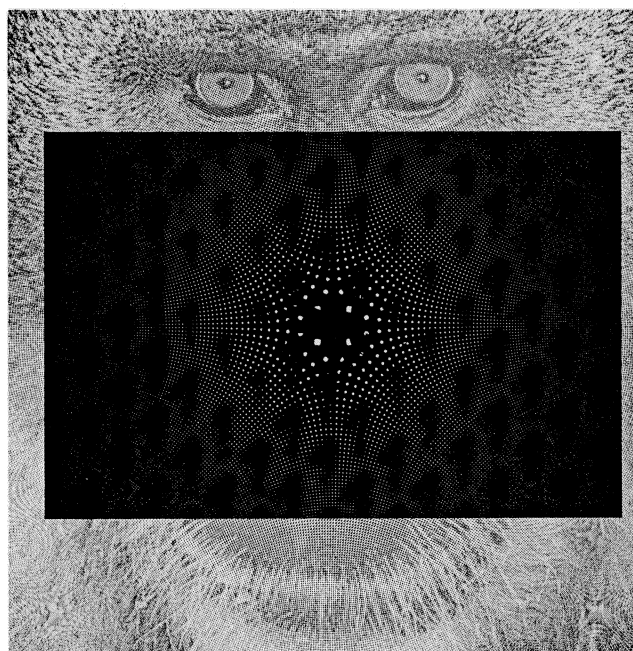


Figure 9: The superposition of the distorted master screen of Fig. 7(b) on the distorted basic screen of Fig. 8 generates inside the halftoned image clearly visible and undistorted “1”-shaped moiré intensity profiles. The original color figure is reproduced here in black-and-white. (Note that the scaling factor is not the same as in Fig. 7.).